Soft Combination Schemes for Cooperative Spectrum Sensing in Cognitive Radio Networks

Bin Shen and Kyung Sup Kwak

This paper investigates linear soft combination schemes for cooperative spectrum sensing in cognitive radio networks. We propose two weight-setting strategies under different basic optimality criteria to improve the overall sensing performance in the network. The corresponding optimal weights are derived, which are determined by the noise power levels and the received primary user signal energies of multiple cooperative secondary users in the network. However, to obtain the instantaneous measurement of these noise power levels and primary user signal energies with high accuracy is extremely challenging. It can even be infeasible in practical implementations under a low signal-to-noise ratio regime. We therefore propose reference data matrices to scavenge the indispensable information of primary user signal energies and noise power levels for setting the proposed combining weights adaptively by keeping records of the most recent spectrum observations. Analyses and simulation results demonstrate that the proposed linear soft combination schemes outperform the conventional maximal ratio combination and equal gain combination schemes and yield significant performance improvements in spectrum sensing.

Keywords: Soft combination, energy detection, cooperative spectrum sensing, cognitive radio network.

Manuscript received Sept. 9, 2008; revised Apr. 7, 2009; accepted Apr. 14, 2009.

This work was supported by Inha University Research Grant.

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I. Introduction

In recent years, cognitive radio (CR) has emerged as a promising paradigm for exploiting the spectrum opportunity, which is restricted by the current rigid spectrum allocation scheme, to solve the spectrum scarcity problem [1], [2]. Since CRs are inherently lower priority or secondary users (SU) who opportunistically access the temporarily unused licensed spectrum exclusively allocated to primary users (PU), the fundamental requirement for them is to avoid interference to the potential PUs in the vicinity.

Among different spectrum sensing schemes [3] for reliably identifying the licensed spectrum status, the energy detector (ED) scheme incurs a very low implementation cost and therefore is widely used. It serves as the optimal method to detect a PU signal with unknown location, structure, and strength, when the detector only knows the power of the received signal. However, the ED scheme is vulnerable to destructive channel effects, such as multipath fading/shadowing [4] and noise-power fluctuation [5], [6], which result in the hidden terminal problem and ambiguity in threshold setting, respectively. These effects may deteriorate the performance of ED and hamper it from operating in a reliable manner.

These drawbacks imply the need for user cooperation in CR networks, where SUs collaborate to perform spectrum sensing to compensate for the degraded sensing performance of a single SU [7]-[10]. Cooperation among SUs is usually coordinated by a fusion center through either hard decision or soft data fusion strategies. Apparently, soft fusion is superior to hard fusion since it imposes a different communication bandwidth requirement on the control channel for conveying the sensing information between the cooperative SUs and the fusion center. In the soft data combination scheme, if each SU transmits the real value of its sensing data to the fusion center, theoretically infinite bits are required, and this will result in a wide communication bandwidth. However, this problem can be solved by quantization of local observations at the expense of additional noise and a signal-to-noise ratio (SNR) loss at the receiver [11], but it is beyond the scope of this paper. In [9], a non-linear optimization enigma is used to formulate the cooperative spectrum sensing problem which might be difficult to implement. In [10], an optimal soft combination scheme is proposed, based on some approximation in the target optimality function and the assumption that cooperative SUs in the network experience independently and identically distributed (i.i.d.) fading effects. It is thereby proved to be identical to a maximal ratio combination (MRC) strategy.

In this paper, we concentrate on a cooperative spectrum sensing scenario, in which a linear soft combination of raw measurements from individual cooperative SUs is performed at the fusion center. We develop two easy-to-implement soft combination schemes. Optimal weights for fusing the collected observations at the fusion center are derived under two different criteria, namely, the Neyman-Pearson criterion and the Minimax criterion. These two criteria yield different philosophies in exploitation of the unused licensed spectrum. The former scheme guarantees minimal interference to the active PU on the basis of desired constant spectrum opportunities for the SUs, while the latter one emphasizes a more aggressive way to harness the unused licensed spectrum by making the probabilities of interfering PU and missing the spectrum opportunity equal. An additional benefit of the proposed soft combination schemes is that it is not necessary to assume the independence condition between the cooperative SUs. However, the derived soft combination weights require highly accurate information of noise power levels and PU signal energies, which is extremely challenging in low SNR regimes. To enable the fusion center to acquire the SNR information about cooperative users, reference matrices are introduced in the weighting procedure which stores the most recent sensing data in either a noise matrix or signal energy matrix according to their corresponding global decisions. This easily implemented strategy greatly reduces the system complexity in practice, and the tradeoff is trivial performance degradation.

The rest of this paper is organized as follows. We begin in section II with the system model and describe the local and cooperative spectrum sensing scenarios in detail. Optimal weight setting strategies are then presented in section III. We propose an implementation algorithm for weight setting in section IV. Simulations are carried out and analyzed in section V, and conclusions are drawn in section VI.

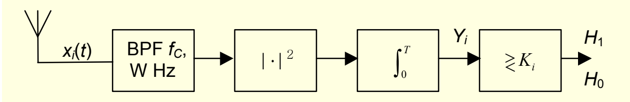

Fig. 1. Block diagram of i-th energy detector.

We use the following key notations: superscript $(\cdot)^T$ and $E[\cdot]$ stand for transpose and expectation operation, respectively; diag$(\cdot)$ represents the diagonal process; and $\|\cdot\|_2$ denotes the Euclidean norm.

II. System Model

1. Energy Detection-Based Local Spectrum Sensing

We consider M SUs are dispersed over a certain geographical area by some upper-layer distributing algorithms. For any arbitrary i-th SU, the sensing task is usually expressed as a binary hypothesis test (H_0, H_1):

$$x_i(t) = \begin{cases} n_i(t), & H_0,\ 0 \le t \le T, \\ h_i s(t) + n_i(t), & H_1,\ i = 1, 2, \cdots, M, \end{cases} \qquad (1)$$

where $s(t)$ is the transmitted complex PU signal, $x_i(t)$ is the received signal at the i-th SU, $n_i(t)$ is the complex additive white Gaussian noise (AWGN) with zero mean and one-sided noise power spectral density $N_{0,i}$, and T is the sensing interval. Here, h_i is the channel gain between PU and the i-th SU, which accommodates any channel effects, such as multipath fading, shadowing, and propagation path loss. In this paper, we consider quasi-static flat fading channels between PU and the cooperative SUs, in which the fading coefficients h_i vary from (observation) period to period, while their probability distribution functions (PDFs) are determined by the fading characteristic of the channels.

As shown in Fig. 1, the local test statistic Y_i of the i-th SU is obtained as the output of the energy integrator:

$$Y_i = \int_0^T |x_i(t)|^2 dt. \qquad (2)$$

The conditional PDFs of test statistic Y_i are given as in [12] as

$$\begin{aligned} H_0 &: f_{Y_i}(y_i) = \frac{1}{N_{0,i}\Gamma(m)} \left(\frac{y_i}{N_{0,i}}\right)^{m-1} e^{-\frac{y_i}{N_{0,i}}} u(y_i), \\ H_1 &: f_{Y_i}(y_i) = \frac{1}{N_{0,i}} \left(\frac{y_i}{E_i}\right)^{\frac{m-1}{2}} e^{-\frac{y_i+E_i}{N_{0,i}}} I_{m-1}\left(\frac{\sqrt{y_i E_i}}{N_{0,i}/2}\right) u(y_i), \end{aligned} \qquad (3)$$

where m is the time-bandwidth product TW, W is the spectrum sensing bandwidth, $\Gamma(\cdot)$ is the Gamma function, $I_{m\text{-}1}(\cdot)$ is the $(m\text{–}1)$th-order modified Bessel function of the first kind, and $u(\cdot)$ is the unit step function. Here, E_i is the captured PU signal energy at the i-th SU, which can be represented by the sum of the squares of $2m$ virtual samples [13]:

$$E_i = \frac{1}{2W}\sum_{k=1}^{2m}\left|s_{i,k}\right|^2 = m\gamma_i N_{0,i} = E_s\left|h_i\right|^2, \tag{4}$$

where $s_{i,k}$ is the sample of the PU signal at instant k, $\gamma_i = P_s\left|h_i\right|^2/(N_{0,i}W)$ is the PU signal-to-noise ratio (SNR) of the i-th SU, and $E_s=P_sT$ is the PU signal energy transmitted during the sensing interval T. The channel gain vector $\mathbf{h}=[h_1, h_2,\cdots,h_M]^T$ is assumed to be constant in each sensing interval T, but it may still vary from burst to burst.

The corresponding probabilities of false alarm and miss detection in the threshold test are given as

$$\begin{cases} P_{\mathrm{FA},i} = \int_{K_i}^{\infty} f_{Y_i}(y_i)dy_i, & H_0, \\ P_{\mathrm{MISS},i} = \int_0^{K_i} f_{Y_i}(y_i)dy_i, & H_1, \end{cases} \tag{5}$$

where K_i is the threshold of the i-th ED, and $P_{\mathrm{D},i}=(1-P_{\mathrm{MISS},i})$ is the probability of detection. For analytical tractability, we utilize a Gaussian approximated model when the time-bandwidth product m is asymptotically large (>100). Based on the central limit theorem (CLT), the statistic Y_i follows normal distributions with mean $\mu_{j,i}$ and variance $\sigma_{j,i}^2$ under hypothesis $H_j\,(j\in\{0,1\})$ as in [13]:

$$\begin{cases} \mu_{0,i} = N_{0,i}m, & \sigma_{0,i}^2 = N_{0,i}^2 m; \\ \mu_{1,i} = N_{0,i}m + E_i, & \sigma_{1,i}^2 = N_{0,i}^2 m + 2N_{0,i}E_i. \end{cases} \tag{6}$$

From (5) and (6), $P_{\mathrm{FA},i}$ and $P_{\mathrm{MISS},i}$ are again given by

$$\begin{cases} P_{\mathrm{FA},i} = Q\left(\dfrac{K_i-\mu_{0,i}}{\sigma_{0,i}}\right), \\ P_{\mathrm{MISS},i} = 1-Q\left(\dfrac{K_i-\mu_{1,i}}{\sigma_{1,i}}\right), \end{cases} \tag{7}$$

where $Q(x) = \int_x^{+\infty}\exp(-t^2/2)dt/\sqrt{2\pi}$.

2. Soft Combination-Based Cooperative Spectrum Sensing

In this subsection, we describe the cooperative spectrum sensing scenario, where M SUs cooperate with a fusion center to enhance the overall spectrum sensing performance in the network as shown in Fig. 2.

To enable collaboration, M SUs first sense the licensed spectrum independently, and then transmit the test statistics $\mathbf{Y}=[Y_1, Y_2 \cdots Y_M]^T$ directly to the fusion center:

$$\mathbf{Z}=\mathbf{Y}+\mathbf{V}, \tag{8}$$

where the control channel noise $\mathbf{V}=[V_1, V_2, \cdots, V_M]^T$ consists of zero-mean, spatially uncorrelated Gaussian variables with variances $\boldsymbol{\delta}=[\delta_1^2, \delta_2^2, \cdots \delta_M^2]^T$. The use of the AWGN channel model in (8) is justified by assumptions regarding analog-forwarding schemes and the slow-changing nature of the channels between the M SUs and the fusion center. We assume that the channel coherence time is much longer than the channel estimation period, such that once the fusion center has estimated the channel gains from the SUs, these channels can be treated as constant AWGN channels [9].

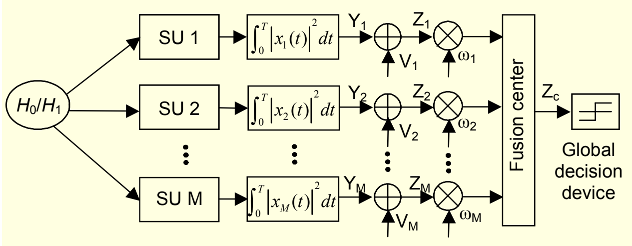

Fig. 2. Block diagram of soft-combination-based cooperative spectrum sensing.

Accordingly, the received statistics $\mathbf{Z}$ at the fusion center are normally distributed variables with means and variances given by

$$\begin{cases} E[Z_{0,i}] = \mu_{0,i}, \mathrm{Var}[Z_{0,i}] = \sigma_{0,i}^2 + \delta_i^2, & H_0, \\ E[Z_{1,i}] = \mu_{1,i}, \mathrm{Var}[Z_{1,i}] = \sigma_{1,i}^2 + \delta_i^2, & H_1. \end{cases} \tag{9}$$

After collecting the sensing data $\mathbf{Z}$ from the M SUs, the fusion center linearly combines them into a global detection statistics Z_c:

$$Z_c = \sum_{i=1}^{M}\omega_i Z_i = \boldsymbol{\omega}^T\mathbf{Z}, \tag{10}$$

where $\boldsymbol{\omega} \triangleq [\omega_1, \omega_2, \cdots, \omega_M]^T$ is the weighting vector satisfying $\|\boldsymbol{\omega}\|_2^2 = 1, \quad \omega_i \geq 0$. The combining weight for the signal from a particular SU represents its contribution to the global decision. Since $\{Z_i\}_{i=1}^M$ are normal random variables, their linear combination is also normal. Consequently, the global decision statistic Z_c has means given by

$$\bar{Z}_c = E[Z_c] = \begin{cases} \boldsymbol{\omega}^T\boldsymbol{\mu}_0, & H_0, \\ \boldsymbol{\omega}^T\boldsymbol{\mu}_1, & H_1. \end{cases} \tag{11}$$

where the mean vectors $\boldsymbol{\mu}_0 = [\mu_{0,1}, \mu_{0,2}, \cdots, \mu_{0,M}]^T$ and $\boldsymbol{\mu}_1 = [\mu_{1,1}, \mu_{1,2}, \cdots, \mu_{1,M}]^T$. It should be noted that $\boldsymbol{\mu}_1 = \boldsymbol{\mu}_0 + E_s\mathbf{H}$, where the vector of channel gain squares $\mathbf{H}=[\left|h_1\right|^2, \left|h_2\right|^2, \cdots, \left|h_M\right|^2]^T$.

Regarding the variance of Z_c, it is true that

$$\mathrm{Var}[Z_c] = \begin{cases} \sum_{j=1}^{M}(\sigma_{0,i}^2+\delta_i^2)\omega_j^2 = \boldsymbol{\omega}^T\sum{}_0\boldsymbol{\omega}, & H_0, \\ \sum_{j=1}^{M}(\sigma_{1,i}^2+\delta_i^2)\omega_j^2 = \boldsymbol{\omega}^T\sum{}_1\boldsymbol{\omega}, & H_1, \end{cases} \tag{12}$$

where $\sum_0$=diag($\boldsymbol{\sigma}_0+\boldsymbol{\delta}$) and $\sum_1$=diag($\boldsymbol{\sigma}_1+\boldsymbol{\delta}$) are diagonal matrices with $\boldsymbol{\sigma}_0=[\sigma_{0,1}^2,\sigma_{0,2}^2,\cdots,\sigma_{0,M}^2]^T$ and $\boldsymbol{\sigma}_1=[\sigma_{1,1}^2,\sigma_{1,2}^2,\cdots,\sigma_{1,M}^2]^T$. It is worth noting that the statistics $\mathbf{Z}$ do not have to be conditionally independent, though we utilize the independent case for illustrative purposes, that is, with Σ_0 and Σ_1 diagonal. If the elements of $\mathbf{Z}$ are correlated with each other, then the covariance matrices Σ_0 and Σ_1 are generally non-diagonal, and the subsequent analysis will continue to hold.

Based on the statistical properties given in (11) and (12), the performance of the proposed cooperative spectrum sensing scheme can be evaluated as

$$P_{\text{FA,C}}=Q\left(\frac{\lambda-\boldsymbol{\mu}_0^T\boldsymbol{\omega}}{\sqrt{\boldsymbol{\omega}^T\sum_0\boldsymbol{\omega}}}\right),\ P_{\text{MISS,C}}=1-Q\left(\frac{\lambda-\boldsymbol{\mu}_1^T\boldsymbol{\omega}}{\sqrt{\boldsymbol{\omega}^T\sum_1\boldsymbol{\omega}}}\right),\quad(13)$$

where λ is the threshold of the fusion center.

III. Optimal Soft Combination Schemes

1. Weight Optimization via Neyman-Pearson Criterion

For a cooperative spectrum sensing algorithm, the main metric of sensing performance is either the maximization of the detection probability for a target false alarm probability or minimization of the false alarm probability for a target detection probability. Setting the threshold λ at the fusion center for a desired probability of false alarm $P_{\text{FA\_DES,C}}$, we obtain the probability of detection within the Neyman-Pearson framework:

$$P_{\text{D,C}}=Q\left(\frac{Q^{-1}(P_{\text{FA\_DES,C}})\sqrt{\boldsymbol{\omega}^T\sum_0\boldsymbol{\omega}}-E_s\mathbf{H}^T\boldsymbol{\omega}}{\sqrt{\boldsymbol{\omega}^T\sum_1\boldsymbol{\omega}}}\right),\quad(14)$$

where $Q^{-1}(\cdot)$ is the inverse function of Q(·).

From (11) and (12), it is clear that the weight vector **ω** plays an important role in determining the PDF of the global test statistic Z_C. To measure the effect of the PDF on the detection performance, we introduce a deflection coefficient (DC) [14]:

$$d_{\text{DC}}^2(\boldsymbol{\omega})=\frac{(E[Z_C|H_1]-E[Z_C|H_0])^2}{\text{Var}[Z_C|H_0]}=\frac{(E_s\mathbf{H}^T\boldsymbol{\omega})^2}{\boldsymbol{\omega}^T\sum_0\boldsymbol{\omega}}.\quad(15)$$

The deflection coefficient $d_{\text{DC}}^2(\boldsymbol{\omega})$ provides a good measure of the detection performance because it characterizes the variance-normalized distance between the centers of two conditional PDFs of Z_C. When we regard $d_{\text{DC}}^2(\boldsymbol{\omega})$ as the optimization target, the optimal weight vector is the one that maximizes it:

$$\boldsymbol{\omega}_{\text{opt,DC}}=\arg\max_{\boldsymbol{\omega}} d_{\text{DC}}^2(\boldsymbol{\omega}).\quad(16)$$

By solving the equation $\partial d_{\text{DC}}^2(\boldsymbol{\omega})/\partial\boldsymbol{\omega}=0$, we obtain

$$\boldsymbol{\omega}_{\text{opt,DC}}^*=\frac{\boldsymbol{\omega}^T\sum_0\boldsymbol{\omega}}{\boldsymbol{\omega}^T\mathbf{H}}\sum\nolimits_0^{-1}\mathbf{H}=\beta_{\text{DC}}\sum\nolimits_0^{-1}\mathbf{H},\quad(17)$$

where β_{DC} is a scaling factor determined by **ω**, but it does not affect either the DC in (15) or the sensing performance in (13). Therefore, by setting β_{DC} to 1 and normalizing each weighting coefficient, we obtain the optimal weighting vector:

$$\boldsymbol{\omega}_{\text{opt,DC}}=\boldsymbol{\omega}_{\text{opt,DC}}^*/\left\|\boldsymbol{\omega}_{\text{opt,DC}}^*\right\|_2.\quad(18)$$

The detection performance is thus given by

$$P_{\text{D,C}}=Q\left(\frac{\eta\sqrt{\mathbf{H}^T\sum_0^{-1}\mathbf{H}}-E_s\mathbf{H}^T\sum_0^{-1}\mathbf{H}}{\sqrt{\mathbf{H}^T\sum_0^{-2}\sum_1\mathbf{H}}}\right),\quad(19)$$

where η is the constant $Q^{-1}(P_{\text{FA\_DES,C}})$.

For a given $P_{\text{FA\_DES,C}}$, $P_{\text{D,C}}$ is maximized in the sense that the distance between the two PDFs of Z_c under hypotheses H_0 and H_1 is enlarged to the maximum by $\boldsymbol{\omega}_{\text{opt,DC}}$. This optimality is achieved in the Neyman-Pearson framework, which maintains a constant false alarm rate (CFAR) in spectrum sensing. As for the constant detection rate (CDR), the obtained optimal weights are still valid, except the fusion center threshold λ is determined by the desired detection probability and the PU signal properties.

Note that $P_{\text{D,C}}$ in (19) is actually a probability conditioned on the channel gains' square vector **H**. Statistically, the average $P_{\text{D,C}}$ is

$$\bar{P}_{\text{D,C}}=\int_{\text{G}_1\subset\mathbb{R}_+^M}Q\left(\frac{\eta\sqrt{\mathbf{X}^T\sum_0^{-1}\mathbf{X}}-E_s\mathbf{X}^T\sum_0^{-1}\mathbf{X}}{\sqrt{\mathbf{X}^T\sum_0^{-2}\sum_1\mathbf{X}}}\right)p_{\mathbf{H}}(\mathbf{X})d\mathbf{X},\quad(20)$$

where $p_{\mathbf{H}}(\mathbf{X})$ is the PDF of the multi-variable **H**, $\mathbf{G}_1$ is the subset of $\mathbb{R}_+^M$ that contains all **H** vectors leading to the H_1 decision, and $\mathbb{R}_+^M$ is the positive M-dimensional vector space.

The proposed optimal weighting vector $\boldsymbol{\omega}_{\text{opt,DC}}$ is similar to an MRC weighting scheme:

$$\boldsymbol{\omega}_{\text{MRC}}=E_s(\sum\nolimits_{\boldsymbol{\mu}_0}+\sum\nolimits_{\boldsymbol{\delta}})^{-1}\mathbf{H},\quad(21)$$

where $\sum_{\mu_0}$ and $\sum_\delta$ are diagonal matrices with entries of $\boldsymbol{\mu}_0$ and **δ** on the diagonal, respectively. SUs with larger SNRs are allocated greater weights; thus, they make greater contributions to the global statistic Z_c. Also, $\boldsymbol{\omega}_{MRC}$ differs from $\boldsymbol{\omega}_{opt,DC}$ mainly in the denominators of each combining branch. In other words, the dimensions of these two weighting coefficient denominators in (17) and (21) are respectively squares and amplitudes of $\{N_{0,i}\}$. In section IV, we will demonstrate that $\boldsymbol{\omega}_{opt,DC}$ is superior to the MRC weights $\boldsymbol{\omega}_{MRC}$ when the noise power levels are not identical at different SUs. Furthermore, we notice that if the noise power levels are all the same among the cooperative SUs, the proposed scheme reduces to the MRC scheme because the identical noise factor changes the co-variance matrix Σ_0 into an identity matrix multiplied by a scalar mN_0^2, and $\mathbf{H}$ is the merely effective parameter in the data combination.

2. Weight Optimization via Minimax Criterion

In this subsection, we consider the problem of minimizing the Bayesian risk R in detecting the PU signal:

$$R = E[C] = \sum_{i=0}^{1}\sum_{j=0}^{1} C_{i,j} P(H_i | H_j) P(H_j), \tag{22}$$

where $C_{0,1}$, $C_{1,0}$, and $C_{1,1}$ correspond to the cost of miss detection, false alarm, and detection, respectively; while $C_{0,0}$ is the cost of correctly identifying the spectrum hole; $P(H_j)$ denotes the probability of PU status indicated by hypothesis H_j; $P(H_i | H_j)$ represents the probability that the fusion center makes a decision of H_i, when hypothesis H_j is true. Without loss of generality, we assume that $C_{0,0}$=$C_{1,1}$=0, and then, R is simplified as

$$R=E[C]=C_{FA}P(H_0)P_{FA,C}+C_{MISS}P(H_1)P_{MISS,C}. \tag{23}$$

Without any *a priori* knowledge of $P(H_0)$ and $P(H_1)$, the optimal solution for minimizing R in (23) can be obtained according to the minimax criterion:

$$\frac{dR}{dP(H_1)} = -C_{\text{FA}}P_{\text{FA,C}} + C_{\text{MISS}}P_{\text{MISS,C}} = 0, \tag{24}$$

where the optimization problem is set by the equation $C_{FA}P_{FA,C}$=$C_{MISS}P_{MISS,C}$.

Due to the intractability of threshold λ in a closed-form according to (24), we assume $P_{FA,C}$=$P_{MISS,C}$ for simplicity, and obtain the threshold λ as

$$\lambda = \frac{\sqrt{\boldsymbol{\omega}^T \sum_1 \boldsymbol{\omega}}\,\boldsymbol{\mu}_0^T \boldsymbol{\omega} + \sqrt{\boldsymbol{\omega}^T \sum_0 \boldsymbol{\omega}}\,\boldsymbol{\mu}_1^T \boldsymbol{\omega}}{\sqrt{\boldsymbol{\omega}^T \sum_0 \boldsymbol{\omega}} + \sqrt{\boldsymbol{\omega}^T \sum_1 \boldsymbol{\omega}}}. \tag{25}$$

The Bayesian risk R is thus

$$\begin{aligned} R &= P(H_0)C_{\text{FA}} Q\left(\frac{\lambda - \boldsymbol{\omega}^T \boldsymbol{\mu}_0}{\sqrt{\boldsymbol{\omega}^T \sum_0 \boldsymbol{\omega}}}\right) + P(H_1)C_{\text{MISS}} Q\left(\frac{\boldsymbol{\omega}^T \boldsymbol{\mu}_1 - \lambda}{\sqrt{\boldsymbol{\omega}^T \sum_1 \boldsymbol{\omega}}}\right) \\ &= [P(H_0)C_{\text{FA}} + P(H_1)C_{\text{MISS}}] Q\left(\frac{(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)^T \boldsymbol{\omega}}{\sqrt{\boldsymbol{\omega}^T \sum_0 \boldsymbol{\omega}} + \sqrt{\boldsymbol{\omega}^T \sum_1 \boldsymbol{\omega}}}\right). \end{aligned} \tag{26}$$

Since $Q(\cdot)$ is a monotonically decreasing function, the Bayesian risk in (26) is minimized if the metric function $J(\boldsymbol{\omega})$ is maximized, where

$$J(\boldsymbol{\omega}) = \frac{(\boldsymbol{\mu}_1 - \boldsymbol{\mu}_0)^T \boldsymbol{\omega}}{\sqrt{\boldsymbol{\omega}^T \sum_0 \boldsymbol{\omega}} + \sqrt{\boldsymbol{\omega}^T \sum_1 \boldsymbol{\omega}}}. \tag{27}$$

Consequently, the optimal weighting vector $\boldsymbol{\omega}_{opt,Risk}$ is expressed as

$$\boldsymbol{\omega}_{\text{opt,Risk}} = \arg\max_{\boldsymbol{\omega}} J(\boldsymbol{\omega}). \tag{28}$$

Setting $\partial J(\boldsymbol{\omega}) / \partial \boldsymbol{\omega} = 0$, we obtain the following result after some algebraic manipulations:

$$\boldsymbol{\omega}_{\text{opt,Risk}}^* = \beta_{\text{Risk}} (\alpha_{\text{Risk}} \Sigma_0 + \Sigma_1)^{-1} \mathbf{H}, \tag{29}$$

where

$$\begin{aligned} \beta_{\text{Risk}} &= \frac{(\sqrt{\boldsymbol{\omega}^{\text{T}} \sum_0 \boldsymbol{\omega}} + \sqrt{\boldsymbol{\omega}^{\text{T}} \sum_1 \boldsymbol{\omega}})\sqrt{\boldsymbol{\omega}^{\text{T}} \sum_1 \boldsymbol{\omega}}}{\boldsymbol{\omega}^T \mathbf{H}}, \\ \alpha_{\text{Risk}} &= \sqrt{\frac{\boldsymbol{\omega}^{\text{T}} \sum_1 \boldsymbol{\omega}}{\boldsymbol{\omega}^{\text{T}} \sum_0 \boldsymbol{\omega}}} = \sqrt{1 + \frac{2E_s \boldsymbol{\omega}^T (\sum_0 - \sum_{\boldsymbol{\delta}})^{1/2} \sum_{\mathbf{H}} \boldsymbol{\omega}}{\sqrt{m}\boldsymbol{\omega}^T \sum_0 \boldsymbol{\omega}}}. \end{aligned} \tag{30}$$

The scaling factor β_{Risk} in (30) is a constant determined by ω, yet it does not affect the risk function R in (26). Thus, we can effectively set β_{Risk}=1, which simplifies the optimal weighting vector. Furthermore, because coefficient α_{Risk} depends on the weighting vector $\boldsymbol{\omega}$, it is actually impossible to derive the optimal coefficients in a closed-form. However, considering that the SUs are subject to low SNRs, we obtain the approximation in (30) as

$$\frac{2E_s \boldsymbol{\omega}^T (\sum_0 - \sum_{\delta})^{1/2} \sum_{\mathbf{H}} \boldsymbol{\omega}}{\sqrt{m}\boldsymbol{\omega}^T \sum_0 \boldsymbol{\omega}} << 1, \tag{31}$$

where $\sum_{\boldsymbol{\delta}}$=diag($\boldsymbol{\delta}$) and $\sum_{\mathbf{H}}$=diag($\mathbf{H}$). The approximation in (31) leads to $\alpha_{\text{Risk}} \approx 1$ in (30). Accordingly, we obtain the approximate optimal weighting coefficients under a low SNR as

$$\boldsymbol{\omega}_{\text{opt,Risk}}^* = (\sum_0 + \sum_1)^{-1} \mathbf{H}. \tag{32}$$

To avoid confusion, we may call these coefficients *near-*

optimal weights, yet still denote them as $\boldsymbol{\omega}_{\text{opt,Risk}}$:

$$\boldsymbol{\omega}_{\text{opt,Risk}} = \boldsymbol{\omega}^*_{\text{opt,Risk}} / \left\|\boldsymbol{\omega}^*_{\text{opt,Risk}}\right\|_2 . \tag{33}$$

Finally, the risk function in (26) is evaluated by

$$R = [P(H_0)C_{\text{FA}} + P(H_1)C_{\text{MISS}}]Q\left(\frac{E_s\mathbf{H}^T(\sum_0+\sum_1)^{-1}\mathbf{H}}{\sqrt{\Delta_0}+\sqrt{\Delta_1}}\right), \tag{34}$$

where

$$\Delta_0 = \mathbf{H}^T(\textstyle\sum_0+\sum_1)^{-2}\sum_0\mathbf{H},$$
$$\Delta_1 = \mathbf{H}^T(\textstyle\sum_0+\sum_1)^{-2}\sum_1\mathbf{H}.$$

If C_{FA} and C_{MISS} are both equal to a constant error detection cost C_{ERR}, we obtain the error detection probability $P_{\text{ERR,C}}$ according to (26) as follows:

$$\begin{aligned} P_{\text{ERR,C}} &= C_{\text{ERR}}P(H_0)P_{\text{FA,C}} + C_{\text{ERR}}P(H_1)P_{\text{MISS,C}} \\ &= C_{\text{ERR}}Q\left(\frac{E_s\mathbf{H}^T(\sum_0+\sum_1)^{-1}\mathbf{H}}{\sqrt{\Delta_0}+\sqrt{\Delta_1}}\right), \end{aligned} \tag{35}$$

where $P_{\text{ERR,C}}$ corresponds to the error probability in a minimal error detection probability criterion.

Similarly, we can obtain the averaged $\bar{P}_{\text{ERR,C}}$ as

$$\bar{P}_{\text{ERR,C}} = \int_{\mathbf{G}_2\subset\mathbb{R}_+^M} C_{\text{ERR}}Q\left(\frac{E_S\mathbf{X}^T(\sum_0+\sum_1)^{-1}\mathbf{X}}{\sqrt{\Delta_0}+\sqrt{\Delta_1}}\right)p_{\mathbf{H}}(\mathbf{X})d\mathbf{X}, \tag{36}$$

where $\mathbf{G}_2$ is the subset of $\mathbb{R}_+^M$, which contains all $\mathbf{H}$ vectors leading to the H_1 decision.

IV. Implementation of Weighting Schemes

1. Implementation of Weight Vectors

The optimal and near-optimal weighting vectors in (18) and (33) are actually determined by PU signal energy quantities $\{E_sH_i\}_{i=1}^M = \{E_s|h_i|^2\}_{i=1}^M$ and the squares of noise power levels $\{mN_{0,i}\}_{i=1}^M$, both of which need to be measured or estimated. For each SU, the PU signal energy component E_sH_i in the sensing observation can be estimated by the fusion center from the corresponding integrator output. Hereafter, we suppose that the noise variance variables $\{\sigma_{0,i}^2\}_{i=1}^M$ in (6) are distributed around an average network noise power level σ_0^2, with a deviation d, which is normally distributed as $N(0, D\sigma_0^2)$, where D indicates the location difference factor in the network area. In other words, the noise variances at different SUs are usually not identical because SUs dispersed over the network may experience different thermal noises and interferences. The actual noise variance at the i-th SU is $\sigma_{0,i}^2 = \sigma_0^2 + d$, where σ_0^2 is the average noise variance in the network.

Similarly, we assume δ_0^2 is the average noise variance on the control channels. Consequently, the average received SNR of the M cooperative SUs at the fusion center is

$$\gamma_{\text{avg}} = E\left[\frac{E_s\mathbf{1}^T(\sum_{\mu_0}+\sum_\delta)^{-1}\mathbf{H}}{M}\right] = \frac{E_s}{\sigma_0^2+\delta_0^2}, \tag{37}$$

where $\mathbf{1}$ is a column vector of all ones.

To obtain the derived optimal weight vector, that is, $\boldsymbol{\omega}_{\text{opt,DC}}$ in (17), we can implement it by using

$$\boldsymbol{\omega}_{\text{est}} = \textstyle\sum_{\text{est}}^{-1}\tilde{\mathbf{E}}, \tag{38}$$

where $\sum_{\text{est}} = \text{diag}(\tilde{\boldsymbol{\sigma}}_0 + \boldsymbol{\delta}_0)$ is a diagonal matrix with $\tilde{\boldsymbol{\sigma}}_0 + \boldsymbol{\delta}_0$ on the diagonal, and $\boldsymbol{\delta}_0 = \delta_0^2\mathbf{1}$ is assumed to be a known constant vector indicating the control channels' noise variance. The elements of vectors $\tilde{\mathbf{E}}$ and $\tilde{\boldsymbol{\sigma}}_0$ are estimates of vectors $\mathbf{E}$ and $\boldsymbol{\sigma}_0$, respectively:

$$\begin{cases} \tilde{\mathbf{E}} \triangleq [\tilde{E}_1, \tilde{E}_2, \cdots, \tilde{E}_M]^T = E_s\tilde{\mathbf{H}}, \\ \tilde{\boldsymbol{\sigma}}_0 \triangleq [\tilde{\sigma}_{0,1}^2, \tilde{\sigma}_{0,2}^2, \cdots, \tilde{\sigma}_{0,M}^2]^T = m\tilde{\mathbf{N}}_0^2. \end{cases} \tag{39}$$

The weights in (38) are actually the estimated $\boldsymbol{\omega}_{\text{opt,DC}}$. It is worth noting that in (39) we can ignore the PU signal energy E_s contained in $\tilde{H}_{i,k}$, since they are rendered ineffective in (19) after the normalization operation.

The $\boldsymbol{\omega}_{\text{opt,Risk}}$ in (33) and $\boldsymbol{\omega}_{\text{MRC}}$ in (21) can also be implemented in a similar way by estimating the parameters needed for weight setting.

2. Recursive Estimation Algorithm for Weight Setting

The method of setting the weights in (38) provides an efficient and easily implemented strategy, in terms of simple implementation complexity. To obtain PU signal energies hidden in the raw sensing data of the cooperative SUs, an efficient and effective method is employed.

We utilize two sensing data matrices to keep records of PU's behavior, in which the current sensing data $\mathbf{Z}$ is categorized and stored in either a *Presence* or *Absence* matrix for future reference according to the current global decision based on it. In other words, if it is decided that the current data $\mathbf{Z}$ contains the PU signal energy, it is stored in an M-by-L *presence* matrix $\mathbf{Z}^{(\text{P})}$ in a first-in-first-out manner. Otherwise, it is stored in an M-by-L *absence* matrix $\mathbf{Z}^{(\text{A})}$, and at the same time, a zero column vector is stacked into $\mathbf{Z}^{(\text{P})}$ because the fusion center has decided that no PU signal energy is contained in $\mathbf{Z}$. The estimates of $\{E_sH_{i,k}\}$ and $\{mN_{0,i,k}\}$ for the current statistic $Z_{c,k}$

are calculated by simple arithmetic operations:

$$\begin{cases} E_s\tilde{H}_{i,k} = \sum_{l=k-L}^{k-1} \xi_l \left| Z_{i,l}^{(P)} - Z_{i,l}^{(A)} \right|, \\ m\tilde{N}_{0,i,k} = \sum_{l=k-L}^{k-1} \xi_l \left| Z_{i,l}^{(A)} \right|, \qquad i = 1, \cdots, M, \end{cases} \tag{40}$$

where k is the time index of the current sensing data; ξ_l is the forgetting factor, which is usually a scalar between 0.1 and 0.9 ($\sum_{l=k-L}^{k-1} \xi_l = 1$); and L is the reference matrix depth, which should be carefully tuned on the basis of estimation or prediction of the channel gain varying velocity. We conclude that this depth should not exceed half of the quasi-static duration of a channel, which implies that the reference truncation window is sufficiently short within the empirical duration of the near-static channels.

Additionally, to be more compact for implementation, the proposed data matrices in (40) can be implemented as

$$\begin{cases} E_s\tilde{H}_{i,k} = \frac{L-1}{L} E_s\tilde{H}_{i,k-2} + \frac{1}{L}\left| Z_{i,k-1}^{(P)} - Z_{i,k-1}^{(A)} \right|, \\ m\tilde{N}_{0,i,k} = \frac{L-1}{L} m\tilde{N}_{0,i,k-2} + \frac{1}{L}\left| Z_{i,k-1}^{(A)} \right|, \qquad i = 1, \cdots, M, \end{cases} \tag{41}$$

where only two forgetting factors, ξ_{k-2} and ξ_{k-1}, are used and set as constants $(L-1)/L$ and $1/L$, respectively.

In summary, the implementation complexity of the proposed recursive estimate algorithm is mainly determined by the number of memory units ($2ML$) consumed in storing the sensing observations collected from the cooperative SUs. Because the estimate algorithm is a simple arithmetic averaging method, the time and power consumed in computing the time means of the sensing observations is quite trivial. Furthermore, because the weight vector $\boldsymbol{\omega}_{\text{opt,Risk}}$ needs an additional estimate of $\boldsymbol{\Sigma}_1$, which is also a simple function of the estimated parameters given in (40), the implementation complexities of setting $\boldsymbol{\omega}_{\text{opt,DC}}$ and $\boldsymbol{\omega}_{\text{opt,Risk}}$ are almost the same.

V. Simulations and Analyses

In this section, the proposed cooperative spectrum sensing schemes are evaluated by simulations. The basic parameters are fixed and set as $T = 1$ ms, $W = 1$ MHz, $M = 10$, $D = 40\%$, and $L = 16$. Each simulation consisted of 10^5 iterations.

The channel gains between each SU and the target PU are generated according to a complex normal distribution $CN(0, 1)$, which suggests that the M SUs are experiencing i.i.d Rayleigh fading. For simplicity, we assume that the PU signal power and the channel gains $\mathbf{h}$ have constant values for each sensing interval T, considering that T is set sufficiently small. This assumption is reasonable. It is satisfied in a realistic scenario,

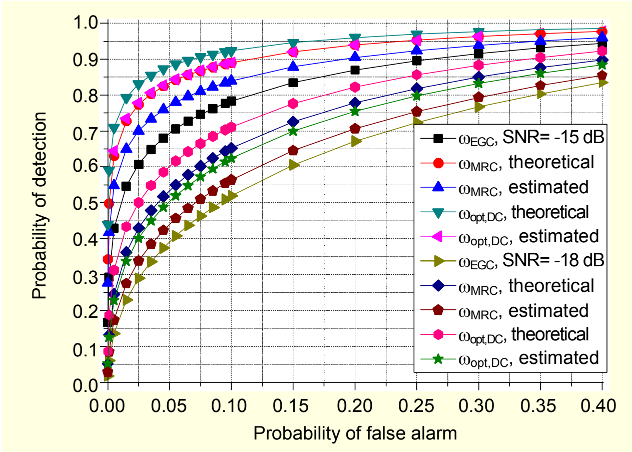

Fig. 3. ROC of Neyman-Pearson-criterion-based weighting scheme.

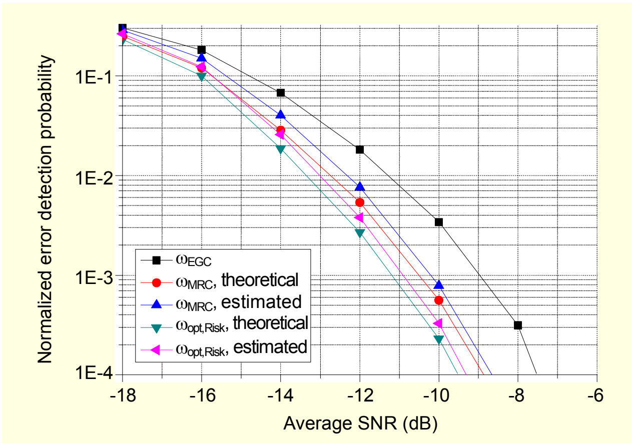

Fig. 4. Normalized error detection probability P'_{ERR} vs. average SNR.

where the variation of the dynamic radio environment is reflected in the variations of the statistical properties of $\mathbf{h}$ over a relatively large time-scale. We also assume that $\boldsymbol{\delta}$ in the simulation is a unit vector, because of the potentialities of the amplify-and-forward capabilities of the SUs.

Figure 3 shows the receiver operating characteristics (ROC) of the proposed cooperative sensing schemes. As seen in the figure, $\boldsymbol{\omega}_{\text{opt,DC}}$ outperforms both $\boldsymbol{\omega}_{\text{MRC}}$ and $\boldsymbol{\omega}_{\text{EGC}}$. With only 10 cooperative SUs, the ROC performance is quite satisfactory in practice. The performance of the estimated $\boldsymbol{\omega}_{\text{opt,DC}}$ is very close to that of the theoretical $\boldsymbol{\omega}_{\text{MRC}}$. Although the estimated $\boldsymbol{\omega}_{\text{opt,DC}}$ degrades the ROC performance compared to the theoretical $\boldsymbol{\omega}_{\text{opt,DC}}$, it can be expected that a small increase in the number of cooperative SUs would compensate the performance deterioration sufficiently.

Figure 4 depicts the normalized error detection probability $P'_{\text{ERR,C}} = P_{\text{ERR,C}} / C_{\text{ERR}}$ in the CR network when C_{FA} and C_{MISS} are equally set to C_{ERR}. We can easily observe that $\boldsymbol{\omega}_{\text{opt,Risk}}$ outperforms both $\boldsymbol{\omega}_{\text{MRC}}$ and $\boldsymbol{\omega}_{\text{EGC}}$. When the average SNR increases, the estimated weights all approach the corresponding theoretical weights.

VI. Conclusion

In this paper, we investigated and analyzed linear soft-combination-based cooperative spectrum sensing schemes in CR networks, consisting of allocation of optimal weights to individual cooperative SUs. An efficient and effective method of practically estimating PU signal energies and noise power levels to implement the proposed optimal weights was also presented. As demonstrated by our analyses and simulations, the proposed soft combination schemes outperform the conventional MRC and equal gain combination schemes and yield significant improvements in spectrum sensing.

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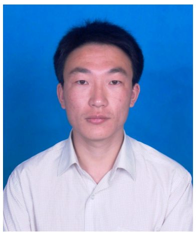

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