시변 지연이 존재하는 선형시스템의 개선된 안정성 판별법

논 문
59-12-27

Improved Stability Criteria for Linear Systems with Time-varying Delay

권 오 민† · 박 주 현* · 이 상 문**
(Oh-Min Kwon · Ju-Hyun Park · Sang-Moon Lee)

Abstract - In this paper, improved stability criteria for linear systems with time-varying delays are proposed. By constructing a new Lyapunov functional, novel stability criteria are established in terms of linear matrix inequalities (LMIs). Two numerical examples are carried out to support the effectiveness of the proposed method.

Key Words : Time-varying delays, linear systems, Lyapunov method, LMI

1. Introduction

It is well known that time delays may deteriorate system performance or even cause instability [1-2]. Since time delays occur in many systems such as networked control systems, neural networks, chemical processes and so on, the subject of stability analysis of systems with time-varying delays has been received considerable attentions during the last two decades. For examples, see [1-11] and references therein.

In the field of stability analysis for systems with time delays, an important index for checking the conservatism of stability criteria is to enlarge the feasible region for guaranteeing the concerned system to be asymptotically stable with a given time-derivative condition of delays. Therefore, how to choose Lyapunov-Krasovskii's functional and obtain the bound of its time-derivative along the trajectories of time-varying systems play important roles to enhance the feasible region of stability criteria. In this regard, He et al. [9] introduced free-weighting techniques and the superiority of proposed methods through numerical examples. In Wei et al. [10], further improved method was proposed by extending the method of free-weighing and consideration of cross terms. However, the inclusion of free-weight matrices increases the computational burden and time cost. Recently, without introducing free-variables, Sun et al. [11] investigated delay-dependent stability criterion of, the following linear systems with time-varying delays:

$$\dot{x}(t) = Ax(t) + A_d x(t-h(t)) \quad (1)$$
$$x(s) = \phi(s),\ s \in [-h_U, 0],$$

where $x(t) \in R^n$ is the state vector, A, $A_d \in R^{n \times n}$ are known system matrices, $\phi(s) \in C_{n,h_U}$ is a given vector valued initial function, $h(t)$ represents time-varying delays which satisfies $0 \le h(t) \le h_U$, $h_{Dl} \le \dot{h}(t) \le h_{Du} < 1$. Here, $C_{n,h_U}([-h_U, 0), R^n)$ denotes the Banach space of continuous vector functions which maps the interval $[-h_U, 0)$ into R^n.

The contribution of the work [11] is the use of triple integral form of Lyapunov-Krasovskii's functional. However, there are rooms for further improvement of stability criteria.

Motivated by the above discussion, improved LMI criteria to find maximum delay bounds for guaranteeing the time-varying delay systems to be asymptotically stable will be proposed based on the methods of [11]. The main contributions of this paper can be summarized as follow:

- Unlike the method of [11], new augmented variables such as $\frac{1}{h(t)}\int_{t-h(t)}^{t} x(s)ds$ and $\frac{1}{h_U - h(t)}\int_{t-h(t)}^{t} x(s)ds$ will be taken to utilize information about time-varying delay. Therefore, the derived stability criteria contain more information about time-varying delays such that the feasible region of stability criteria are enlarged. To the best of author's knowledge, this idea has not been proposed. The detail explanation of this contribution will be discussed in Remark 1.

- In this paper, we propose a new Lyapunov-

† 교신저자, 정회원 : 충북대학교 전기공학부 조교수
E-mail : madwind@chungbuk.ac.kr
* 정 회 원 : 영남대학교 전기공학과 부교수
** 정 회 원 : 대구대학교 전자공학부 전임강사
접수일자 : 2010년 8월 9일
최종완료 : 2010년 11월 15일

Krasovskii's functional

$$V(t)=h(t)\begin{bmatrix}x(t)\\x(t-h(t))\end{bmatrix}^T\begin{bmatrix}P_{1,11} & P_{1,12}\\ \star & P_{1,22}\end{bmatrix}\begin{bmatrix}x(t)\\x(t-h(t))\end{bmatrix}+(h_U-h(t))\begin{bmatrix}x(t)\\x(t-h(t))\end{bmatrix}^T\begin{bmatrix}P_{2,11} & P_{2,12}\\ \star & P_{2,22}\end{bmatrix}\begin{bmatrix}x(t)\\x(t-h(t))\end{bmatrix} \tag{2}$$

where $\begin{bmatrix}P_{1,11} & P_{1,12}\\ \star & P_{1,22}\end{bmatrix}$ and $\begin{bmatrix}P_{2,11} & P_{2,12}\\ \star & P_{2,22}\end{bmatrix}$ are positive matrices. To the best of author's knowledge, the Lyapunov - Krasovskii's functional of the form (2) has not been proposed in any literature. The motivation of Eq. (2) and some discussions will be introduced in Remark 2. The enhancement of feasible region of stability criteria by considering Eq. (2) will be shown in Section 4 through numerical examples.

Up to now, many researchers have studied the methods and techniques to improve the feasible region of stability criteria as mentioned before. Therefore, these new considerations discussed above will contribute to the research of stability and stabilization for systems with time-varying delays. With these considerations and by utilizing convex-hull properties, sufficient conditions are derived in terms of LMIs which can be easily solved by various efficient convex optimization algorithms [12]. Through two numerical examples, improved results are shown by comparing our obtained results with the recent ones in [9]-[11].

Throughout this paper, $\star$ represents the elements below the main diagonal of a symmetric matrix. The notation $X>Y$, where X and Y are matrices of same dimensions, means that the matrix $X-Y$ is positive definite, I denotes the identity matrix whose dimensions can be determined from the context. R^n is the n-dimensional Euclidean space, $R^{m\times n}$ denotes the set of $m\times n$ real matrix.

2. Problem Statements

The objective of this paper is to develope a delay-dependent stability criteria for system (1) which was introduced in Section Introduction.

For the condition $0\le h(t)\le h_U$, let us define Δ_h in the following set

$$\Phi_h:=\{\Delta_h|\Delta_h\in Co\{\Delta_h^1,\ \Delta_h^2\}\}, \tag{3}$$

where Co denotes the convex hull and $\Delta_h^1=0$ and $\Delta_h^2=h_U$. Then, there exist parameters $\alpha_i\,(i=1,2)$ where $\alpha_i\ge 0$ and $\sum_{i=1}^{2}\alpha_i=1$ such that $h(t)$ can be expressed as a convex combination of the vertex values as follows:

$$h(t)=\sum_{i=1}^{2}\alpha_i\Delta_h^i. \tag{4}$$

If the elements of matrix $G(h(t))$ are affinely dependent on $h(t)$, then $G(h(t))$ whose elements include $h(t)$ can be expressed as a convex combination of the vertex values

$$G(h(t))=\sum_{i=1}^{2}\alpha_i G(\Delta_h^i). \tag{5}$$

Before deriving the main result, we need the following fact and lemma.

Lemma 1 (Finsler's Lemma). [13] Let $\zeta\in R^n$, $\Phi=\Phi^T\in R^{n\times n}$, and $B\in R^{m\times n}$ such that $rank(B)<n$. The following statement is equivalent:

i) $\zeta^T\Phi\zeta<0,\ \forall B\zeta=0,\ \zeta\ne 0,$

ii) $(B^\perp)^T\Phi B^\perp<0$

where $B^\perp$ is a right orthogonal complement of B.

Lemma 2. [11] For any constant matrix $Q=Q^T$ and a scalar function $h(t)>0$ such that the following integrations are well defined, then

1) $-\int_{t-h(t)}^{t}x^T(s)Qx(s)ds \le -(1/h(t))\left(\int_{t-h(t)}^{t}x(s)ds\right)^T Q\left(\int_{t-h(t)}^{t}x(s)ds\right)$

2) $-\int_{t-h(t)}^{t}\int_{s}^{t}x^T(u)Qx(u)duds \le -(2/h^2(t))\left(\int_{t-h(t)}^{t}\int_{s}^{t}x(u)duds\right)^T Q\left(\int_{t-h(t)}^{t}\int_{s}^{t}x(u)duds\right)$

3. Main Results

For simplicity of matrix expression, let us define the following notations.

$$\zeta(t)=\begin{bmatrix}x(t)\\ x(t-h(t))\\ x(t-h_U)\\ \dot{x}(t)\\ \dot{x}(t-h_U)\\ (1/h(t))\int_{t-h(t)}^{t}x(s)ds\\ (1/(h_U-h(t)))\int_{t-h_U}^{t-h(t)}x(s)ds\\ (1-\dot{h}(t))\dot{x}(t-h(t))\end{bmatrix},$$

$\Sigma_1^{(i)}=[\Sigma_{1(m,n)}^{(i)}],\ m=1,\ldots,8,\ n=1,\ldots,8,$

$\Sigma_{1(1,1)}^{(i)}=R_{11}+R_{11}^T+N_{11}+G_{11}+(-2+h_U^{-1}\Delta_h^i)Q_1+h_UQ_2-2Q_3,$

$\Sigma_{1(1,2)}^{(i)}=R_{34}-(-2+h_U^{-1}\Delta_h^i)Q_1,\ \ \Sigma_{1(1,3)}^{(i)}=-R_{13}+R_{23}^T,$

$\Sigma_{1(1,4)}^{(i)}=R_{11}+N_{12}+G_{12},\ \ \Sigma_{1(1,5)}^{(i)}=R_{12},\ \ \Sigma_{1(1,6)}^{(i)}=\Delta_h^iR_{33}+2Q_3,$

$\Sigma_{1(1,7)}^{(i)}=(h_U-\Delta_h^i)R_{33},\ \ \Sigma_{1(1,8)}^{(i)}=R_{14},$

$\Sigma_{1(2,2)}^{(i)}=-(1-h_{Du})G_{11}+(-2+h_U^{-1}\Delta_h^i)Q_1+(-1-h_U^{-1}\Delta_h^i)Q_1-2Q_3,$

$\Sigma_{1(2,3)}^{(i)}=-R_{34}^T-(-1-h_U^{-1}\Delta_h^i)Q_1,\ \ \Sigma_{1(2,4)}^{(i)}=R_{14}^T,\ \ \Sigma_{1(2,5)}^{(i)}=R_{24}^T,$

$\Sigma_{1(2,6)}^{(i)}=0,\ \ \Sigma_{1(2,7)}^{(i)}=2Q_3,\ \ \Sigma_{1(2,8)}^{(i)}=R_{44}-G_{12},$

$\Sigma_{1(3,3)}^{(i)}=-R_{23}-R_{23}^T-N_{11}+(-1-h_U^{-1}\Delta_h^i)Q_1,\ \ \Sigma_{1(3,4)}^{(i)}=R_{12}^T,$

$\Sigma_{1(3,5)}^{(i)}=R_{22}-N_{12},\ \ \Sigma_{1(3,6)}^{(i)}=-\Delta_h^iR_{33},\ \ \Sigma_{1(3,7)}^{(i)}=-(h_U-\Delta_h^i)R_{33},$

$\Sigma_{1(3,8)}^{(i)}=R_{24},\ \ \Sigma_{1(4,4)}^{(i)}=N_{22}+G_{22}+h_U^2Q_1+(h_U^2/2)Q_3,\ \ \Sigma_{1(4,5)}^{(i)}=0,$

$\Sigma_{1(4,6)}^{(i)}=\Delta_h^iR_{13},\ \ \Sigma_{1(4,7)}^{(i)}=(h_U-\Delta_h^i)R_{13},\ \ \Sigma_{1(4,8)}^{(i)}=0,$

$\Sigma^{(i)}_{1(5,5)} = -N_{22},\ \Sigma^{(i)}_{1(5,6)} = \Delta^i_h R_{23},\ \Sigma^{(i)}_{1(5,7)} = (h_U - \Delta^i_h) R_{23},$
$\Sigma^{(i)}_{1(5,8)} = 0,\ \Sigma^{(i)}_{1(6,6)} = -\Delta^i_h Q_2 - 2Q_3,\ \Sigma^{(i)}_{1(6,7)} = 0,$
$\Sigma^{(i)}_{1(6,8)} = \Delta^i_h R_{34},\ \Sigma^{(i)}_{1(7,7)} = -(h_U - \Delta^i_h) Q_2 - 2Q_3,$
$\Sigma^{(i)}_{1(7,8)} = (h_U - \Delta^i_h) R_{34},\ \Sigma^{(i)}_{1(8,8)} = -(1/(1-h_{Dl}))G_{22},$
$$\Gamma = \begin{bmatrix} A & A_d & 0 & -I & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{6}$$

Then, we have the following theorem.

Theorem 1. For given $h_U > 0$, and $h_{Dl} < h_{Du} < 1$, the system (1) is asymptotically stable for $0 \le h(t) \le h_U$ and $h_{Dl} \le \dot{h}(t) \le h_{Du}$ if there exist positive matrices $\boldsymbol{R} = [R_{ij}]_{5\times 5}$ $\boldsymbol{N} = [N_{ij}]_{2\times 2}, \boldsymbol{G} = [G_{ij}]_{2\times 2}, Q_i (i=1,2,3)$, such that the following LMIs hold

$$(\Gamma^\perp)^T \Sigma^{(i)}_1 \Gamma^\perp < 0 \quad (i=1,2) \tag{7}$$

where $\Sigma^{(i)}_1$ and Γ are defined in (6), and $\Gamma^\perp$ is the right orthogonal complement of Γ.

Proof. For positive matrices $\boldsymbol{R} = [R_{ij}]_{5\times 5}$, $\boldsymbol{N} = [N_{ij}]_{2\times 2}$, $\boldsymbol{G} = [G_{ij}]_{2\times 2}$, $Q_i (i=1,2,3)$, let us consider the following Lyapunov function defined by

$$V(t) = \sum_{i=1}^{6} V_i(t) \tag{8}$$

where

$$V_1(t) = \begin{bmatrix} x(t) \\ x(t-h_U) \\ \int_{t-h_U}^{t} x(s)ds \\ x(t-h(t)) \end{bmatrix}^T \boldsymbol{R} \begin{bmatrix} x(t) \\ x(t-h_U) \\ \int_{t-h_U}^{t} x(s)ds \\ x(t-h(t)) \end{bmatrix},$$
$$V_2(t) = \int_{t-h_U}^{t} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T \boldsymbol{N} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix},$$
$$V_3(t) = \int_{t-h(t)}^{t} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix}^T \boldsymbol{G} \begin{bmatrix} x(s) \\ \dot{x}(s) \end{bmatrix},$$
$$V_4(t) = h_U \int_{t-h_U}^{t} \int_{s}^{t} \dot{x}^T(u) Q_1 \dot{x}(u) du ds,$$
$$V_5(t) = \int_{t-h_U}^{t} \int_{s}^{t} x^T(u) Q_2 x(u) du ds,$$
$$V_6(t) = \int_{t-h_U}^{t} \int_{s}^{t} \int_{u}^{t} \dot{x}^T(v) Q_3 \dot{x}(v) dv du ds. \tag{9}$$

First, the time derivative of $V_1(t)$ along the solution of Eq. (1) is obtained by

$$\dot{V}_1(t) = 2\begin{bmatrix} x(t) \\ x(t-h_U) \\ \int_{t-h_U}^{t} x(s)ds \\ x(t-h(t)) \end{bmatrix}^T \boldsymbol{R} \begin{bmatrix} \dot{x}(t) \\ \dot{x}(t-h_U) \\ x(t)-x(t-h_U) \\ (1-\dot{h}(t))\dot{x}(t-h(t)) \end{bmatrix}$$
$$= 2\begin{bmatrix} x(t) \\ x(t-h_U) \\ \eta_1(t) \\ x(t-h(t)) \end{bmatrix}^T \boldsymbol{R} \begin{bmatrix} \dot{x}(t) \\ \dot{x}(t-h_U) \\ x(t)-x(t-h_U) \\ (1-\dot{h}(t))\dot{x}(t-h(t)) \end{bmatrix}$$
$$= 2\begin{bmatrix} x(t) \\ x(t-h_U) \\ \left(\frac{1}{h(t)}\int_{t-h(t)}^{t} x(s)ds\right) \\ \left(\frac{1}{h_U-h(t)}\int_{t-h_U}^{t-h(t)} x(s)ds\right) \\ x(t-h(t)) \end{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & h(t)I & 0 \\ 0 & 0 & (h_U-h(t))I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$$
$$\times \boldsymbol{R} \begin{bmatrix} \dot{x}(t) \\ \dot{x}(t-h_U) \\ x(t)-x(t-h_U) \\ (1-\dot{h}(t))\dot{x}(t-h(t)) \end{bmatrix} \tag{10}$$

where

$$\eta_1(t) = \left(\frac{1}{h(t)}\int_{t-h(t)}^{t} x(s)ds\right)h(t) + \left(\frac{1}{h_U-h(t)}\int_{t-h_U}^{t-h(t)} x(s)ds\right)(h_U-h(t)).$$

Second, differentiating $V_2(t)$ leads to

$$\dot{V}_2(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \boldsymbol{N} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} - \begin{bmatrix} x(t-h_U) \\ \dot{x}(t-h_U) \end{bmatrix}^T \boldsymbol{N} \begin{bmatrix} x(t-h_U) \\ \dot{x}(t-h_U) \end{bmatrix} \tag{11}$$

Third, the upper bound of $\dot{V}_3(t)$ can be obtained as

$$\dot{V}_3(t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \boldsymbol{G} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} - (1-\dot{h}(t)) \begin{bmatrix} x(t-h(t)) \\ \dot{x}(t-h(t)) \end{bmatrix}^T \boldsymbol{G} \begin{bmatrix} x(t-h(t)) \\ \dot{x}(t-h(t)) \end{bmatrix}$$
$$= \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} G_{11} & G_{12} \\ \star & G_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}$$
$$- \begin{bmatrix} x(t-h(t)) \\ (1-\dot{h}(t))\dot{x}(t-h(t)) \end{bmatrix}^T \begin{bmatrix} (1-\dot{h}(t))G_{11} & G_{12} \\ \star & \frac{1}{1-\dot{h}(t)}G_{22} \end{bmatrix}$$
$$\times \begin{bmatrix} x(t-h(t)) \\ (1-\dot{h}(t))\dot{x}(t-h(t)) \end{bmatrix}$$
$$\le \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} G_{11} & G_{12} \\ \star & G_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}$$
$$- \begin{bmatrix} x(t-h(t)) \\ (1-\dot{h}(t))\dot{x}(t-h(t)) \end{bmatrix}^T \begin{bmatrix} (1-h_{Du})G_{11} & G_{12} \\ \star & \frac{1}{1-h_{Dl}}G_{22} \end{bmatrix}$$
$$\times \begin{bmatrix} x(t-h(t)) \\ (1-\dot{h}(t))\dot{x}(t-h(t)) \end{bmatrix} \tag{12}$$

Fourth, the time derivative of $\dot{V}_4(t)$ leads to

$$\dot{V}_4(t) = h_U^2 \dot{x}^T(t) Q_1 \dot{x}(t) - h_U \int_{t-h_U}^{t} \dot{x}^T(s) Q_1 \dot{x}(s) ds \tag{13}$$

Note that

$$-h_U \int_{t-h_U}^{t} \dot{x}^T(s) Q_1 \dot{x}(s) ds \tag{14}$$
$$= -h_U \int_{t-h(t)}^{t} \dot{x}^T(s) Q_1 \dot{x}(s) ds - h_U \int_{t-h(t)}^{t-h_U} \dot{x}^T(s) Q_1 \dot{x}(s) ds.$$

With $-h_U = -(h_U - h(t)) - h(t)$, $0 \le h(t) \le h_U$, and Lemma 2, an upper bound of the first integral term in the right side of Eq. (14) can be estimated as

$$-h_U \int_{t-h(t)}^{t} \dot{x}^T(s) Q_1 \dot{x}(s) ds$$
$$= -(h_U - h(t)) \int_{t-h(t)}^{t} \dot{x}^T(s) Q_1 \dot{x}x(s) ds$$
$$-h(t) \int_{t-h(t)}^{t} \dot{x}^T(s) Q_1 \dot{x}(s) ds$$
$$\le -h_U^{-1}(h_U - h(t))h(t) \int_{t-h(t)}^{t} \dot{x}^T(s) Q_1 \dot{x}(s) ds$$
$$-h(t) \int_{t-h(t)}^{t} \dot{x}^T(s) Q_1 \dot{x}(s) ds$$
$$\le (-2 + h_U^{-1}h(t))[x(t) - x(t-h(t))]^T Q_1 [x(t) - x(t-h(t))]. \tag{15}$$

With the similar method in (15), an upper bound of the second integral term in the right side of Eq. (14) can be obtained as

$$
\begin{aligned}
&-h_U\int_{t-h_U}^{t-h(t)}\dot{x}^T(s)Q_1\dot{x}(s)ds\\
&= -(h_U-h(t))\int_{t-h_U}^{t-h(t)}\dot{x}^T(s)Q_1\dot{x}x(s)ds\\
&\quad -h(t)\int_{t-h_U}^{t-h(t)}\dot{x}^T(s)Q_1\dot{x}(s)ds\\
&\le -(h_U-h(t))\int_{t-h_U}^{t-h(t)}\dot{x}^T(s)Q_1\dot{x}(s)ds\\
&\quad -h_U^{-1}h(t)(h_U-h(t))\int_{t-h_U}^{t-h(t)}\dot{x}^T(s)Q_1\dot{x}(s)ds\\
&\le (-1-h_U^{-1}h(t))\left[x(t-h(t))-x(t-h_U)\right]^TQ_1\\
&\quad \times\left[x(t-h(t))-x(t-h_U)\right],
\end{aligned}
\tag{16}
$$

where $-h_U\le-(h_U-h(t))$ and Lemma 1 were utilized in (16). Then, from (15) and (16), an upper bound of $\dot{V}_4(t)$ can be obatined as

$$
\begin{aligned}
\dot{V}_4(t)\le\ & h_U^2\dot{x}^T(t)Q_1\dot{x}(t)\\
&+(-2+h_U^{-1}h(t))\left[x(t)-x(t-h(t))\right]^T\\
&\times Q_1\left[x(t)-x(t-h(t))\right]\\
&+(-1-h_U^{-1}h(t))\left[x(t-h(t))-x(t-h_U)\right]^TQ_1\\
&\times\left[x(t-h(t))-x(t-h_U)\right].
\end{aligned}
\tag{17}
$$

Fifth, by calculating $\dot{V}_5(t)$, we have

$$
\begin{aligned}
\dot{V}_5(t)&= h_Ux^T(t)Q_2x(t)-\int_{t-h_U}^{t}x^T(s)Q_2x(s)ds\\
&= h_Ux^T(t)Q_2x(t)-\int_{t-h(t)}^{t}x^T(s)Q_2x(s)ds\\
&\quad -\int_{t-h_U}^{t-h(t)}x^T(s)Q_2x(s)ds\\
&\le h_Ux^T(t)Q_2x(t)-\left(\frac{1}{h(t)}\int_{t-h(t)}^{t}x(s)ds\right)^T(h(t)Q_2)\\
&\times\left(\frac{1}{h(t)}\int_{t-h(t)}^{t}x(s)ds\right)\\
&-\left(\frac{1}{h_U-h(t)}\int_{t-h_U}^{t-h(t)}x(s)ds\right)^T((h_U-h(t))Q_2)\\
&\times\left(\frac{1}{h_U-h(t)}\int_{t-h_U}^{t-h(t)}x(s)ds\right)
\end{aligned}
\tag{18}
$$

Lastly, calculating the time derivative of $V_6(t)$ leads to

$$
\begin{aligned}
&\dot{V}_6(t)\\
&= (h_U^2/2)\dot{x}^T(t)Q_3\dot{x}(t)-\int_{t-h_U}^{t}\int_{s}^{t}\dot{x}^T(u)Q_3x(u)duds\\
&= (h_U^2/2)\dot{x}^T(t)Q_3\dot{x}(t)-\int_{t-h(t)}^{t}\int_{s}^{t}\dot{x}^T(u)Q_3x(u)duds\\
&\quad -\int_{t-h_U}^{t-h(t)}\int_{s}^{t}\dot{x}^T(u)Q_3x(u)duds\\
&\le (h_U^2/2)\dot{x}^T(t)Q_3\dot{x}(t)-\int_{t-h(t)}^{t}\int_{s}^{t}\dot{x}^T(u)Q_3x(u)duds\\
&\quad -\int_{t-h_U}^{t-h(t)}\int_{s}^{t-h(t)}\dot{x}^T(u)Q_3x(u)duds\\
&\le (h_U^2/2)\dot{x}^T(t)Q_3\dot{x}(t)\\
&\quad -(h^2(t)/2)^{-1}\left(h(t)x(t)-\int_{t-h(t)}^{t}x(s)ds\right)^T\\
&\quad \times Q_3\left(h(t)x(t)-\int_{t-h(t)}^{t}x(s)ds\right)\\
&\quad -((h_U-h(t))/2)^{-1}\left((h_U-h(t))x(t-h(t))-\int_{t-h_U}^{t}x(s)ds\right)^T\\
&\quad \times Q_3\left((h_U-h(t))x(t-h(t))-\int_{t-h_U}^{t}x(s)ds\right)\\
&\le (h_U^2/2)\dot{x}^T(t)Q_3\dot{x}(t)\\
&\quad -2\left(x(t)-\frac{1}{h(t)}\int_{t-h(t)}^{t}x(s)ds\right)^TQ_3\left(x(t)-\frac{1}{h(t)}\int_{t-h(t)}^{t}x(s)ds\right)\\
&\quad -2\left(x(t-h(t))-\frac{1}{h_U-h(t)}\int_{t-h_U}^{t-h(t)}x(s)ds\right)^T\\
&\quad \times Q_3\left(x(t-h(t))-\frac{1}{h_U-h(t)}\int_{t-h_U}^{t-h(t)}x(s)ds\right).
\end{aligned}
\tag{19}
$$

From (7)-(19), an upper bound of $\dot{V}(t)=\sum_{i=1}^{6}\dot{V}_i(t)$ can be $\dot{V}(t)\le\zeta^T(t)\{\Sigma_1(\mathrm{h(t)})\}\zeta(\mathrm{t})$

where

$$
\begin{aligned}
&\Sigma_1(h(t))=[\Sigma_{1(m,n)}],\ m=1,\ldots,8,\ n=1,\ldots,8,\\
&\Sigma_{1(1,1)}=R_{11}+R_{11}^T+N_{11}+G_{11}+(-2+h_U^{-1}h(t))Q_1+h_UQ_2-2Q_3,\\
&\Sigma_{1(1,2)}=R_{34}-(-2+h_U^{-1}h(t))Q_1,\ \Sigma_{1(1,3)}=-R_{13}+R_{23}^T,\\
&\Sigma_{1(1,4)}=R_{11}+N_{12}+G_{12},\ \Sigma_{1(1,5)}=R_{12},\ \Sigma_{1(1,6)}=h(t)R_{33}+2Q_3,\\
&\Sigma_{1(1,7)}=(h_U-h(t))R_{33},\ \Sigma_{1(1,8)}=R_{14},\\
&\Sigma_{1(2,2)}=-(1-h_{Du})G_{11}+(-2+h_U^{-1}h(t))Q_1+(-1-h_U^{-1}h(t))Q_1\\
&\qquad -2Q_3,\\
&\Sigma_{1(2,3)}=-R_{34}^T-(-1-h_U^{-1}h(t))Q_1,\ \Sigma_{1(2,4)}=R_{14}^T,\ \Sigma_{1(2,5)}=R_{24}^T,\\
&\Sigma_{1(2,6)}=0,\ \Sigma_{1(2,7)}=2Q_3,\ \Sigma_{1(2,8)}=R_{44}-G_{12},\\
&\Sigma_{1(3,3)}=-R_{23}-R_{23}^T-N_{11}+(-1-h_U^{-1}h(t))Q_1,\ \Sigma_{1(3,4)}=R_{12}^T,\\
&\Sigma_{1(3,5)}=R_{22}-N_{12},\ \Sigma_{1(3,6)}=-h(t)R_{33},\ \Sigma_{1(3,7)}=-(h_U-h(t))R_{33},\\
&\Sigma_{1(3,8)}=R_{24},\ \Sigma_{1(4,4)}=N_{22}+G_{22}+h_U^2Q_1+(h_U^2/2)Q_3,\ \Sigma_{1(4,5)}=0,\\
&\Sigma_{1(4,6)}=h(t)R_{13},\ \Sigma_{1(4,7)}=(h_U-h(t))R_{13},\ \Sigma_{1(4,8)}=0,\\
&\Sigma_{1(5,5)}=-N_{22},\ \Sigma_{1(5,6)}=h(t)R_{23},\ \Sigma_{1(5,7)}=(h_U-h(t))R_{23},\\
&\Sigma_{1(5,8)}=0,\ \Sigma_{1(6,6)}=-h(t)Q_2-2Q_3,\ \Sigma_{1(6,7)}=0,\\
&\Sigma_{1(6,8)}=h(t)R_{34},\ \Sigma_{1(7,7)}=-(h_U-h(t))Q_2-2Q_3,\\
&\Sigma_{1(7,8)}=(h_U-h(t))R_{34},\ \Sigma_{1(8,8)}=-(1/(1-h_{Dl}))G_{22}.
\end{aligned}
\tag{20}
$$

Note that $0=\Gamma\zeta(t)$ where Γ was defined in (5).
From Lemma 1 and the properties of convex-hull, if LMI (6) holds, then system (1) is asymptotically stable for $0\le h(t)\le h_U$ and $h_{Dl}\le\dot{h}(t)\le h_{Du}$. This completes the proof. ■

Remark 1. In [11], by taking $\int_{t-h(t)}^{t}x(s)ds$ and $\int_{t-h_U}^{t-h(t)}x(s)ds$ as augmented vectors, the time-derivative of $V_6(t)=\int_{t-h_U}^{t}\int_{s}^{t}\int_{u}^{t}\dot{x}(v)Q_3\dot{x}(v)dvduds$ was estimated as

$$
\begin{aligned}
\dot{V}_6(t) \le\ & (h_U^2/2)\dot{x}^T(t)Q_3\dot{x}(t) - \int_{t-h_U}^{t}\int_s^t \dot{x}^T(u)Q_3\dot{x}(u)duds \\
\le\ & (h_U^2/2)\dot{x}^T(t)Q_3\dot{x}(t) \\
& -(2/h_U^2)\left(\int_{t-h_U}^{t}\int_s^t \dot{x}(u)duds\right)^T Q_3\left(\int_{t-h_U}^{t}\int_s^t \dot{x}(u)duds\right) \\
\le\ & (h_U^2/2)\dot{x}^T(t)Q_3\dot{x}(t) \\
& -(2/h_U^2)\left(h_U x(t) - \int_{t-h_U}^{t} x(s)ds\right)^T \\
& \times Q_3\left(h_U x(t) - \int_{t-h_U}^{t} x(s)ds\right) \\
\le\ & (h_U^2/2)\dot{x}^T(t)Q_3\dot{x}(t) \\
& -(2/h_U^2)\left(h_U x(t) - \int_{t-h(t)}^{t} x(s)ds - \int_{t-h_U}^{t-h(t)} x(s)ds\right)^T \\
& \times Q_3\left(h_U x(t) - \int_{t-h(t)}^{t} x(s)ds - \int_{t-h_U}^{t-h(t)} x(s)ds\right).
\end{aligned} \tag{21}
$$

However, unlike the method mentioned above in (21), before taking the upper bound of $-\int_{t-h_U}^{t}\int_s^t \dot{x}^T(u)Q_3\dot{x}(u)duds$ as $-(2/h_U^2)\left(\int_{t-h_U}^{t}\int_s^t \dot{x}(u)duds\right)^T Q_3\left(\int_{t-h_U}^{t}\int_s^t \dot{x}(u)duds\right)$, we divide the integral term $-\int_{t-h_U}^{t}\int_s^t \dot{x}^T(u)Q_3\dot{x}(u)duds$ as

$$
\begin{aligned}
& -\int_{t-h_U}^{t}\int_s^t \dot{x}^T(u)Q_3\dot{x}(u)duds \\
&= -\int_{t-h(t)}^{t}\int_s^t \dot{x}^T(u)Q_3\dot{x}(u)duds - \int_{t-h_U}^{t-h(t)}\int_s^t \dot{x}^T(u)Q_3\dot{x}(u)duds.
\end{aligned} \tag{22}
$$

In the integral term $-\int_{t-h_U}^{t-h(t)}\int_s^t \dot{x}^T(u)Q_3\dot{x}(u)duds$, since $t-h_U \le s \le t-h(t)$ and $s \le u \le t$, it can be estimated as

$$
\begin{aligned}
& -\int_{t-h_U}^{t-h(t)}\int_s^t \dot{x}^T(u)Q_3\dot{x}(u)duds \\
&= -\int_{t-h_U}^{t-h(t)}\int_s^{t-h(t)} \dot{x}^T(u)Q_3\dot{x}(u)duds \\
&\quad -\int_{t-h_U}^{t-h(t)}\int_{t-h(t)}^{t} \dot{x}^T(u)Q_3\dot{x}(u)duds \\
&\le -\int_{t-h_U}^{t-h(t)}\int_s^{t-h(t)} \dot{x}^T(u)Q_3\dot{x}(u)duds.
\end{aligned} \tag{23}
$$

With the new augmented vectors $\frac{1}{h(t)}\int_{t-h(t)}^{t} x(s)ds$ and $\frac{1}{h_U-h(t)}\int_{t-h_U}^{t-h(t)} x(s)ds$, au upper bound of $\dot{V}_6(t)$ can be

$$
\begin{aligned}
& \dot{V}_6(t) \\
& \le (h_U^2/2)\dot{x}^T(t)Q_3\dot{x}(t) \\
& \quad -2\left(x(t) - \frac{1}{h(t)}\int_{t-h(t)}^{t} x(s)ds\right)^T Q_3\left(x(t) - \frac{1}{h(t)}\int_{t-h(t)}^{t} x(s)ds\right) \\
& \quad -2\left(x(t-h(t)) - \frac{1}{h_U-h(t)}\int_{t-h_U}^{t} x(s)ds\right)^T \\
& \quad \times Q_3\left(x(t-h(t)) - \frac{1}{h_U-h(t)}\int_{t-h_U}^{t} x(s)ds\right).
\end{aligned} \tag{24}
$$

This consideration may lead to less conservative stability region of system (1). Through numerical examples, we will show the improvement of stability region of system (1).

Remark 2. In delay-dependent stability analysis, as mentioned in Section Introduction, an important index for checking of conservatism of stability criterion is how to choose Lyapunov-Krasovskii's functional and derive its time derivative upper bound to reduce the conservatism of stability region. Therefore, the utilization of information of time-varying delay plays a key role to increase the stability region of time-delay systems. Also, the proposed method utilizes the convex-hull properties of time-varying delays $h(t)$. Can a Lyapunov-Krasovskii's functional which is dependent on $h(t)$ enhance the feasible region of stability criterion for time-varying systems? With this regard, the authors propose the following new Lyapunov-Krasovskii's functional which has not been considered in other literature

$$
\begin{aligned}
V_7(t) = h(t)&\begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \begin{bmatrix} P_{1,11} & P_{1,12} \\ \star & P_{1,22} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix} \\
&+ (h_U - h(t))\begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}^T \begin{bmatrix} P_{2,11} & P_{2,12} \\ \star & P_{2,22} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-h(t)) \end{bmatrix}
\end{aligned} \tag{25}
$$

With the new Lyapunov-Krasovskii's functional in (25), a new delay-dependent stability criterion will be introduced as Theorem 2.

Before introducing Theorem 2, let us define the following notations for simplicity.

$$
\begin{aligned}
& \mathcal{P}_1 = [P_{1,ij}]_{2\times 2},\ \mathcal{P}_2 = [P_{2,ij}]_{2\times 2}, \\
& \Pi_1 = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \Pi_2 = \begin{bmatrix} 0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}. \\
& \Sigma_2^{(i)} = \Pi_1^T(h_{Du}\mathcal{P}_1 - h_{Dl}\mathcal{P}_2)\Pi_1 + \Pi_1^T\left[\Delta_h^i \mathcal{P}_1 + (h_U - \Delta_h^i)\mathcal{P}_2\right]\Pi_2 \\
& \qquad + \Pi_2^T\left[\Delta_h^i \mathcal{P}_1 + (h_U - \Delta_h^i)\mathcal{P}_2\right]\Pi_1,
\end{aligned} \tag{26}
$$

Theorem 2. For given $h_U > 0$, and $h_{Dl} < h_{Du} < 1$, the system (1) is asymptotically stable for $0 \le h(t) \le h_U$ and $h_{Dl} \le \dot{h}(t) \le h_{Du} < 1$ if there exist positive matrices $\mathcal{P}_1 = [P_{1,ij}]_{2\times 2}$, $\mathcal{P}_2 = [P_{2,ij}]_{2\times 2}$, $\mathcal{R} = [R_{ij}]_{5\times 5}$ $\mathcal{N} = [N_{ij}]_{2\times 2}$, $\mathcal{G} = [G_{ij}]_{2\times 2}$, $Q_i\,(i=1,2,3)$, such that the following LMIs hold

$$
(\Gamma^{\perp})^T(\Sigma_1^{(i)} + \Sigma_2^{(i)})\Gamma^{\perp} < 0 \quad (i=1,2) \tag{27}
$$

where $\Gamma^{\perp}$ and $\Sigma_1^{(i)}$ are the same ones in Theorem 1, and $\Sigma_2^{(i)}$ is defined in (26).

Proof. For positive matrices $\mathcal{P}_1 = [P_{1,ij}]_{2\times 2}$, $\mathcal{P}_2 = [P_{2,ij}]_{2\times 2}$ $\mathcal{R} = [R_{ij}]_{5\times 5}$, $\mathcal{N} = [N_{ij}]_{2\times 2}$, $\mathcal{G} = [G_{ij}]_{2\times 2}$, $Q_i\,(i=1,2,3)$, let us consider the following Lyapunov function defined by

$$
V(t) = \sum_{i=1}^{7} V_i(t) \tag{28}
$$

where $V_i(t)(i=1,...,6)$ are the same ones in (8) and $V_7(t)$ is in (25). By taking the time-derivative of $V_7(t)$, we have

$$\begin{aligned}
&\dot{V}_7(t)\\
&= \dot{h}(t)\begin{bmatrix} x(t) \\ x(t-h(t))\end{bmatrix}^T\begin{bmatrix} P_{1,11} & P_{1,12} \\ \star & P_{1,22}\end{bmatrix}\begin{bmatrix} x(t) \\ x(t-h(t))\end{bmatrix}\\
&\quad +2h(t)\begin{bmatrix} x(t) \\ x(t-h(t))\end{bmatrix}^T\begin{bmatrix} P_{1,11} & P_{1,12} \\ \star & P_{1,22}\end{bmatrix}\begin{bmatrix} \dot{x}(t) \\ (1-\dot{h}(t))\dot{x}(t-h(t))\end{bmatrix}\\
&\quad -\dot{h}(t)\begin{bmatrix} x(t) \\ x(t-h(t))\end{bmatrix}^T\begin{bmatrix} P_{2,11} & P_{2,12} \\ \star & P_{2,22}\end{bmatrix}\begin{bmatrix} x(t) \\ x(t-h(t))\end{bmatrix}\\
&\quad +2(h_U-h(t))\begin{bmatrix} x(t) \\ x(t-h(t))\end{bmatrix}^T\begin{bmatrix} P_{2,11} & P_{2,12} \\ \star & P_{2,22}\end{bmatrix}\begin{bmatrix} \dot{x}(t) \\ (1-\dot{h}(t))\dot{x}(t-h(t))\end{bmatrix}\\
&\le h_{Du}\begin{bmatrix} x(t) \\ x(t-h(t))\end{bmatrix}^T\begin{bmatrix} P_{1,11} & P_{1,12} \\ \star & P_{1,22}\end{bmatrix}\begin{bmatrix} x(t) \\ x(t-h(t))\end{bmatrix}\\
&\quad +2h(t)\begin{bmatrix} x(t) \\ x(t-h(t))\end{bmatrix}^T\begin{bmatrix} P_{1,11} & P_{1,12} \\ \star & P_{1,22}\end{bmatrix}\begin{bmatrix} \dot{x}(t) \\ (1-\dot{h}(t))\dot{x}(t-h(t))\end{bmatrix}\\
&\quad -h_{Dl}\begin{bmatrix} x(t) \\ x(t-h(t))\end{bmatrix}^T\begin{bmatrix} P_{2,11} & P_{2,12} \\ \star & P_{2,22}\end{bmatrix}\begin{bmatrix} x(t) \\ x(t-h(t))\end{bmatrix}\\
&\quad +2(h_U-h(t))\begin{bmatrix} x(t) \\ x(t-h(t))\end{bmatrix}^T\begin{bmatrix} P_{2,11} & P_{2,12} \\ \star & P_{2,22}\end{bmatrix}\begin{bmatrix} \dot{x}(t) \\ (1-\dot{h}(t))\dot{x}(t-h(t))\end{bmatrix}.\\
&= \zeta^T(t)\{\Pi_1^T(h_{Du}\boldsymbol{P_1}-h_{Dl}\boldsymbol{P_2})\Pi_1+\Pi_1^T[h(t)\boldsymbol{P_1}+(h_U-h(t))\boldsymbol{P_2}]\Pi_2\\
&\quad +\Pi_2^T[h(t)\boldsymbol{P_1}+(h_U-h(t))\boldsymbol{P_2}]\Pi_1\}\zeta(t)
\end{aligned} \tag{29}$$

By using the similar method of the proof of Theorem 1, it is straightforward that if LMI (27) holds, then system (1) is asymptotically stable for $0 \le h(t) \le h_U$ and $h_{Dl} \le \dot{h}(t) \le h_{Du} < 1$. This completes our proof. ■

As a special case, when $h(t)=h_U$, Theorem 1 is simplified as Corollary 1. Before introducing Corollary 1, we define the following notations.

$$\begin{aligned}
&\zeta_c^T = \left[x^T(t) \;\; x^T(t-h_U) \;\; \dot{x}^T(t) \;\; \dot{x}^T(t-h_U) \;\; \frac{1}{h_U}\int_{t-h_U}^t x^T(s)ds\right],\\
&\Sigma_c = [\Sigma_{c(m,n)}],\ m=1,...,5,\ n=1,...,5,\\
&\Sigma_{c(1,1)} = R_{13}+R_{13}^T+N_{11}-Q_1+h_UQ_2-2Q_3,\\
&\Sigma_{c(1,2)} = -R_{13}+R_{23}^T+Q_1,\ \Sigma_{c(1,3)} = R_{11}+N_{12},\ \Sigma_{c(1,4)} = R_{12},\\
&\Sigma_{c(1,5)} = h_UR_{33}+2Q_3,\ \Sigma_{c(2,2)} = -R_{23}-R_{23}^T-N_{11}-Q_1,\\
&\Sigma_{c(2,3)} = R_{12}^T,\ \Sigma_{c(2,4)} = R_{22}-N_{12},\ \Sigma_{c(2,5)} = -h_UR_{33},\\
&\Sigma_{c(3,3)} = N_{22}+(h_U^2/2)Q_3,\ \Sigma_{c(3,4)} = 0,\ \Sigma_{c(3,5)} = h_UR_{13},\\
&\Sigma_{c(4,4)} = -N_{22}+h_U^2Q_1,\ \Sigma_{c(4,5)} = h_UR_{23},\ \Sigma_{c(5,5)} = -h_UQ_2-2Q_3,\\
&\Gamma_c = [A \;\; A_d \;\; -I \;\; 0 \;\; 0].
\end{aligned} \tag{30}$$

Corollary 1. For given $h_U > 0$, the system $\dot{x}(t) = Ax(t)+Ax(t-h_U)$ is asymptotically stable if there exist positive matrices $\boldsymbol{R}=[R_{ij}]_{3\times 3}$, $\boldsymbol{N}=[N_{ij}]_{2\times 2}$, $Q_i(i=1,2,3)$, such that the following LMIs hold

$$(\Gamma_c^\perp)^T(\Sigma_c)\Gamma_c^\perp < 0 \tag{31}$$

where Σ_c and Γ_c are defined in (30), and $\Gamma_c^\perp$ is the right orthogonal complement of Γ_c.

Proof. Let us consider the following Lyapunov-Krasovskii's functional

$$V(t) = \sum_{i=1}^5 V_i(t), \tag{32}$$

where

$$\begin{aligned}
&V_1(t) = \begin{bmatrix} x(t) \\ x(t-h_U) \\ \int_{t-h_U}^t x(s)ds\end{bmatrix}^T \boldsymbol{R} \begin{bmatrix} x(t) \\ x(t-h_U) \\ \int_{t-h_U}^t x(s)ds\end{bmatrix},\\
&V_2(t) = \int_{t-h_U}^t \begin{bmatrix} x(s) \\ \dot{x}(s)\end{bmatrix}^T \boldsymbol{N} \begin{bmatrix} x(s) \\ \dot{x}(s)\end{bmatrix} ds,\\
&V_3(t) = h_U\int_{t-h_U}^t\int_s^t \dot{x}^T(u)Q_1\dot{x}(u)duds,\\
&V_4(t) = \int_{t-h_U}^t\int_s^t x^T(u)Q_2x(u)duds,\\
&V_5(t) = \int_{t-h_U}^t\int_s^t\int_u^t \dot{x}^T(v)Q_3\dot{x}(v)dvduds.
\end{aligned} \tag{33}$$

With the defined augmented vector $\zeta(t)$, the time derivative of $V(t)$ can be estimated as

$$\dot{V}(t) \le \zeta_c^T(t)\Sigma_c\zeta_c(t). \tag{34}$$

Therefore, by Lemma 1, $\zeta_c^T(t)\Sigma_c\zeta_c(t) < 0$ with $0=\Gamma_c\zeta_c(t)$ is equivalent to (31). Therefore, if LMI (32) hold, then system $\dot{x}(t) = Ax(t)+Ax(t-h_U)$ is asymptotically stable. This completes our proof. ■

4. Numerical Examples

Example 1. Consider the following systems

$$\dot{x}(t) = \begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9\end{bmatrix}x(t)+\begin{bmatrix} -1.1 & 0.2 \\ -0.1 & -1.1\end{bmatrix}x(t-h(t)) \tag{35}$$

When $-h_D \le \dot{h}(t) \le h_D < 1$, by applying Theorem 1 and 2 to system (35), maximum delay bounds for guaranteeing system (35) to be asymptotically stable are listed in Table 1. For different h_D, obtained results from Theorem 1 are larger than those of [9]-[11]. When time-varying delay dependent Lyapunov-Kraosvskii's functional shown in (25) is included in Theorem 2, one can see further improved results are obtained from Table 1. Therefore, our proposed method gives larger delay bounds than the ones in [9]-[11].

Example 2. Consider the following systems

$$\dot{x}(t) = \begin{bmatrix} -2 & 0 \\ 0 & -0.9\end{bmatrix}x(t)+\begin{bmatrix} -1 & 0 \\ -1 & -1\end{bmatrix}x(t-h(t)) \tag{36}$$

Assume $-h_D \le \dot{h}(t) \le h_D < 1$. Table 2 contain maximum delay bounds obtained from Theorem 1 and 2. Except the result comparing with the one in [11] when $h_D = 0.1$, the results of Theorem 1 gives larger delay bounds than the ones in [9]-[11]. However, the results obtained from Theorem 2 provide improved delay bounds for $h_D = 0.1$, 0.5, and 0.9.

표 1 예제 1에서 h_D 변화에 따른 안정성을 보장하는 h_U의 상한 값

Table 1 Upper bounds of h_U for guaranteeing stability in Example 1 with different h_D.

h_D	0.2	0.4	0.6	0.8
Ref. [9]	1.6070	1.4119	1.2430	1.1077
Ref. [11]	1.6412	1.5124	1.4264	1.3640
Ref. [10]	2.1150	1.8663	1.6666	1.4951
Theorem 1	2.1770	1.9855	1.8538	1.7573
Theorem 2	2.1804	1.9869	1.8553	1.7603

표 2 예제 2에서 h_D 변화에 따른 안정성을 보장하는 h_U의 상한 값

Table 2 Upper bounds of h_U for guaranteeing stability in Example 2 with different h_D.

h_D	0.1	0.5	0.9
Ref.[9]	3.6053	2.0439	1.3789
Ref.[10]	3.6224	2.2038	1.6064
Ref. [11]	3.7304	2.5021	2.0982
Theorem 1	3.6998	2.5167	2.1276
Theorem 2	3.7454	2.5184	2.1313

5. Conclusions

In this paper, new delay-dependent stability criteria for linear systems with time-varying delays were proposed. By introducing new augmented vectors including time-varying delay functional and utilizing convex-hull properties, improved stability criteria were proposed in Theorem 1. Also, new Lyapunov-Krasovskii's functional which contains time-varying delays was introduced in Theorem 2 and its improvement was shown through two numerical examples.

Acknowledgement

This work was supported by the research grant of the Chungbuk National University in 2010.

References

[1] J. Hale, and S.M.V. Lunel, Introduction to Functional Differential Equatins, Springer-Verlag, New York, 1993.

[2] V.B. Kolmanovskii, and A. Myshkis, Applied Theory to Functional Differential Equations, Kluwer Academic Publishers, Boston, 1992.

[3] J.-H. Kim, "Delay and its time-derivative dependent robust stability of time-delayed linear systems with uncertainty", IEEE Transactions on Automatic Control, vol.46, pp.789-792, 2001.

[4] J.-P. Richard, TIme-delay systems: an overview of some recent advances and open problems, Automatica, vol.39, pp.1667-1694, 2003.

[5] S. Xu, and J. Lam, A survey of linear matrix inequality techniques in stability analysis of delay systems, International Journal of Systems Sciences, 39, pp.1095-1113, 2008.

[6] J.-H. Kim, Y.-G. Yi, "Delay-dependent robust stability of uncertain time-delayed linear systems", Trans. KIEE, 55D, pp.147-156, 2006.

[7] O.M. Kwon, and J.H. Park, "An improved delay-dependent robust control for uncertain time-delay systems", IEEE Transactions on Automatic Control, vol. 49, No. 11, pp.1991-1995, 2004.

[8] M.J. Park, O.M. Kwon, J.H. Park, and S.M. Lee, New delay-dependent stability criterion for neural networks with discrete and distributed time-varying delays, Trans. KIEE, vol.58, no.6, pp.1809-1814

[9] Y. He, Q.-G. Wang, L. Xie, and C. Lin, Further improvemet of free-weighting matrices technique for systems with time-varyig delay, IEEE Transactions on Automatic Control, vol.52, no.2, pp.293-299, 2007.

[10] W.Qian, S. Cong, Y. Sun, and S. Fei, Novel robust stability criteria for uncertain systems with time-varying delay, Applied Mathematics and Computation, vol.215, pp.866-872, 2009.

[11] J. Sun, G.P. Liu, J. Chen, Further results on stability criteria for linear systems with time-varying delay, 2008 IEEE International Conference on Systems, Man and Control (SMC 2008), pp.2736-2740

[12] S. Boyd, L.E. Ghaoui, E. Feron, V. Balakrishnan, Linear Matrix Inequalities in Systems and Control Theory, Philadelphia, SIAM, 1994.

[13] R.E. Skelton, T. Iwasaki, and K.M. Grigoradis, A unified algebraic approach to linear control design, New York, Talyor and Francis, 1997.

저 자 소 개

권 오 민 (權 五 珉)

1974년 7월 13일생. 1997년 경북대학교 전자공학과 졸업(공학). 2004년 포항공과대학교 전기전자공학부 졸업(공박). 현재 충북대학교 전기공학과 조교수.

Tel : 043-261-2422

Fax : 043-263-2419

E-mail : madwind@chungbuk.ac.kr

박 주 현 (朴 柱 炫)

1968년 1월 11일생. 1990년 경북대학교 전자공학과 졸업(공학). 1997년 포항공과대학교 전기전자공학부 졸업(공박). 현재 영남대학교 전기공학과 부교수

Tel : 053-810-2491

Fax : 053-810-4767

E-mail : jessie@ynu.ac.kr

이 상 문 (李 相 文)

1973년 6월 15일생. 1999년 경북대학교 전자공학과 졸업(공학). 2006년 포항공과대학교 전기전자공학부 졸업(공박). 현재 대구대학교 전자공학부 전임강사.

E-mail : moony@daegu.ac.kr