Bull. Korean Math. Soc. **47** (2010), No. 6, pp. 1121–1129
DOI 10.4134/BKMS.2010.47.6.1121

ON (σ,τ)-LIE IDEALS WITH GENERALIZED DERIVATION

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Abstract. In the present paper, we extend some well known results concerning derivations of prime rings to generalized derivations for (σ,τ)-Lie ideals.

1. Introduction

Let R be an associative ring with center Z and σ,τ two mappings from R into itself. For any $x,y\in R$, we write $[x,y]$ and $[x,y]_{\sigma,\tau}$, for $xy-yx$ and $x\sigma(y)-\tau(y)x$ respectively and make extensive use of basic commutator identities:

1) $[xy,z]_{\sigma,\tau}=x[y,z]_{\sigma,\tau}+[x,\tau(z)]y=x[y,\sigma(z)]+[x,z]_{\sigma,\tau}y$,

2) $[x,yz]_{\sigma,\tau}=\tau(y)[x,z]_{\sigma,\tau}+[x,y]_{\sigma,\tau}\sigma(z)$.

Let U be an additive subgroup of R. The definition of (σ,τ)-Lie ideal of R is given in [15] as follows:

(i) U is a (σ,τ)-right Lie ideal of R if $[U,R]_{\sigma,\tau}\subset U$.

(ii) U is a (σ,τ)-left Lie ideal of R if $[R,U]_{\sigma,\tau}\subset U$.

(iii) U is a (σ,τ)-Lie ideal of R, if U is both a (σ,τ)-right Lie ideal and (σ,τ)-left Lie ideal of R.

It is clear that every Lie ideal of R is a $(1,1)$-left (right) Lie ideal of R, where $1:R\to R$ is an identity map.

Recall that a ring R is prime if for any $x,y\in R, xRy=\{0\}$ implies that $x=0$ or $y=0$. An additive mapping $d:R\to R$ is called a derivation if $d(xy)=d(x)y+xd(y)$ holds for all $x,y\in R$. For a fixed $a\in R$, the mapping $I_a:R\to R$ given by $I_a(x)=[a,x]$ is a derivation which is said to be an inner derivation.

The commutativity of prime rings with derivation was initiated by E. C. Posner [17]. Over the last two decades, a great deal of work has been done on this subject. A function $f_{a,b}:R\to R$ is called a generalized inner derivation if $f_{a,b}(x)=ax+xb$ for some fixed $a,b\in R$. It is straightforward to note that

Received December 16, 2008; Revised December 3, 2009.
2000 *Mathematics Subject Classification.* 16N60, 16W25, 16U80.
Key words and phrases. derivations, (σ,τ)-Lie ideals, generalized derivations.

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$f_{a,b}$ is a generalized inner derivation, then for any $x, y \in R$

$$\begin{aligned} f_{a,b}(xy) &= f_{a,b}(x)y + x[y, b] \\ &= f_{a,b}(x)y + xI_b(y) \end{aligned}$$

where I_b is an inner derivation. In view of the above observation, the concept of generalized derivation is introduced in [14] and [8] as follows:

An additive mapping $f : R \to R$ is called a generalized derivation associated with a derivation d if

$$f(xy) = f(x)y + xd(y) \quad \text{for all } x, y \in R.$$

One may observe that the concept of generalized derivation includes the concept of derivations and generalized inner derivations, also of the left multipliers when $d = 0$. Hence it should be interesting to extend some results concerning these notions to generalized derivations. Some recent results were shown on generalized derivation in [8], [14] and [1]. Furthermore, some authors have also studied generalized derivation in the theory of operator algebras and C^*-algebras (see for example [14]).

On the other hand, in [10, Definition 1], Gölbaşı and Kaya introduced the notation of right generalized derivation and left generalized derivation with associated derivation d as follows:

An additive mapping $f : R \to R$ is said to be right generalized derivation with associated derivation d if

$$f(xy) = f(x)y + xd(y) \quad \text{for all } x, y \in R \tag{1.1}$$

and f is said to be left generalized derivation with associated derivation d if

$$f(xy) = d(x)y + xf(y) \quad \text{for all } x, y \in R. \tag{1.2}$$

f is said to be a generalized derivation with associated derivation d if it is both a left and right generalized derivation with associated derivation d. Of course, every derivation is generalized derivation and also, the definition of generalized derivation given in Bresar [8] is a right generalized derivation with associated derivation d according to above definition. In this context, we mention the definition of generalized derivation that means two sided generalized derivation.

In [1], Argaç and Albaş proved that if a prime ring R has (d, α), (g, β) nonzero generalized derivations such that $ad(x) = g(x)a$ for all $x \in R$, then one of the following possibilities holds; (i) $a \in C$ (extended centroid). (ii) There exist $p, q \in Q_r(R_C)$ (a right Martindale ring of quotients) such that $\alpha(x) = [x, p]$, $\beta(x) = [q, x]$, $qa \in C$, $p = \lambda a$, where $\lambda \in C$, for all $x \in R$. In this paper, one of our first objectives is to show that this result satisfies for generalized derivations on (σ, τ)-left Lie ideal of R.

In [12], Herstein showed that if R is a prime ring of characteristic different from two and d is a nonzero derivation such that $d(R) \subset Z$, then R must be commutative. Several authors investigated this result for Lie ideals or (σ, τ)-Lie ideals of a prime ring admitting derivation or generalized derivation (see

[7], [6], [5], [10]). We prove this theorem for (σ,τ)-left Lie ideals of prime ring and a mapping such that $f(x) = xa - bx$ for all $x \in R$. Thus we extend [11, Lemma 1] to (σ,τ)-Lie ideals of a prime ring of R.

On the other hand, in [9], Daif and Bell proved that if a semiprime ring R has a derivation d satisfying the following condition, then I is a central ideal;

$$\text{there exists a nonzero ideal } I \text{ of } R \text{ such that}$$
$$\text{either } d([x,y]) = [x,y] \text{ for all } x,y \in I, \text{ or } d([x,y]) = -[x,y] \text{ for all } x,y \in I.$$

Latter on, the author proved this result generalized derivations of semiprime rings in [2]. Finally, we prove corresponding result for (σ,τ)-Lie ideal of a prime ring with generalized derivation.

Throughout the present paper, we assume that R be a prime ring with characteristic not two, σ and τ two automorphisms and U a nonzero (σ,τ)-Lie ideal of R. We denote a generalized derivation $f : R \to R$ determined by derivation d of R by (f,d). If $d = 0$, then $f(xy) = f(x)y$ for all $x,y \in R$ and there exists $q \in Q_r(R_C)$ such that $f(x) = qx$ for all $x \in R$ by [14, Lemma 2]. So, we assume that $d \neq 0$.

2. Results

In the view of the definition of generalized derivation, one can easily notice that the following remark.

Remark 1. Let (f,d) be a generalized derivation of R. If $f\sigma = \sigma f$, $f\tau = \tau f$, then $f([x,y]_{\sigma,\tau}) = [d(x),y]_{\sigma,\tau} + [x,f(y)]_{\sigma,\tau}$ for all $x,y \in R$.

Lemma 1. [4, Lemma 3] *Let R be a prime ring with* char$R \neq 2$, $a \in R$ *and* $aU = 0$ (*or* $Ua = 0$).

i) *If U is a (σ,τ)-right Lie ideal of R, then $a = 0$ or $U \subset C_{\sigma,\tau}$.*

ii) *If U is a (σ,τ)-left Lie ideal of R, then $a = 0$ or $U \subset Z$.*

Lemma 2. [3, Lemma 6] *Let R be a prime ring with* char$R \neq 2$ *and U a (σ,τ)-left Lie ideal of R. Suppose there exists $a \in R$ such that $[a,U] = 0$. Then $a \in Z$ or $\sigma(u) + \tau(u) \in Z$ for all $u \in U$.*

Lemma 3. [16, Theorem 2] *Let R be a prime ring with* char$R \neq 2$ *and U a noncentral (σ,τ)-left Lie ideal of R. Then there exist a nonzero ideal M of R such that $[R,M]_{\sigma,\tau} \subset U$ and $[R,M]_{\sigma,\tau} \not\subseteq C_{\sigma,\tau}$ or $\sigma(u) + \tau(u) \in Z$ for all $u \in U$.*

Lemma 4. *Let R be a prime ring with* char$R \neq 2$, *(f,d) a generalized derivation of R and U a noncentral (σ,τ)-left Lie ideal of R. If $f(U) = 0$, then $\sigma(u) + \tau(u) \in Z$ for all $u \in U$.*

Proof. Suppose to the contrary that $\sigma(u) + \tau(u) \notin Z$ for some $u \in U$. By Lemma 3, there exists a nonzero ideal M of R such that $[R,M]_{\sigma,\tau} \subset U$ but $[R,M]_{\sigma,\tau} \not\subseteq C_{\sigma,\tau}$. For any $x \in R$ and $m \in M$,

$$[x,m]_{\sigma,\tau}\sigma(m) = [x\sigma(m),m]_{\sigma,\tau} \in U.$$

Then

$$0 = f([x,m]_{\sigma,\tau}\sigma(m)) = f([x,m]_{\sigma,\tau})\sigma(m) + [x,m]_{\sigma,\tau}d(\sigma(m))$$

and so

$$[x,m]_{\sigma,\tau}d(\sigma(m)) = 0 \text{ for all } x \in R, m \in M. \tag{2.1}$$

Replacing x by $xy, y \in R$ in (2.1) and applying (2.1), we get

$$0 = [xy,m]_{\sigma,\tau}d(\sigma(m)) = x[y,m]_{\sigma,\tau}d(\sigma(m)) + [x,\tau(m)]yd(\sigma(m)).$$

That is

$$[x,\tau(m)]Rd(\sigma(m)) = 0 \text{ for all } x \in R, m \in M.$$

Since R is a prime ring, it follows that

$$m \in Z \text{ or } d(\sigma(m)) = 0 \text{ for all } m \in M.$$

We set $K = \{m \in M \mid m \in Z\}$ and $L = \{m \in M \mid d(\sigma(m)) = 0\}$. Clearly each of K and L is additive subgroup of M. Moreover, M is the set-theoretic union of K and L. But a group can not be the set-theoretic union of its two proper subgroups, hence $K = M$ or $L = M$. In the former case, $M \subset Z$ which forces R to be commutative. This is impossible because of $U \nsubseteq Z$. In the latter case, $d(\sigma(M)) = 0$. Since R is a prime ring and $\sigma(M)$ a nonzero ideal of R, we get $d = 0$, a contradiction. This completes the proof. □

Lemma 5. *Let R be a prime ring with* char$R \neq 2$, *(f,d) a generalized derivation of R and U a noncentral (σ,τ)-left Lie ideal of R. If $d(Z) \neq 0$ and $[f(U),a]_{\sigma,\tau} = 0$, then $a \in Z$ or $\sigma(u) + \tau(u) \in Z$ for all $u \in U$.*

Proof. Choose $\alpha \in Z$ such that $d(\alpha) \neq 0$. It is easily seen that $d(\alpha) \in Z$. For all $x \in R, u \in U$, we get

$$\begin{aligned} 0 &= [f([x,u]_{\sigma,\tau}\alpha),a]_{\sigma,\tau} \\ &= [f([x,u]_{\sigma,\tau})\alpha + [x,u]_{\sigma,\tau}d(\alpha),a]_{\sigma,\tau} \\ &= [f([x,u]_{\sigma,\tau}),a]_{\sigma,\tau}\alpha + f([x,u]_{\sigma,\tau})[\alpha,\sigma(a)] + [[x,u]_{\sigma,\tau},a]_{\sigma,\tau}d(\alpha) \\ &\quad + [x,u]_{\sigma,\tau}[d(\alpha),\sigma(a)] \end{aligned}$$

and so

$$[[x,u]_{\sigma,\tau},a]_{\sigma,\tau}d(\alpha) = 0 \text{ for all } x \in R, u \in U.$$

Since R is prime and $0 \neq d(\alpha) \in Z$, we see that

$$[[x,u]_{\sigma,\tau},a]_{\sigma,\tau} = 0 \text{ for all } x \in R, u \in U. \tag{2.2}$$

Substituting $x\sigma(u)$ for x in (2.2) and using this equation, we obtain

$$[x,u]_{\sigma,\tau}\sigma([u,a]) = 0 \text{ for all } x \in R, u \in U.$$

Now, taking xy instead of x in the last equation, we obtain

$$[R,\tau(u)]R\sigma([u,a]) = 0 \text{ for all } u \in U.$$

Since R is a prime ring, it follows either $u \in Z$ or $[u,a] = 0$ for all $u \in U$. By a standard argument one of these must hold for all $u \in U$. If $u \in Z$ for all $u\in U$, then $U\subset Z$, and so $\sigma(u)+\tau(u)\in Z$ for all $u\in U$. If $[U,a]=0$, then $a\in Z$ or $\sigma(u)+\tau(u)\in Z$ for all $u\in U$ by Lemma 2. Thus the proof is completed. □

Theorem 1. *Let R be a prime ring with* $\mathrm{char}R\neq 2$, $(f,d),(g,h)$ *two generalized derivations of R and U a noncentral (σ,τ)-left Lie ideal of R. If $f(u)v=ug(v)$ for all $u,v\in U$, then $\sigma(u)+\tau(u)\in Z$ for all $u\in U$.*

Proof. Suppose that $\sigma(u)+\tau(u)\notin Z$ for some $u\in U$. Then there exists a nonzero ideal M of R such that $[R,M]_{\sigma,\tau}\subset U$ and $[R,M]_{\sigma,\tau}\nsubseteq C_{\sigma,\tau}$ by Lemma 3. For any $x\in R$ and $m\in M$, $\tau(m)[x,m]_{\sigma,\tau}=[\tau(m)x,m]_{\sigma,\tau}\in U$. Taking $\tau(m)[x,m]_{\sigma,\tau}$ instead of u in the hypothesis, we get

$$f\left(\tau(m)[x,m]_{\sigma,\tau}\right)v=\tau(m)[x,m]_{\sigma,\tau}g(v),$$

$$d(\tau(m))[x,m]_{\sigma,\tau}v+\tau(m)f\left([x,m]_{\sigma,\tau}\right)v=\tau(m)[x,m]_{\sigma,\tau}g(v).$$

Using the hypothesis in the above relation, we arrive at

$$d(\tau(m))[x,m]_{\sigma,\tau}v=0 \text{ for all } m\in M, v\in U, x\in R.$$

That is

$$d(\tau(m))[x,m]_{\sigma,\tau}U=(0) \text{ for all } m\in M,\ x\in R.$$

By Lemma 1, we obtain that

$$d(\tau(m))[x,m]_{\sigma,\tau}=0 \text{ for all } m\in M,\ x\in R. \tag{2.3}$$

Replacing x by xy, $y\in R$ in (2.3) and using (2.3), we have

$$d(\tau(m))x[y,\sigma(m)]=0$$

and so

$$d(\tau(m))R[y,\sigma(m)]=0 \text{ for all } m\in M,\ y\in R.$$

Since R is a prime ring, it follows that

$$m\in Z \text{ or } d(\tau(m))=0 \text{ for all } m\in M.$$

Let $L=\{m\in M\mid m\in Z\}$ and $K=\{m\in M\mid d(\tau(m))=0\}$. By the same method in Lemma 4, we get $d=0$, a contradiction. This completes the proof. □

An immediately results of Theorem 1 we give the following corollaries.

Corollary 1. *Let R be a prime ring with* $\mathrm{char}R\neq 2$, $(f,d),(g,h)$ *two generalized derivations of R and U a noncentral (σ,τ)-left Lie ideal of R. If $f(u)u=ug(u)$ for all $u\in U$, then $\sigma(u)+\tau(u)\in Z$ for all $u\in U$.*

In particular if we take $f=g$, then we have the following corollary, which is a generalization of [13, Theorem] for the case when characteristic of underlying ring is different from two.

Corollary 2. *Let R be a prime ring with* $\mathrm{char}R \neq 2$, (f,d) *a generalized derivation of R and U a noncentral (σ,τ)-left Lie ideal of R. If* $[u, f(u)] = 0$ *for all $u \in U$, then* $\sigma(u) + \tau(u) \in Z$ *for all $u \in U$.*

Corollary 3. *Let R be a prime ring with* $\mathrm{char}R \neq 2$, *d, h two nonzero derivations of R and U a noncentral (σ,τ)-left Lie ideal of R. If* $d(u)v = uh(v)$ *for all $u, v \in U$, then* $\sigma(u) + \tau(u) \in Z$ *for all $u \in U$.*

Theorem 2. *Let R be a prime ring with* $\mathrm{char}R \neq 2, U$ *a nonzero (σ,τ)-left Lie ideal of R. Let $a, b \in R$ and $f : R \to R$ be a mapping such that $f(x) = xa - bx$ for all $x \in R$. If $f(U) \subset U$ and $f(U) \subset Z$, then $\sigma(u)+\tau(u) \in Z$ for all $u \in U$.*

Proof. By the hypothesis, for all $u \in U$, we have $f(u) = ua - bu \in Z$. Commuting this element by u, we obtain that,

$$u[a,u] = [b,u]u \text{ for all } u \in U. \tag{2.4}$$

A linearization of (2.4) yields that

$$u[a,v] + v[a,u] = [b,v]u + [b,u]v.$$

Taking $f(u)$ instead of u in the above equation, we find that

$$f(u)[a,v] + v[a,f(u)] = [b,v]f(u) + [b,f(u)]v.$$

Using $f(u) \in Z$ in the last equation, we have

$$f(u)([a,v] - [b,v]) = 0 \text{ for all } u, v \in U. \tag{2.5}$$

Using the primeness of R and $f(u) \in Z$ in (2.5), we conclude that

$$f(U) = 0 \text{ or } [a-b, U] = (0).$$

If $[a-b, U] = (0)$, then $a - b \in Z$ or $\sigma(u) + \tau(u) \in Z$ for all $u \in U$ by Lemma 2. Now, we assume that $a - b \in Z$. From (2.4), we obtain

$$u^2a - uau = ubu - bu^2 \text{ for all } u \in U. \tag{2.6}$$

Let $a - b = \alpha$, $\alpha \in Z$. Writing $a = b + \alpha$ in (2.6), we have

$$u^2b + u^2\alpha - ubu - u\alpha u = ubu - bu^2,$$

$$u^2b + u^2\alpha + bu^2 = 2ubu + u\alpha u$$

Using $\alpha \in Z$ in the last equation, we get

$$u^2b + bu^2 - 2ubu = 0,$$

$$u^2b - ubu = ubu - bu^2$$

and so

$$u[u,b] = [u,b]u.$$

That is

$$[u,[u,b]] = 0 \text{ for all } u \in U.$$

This yields that $[u, d_b(u)] = 0$, where $d_b : R \to R$, $d_b(x) = [x,b]$ is an inner derivation of R. Therefore $\sigma(u)+\tau(u) \in Z$ for all $u \in U$ or $d_b = 0$ by Corollary 2.

If $d_b = 0$, then $b \in Z$. We have $f(u) = ua - bu = u(a-b) \in Z$, by the hypothesis. Since $a - b \in Z$ and R is a prime ring, we obtain that $U \subset Z$ or $a - b = 0$.

Now, we assume that $a = b$. Using $b \in Z$, we get

$$f(x) = xa - bx = xb - bx = 0 \text{ for all } x \in R.$$

As a result $f = 0$, and so $f(U) = 0$. Hence $\sigma(u) + \tau(u) \in Z$ for all $u \in U$ according to Lemma 4. This completes the proof. □

Corollary 4. *Let R be a prime ring with* char$R \neq 2, U$ *a nonzero (σ,τ)-Lie ideal of R and $a \in R$. If $[U,a]_{\sigma,\tau} \subset Z$, then $a \in Z$ or $\sigma(u) + \tau(u) \in Z$ for all $u \in U$.*

Proof. Let f be a mapping such that $f(x) = [x,a]_{\sigma,\tau} = x\sigma(a) - \tau(a)x$ for all $x \in R$. Since U is a (σ,τ)- Lie ideal, we have $f(U) \subset U$. By Theorem 2, we get $\sigma(u) + \tau(u) \in Z$ for all $u \in U$. □

We can give a following corollary in view of Corollary 4, which is a generalization of [11, Lemma 1].

Corollary 5. *Let R be a prime ring with* char$R \neq 2, U$ *a nonzero (σ,τ)-Lie ideal of R. If $[U,U]_{\sigma,\tau} \subset Z$, then $\sigma(u) + \tau(u) \in Z$ for all $u \in U$.*

Theorem 3. *Let R be a prime ring with* char$R \neq 2, U$ *a nonzero (σ,τ)-Lie ideal of R, (f,d) a generalized derivation of R such that $f\sigma = \sigma f$, $f\tau = \tau f$ and $d(Z) \neq 0$. If (f,d) satisfies one of the following conditions, then $\sigma(u) + \tau(u) \in Z$ for all $u \in U$.*

i) $f([u,v]_{\sigma,\tau}) = [u,v]_{\sigma,\tau}$ *for all $u, v \in U$.*

ii) $f([u,v]_{\sigma,\tau}) = -[u,v]_{\sigma,\tau}$ *for all $u, v \in U$.*

iii) *For each $u, v \in U$, either $f([u,v]_{\sigma,\tau}) = [u,v]_{\sigma,\tau}$ or $f([u,v]_{\sigma,\tau}) = -[u,v]_{\sigma,\tau}$.*

Proof. i) By the hypothesis, we obtain

$$f([u,v]_{\sigma,\tau}) = [d(u),v]_{\sigma,\tau} + [u,f(v)]_{\sigma,\tau} = [u,v]_{\sigma,\tau} \text{ for all } u,v \in U. \tag{2.7}$$

Replacing v by $[v,w]_{\sigma,\tau}$, $w \in U$ in (2.7), we get

$$[d(u),[v,w]_{\sigma,\tau},]_{\sigma,\tau} + [u,f([v,w]_{\sigma,\tau},)]_{\sigma,\tau} = [u,[v,w]_{\sigma,\tau},]_{\sigma,\tau}.$$

Using the hypothesis, we have

$$[d(u),[v,w]_{\sigma,\tau},]_{\sigma,\tau} + [u,[v,w]_{\sigma,\tau},]_{\sigma,\tau} = [u,[v,w]_{\sigma,\tau},]_{\sigma,\tau}$$

and so

$$[d(u),[v,w]_{\sigma,\tau},]_{\sigma,\tau} = 0 \text{ for all } u,v,w \in U.$$

That is

$$\Big[d(U),[U,U]_{\sigma,\tau}\Big]_{\sigma,\tau} = (0).$$

By Lemma 5, we conclude that, $[U,U]_{\sigma,\tau} \subset Z$ or $\sigma(u) + \tau(u) \in Z$ for all $u \in U$. If $[U,U]_{\sigma,\tau} \subset Z$, then we have already $\sigma(u) + \tau(u) \in Z$ for all $u \in U$ by Corollary 5.

ii) can be proved by using the same techniques.

iii) For each $w \in U$, we put

$$U_w = \{v \in U \mid f([w,v]_{\sigma,\tau}) = [w,v]_{\sigma,\tau}\}$$

and

$$U_w^* = \{v \in U \mid f([w,v]_{\sigma,\tau}) = -[w,v]_{\sigma,\tau}\}.$$

Then additive group U is the union of its two subgroups U_w and U_w^*. But a group cannot be the union of its proper subgroups, hence $U = U_w$ or $U = U_w^*$. By using the same method as used in (i) or (ii), we get the required result. □

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