Sum-Rate Capacity with Fairness in Correlated MIMO Broadcast Channels

Seung Hwan Lee · Jin-Up Kim

Abstract

Although the maximum sum-rate capacity of multiple-input multiple output(MIMO) broadcast channels(BCs) can be achieved by dirty-paper coding(DPC), the results were obtained without fairness considerations in uncorrelated MIMO channels. In this paper, we propose new multiuser scheduling algorithms, which find a best user set for approaching the maximum sum-rate capacity while maintaining fairness among users. We analyze the performance of the proposed algorithms using zero-forcing dirty paper coding(ZF-DPC) in the correlated MIMO BCs for throughput and delay fairness, respectively. Numerical results demonstrate that a large time window can reduce the average throughput difference between users, but it increases head-of-line(HOL) delay jitters in the case of delay fairness.

Key words : Multiple Input Multiple Output(MIMO), Broadcast Channel(BC), Sum-Rate Capacity, Fairness.

Ⅰ. Introduction

It is well known that the single user capacity of a MIMO system increases linearly with the minimum number of transmit and receive antennas. In multiuser MIMO broadcast channels(BCs or downlinks, i.e., channels from the base station to mobile users), dirty-paper coding (DPC)[1] can achieve the sum-rate capacity[2]. However, the results are not directly applicable to practical scenarios where a MIMO BC correlated in time and fairness between users is required in the MIMO BC. If the MIMO BC is independent identically distributed(*i.i.d.*) in time, it can provide a statistical fairness to users automatically without considering any fairness. In real channel models, there exist time-varying temporal correlations in every channel realization. Various channel models have been proposed to describe the characteristics of the real channels. Among them, the Jakes model[3] has been widely used due to its simplicity. Because the deterministic Jakes model is not wise-sense stationary, many different approaches have been studied for the improvement of this model. In [4], the authors have proposed a sum-of-sinusoids statistical simulation model for Rayleigh fading. In this case, the power spectrum of the model is still the same as for the Jakes model, which is a function of the maximum Doppler frequency and the carrier frequency. In addition to Rayleigh fading due to the vector sum of multipath components, users in real channel environments experience shadowing due to the terrain features between the base station and the users. In the correlated MIMO BC due to Rayleigh fading and shadowing, users are likely to have different channel conditions and temporal variations. Let us assume a scenario where throughput fairness should be provided in a temporal correlated MIMO BC between users. In this scenario, some users with small Doppler spread may stay in bad channel conditions for a certain period of time longer than temporal fairness requirements. In this case, these users receive fewer amounts of data than other users during the period, if there are no fairness considerations. Conventional multiuser selection algorithms, combined with practical MIMO precoding techniques like the maximum sum-rate rule with linear beamforming, [5] ignore the unfair situation due to different channel conditions and only select users who can maximize the sum-rate capacity of the system.

In this paper, we propose new multiuser scheduling algorithms, which try to maximize the sum-rate capacity while providing fairness between users in the correlated MIMO BC. We investigate the fairness in view of both throughput and delay. These algorithms can be combined with any MIMO precoding techniques such as zero-forcing dirty paper coding(ZF-DPC)[6] and Tomlinson-Harashima precoding(THP)[7], which use QR decomposition at the transmitter. The sum-rate capacity of the MIMO BC can be obtained by the sum of each capacity of independent spatial channels created by the QR decomposition. For the proposed algorithms, we introduce fairness factors that determine the effective data rate according to the difference between fairness requirements and achievements. The effective sum-rate capacity becomes identical to the maximum sum-rate capacity, if

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Manuscript received June 19, 2009 ; revised August 28, 2009. (ID No. 20090619-025J)

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perfect fairness between users is achieved. In the above scenario, users in bad channel conditions have a higher effective data rate than actual individual rates and are likely to be included in the selected user set for transmission because of their higher effective data rates. In this case, throughput fairness can be achieved but the actual sum-rate capacity may be reduced. The same principle can be applied to delay-fairness cases. Any user with a large head-of-line(HOL) delay has high effective data rate and the maximum sum-rate rule finds a user set that maximizes the effective sum-rate capacity. In this way, the maximum sum-rate capacity rule with the effective data rate can always find a best user set, which is then able to achieve the maximum sum-rate capacity while considering throughput or delay fairness.

The rest of this paper is organized as follows. In section Ⅱ, the system model is introduced. In section Ⅲ, we review the ZF-DPC technique and the maximum sum-rate rule in brief. Section Ⅳ explains the proposed algorithm. Numerical results are presented in section Ⅴ, and conclusions are drawn in section Ⅵ.

Ⅱ. System Model

We use boldface type to denote matrices and vectors. For any general matrix, **A**, $\mathbf{A}^T$ denotes the transpose. For any general set, B, $|B|$ denotes the cardinality of the set.

Consider a correlated MIMO broadcast channel with M_T transmit antennas at a base station and K users, each with a single receive antenna. Let $\mathbf{h}_k(t) \in \mathbb{C}^{M_T \times 1}$ denote the channel at time t between the transmit antenna arrays and the receive antenna for user k. Then the downlink channel at time instant t can be represented as follows:

$$y_k(t) = \mathbf{h}_k^T(t)\mathbf{x}(t) + z_k(t), \qquad k = 1, \cdots, K, \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{C}^{M_T \times 1}$ is the transmit signal vector, $y_k(t)$ is the received signal for user k and $z_k(t)$ is the complex additive white Gaussian noise(AWGN) with zero mean and unit variance for user k. The transmit signals are assumed to experience Rayleigh fading and log-normal shadowing. Then the channel $\mathbf{h}_k(t)$ can be expressed as follows:

$$\mathbf{h}_k(t) = \sqrt{SNR_0} \cdot w_k(t)\ \mathbf{g}_k(t), \tag{2}$$

where SNR_0 denotes the average signal-to-noise ratio (SNR) of all users, $w_k(t)$ denotes the shadowing of user k. The vector $\mathbf{g}_k(t) = [g_{k1}(t), g_{k2}(t), \cdots, g_{kM_T}(t)]^T$ denotes the Rayleigh-distributed column vector between the transmit antenna array and user k with zero mean and unit variance. The effect of shadowing is modeled as a log-normal process with a standard deviation of σ_s, which is given by [8]:

$$w_k(t) = 10^{[\sigma_s \lambda_k(t)]/20}, \tag{3}$$

where $\lambda_k(t)$ is a sum-of-sinusoid(SOS) random process of user k, which determines the characteristics of shadowing. For Rayleigh fading, the sum-of-sinusoids statistical model[4] is applied to each entry of the vector $\mathbf{g}_k(t)$ independently. Because the long-term channel variation due to shadowing can be ignored compared with the Rayleigh fading for a certain fairness window time, the temporal auto-correlation function of the correlated MIMO BC can be described as the Jakes power spectrum as follows:

$$\rho_k = \sigma_m^2 J_0(2\pi f_k^d \tau_k), \tag{4}$$

where σ_m^2 denotes the mean channel gain due to shadowing, τ_k denotes the time difference between two Rayleigh fading variables of user k and $J_0(\cdot)$ denotes the zeroth-order Bessel function of the first kind.

Ⅲ. Review of ZF-DPC and Maximum Sum-Rate Rule

Zero-forcing dirty paper coding(ZF-DPC) exploits the DPC principle by which the capacity of a system with known interference at the transmitter is the same as if there was no interference present. It decomposes the MIMO channel into transmit beamforming matrix **F** and lower triangular matrix **B** using the QR decomposition. Because of the lower triangular matrix **B**, the first encoded user has no interference, the second encoded user experiences interference only from the first encoded user and so on. In other words, any interference caused by users $j>i$ on each user i is forced to zero by the pre-subtraction at the transmitter. If we ignore the encoding order of ZF-DPC though it affects the capacity, the sum-rate capacity of ZF-DPC is given by

$$C^{ZF-DPC} = \sum_{k=1}^{M_T} \log_2\left(1 + b_{kk}^2 P_k\right), \tag{5}$$

where b_{kk} is the kth diagonal entry of **B**, b_{kk}^2 is the effective channel gain to kth user and P_k is the signal power allocated to kth user with a power constraint of P by $\sum_{k=1}^{M_T} P_k \le P$. In this paper, we assume the encoding order of ZF-DPC so that the effective channel gain b_{kk}^2 decreases as the user index increases. The second term in the logarithm function, $b_{kk}^2 P_k$, is the effective SNR of user k at the receiver. It is useful to note that the total power of the ZF-DPC is not constrained in this paper. There may be a possible power increase in the ZF-DPC due to the pre-subtraction regardless of the input signal power. However, the power increase at the transmitter does not affect the effective SNR at the receiver because it is cancelled through the MIMO channel.

If we assume the number of selected users for transmission is the same as that for transmit antennas, the maximum sum-rate rule finds a user set $S_{\max}$ from all possible user groups S with the sum-rate capacity of ZFDPC in (5)

$$S_{\max} = \arg \max_{S \subset \{1,\dots K\}, |S| = M_T} C^{ZF\text{-}DPC}(S). \tag{6}$$

The maximum sum-rate rule is used for the proposed algorithm described in the next section.

Ⅳ. Multiuser Scheduling Algorithm

In this section, we explain the proposed multiuser scheduling algorithm. We assume two fairness requirements: throughput fairness and delay fairness. In the case of throughput fairness, the required data rate of all users is assumed to be the same and the throughput-fairness scheduling algorithm finds a user set that makes the average data rate of all users identical as well as the sum-rate capacity by the selected users approach the maximum sum-rate capacity. Similarly, in the case of delay fairness, the required average HOL delay is assumed to be the same and the delay-fairness scheduling algorithm finds a user set that makes the average HOL delays in the queues of all users identical and the sum-rate capacity by the selected users approach the maximum sum-rate capacity. In both cases, user selection algorithms operate once every certain period of time and ZF-DPC is applied in the MIMO BC.

4-1 Multiuser Selection for Throughput Fairness

For throughput fairness considerations in the correlated fading MIMO BC, we define an effective data rate $R_k^{eff}(n)$ of kth user(k=1$\cdots$$K$) at a certain discrete time n as follows:

$$R_k^{eff}(n) = R_k(n) \cdot \exp\left(\frac{R_M(n) - \overline{R_k}(n)}{1 + \sqrt{R_M(n)}} \right), \tag{7}$$

where $R_k(n)$ is the actual data rate determined by the channel condition of user k; $\overline{R_k}(n)$ is the average data rate of user k for a period of time window, T_R, which is related to throughput fairness; and $R_M(n)$ is the mean of the average data rates of all users, which is given as follows:

$$R_M(n) = \frac{1}{K} \sum_{k=1}^{K} \overline{R_k}(n) \quad . \tag{8}$$

The average data rate of user k is updated according to (9):

$$\overline{R_k}(n+1) = \left(1 - \frac{1}{T_R} \right) \overline{R_k}(n) + \frac{1}{T_R} R_k(n). \tag{9}$$

Using the Shannon capacity formula, we can obtain the effective SNR corresponding to the effective data rate at time index n, which is given by:

$$SNR_k^{eff}(n) = \{1 + SNR_k(n)\}^{\eta_k^T(n)} - 1, \tag{10}$$

where $SNR_k(n)$ is the measured SNR of user k determined by the real channel condition and $\eta_k^T(n)$ represents the throughout fairness factor of user k given by:

$$\eta_k^T(n) = \exp\left(\frac{a_k R_M(n) - \overline{R_k}(n)}{1 + \sqrt{R_M(n)}} \right), \tag{11}$$

where the scalars a_k are constants to allow different throughput requirements among users.

Therefore, according to (5) and (10), the effective sum-rate capacity with ZF-DPC, including throughput fairness at time index n, is given by:

$$C_T^{ZF\text{-}DPC}(n) = \sum_{k=1}^{M_T} \eta_k^T(n) \log_2 \{1 + SNR_k(n)\}, \tag{12}$$

The proposed scheduler calculates the effective sum-rate capacity $C_T^{ZF\text{-}DPC}(n)$ with all possible user sets with M_T users and selects a user set according to the maximum sum-rate rule (6). Any user with small average data rate compared with the required data rate has large exponent values and makes its effective data rate higher than the actual data rate due to the exponent term.

4-2 Multiuser Selection for Delay Fairness

Similar to the case of throughput fairness, we define an effective delay of user k as follows:

$$\eta_k^D(n) = \exp\left(\frac{\overline{D_k}(n) - D_M(n)}{1 + \sqrt{D_M(n)}} \right), \tag{13}$$

where $\overline{D_k}(n)$ is the average HOL delay of user k for a period of time window, T_D, which is related to delay fairness, and $D_M(n)$ is the mean of the average HOL delays of all users, which is given by:

$$D_M(n) = \frac{1}{K} \sum_{k=1}^{K} \overline{D_k}(n). \tag{14}$$

The average delay of user k is updated with the HOL delay $D_k^{HOL}(n)$ at the time index n according to (15):

$$\overline{D}_k(n+1) = \left(1 - \frac{1}{T_D}\right)\overline{D}_k(n) + \frac{1}{T_D} D_k^{HOL}(n). \tag{15}$$

Then the effective sum-rate capacity with ZF-DPC including delay fairness at time index n is given by:

$$C_D^{ZF\text{-}DPC}(n) = \sum_{k=1}^{M_T} \eta_k^D(n) \log_2\{1 + SNR_k(n)\}. \tag{16}$$

We can also apply (16) to the maximum sum-rate rule (6) for achieving the possible maximum sum-rate capacity while making HOL delays of all users the same. If an HOL delay of any user is smaller than the mean of average HOL delays, $D_M(n)$, the user has a smaller effective data rate than the actual data rate. This means that the user may be excluded by the sum-rate rule despite possible high channel gain. On the contrary, any user with a large HOL delay may be included in the selection regardless of the physical channel gain to reduce the HOL delay difference between users.

V. Numerical Results

In this section, numerical results for the performance of the proposed algorithms are presented. Throughout the simulations, we assume K=8, M_T=4 for multiuser configurations. For correlated fading, method of equal areas (MEA)[8] is used with the standard deviation σ_s=4.3 dB and the decorrelation distance D=8.3 m for generating shadowing of urban environment, and the carrier frequency f_c=2 GHz and the velocity of all users v=10 km/h are assumed. The proposed scheduling algorithms are assumed to operate once per every slot time T_s=10 ms. We also assume that the total input power P=1 and each user has the same input power as $P_1 = P_2 = \cdots = 1/M_T$. For the analysis of delay fairness, the total arrival rate of Poisson process is set to be the same as the total average sum-rate capacity achieved so that no queue overflow occurs. Each user has the same arrival rate, which is obtained by dividing the total arrival rate by the number of users.

Fig. 1 shows the sum-rate capacity with throughput fairness versus the average SNR for different fairness time windows. First, we noticed that the sum-rate capacity of no fairness is between the ergodic capacity of (M_T=4, K=4) and (M_T=4, K=8). This is because the scheduler selects the most orthogonal 4 users with favorable channel conditions from 8 users by the maximum sum-rate rule. In this case, its performance is similar to the performance of the receive antenna selection in single user scenarios. The sum-rate capacity with no fairness is the maximum achievable sum-rate capacity without fairness considerations.

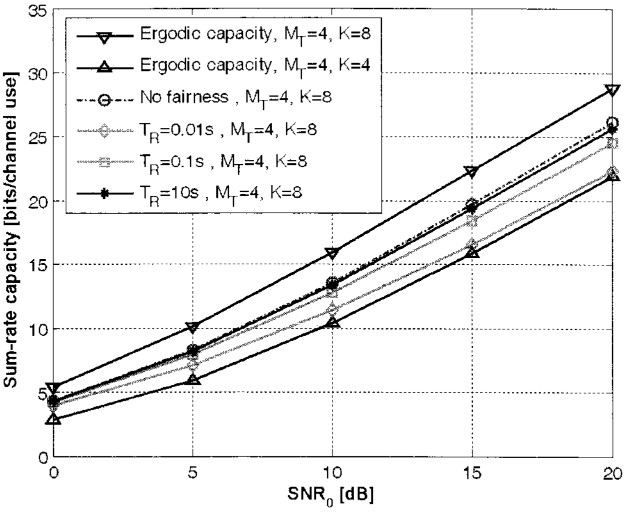

Fig. 1. The sum-rate capacity with throughput fairness for different time windows.

When the time window is T_R=0.01s, the sum-rate capacity has poorer performance than for the case of no fairness because the scheduler tends to select users whose effective data rates are high, rather than users with favorable channel conditions, to overcome the throughput difference between users. At high SNRs, the capacity degradation is more evident than at low SNRs. This is because the throughput fairness factor has larger values at high SNRs than at low SNRs. However, with the time window T_R=10s, the sum-rate capacity almost approaches the maximum sum-rate capacity. This is because when the size of time window is sufficiently large compared to the coherent time due to Doppler spread, the correlated fading channel can be considered as an uncorrelated fading channel within that time window.

Fig. 2 shows the average throughput per user and the throughput difference due to throughput fairness versus the size of time window when SNR_0=10 dB.

We notice that the throughput per user approaches the maximum sum-rate capacity as the size of the time window increases, which is also shown in Fig. 1. The throughput difference, defined as the standard deviation of each user's throughput during the time window period, also decreases as the size of time window increases. This is because a larger time window means a less strict fairness requirement in time, which reduces the effect of the correlated fading.

Fig. 3 shows the sum-rate capacity with delay fairness versus the average SNR for different fairness time windows.

Similar to the case of throughput fairness, the sum-rate capacity with delay fairness approaches the maximum sum-rate capacity with no fairness as the size of time window, T_D, increases. When the time window T_D

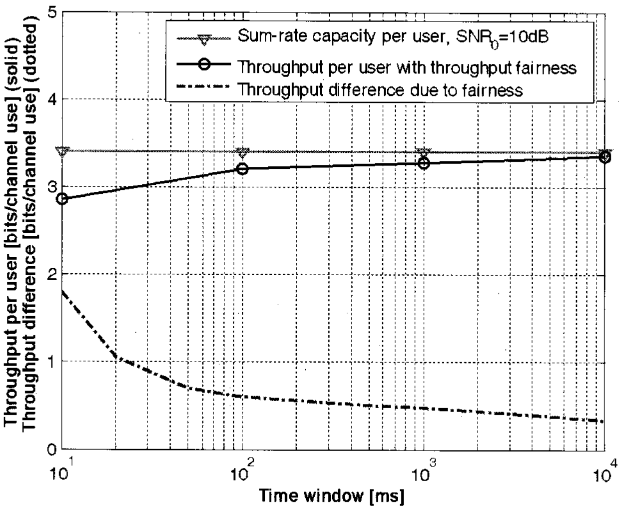

Fig. 2. Throughput per user with different time windows and throughput differences due to fairness.

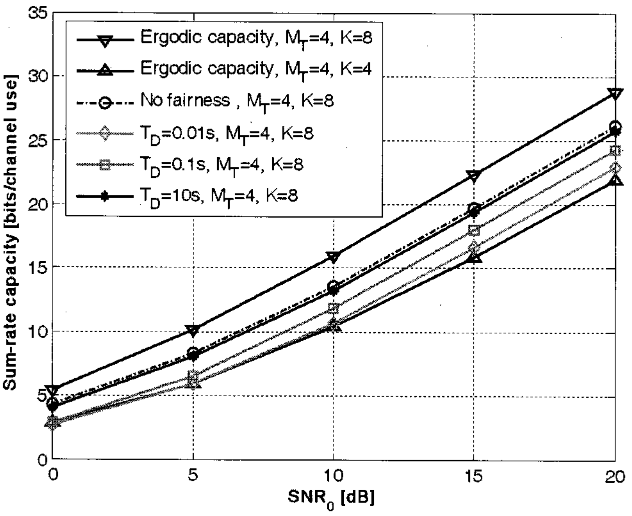

Fig. 3. The sum-rate capacity with delay fairness for different time windows.

=10, the sum-rate capacity with delay fairness is almost the same as that with no fairness because the correlated fading channel can be considered as the uncorrelated fading channel with a sufficiently large size of time window compared to the coherence time of the channel, which is an identical result to the case of throughput fairness. However, unlike the case of throughput fairness, the capacity degradation at low SNR values is more evident than that at high SNRs because small values of the actual data rates at low SNRs are more sensitive to the same delay fairness factor in calculating the sum of effective data rates.

Fig. 4 shows the average throughput per user and the delay jitter due to delay fairness versus the size of time window when SNR_0=20 dB. As the size of time window

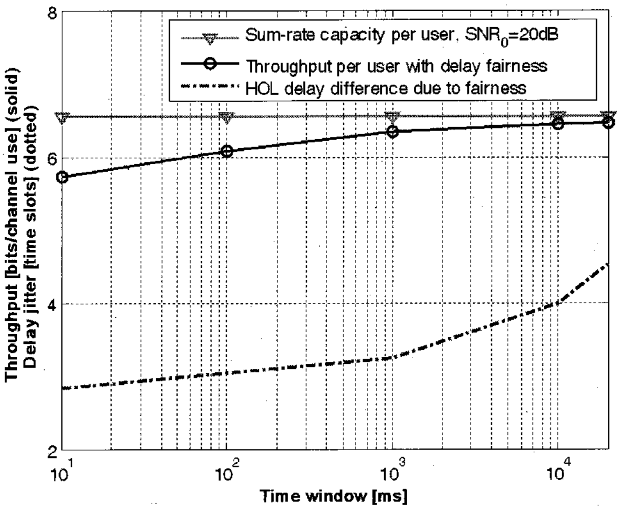

Fig. 4. Throughput per user with different sized time windows and delay jitter due to fairness.

increases, the throughput per user approaches the maximum sum-rate capacity. However, the delay jitter increases with the size of the time window. This is because a large time window means less strict delay fairness, so that the delay fairness factor has less effect on selecting users by the scheduler, which allows large delay jitters in each user selection procedure. Note that the delay jitter is obtained by the standard deviation of HOL delay of each user's queue.

Ⅵ. Conclusion

In this paper, we proposed new multiuser scheduling algorithms that can achieve the possible maximum sum-rate capacity while maintaining fairness between users in the correlated MIMO broadcast channels. Two multiuser selection algorithms were introduced for the cases of throughput fairness and delay fairness, respectively. Each of the proposed multiuser selection algorithms uses the new concept of the effective data rate, which can reflect the status of fairness among users. This effective data rate can be combined with the sum-rate capacity of ZF-DPC, which represents the effective sum-rate capacity with fairness considerations.

Numerical results showed that the performance of the proposed algorithms was determined by the amount of fairness requirements. With a relatively large window size compared to the coherence time due to Doppler spread, the sum-rate capacity even with fairness considerations can achieve the maximum sum-rate capacity without fairness considerations. On the contrary, if the window size is small compared to the coherence time, the scheduler tends to select users not by maximizing the sum-rate capacity but by satisfying the fairness requirements.

Further work should be performed to consider both throughput and delay fairness simultaneously in selecting users according to QoS requirements.

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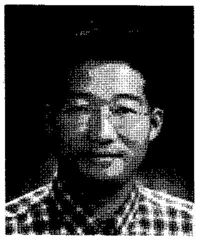

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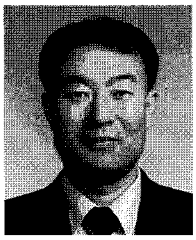

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