One-to-All Broadcasting in Petersen-Torus Networks for SLA and MLA Models

Jung-Hyun Seo and Hyeong-Ok Lee

ABSTRACT—In a network, broadcasting is the dissemination of a message from a source node holding a message to all the remaining nodes through a call. This letter proposes a one-to-all broadcasting algorithm in the Petersen-torus network PT(n, n) for the single-link-available and multiple-link-available models. A PT(n, n) is a regular network whose degree is 4 and number of nodes is $10n^2$, where the Petersen graph is set as a basic module, and the basic module is connected in the form of a torus. A broadcasting algorithm is developed using a divide-and-conquer technique, and the time complexity of the proposed algorithm approximates n+4, the diameter of PT(n, n), which is the lower bound of the time complexity of broadcasting.

Keywords—Broadcasting, Petersen torus, interconnection network, parallel processing.

I. Introduction

In a network, broadcasting is the dissemination of a message from the originating node holding the message to all the rest of nodes through a call, that is, successful data transmission between nodes. Broadcasting algorithms can be divided into two types: one-to-all broadcasting, in which a message is transmitted from one node holding the message to all the rest of nodes, and all-to-all broadcasting, in which a message is transmitted from all nodes holding a particular message to all the rest of nodes. A broadcasting algorithm can be applied in many ways in several fields, such as management of shared variables for distributed programming, image processing, data copying in the databases of large-scale networks, and so on. For this, an effective broadcasting algorithm is necessary [1].

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An interconnection network consists of a set of processors, local memory, and a communication link between processors for data transfer. An interconnection network can be modeled as an undirected graph G=(V, E), where the node-set V represents the processors, and the edge-set E represents the communication links. Each processor P_i is an element of node set V, and two processors P_i and P_j are connected by communication links (P_i, P_j). The number of nodes adjacent to the node P_i is defined as degree of the node.

Broadcasting is performed by successive calling to transmit a message, and it has several limitations. First, just two nodes participate in a call. Second, a call is fully done in unit time. Third, each node may take part in one call in unit time. Fourth, a call appears between the neighboring two nodes. The SLA model means that one node holding a message in unit time transmits the message just to one neighboring node, while the MLA model means that one node holding a message in unit time transmits the message to all the neighboring nodes. This letter proposes a one-to-all broadcasting algorithm in the SLA and MLA models. It is assumed that a communication link is a half-duplex communication where only unidirectional communication is available at a given time [2].

The time complexity of various one-to-all broadcasting algorithms of the MLA model in a network may be described as follows. The broadcasting algorithm of [3] in a $4^k \times 4^k$ torus is $2k$. That of the algorithms of [4] and [5] in a $5^k \times 5^k$ torus is $2k$ and $2k+1$, respectively, and that of the broadcasting algorithm of [6] in a $k \times k$ torus is $2k+1$.

The proposed one-to-all broadcasting algorithm uses a divide-and-conquer technique in the Petersen torus. This letter is composed as follows. Section II introduces the Petersen-torus network and analyzes a broadcasting algorithm in the Petersen graph. Section III proposes a broadcasting algorithm, and finally, the conclusion is given.

II. Related Work

1. Petersen Torus

The Petersen torus $PT(m, n)$ $(m, n \geq 2)$ sets the Petersen graph (Fig. 1.(b)) as a basic module, arranges m (x axis)$\times n$ (y axis) modules on grid points, and connects them under an edge definition. The Petersen-torus network is defined as $PT(m, n)=(V_{pt}, E_{pt})$. The node definition of the $PT(m, n)$ is

$$V_{pt}=\{(x, y, p), 0 \leq x < m, 0 \leq y < n, 0 \leq p \leq 9\}.$$

The edges of $PT(m, n)$ are divided into internal edges and external edges. The edges connecting the nodes belonging to the same basic module are called internal edges, in which the edges of the Petersen graph are used as they are. The edges connecting the nodes belonging to other basic modules are called external edges. Edges are defined in [7].

2. Broadcasting in the Petersen Graph

This section examines the broadcasting algorithm of the Petersen graph, which is the basis of the broadcasting algorithm of the Petersen torus.

Lemma 1. The one-to-all broadcasting time of the Petersen graph is 4 in the SLA model and 2 in the MLA model.

Proof. The process of receiving a message in all nodes within a broadcasting time of 4 is as shown in Fig. 2(a), with node 0 in

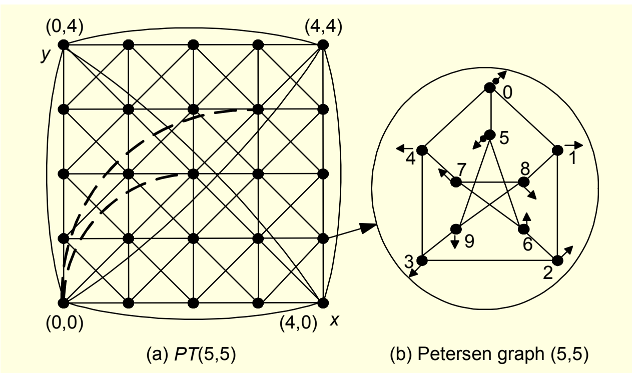

Fig. 1. Petersen torus $PT(5, 5)$.

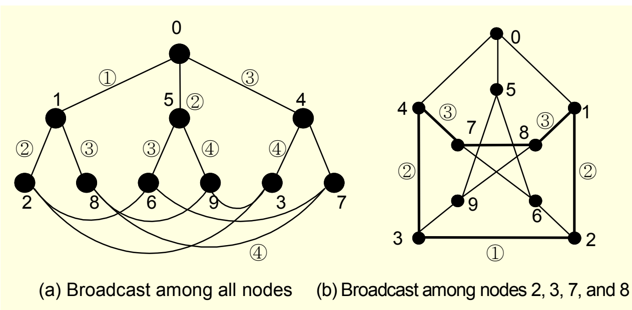

Fig. 2. Broadcast time in the Petersen graph.

the Petersen graph as a start node. Figure 2(a) shows the Petersen graph in the form of a spanning tree. The Petersen graph is a node- and edge-symmetric graph; therefore, the broadcasting time in the SLA model from a certain node to all nodes of the Petersen graph is 4. In the MLA model, a message is transmitted from the root node to child nodes and the message is transmitted from three child nodes to each terminal node as shown in Fig. 2(a); therefore, the broadcasting time is 2. □

Lemma 2. In the Petersen graph in Fig. 2(b), if any of nodes 2, 3, 7, or 8 has a message, the broadcasting time to transmit the message to the other three nodes is 3.

Proof. As shown in the Petersen graph of Fig. 2(b), the nodes 1, 2, 3, 4, 7, and 8 form a ring, and the broadcasting time in the ring having six nodes is 3. □

III. Broadcasting Algorithm in the Petersen Torus

One-to-all broadcasting uses a divide-and-conquer technique. The source node is $U(x, y, p)$, the destination node is $V(x', y', p')$, and the x- and y-axis distances from node U to node V are $d_x=|x-x'|$ and $d_y=|y-y'|$. With the basic module with the initial message as the center, the graph is divided into four areas, and the message is transmitted to each basic module located in the center of each of the four areas. With each of the four basic modules with the message as a center, each area is divided again into four subareas, and the message is transmitted to basic modules located in the center of each subarea. Division and message transmission are repeated until $d_x<1$ and $d_y<1$. Once division is finished, a message is transmitted from a basic module having a message to eight neighboring basic modules. This process is as shown in Fig. 3. The circled numbers indicate the order in which the message is transmitted from one basic module to another basic module.

Theorem 1. When PT(n, n) is used in the SLA model, the

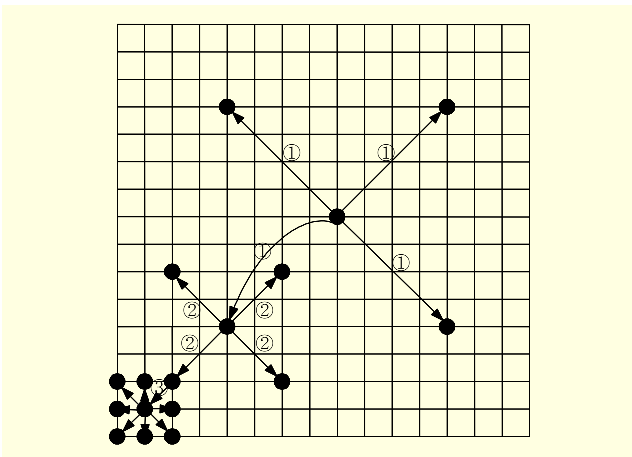

Fig. 3. Stage of broadcast in the Petersen torus.

broadcasting time is n+2log$_2n$+13. In the MLA model, the broadcasting time is n+ log$_2n$ +7.

Proof. A message is transmitted from a node having the initial message to all nodes of the basic module to which it belongs. As described in lemma 1, this broadcasting time is 4. The message is transmitted from the basic module having the message to four basic modules with d_x=d_y=$\lfloor n/4 \rfloor$. The length of the external path from the node U to V of four nodes is $\lfloor n/4 \rfloor$, and that of the internal path is $\lfloor n/4 \rfloor$–1. The nodes 2 and 3 are adjacent as are nodes 7 and 8. Therefore, the length of the internal path is 1 for a route using a continual diagonal edge and a reverse diagonal edge. This broadcasting time is $\lfloor n/4 \rfloor$×2–1. The time of message transmission from the nodes of four basic modules having a message to the next four basic modules is 3 as described in lemma 2. Initially, d_x=d_y=$\lfloor n/4 \rfloor$. In the next stage, it is halved, that is, d_x=d_y=$\lfloor n/8 \rfloor$. To meet d_x<1 and d_y<1, the division time should be done in $\lfloor \log_2(n-1) \rfloor - 1$. When d_x<1 and d_y<1, division should stop and a message should be transmitted to the eight neighboring basic modules. To transmit the message to the neighboring basic modules, the message should be transmitted to all nodes of the basic modules to which the node having the message belongs. As described in lemma 1, the time is 4 and the broadcasting time to transmit a message to the neighboring basic modules is 1. Then, one node of ten nodes has the message in all basic modules of the PT network. The message is transmitted from all nodes having the message to all nodes of the basic module to which it belongs. As described in lemma 1, the broadcasting time is 4. Adding up the broadcasting time, 4+$\sum_{k=1}^{\lfloor \log_2(n-1) \rfloor -1}(\frac{n}{4k}\times 2-1+3)$ +1+4+4, it can be arranged into this numerical formula, $\sum_{k=1}^{\lfloor \log_2(n-1) \rfloor -1}(\frac{n}{2k}+2)$ +13, where $\sum_{k=1}^{\lfloor \log_2(n-1) \rfloor -1}(\frac{n}{2k}) = \frac{n}{2}+\frac{n}{4}+\frac{n}{8}+\cdots+\frac{n}{\log_2(n-1)-1} < n$, and $\lfloor \log_2(n-1) \rfloor$ is less than log$_2n$+1. Therefore, the time complexity of broadcasting is less than n+2log$_2n$+13. In the MLA model, when a certain node has a message, the time of 2 is taken to transmit the message to all nodes of the basic module to which the node belongs as described in lemma 1. This does not influence the broadcasting time arising out of division, with 2 fewer times taken for transmission in the basic module than in SLA. In the MLA model, the time complexity of broadcasting is 2+$\sum_{k=1}^{\lfloor \log_2(n-1) \rfloor -1}(\frac{n}{4k}\times 2-1+2)$ +1+2+2, which is arranged into $\sum_{k=1}^{\lfloor \log_2(n-1) \rfloor -1}(\frac{n}{2k}+1)$ +7. Therefore, the time complexity of broadcasting is less than n+log$_2n$+7. □

The diameter of PT(n, n) is n+4 [8]. In the SLA model, the time complexity of broadcasting is n+2log$_2n$+13, and in the MLA model, the time complexity of broadcasting is n+log$_2n$+7; therefore, the algorithm is near optimal in approximating the diameter.

IV. Conclusion

This letter proposed a one-to-all broadcasting algorithm based on a divide-and-conquer technique using the attribute of a PT network which is repeatedly divided into four areas. In PT(n, n), the time complexity of a one-to-all broadcasting algorithm is n+2log$_2n$+13 in the SLA model and n+log$_2n$+7 in the MLA model. The time complexity of the proposed one-to-all broadcasting algorithm is $O(\sqrt{N})$; thus, the algorithm is near optimal in approximating the diameter of the Petersen-torus network.

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