Analysis on the Calculation of Plasma Medium with Parallel SO-FDTD Method

Xule Duan, Hong Wei Yang, Xiangkun Kong, and Han Liu

This paper introduces a novel parallel shift operator finite-difference time-domain (SO-FDTD) method for plasma in the dispersive media. We calculate the interaction between the electromagnetic wave of various frequencies and non-magnetized plasma by using the parallel SO-FDTD method. Then, we compare the results, which are calculated with serial and parallel SO-FDTD executions to obtain the speedup ratio and validate the parallel execution. We conclude that the parallel SO method has almost the same precision as the serial SO method, while the parallel approach expands the scope of memory and reduces the CPU time.

Keywords: Parallel SO-FDTD method, dispersive media, plasma, electromagnetic wave.

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I. Introduction

The calculation of electromagnetic (EM) wave propagation in dispersive media by time-domain methods has made great progress, among which, the finite-difference time-domain (FDTD) method is well established and comparatively efficient [1]. In recent years, a large number of new FDTD EM simulation methods have appeared, including recursive convolution [2], the auxiliary differential equation method [3], the Z-transform method [4], piecewise linear recursive convolution [5], current density convolution [6], piecewise linear current density recursive convolution [7], the shift operator (SO) method [8], and so on [9]-[11].

Recently, several studies analyzing plasma-covered stealth targets have been conducted. Simulating modeling using an FDTD method leads to large-scale EM calculations which have a large memory requirement. Due to expensive super-computer and cluster-specific equipments, researchers face the problems of computational efficiency and computational scope and the difficulties of calculation. Therefore, it is rare to find studies on plasma modeling using parallel methods. A parallel FDTD method can be run on an economical high-performance PC connected by LAN. Fulfilling the large memory requirement of using network technology can expand the scope of applications of numerical simulation methods. In this study, a parallel implementation of the SO-FDTD method with second-order accuracy is used in calculations of the EM field in an iterative process. The interaction between EM wave and homogeneous non-magnetized plasma is simulated at different frequencies. Finally, the results obtained with parallel and serial algorithms are compared to analyze the speedup ratio and parallel efficiency and to estimate computational time savings.

II. Calculation Method of Homogeneous Non-magnetized Plasma

With collision cold plasma in the dispersive media, the well-known Maxwell's equations and related equations are given as

$$\frac{\partial \mathbf{D}}{\partial t} = \nabla \times \mathbf{H}, \tag{1}$$

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu_0} \nabla \times \mathbf{E}, \tag{2}$$

$$\mathbf{D}(\omega) = \varepsilon_0 \varepsilon_r(\omega) \mathbf{E}(\omega). \tag{3}$$

We can write formulas (1) and (2) into inferential equations for an iterative solution. Equation (3) is a function in the frequency domain. As for cold plasma, the relative dielectric constant is

$$\varepsilon_r(\omega) = 1 + \frac{\omega_{\mathrm{p}}^2}{\omega(j\nu_{\mathrm{c}} - \omega)}, \tag{4}$$

where ω_{p} is the plasma frequency, and ν_{c} is the fixed value of the plasma collision frequency.

1. Shift Operator (SO) Method

The transitional relationships between the dispersive time domain and the time domain are calculated by applying SO. The constitutive relationship in the frequency domain can be written as rational fractional functions, transformed to the time domain, and then transformed to the dispersive time domain. Recursive formulations from **D** to **E** can be deduced [11].

Dielectric constant $\varepsilon_r(\omega)$ can be written as rational fractional function:

$$\varepsilon_r(\omega) = \frac{\sum_{n=0}^{N} p_n (j\omega)^n}{\sum_{n=0}^{N} q_n (j\omega)^n}. \tag{5}$$

Therefore, according to (5), $\varepsilon_r(\omega)$ in (3) can be written as $\varepsilon_r(j\omega)$. According to the inverse Fourier transform $f(t) = T^{-1}[F(\omega)]$ or the transition relationship from the frequency domain to the time domain $j\omega \to \partial/\partial t$ in electromagnetics, (3) can be written as

$$\mathbf{D}(t) = \varepsilon_0 \varepsilon_r \left(\frac{\partial}{\partial t}\right) \mathbf{E}(t). \tag{6}$$

Assuming the function $y(t) = \dfrac{\partial f(t)}{\partial t}$, its central difference in $(n+0.5)\Delta t$ can approximately be given as

$$\frac{y^{n+1} + y^n}{2} = \frac{f^{n+1} - f^n}{\Delta t}. \tag{7}$$

Define

$$z_t f^n = f^{n+1}, \tag{8}$$

where z_t is the SO [8], [9], and combine (7) with (8):

$$y^n = \left(\frac{2}{\Delta t} \frac{z_t - 1}{z_t + 1}\right) f^n. \tag{9}$$

After comparison,

$$\frac{\partial}{\partial t} \to \left(\frac{2}{\Delta t} \frac{z_t - 1}{z_t + 1}\right). \tag{10}$$

The constitutive relationship in the dispersive time domain can be given as

$$\left[\sum_{l=0}^{N} q_l \left(\frac{2}{\Delta t} \frac{z_t - 1}{z_t + 1}\right)^l\right] \mathbf{D}^n = \varepsilon_0 \left[\sum_{l=0}^{N} p_l \left(\frac{2}{\Delta t} \frac{z_t - 1}{z_t + 1}\right)^l\right] \mathbf{E}^n. \tag{11}$$

Suppose N=2, and inset it into (11):

$$\left\{ \begin{aligned} &\left[q_0 + q_1 \frac{2}{\Delta t} + q_2 \left(\frac{2}{\Delta t}\right)^2\right] z_t^2 + \left[2q_0 - 2q_2 \left(\frac{2}{\Delta t}\right)^2\right] z_t \\ &+ \left[q_0 - q_1 \frac{2}{\Delta t} + q_2 \left(\frac{2}{\Delta t}\right)^2\right] \end{aligned} \right\} \mathbf{D}^n$$

$$= \left\{ \begin{aligned} &\left[p_0 + p_1 \frac{2}{\Delta t} + p_2 \left(\frac{2}{\Delta t}\right)^2\right] z_t^2 + \left[2p_0 - 2p_2 \left(\frac{2}{\Delta t}\right)^2\right] z_t \\ &+ \left[p_0 - p_1 \frac{2}{\Delta t} + p_2 \left(\frac{2}{\Delta t}\right)^2\right] \end{aligned} \right\} \varepsilon_0 \mathbf{E}^n. \tag{12}$$

According to (8), (12) can be written in another form as

$$\mathbf{E}^{n+1} = \frac{1}{b_0} \left[a_0 \left(\frac{\mathbf{D}^{n+1}}{\varepsilon_0}\right) + a_1 \left(\frac{\mathbf{D}^n}{\varepsilon_0}\right) + a_2 \left(\frac{\mathbf{D}^{n-1}}{\varepsilon_0}\right) - b_1 \mathbf{E}^n - b_2 \mathbf{E}^{n-1} \right]. \tag{13}$$

Note that

$$a_0 = q_0 + q_1 \frac{2}{\Delta t} + q_2 \left(\frac{2}{\Delta t}\right)^2,$$

$$a_1 = 2q_0 - 2q_2 \left(\frac{2}{\Delta t}\right)^2,$$

$$a_2 = q_0 - q_1 \frac{2}{\Delta t} + q_2 \left(\frac{2}{\Delta t}\right)^2,$$

$$b_0 = p_0 + p_1 \frac{2}{\Delta t} + p_2 \left(\frac{2}{\Delta t}\right)^2,$$

$$b_1 = 2p_0 - 2p_2\left(\frac{2}{\Delta t}\right)^2,$$

$$b_2 = p_0 - p_1\frac{2}{\Delta t} + p_2\left(\frac{2}{\Delta t}\right)^2. \quad (14)$$

Compare with the relative dielectric constant in (4), $p_0 = \omega_p^2$, $p_1 = \nu_c$, $p_2 = 1$, $q_0 = 0$, $q_1 = \nu_c$, and $q_2 = 1$. According to (13), **E** can be calculated. Equations (1) and (2) are transformed by using the FDTD method, and **H** can be calculated from the electrical field's values. Then, **D** is calculated at the end of the iterative process.

2. Parallel FDTD Method

In this paper, to improve the computational efficiency, the iterative process has been written to parallel process through the whole calculation region of the EM field. As the result, the single-node program with serial process has been transformed into a multi-node, multi-process program. Therefore, it is easiest to improve the calculation scope of the method, to expand the scope of computer memory, and to reduce the CPU time when the computer hardware condition has been limited [12]-[14].

In the parallel computational process, one or more regions is calculated in each processor. They transmit the EM field values at the interface to ensure the calculation is successful. As shown in Fig. 1, the same program runs in each processor with the parallel model of single-program-stream/multiple-data-stream [15].

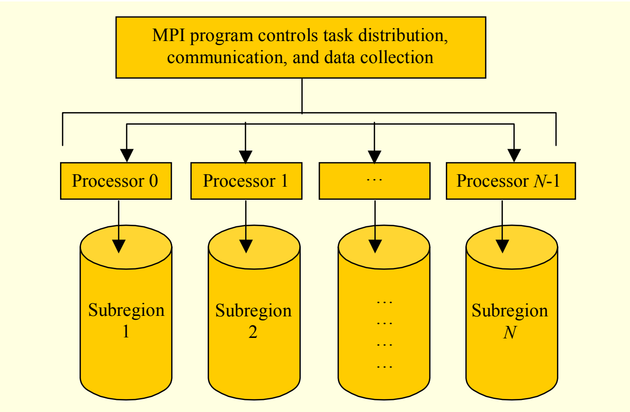

Fig. 1. Distribution of the calculation region.

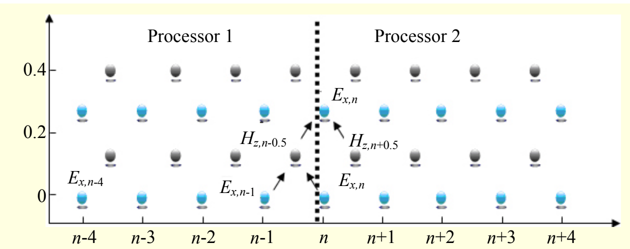

Fig. 2. Data communication at the interface.

Figure 2 shows the parallel process with eight cells and two processors using the FDTD method. As shown in Fig. 2, the whole computational process is divided into two processes. Processor 2 needs $H_{z,n\text{-}0.5}$ and $H_{z,n+0.5}$ to calculate $E_{x,n}$ because $H_{z,n\text{-}0.5}$ is written in the memory of the computer that processor 1 runs on. Therefore, we can call $H_{z,n+0.5}$ from the local memory, and $H_{z,n\text{-}0.5}$ should be sent from processor 1 to processor 2. Similarly, $E_{x,n}$ should be sent from processor 2 to processor 1 when $H_{z,n\text{-}0.5}$ has been calculated in processor 1. The same principle can be used in a multi-processor situation to properly handle the EM field values' communication transfer process at the interface.

III. Validity and Efficiency of the Parallel Method

We simulate the interaction between the EM waves of two frequencies (7 GHz and 50 GHz) and non-magnetized plasma. We use the properties of silver: $f_p = 28.7$ GHz ($\omega_p = 2\pi f_p$), v_c=20 GHz. In the course of this study, it was necessary to simulate an EM wave of 50 GHz. At this frequency, the $\lambda_{\min} = 3\times10^8/5\times10^{10}$. Following the rule of the well-known *Courant condition*, we used a cell size of $\Delta x \le \lambda_{\min}/10$ and a time step of $\Delta t = \Delta x/2c_0$, where c_0 is the light speed in free space. The incident pulse generated in the fifth cell is a sine wave inside a Gaussian envelope, $\sin(\omega t)\exp\left[2\pi(\frac{t-t_0}{\tau})^2\right]$. Mur absorbing boundary conditions were used in the calculation of domain boundaries.

As shown in Fig. 3, the computational domain is subdivided into 5,000 cells. The points from the 3,000th cell to the 4,000th cell are plasma, while other cells are free space. Because the frequency of the EM wave f=7 GHz is well below the plasma frequency f_p=28.7 GHz, the plasma almost completely reflects the EM wave as if it was a metal barrier.

As shown in Fig. 4, the computational domain is subdivided into more cells. Because the frequency of the EM wave f=50 GHz is well above the plasma frequency f_p=28.7 GHz, a small portion of the EM wave is reflected, but the majority of the pulse passes through the plasma.

In Fig. 4, we increase the number of cells from 5,000 to 10,000 and add the plasma from the 6,000th cell to the 8000th cell (Figs. 4(a), (b), and (c)). To analyze the effect on the computational efficiency, we increase the number of cells from 10,000 to 20,000, and add the plasma from the 12,000th cell to the 16,000th cell (Figs. 4(d) and (e)).

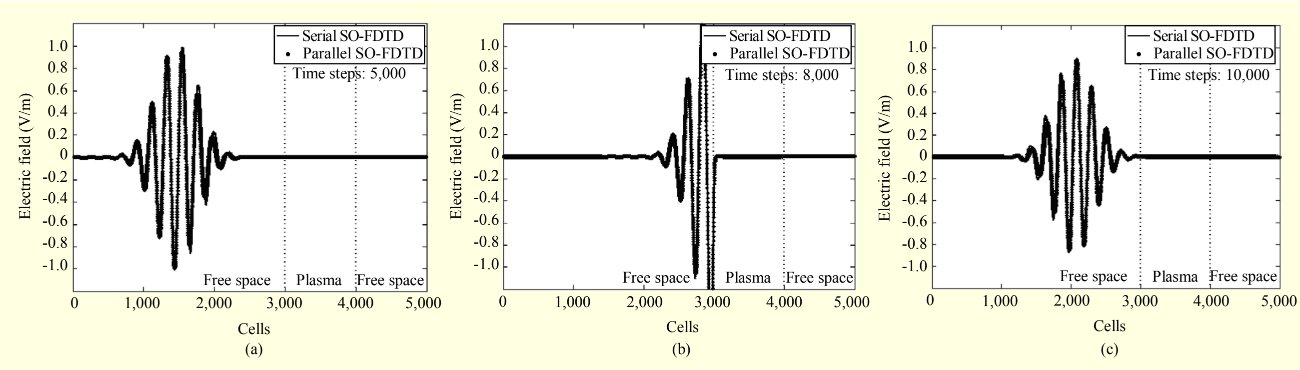

Fig. 3. Simulation of interaction between EM waves and plasma medium. The frequency of EM wave is 7 GHz.

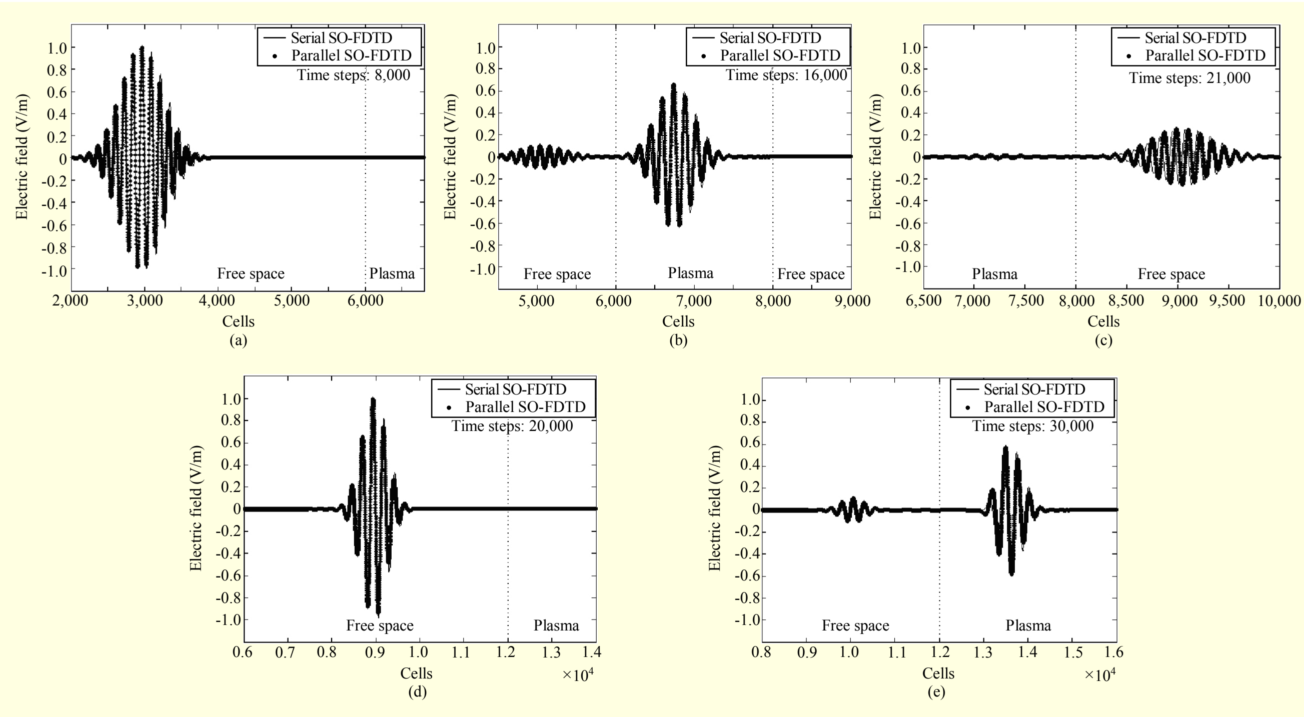

Fig. 4. Simulation of interaction between EM waves and plasma medium. The frequency of EM wave is 50 GHz.

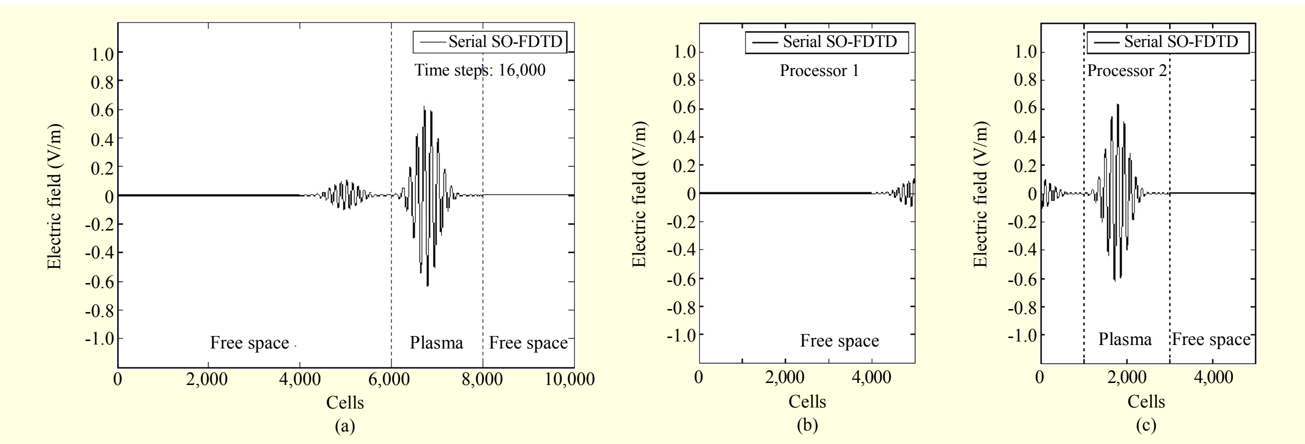

Fig. 5. Division effect chart of the electromagnetic wave for plasma at frequency f=50 GHz.

Table 1. Comparison of parallel SO-FDTD and serial-node method in terms of time and parallel efficiency.

Frequencies and cells	Time steps	SO-FDTD computational times (s)	Parallel SO-FDTD computational times (s)	Parallel speedup ratio	Parallel efficiency	Time saved (s)	Percentage of time saved
7 GHz (5×10^3 cells)	5,000	6.6318	4.9486	1.3408	67.04%	1.6832	25.38%
	8,000	11.1291	8.1469	1.3661	68.30%	2.9822	26.80%
	10,000	14.3221	10.1102	1.4166	70.83%	4.2119	29.41%
50 GHz (1×10^4 cells)	8,000	19.3445	12.0130	1.6103	80.51%	7.3315	37.90%
	16,000	39.2122	24.0686	1.6292	81.46%	15.1435	38.62%
	21,000	51.3848	30.9846	1.6584	82.92%	20.4003	39.70%
50 GHz (2×10^4 cells)	20,000	81.0574	45.1989	1.7933	89.67%	35.8585	44.24%
	30,000	124.9685	68.7555	1.8176	90.88%	56.2130	44.98%
	38,000	159.3009	86.2069	1.8479	92.39%	73.0940	45.88%

A comparison of these results demonstrates that the results of the parallel SO-FDTD method are close to those of the serial SO-FDTD method. Therefore, the validity of the parallel method is verified. Figure 5 shows the parallel process with time steps of 16,000. Figure 5(a) shows serial computational results, and Figs. 5(b) and (c) show subregions under parallel computing.

Table 1 shows the results of parallel simulation run on a computer. The Fortran 95 language was applied with the computer configuration of 256M memory, P4 (2.8 GHz) CPU, a 10M/100M auto-adapted network interface card, and a 10M/100M auto-adapted hub. As seen in Table 1, the parallel speedup ratio and parallel efficiency were analyzed, and the percentage of time saved is also counted.

Parallel speedup ratio is the speed of the parallel CPU process divided by the speed of the serial CPU process. In principle, the parallel speedup ratio ought to be the CPU number. For example, the calculation speedup with 2 CPUs is twice that with 1 CPU. There are many factors that influence the parallel speedup ratio, such as the design of the parallel program. Because there are more calculations in the main processor, the whole program cannot be paralleled completely. Other factors include the network situation, specifically, the propagation speed in the net and the network time-lag effect on the total processing speed of multi-CPUs; the number of running processes and occupied memory on each computer involved in the parallel calculation; as well as the communication model and efficiency of the parallel method. Therefore, the speedup data in the simulation is lower than the speedup data in theory.

Parallel efficiency is the parallel speedup ratio divided by the number of nodes, which indicates calculation efficiency in the parallel method.

As shown in Table 1, as the number of interactive steps and number of cells increase, the unknown numbers and computational complexities of the computer also increase, which directly affects the CPU time and algorithmic efficiency, and higher parallel speedup ratio and parallel efficiency are obtained. When the calculation is complex enough, a parallel efficiency of over 92.39% can be achieved. Compared to the single algorithm, 45.88% of calculation time can be saved. With program and network development, the parallel speedup ratio can be increased, but not dramatically.

IV. Conclusion

This paper introduced a novel parallel SO-FDTD method for plasma in the dispersive media. We calculated the interaction between the EM wave of different frequencies and non-magnetized plasma by using a parallel SO-FDTD method. The results, which were calculated with serial and parallel SO-FDTD executions to obtain speedup ratios, demonstrated that the parallel SO method has almost the same precision as the serial SO method. The parallel approach expands the scope of memory and reduces the CPU time. The results obtained from simulations using parallel and serial SO methods were compared in terms of speedup ratio and parallel efficiency to estimate computational time savings. The calculations and analysis in this paper contribute to the calculation of electrically large bodies with plasma which are found in clusters of ordinary computers.

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