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Burn-in Models: Recent Issues, Developments and Future Topics

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Abstract

Recently, there has been much development on burn-in models in reliability area. Especially, the previous burn-in models have been extended to more general cases. For example, (i) burn-in procedures for repairable systems have been developed (ii) an extended assumption on the failure rate of the system has been proposed and (iii) a stochastic model for burn-in procedure in accelerated environment has been developed. In this paper, recent extensions and advances in burn-in models are introduced and some issues to be considered in the future study are discussed.

Keywords: Burn-in procedure, general failure model, eventually increasing failure rate function, accelerated burn-in.

1. Introduction

Burn-in is a method used to eliminate initial failures in field use. Burning-in a component or system means to subject it to a period of use prior to being used in field. Those components which fail during the burn-in procedure will be scrapped or repaired and only those which survive the burn-in procedure will be considered to be of good quality. Due to the high failure rate in the early stages of system life, which is very common phenomenon for manufactured systems, burn-in procedures have been widely accepted as a method of screening out early failures before systems are actually used in field. An introduction to this important area of reliability can be found in Jensen and Petersen (1982) and Kuo and Kuo (1983). Too short burn-in process still leaves weak or defective components in the population of components, whereas too long burn-in process eliminates not only weak components but also strong ones from the population and it shortens the actual lifetime of the burned-in component. Since excessive or insufficient burn-in is harmful to the performance of a system and burn-in is costly, one of the major problems is to decide how long the procedure should be. The best time to stop the burn-in procedure for a given criterion is called the optimal burn-in time. In the literature, certain cost structures have been proposed, and the corresponding problem of finding the optimal burn-in time has been considered. See, for example, Cha (2000, 2001, 2003), Clarotti and Spizzichino (1991), Mi (1994a, 1996, 1997) and Nguyen and Murthy (1982). Some other performance-based criteria, for example, the mean residual life, the reliability of performing a given mission, or the mean number of failures, have also been considered. See, for instance, Block *et al.* (1994, 2002) and Mi (1991, 1994b). An excellent survey of various research in burn-in can be found in Block and Savits (1997). In Block and Savits (1997), a survey of burn-in research with emphasis on mixture models (which are used to describe population with weak and strong components), criteria for optimal burn-in and whether it

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is better to burn-in at the system or component level has been given. After the paper of Block and Savits (1997), there has been much development on burn-in models. Especially, the previous burn-in models have been extended to more general cases: (i) burn-in procedures for repairable systems have been developed (ii) a generalized assumption on the failure rate of the system has been proposed and (iii) a stochastic model for burn-in procedure in accelerated environment has been developed. In this paper, recent advances and developments in burn-in models are surveyed in detail and some issues to be studied in the future study will be briefly discussed. In Section 2, some recent developments mentioned above will be introduced in detail. In Section 3, some other issues and research topics that can be investigated in the future study will be discussed. Finally, in Section 4, concluding remarks are discussed.

2. Recent Developments in Burn-in Models

2.1. Burn-in procedures for repairable systems

Recently various burn-in procedures for repairable systems are proposed and studied. Mi (1994a) proposed the following burn-in procedure:

(1) Burn-in Procedure A (Mi, 1994a): Consider a fixed burn-in time b and begin to burn-in a new component. If the item fails before a fixed burn-in time b, replace it with shop replacement cost c_s, and continue the burn-in procedure for the replaced item. If the item survives the burned-in time b then the item is to be put into field operation. The cost for burn-in is assumed to be proportional to the total burn-in time with proportionality constant c_0.

Adopting the above burn-in procedure, in field operation, Mi (1994a) considered two kinds of replacement policies for non-repairable and repairable components, respectively: for non-repairable component, age replacement policy is considered and, for repairable component, block replacement policy is considered. By age replacement policy, a component is always replaced at the time of failure or T hours after its installation, where T is a fixed number, whichever occurs first. On the other hand, by block replacement policy, the component is replaced at time $kT(k = 1, 2, \ldots)$, where T is also a fixed number. For intervening failures, only minimal repairs are performed. Considering these burn-in and replacement models, Mi (1994a) obtained the long-run average cost rate $c_1(b, T)$ and studied the properties of the optimal (b^*, T^*) which minimizes $c_1(b, T)$.

In the model of Mi (1994a), the above Burn-in Procedure A is applied even for repairable components. In many cases, by the practical limitation, the products which failed during the burn-in are just scraped regardless of whether the products are repairable or not. In this case, Burn-in Procedure A proposed by Mi (1994a) can be applied. However, when dealing with an expensive product or device of some complexity, the complete product will not be discarded on account of the failure during a burn-in, but rather a repair can be performed. Cha (2000) proposed the following burn-in procedure for repairable component:

(2) Burn-in Procedure B (Cha, 2000): Consider a fixed burn-in time b and begin to burn-in a new component. On each component failure, only minimal repair is done with shop minimal repair cost c_{sm} (with $c_{sm} < c_s$), and continue the burn-in procedure for the repaired component. Immediately after the fixed burn-in time b, the component is put into field operation.

Note that the total burn-in time of this burn-in procedure is a constant b. Adopting the above Burn-in Procedure B, block replacement policy with minimal repair at failure is adopted as it was in Mi (1994a). First, Cha (2000) obtained the long-run average cost rate $c_2(b, T)$ and showed that

$$c_2(b,T) \le c_1(b,T), \quad \forall 0 < b < \infty,\ 0 < T < \infty, \tag{2.1}$$

where $c_1(b,T)$ is the cost rate function in the model of Mi (1994a). It is easy to see that the above inequality (2.1) implies that Burn-in Procedure B is always preferable to Burn-in Procedure A when minimal repair method is applicable during burn-in process. And then the properties of the optimal (b^*, T^*) which minimizes $c_2(b,T)$ are investigated.

In Cha (2001), general failure model is considered and burn-in procedures for general failure model are proposed. In general failure model, when the unit fails, Type I failure occurs with probability $1-p$ and Type II failure occurs with probability p, $0 \le p \le 1$. It is assumed that Type I failure is a minor one thus can be removed by a minimal repair or a complete repair (or a replacement), whereas Type II failure is a catastrophic one thus can be removed only by a complete repair. Such models have been considered in the literature. See, for example, Beichelt and Fischer (1980). Cha (2001) proposed the following two types of burn-in procedures for the general failure model:

(3) Burn-in Procedure C (Cha, 2001): Consider a fixed burn-in time b and begin to burn-in a new component. If the component fails before burn-in time b, then repair it completely regardless of the type of failure with shop complete repair cost c_s, and then burn-in the repaired component again, and so on.

(4) Burn-in Procedure D (Cha, 2001): Consider a fixed burn-in time b and begin to burn-in a new component. On each component failure, only minimal repair is done for the Type I failure with shop minimal repair cost c_{sm} (with $c_{sm} < c_s$), whereas a complete repair is performed for the Type II failure with shop complete repair cost c_s. And continue the burn-in procedure for the repaired component.

Note that the Procedure C stops when there is no failure during a fixed burn-in time $(0,b]$ at the first time, whereas the Procedure D stops when there is no Type II failure during a fixed burn-in time $(0,b]$ at the first time. In the field use the component is replaced by a new burned-in component at the 'field use age' T or at the time of the first Type II failure, whichever occurs first. For each Type I failure occurring during field use, only minimal repair is done. Note that these burn-in and replacement models are generalizations of those in Cha (2000) and Mi (1994a). For these burn-in and replacement models, Cha (2001) obtained the long-run average cost rates $c_i(b,T;p)$, $i = 3,4$, and showed that

$$c_4(b,T;p) \le c_3(b,T;p), \quad \forall 0 < b < \infty,\ 0 < T < \infty,\ 0 < p < 1, \tag{2.2}$$

where $c_3(b,T;p)$ is the cost rate function in the model with Burn-in Procedure C and $c_4(b,T;p)$ is that in the model with Burn-in Procedure D, when the Type II failure probability is given by p. The above inequality (2.2) implies that Burn-in Procedure D is always preferable to Burn-in Procedure C when minimal repair method is applicable during burn-in process. The properties for the optimal (b^*, T^*) which minimizes $c_i(b,T;p), i = 3,4$, are also studied.

In Cha (2003) the generalized burn-in and replacement model considered by Cha (2001) is further extended to the case in which the probability of Type II failure is time-dependent. In the extended model here, when the unit fails at its age t, Type I failure occurs with probability $1 - p(t)$ and Type II failure occurs with probability $p(t)$, $0 \le p(t) \le 1$. As in Cha (2001), the two types of burn-in procedure (Procedure C and Procedure D) are considered. It is also shown that Burn-in Procedure D is always preferable to Burn-in Procedure C when minimal repair method is applicable during burn-in process. The properties of the optimal (b^*, T^*) which minimizes the long-run average cost rate $c(b,T)$ are also investigated assuming some mild conditions on the function $p(t)$.

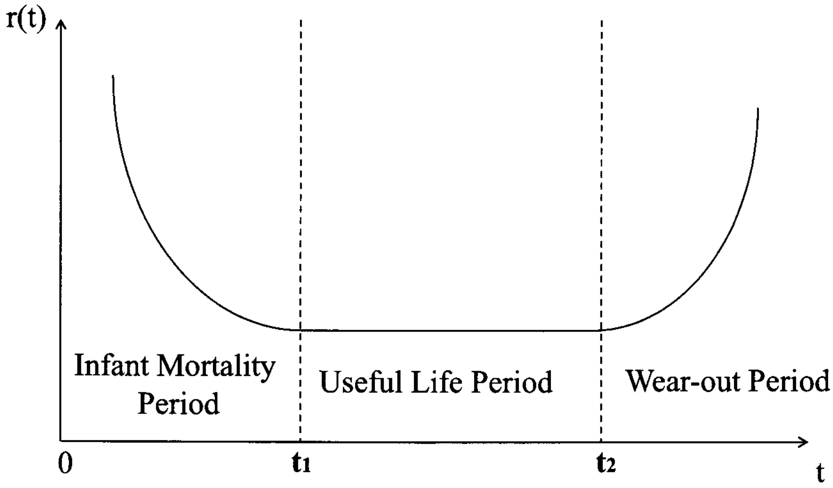

Figure 1: *Bathtub shaped failure rate*

2.2. Generalized assumption on the failure rate function

It is widely believed that many products, particularly electronic products or devices such as silicon integrated circuits, exhibit bathtub shaped failure rate functions. This belief is supported by much experience and extensive data collected by practitioners and researchers in different industries. Hence much research on burn-in has been done under the assumption of bathtub shaped failure rate function. See, for example, Cha (2000, 2001, 2003), Mi (1991, 1994a, 1994b, 1996, 1997), Nguyen and Murthy (1982), Park (1985) and Weiss and Dishon (1971). The definition of bathtub shaped failure rate function is as follows.

Definition 1. *A failure rate function is said to have a bathtub shape on* $[s_0, \infty)$ *if there exist* $0 \le s_0 \le t_1 \le t_2 \le \infty$ *such that*

$$r(t) = \begin{cases} \text{strictly decreases}, & \text{if } s_0 \le t < t_1, \\ \text{is a constant, say } \lambda_0, & \text{if } t_1 \le t < t_2, \\ \text{strictly increases}, & \text{if } t_2 \le t, \end{cases}$$

where t_1 *and* t_2 *are called the change points of* $r(t)$ *on the interval* $[s_0, \infty)$.

In the following it is assumed that $s_0 = 0$ unless special notice is made. An example of bathtub shaped failure rate function is presented in Figure 1.

The time interval $[0, t_1)$ is called the infant mortality period; the interval $[t_1, t_2)$, where $r(t)$ is flat and attains its minimum value, is called the normal operating life or the useful life; the interval $[t_2, \infty)$ is called the wear-out period. Recently, there has been much research on the shape of failure rate functions of mixture distributions. For instance, in Block *et al.* (2003a, 2003b) and Klutke *et al.* (2003), the shape of failure rate functions of mixture distributions which is neither of the traditional bathtub shape nor of the modified bathtub shape are investigated, where Klutke *et al.* (2003) pointed out that the assumption of the traditional bathtub shaped failure rate function could be rather a restrictive assumption for the research on burn-in procedures. Especially, Kececioglu and Sun (1995) asserted that the bathtub shaped failure rate function describes only 10% to 15% of applications.

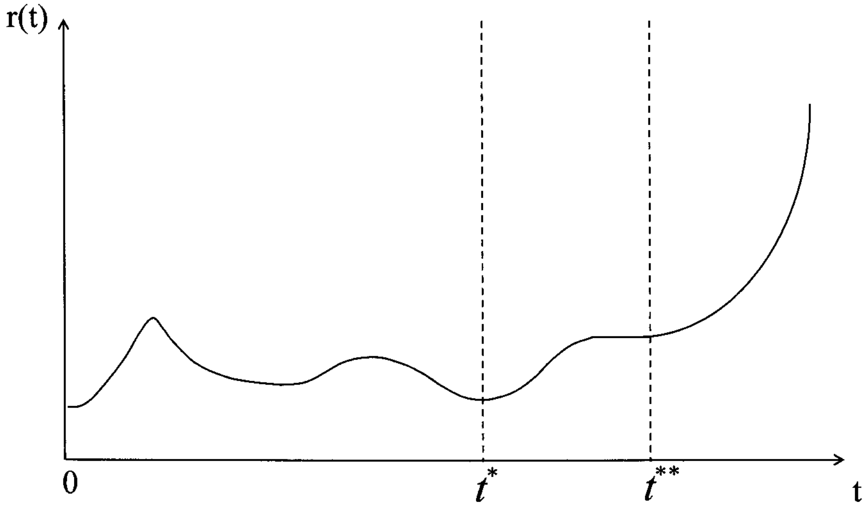

Figure 2: *Eventually increasing failure rate function*

In Mi (2003), more general model for the failure rate function, *i.e.* eventually increasing failure rate function, is proposed and optimal burn-in time has been studied assuming the general failure rate model. It can be seen that this general assumption includes the traditional bathtub shaped failure rate function as special case. The following is the definition of eventually increasing failure rate function.

Definition 2. *A failure rate function* $r(x)$ *is eventually increasing if there exists* $0 \leq x_0 < \infty$ *such that* $r(x)$ *strictly increases in* $x > x_0$. *For an eventually increasing failure rate function* $r(x)$ *the first and second wear-out points* t^* *and* t^{**} *are defined by*

$$t^* = \inf\{t \geq 0 : r(x) \textit{ is nondcreasing in } x \geq t\},$$
$$t^{**} = \inf\{t \geq 0 : r(x) \textit{ is strictly increasing in } x \geq t\}.$$

Obviously $0 \leq t^* \leq t^{**} \leq x_0 < \infty$ if $r(x)$ is eventually increasing. An example of eventually increasing failure rate function is presented in Figure 2.
Observe that the failure rate function in Figure 2 is not of the bathtub shaped failure rate. Note also that if $r(x)$ has a bathtub shape with change points $t_1 \leq t_2 \leq \infty$, then it is eventually increasing with $t^* = t_1$ and $t^{**} = t_2$. Therefore, the eventually increasing failure rate function includes the traditional bathtub shaped failure rate function as special case.

For more detailed discussions about general assumptions for the shape of failure rate function in burn-in model, see also Cha and Mi (2005). Recently, Cha (2006a, 2006b) considered optimal burn-in under the assumption of eventually increasing failure rate function and it has been shown that t^* in eventually increasing failure rate function plays the same role as t_1 in the bathtub shaped failure rate function.

2.3. Burn-in procedures in accelerated environment

Burn-in is generally considered to be expensive and the length of burn-in is typically limited. Furthermore, for today's highly reliable products, many latent failures or weak components require a long time to detect or identify. Thus, as remarked in Section 8 of Block and Savits (1997), burn-in is most often accomplished in an accelerated environment in order to shorten the burn-in process. However, nevertheless, as listed above, much research has been done only on the burn-in procedures performed in the usual level of environment and there have been few probabilistic or stochastic approaches to the burn-in procedures in accelerated environment. Recently, Cha (2006b) proposed a new failure rate model for accelerated burn-in procedure and considered optimal accelerated burn-in time.

Now, the new failure rate model for accelerated burn-in procedure proposed by Cha (2006b) will be illustrated. The probabilistic frame for accelerated burn-in procedure in Cha (2006b) employs the basic statistical property commonly used in accelerated life tests(ALT). Accelerated tests are used to obtain timely information on the life distribution or performance over time of products. During ALT, test units are used more frequently than usual or are subjected to higher than usual levels of stress or stresses like temperature and voltage. Then the information obtained from the test performed in higher level of environment is used to predict actual product performance in usual level of environment. Nelson (1990) provides an extensive and comprehensive source for background material, practical methodology, basic theory, and examples for accelerated testing. Meeker and Escobar (1993) provides a good review of recent research and current issues in ALT.

Let the random variable X be the lifetime of a component used under the usual level of environment and $F(t)$ be the distribution function of X. Let us assume X has density $f(t)$ then its failure rate $r(t)$ is given by $r(t) = f(t)/\bar{F}(t)$, where $\bar{F}(t) = 1 - F(t)$ is the survival function of X. Also denote random variable $X_{\mathcal{A}}$ the lifetime of a component operated in the accelerated level of environment and $F_{\mathcal{A}}(t)$, $r_{\mathcal{A}}(t)$, the corresponding distribution function and failure rate function, respectively. The 'Accelerated Failure Time(AFT)' regression model is the most widely used parametric failure time regression model in ALT. Under this model higher stress has the effect of shrinking time through a scale factor. This can be expressed as

$$F_{\mathcal{A}}(t) = F(\rho \cdot t), \quad \forall t \geq 0, \tag{2.3}$$

where ρ is a constant which depends on the accelerated stresses. As given in Section 3 of Meeker and Escobar (1993), a more general model can be expressed as

$$F_{\mathcal{A}}(t) = F(\rho(t)), \quad \forall t \geq 0, \tag{2.4}$$

where $\rho(t)$ depends on the accelerated environment. Since the accelerated environment gives rise to higher stresses than usual environment, reasonable assumptions are $\rho \geq 1$ for the model (2.3) and $\rho(t) \geq t$, $\forall t \geq 0$, $\rho(0) = 0$ and for the model (2.4). Furthermore we assume that $\rho(t)$ in the model (2.4) is strictly increasing, continuous and differentiable. Then, from the model (2.4), the failure rate function in accelerated environment is given by

$$r_{\mathcal{A}}(t) = \frac{\rho'(t) f(\rho(t))}{1 - F(\rho(t))} = \rho'(t) r(\rho(t)).$$

On the other hand, right after a new component has been burned-in during a fixed burn-in time b in accelerated environment, the 'virtual age', which is transformed to the usual level of environment, of the component would be not less than b. Thus we assume that the burned-in component with accelerated burn-in time b and 'field use age' u has failure rate

$$r_b(u) \equiv r(a(b) + u), \quad \forall u \geq 0, \tag{2.5}$$

where $a(b)$ satisfies $a(b) \geq b$, $\forall b \geq 0$, $a(0) = 0$ and is assumed to be strictly increasing, continuous and differentiable function. This assumption means that the survival function of the burned-in component with accelerated burn-in time b, which is operated in the usual level of environment, is given by

$$\exp\left(-\int_0^t r(a(b)+u)du\right) = \frac{\bar{F}(a(b)+t)}{\bar{F}(a(b))} = P(X > a(b)+t | X > a(b)), \quad \forall t \geq 0.$$

Thus the meaning of the assumption (2.5) is that the performance of a component burned-in under accelerated environment during $(0, b]$ is the same as it were operated in the usual environment during $(0, a(b)]$. Hence the function $a(b)$ represents *the accelerated ageing process* induced by the accelerated burn-in procedure with burn-in time b.

Now, combining the accelerated burn-in phase and the field use phase, the failure rate function of component with accelerated burn-in time b, which is denoted by $\lambda_b(t)$, can be expressed as

$$\lambda_b(t) = \begin{cases} \rho'(t)r(\rho(t)), & \text{if } 0 \leq t \leq b, \quad \text{(Burn-in Phase)} \\ r(a(b)+(t-b)), & \text{if } t \geq b, \quad \text{(Field Use Phase).} \end{cases} \tag{2.6}$$

Since the assumptions on the functions $\rho(t)$ and $a(b)$ are minimal ones, the failure rate model in (2.6) can be applied to a wide range of applications. Also note that for the burn-in procedure performed in the usual level of environment (in this case, $\rho(t) = t$, $\forall t \geq 0$ and $a(b) = b$, $\forall b \geq 0$), the relationship, $\lambda_b(t) = r(t)$, $\forall t \geq 0$, holds. Therefore, the accelerated burn-in model proposed in Cha (2006b) is a generalization of the burn-in model in the usual level of environment. Recently Cha and Na (2009) considered the problem of determining the optimal accelerated burn-in time and replacement policy under the accelerated burn-in model proposed in Cha (2006b).

3. Topics for Developments

In this section, some issues or topics that can be studied in the future research are briefly introduced.

(1) Optimal burn-in based on stochastically ordered subpopulations

Due to high initial failure rate which appears in the early stages of component life, burn-in has been considered as an essential procedure. In Jensen and Petersen (1982), based on various sets of field data, it is observed that the population of produced items is composed of two subpopulations - the strong subpopulation which contains items with normal length of lifetimes and the weak subpopulation which includes items with shorter length of lifetimes. In practice, weak items may be produced along with strong items due to, for example, defective resources and components, error of workers, unstable production environment caused by uncontrolled significant quality factors, and so on. Mixture of these two subpopulations results in a bimodal distribution as illustrated in Jensen and Petersen (1982). According to Jensen and Petersen (1982), the infant mortality period of the life cycle, which exhibits high failure rate, results from failures in a weak subpopulation of a bimodal component lifetime distribution. This can be well understood if we observe the fact that weak items tend to fail earlier than strong items when items are used without burn-in. Thus, in view of this context, it can be said that one of the main purposes of burn-in procedure is to eliminate the weak subpopulation from mixed population.

Following the observations and ideas given in Jensen and Petersen (1982), it can be assumed that the population is a mixture of two ordered subpopulations - the strong subpopulation and the weak subpopulation. Let the lifetime of a component from the strong subpopulation be denoted by X_S and its absolutely continuous distribution function be $F_S(t)$. Similarly, the lifetime of a weak component is denoted by X_W and let the corresponding distribution function be $F_W(t)$. Then, for modelling the relationship between the distribution of strong component and that of weak component, it is reasonable to assume that these lifetimes are ordered as:

$$X_W \leq_{st} X_S, \tag{3.1}$$

which means that (reference, *e.g.*, Ross (1996))

$$F_S(t) \leq F_W(t), \quad t \geq 0.$$

These equations define a general stochastic ordering between two random variables. Another important ordering in reliability applications is the failure rate ordering, which is defined as

$$r_S(t) \leq r_W(t), \quad t \geq 0, \tag{3.2}$$

where $r_S(t)$ and $r_W(t)$ are failure rate functions of X_S and X_W, respectively. The relationship defined in (3.2) is denoted by $X_W \leq_{fr} X_S$ and called 'failure rate ordering' between two random variables. Note that the condition for the relationship $X_W \leq_{fr} X_S$ is stronger than that for $X_W \leq_{st} X_S$, that is, $X_W \leq_{fr} X_S$ implies $X_W \leq_{st} X_S$.

Then the the distribution of total population is given by

$$G(t) = pF_S(t) + (1 - p)F_W(t).$$

In this case, some parametric models for (3.1) and (3.2) could be developed and the relationship between this mixture population and the shape of failure rate function could be investigated. Then optimal burn-in can be studied based on this mixture population model.

In Tseng *et al.* (2003), it is assumed that the product failure corresponds to the first passage time of the degradation path beyond a critical value and a Wiener process is used to describe the continuous degradation path of the quality characteristic of the product. Assuming different degradation patterns for weak and strong populations, the problem of determining the optimal burn-in time has been considered. The approach adopted in Tseng *et al.* (2003) could be understood as a specific parametric model for (3.1) or (3.2).

(2) Some other topics for developments

- **Adopting shock as a burn-in operation:** In Finkelstein and Esaulova (2006), the non-asymptotic and asymptotic properties of mixture failure rates in heterogeneous populations are studied. It is pointed out that, in a specific setting, a shock performs a kind of burn-in operation. Thus a burn-in procedure which incorporates shock operation can be developed under the model proposed by Finkelstein and Esaulova (2006).

- **Optimal burn-in in multi-dimensional optimization problems:** Since the failure rate function of a system used in field operation depends on the burn-in procedure it experiences, it is thus natural to take burn-in and other operating characteristics (*e.g.* maintenance policy) into consideration all together at the same time. Recently, there has been some research which considers multi-dimensional optimization problem with burn-in time as one design variable. For example, Cha and Mi (2008) considered three-dimensional optimization problem, where optimal burn-in time, optimal work size and optimal replacement policy are determined for a data processing system. Similar topics can be developed for different optimization models in reliability area.

4. Concluding Remarks

In most cases, produced product generally have high initial failure rate and thus burn-in is very important research area of reliability. In this paper, recent advances and developments in burn-in models have been surveyed. Furthermore, some issues and research topics which can be developed through future study have been suggested. There have also been a few applications of ideas or techniques in burn-in area for finding the time at which to stop testing the software. The issues and ideas suggested in this paper could also be applied to the problem of determining optimal software testing procedure.

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