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A General Procedure for Estimating the General Parameter Using Auxiliary Information in Presence of Measurement Errors

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Abstract

This article addresses the problem of estimating a family of general population parameter $\theta_{(\alpha,\beta)}$ using auxiliary information in the presence of measurement errors. The general results are then applied to estimate the coefficient of variation C_Y of the study variable Y using the knowledge of the error variance σ_U^2 associated with the study variable Y. Based on large sample approximation, the optimal conditions are obtained and the situations are identified under which the proposed class of estimators would be better than conventional estimator. Application of the main result to bivariate normal population is illustrated.

Keywords: Study variate, auxiliary variate, measurement errors, coefficient of variation(CV), bias and mean square error.

1. Introduction

Statistical procedures for the analysis of the data generally presuppose that the observations are the correct measurements on the characteristics being studied. This is indeed far from reality in many practical situations and error ridden observations are a rule rather than an exception. For example in the standard manometer used for measuring blood pressure by a sleeve wrapped round the arm instead of a direct measurement of intra-arterial blood pressure, see Cochran (1968, p.638). For more details and illustrations, the reader is refereed to Fuller (1987), Shalabh (1997) and Sud and Srivastava (2000). When the measurement errors are negligibly small, the statistical inferences based on observed data continue to remain valid. On the contrary, when they are not appreciably small and are not negligible, the inference may not be simply invalid and inaccurate but may often lead to unexpected, undesirable and unfortunate consequences, see Shalabh (2000).

It is well known that the use of auxiliary information is a common phenomenon in sampling theory of surveys. This information is used at planning stage of a surveys thereby leading to a better choice of sample design or it is used at the estimation stage thereby leading to a better choice of estimator. Out of many ratio, product and regression methods of estimation are good examples in this context.

Coefficient of variation holds an important place in sampling theory, biological studies, biometric and agricultural experiments and economic studies for measuring the fluctuations, stability and inequality. When population mean μ_X of the auxiliary character is known a large number of modified ratio and product estimators for estimating the population mean μ_Y of the study variable Y have been proposed and studied by various authors, see Singh (1986). However, for improving the performance of survey estimates Searles (1964) used the known coefficient of variation(CV) C_Y of the study variate Y.

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Further Das and Tripathi (1981) pointed out that in many practical situations; information on auxiliary variable is available for all units in the population. Thus various parameters such as population mean μ_X, coefficient of variation C_X, variance σ_X^2, moment ratio $\beta_2(X)$ (coefficient of kurtosis) and $\beta_1(X)$ (coefficient of skewness) for auxiliary variable X can be known easily, see Srivastava and Jhajj (1980, 1981) and Upadhyaya and Singh (1999).

It is desired to estimate the general population parameter $\theta_{(\alpha,\beta)}$:

$$\theta_{(\alpha,\beta)} = \mu_Y^{\alpha}\sigma_Y^{2\beta}, \tag{1.1}$$

where

$$(\alpha,\beta) = \left(-1,\frac{1}{2}\right),\ (0,1),\ \left(0,\frac{1}{2}\right),\ (1,0).$$

We mention that:

(i) $\theta_{(-1,1/2)} = C_Y$ (population coefficient of variation), for $(\alpha,\beta) = (-1, 1/2)$,

(ii) $\theta_{(0,1)} = \sigma_Y^2$ (population variance), for $(\alpha,\beta) = (0, 1)$,

(iii) $\theta_{(0,1/2)} = \sigma_Y$ (population standard deviation), for $(\alpha,\beta) = (0, 1/2)$,

(iv) $\theta_{(1,0)} = \mu_Y$ (population mean), for $(\alpha,\beta) = (1, 0)$.

The problems of estimating the population mean(μ_Y) and variance(σ_Y^2) have been dealt by various authors such as Shalabh (1997), Manisha and Singh (2001), Maneesha and Singh (2002), Srivastava and Shalabh (2001), Allen *et al.* (2003) and Singh and Karpe (2008a, 2008b, 2009) in the presence of measurement errors. We further note that the problem of estimating the population standard deviation σ_Y using auxiliary information has been considered by Upadhyaya and Singh (2001) in simple random sampling when the data are recorded without error. In this paper we have made an effort to the estimation of general population parameter $\theta_{(\alpha,\beta)}$ using auxiliary information when the observations are subject to errors. In particular we have discussed the problem of estimating the coefficient of variation using auxiliary information when the observations are subject to errors.

2. Notations and the Conventional Estimator

Consider a finite population $U = \{U_1, U_2 \ldots, U_N\}$ of N units. Let (Y, X) be the study variate and the auxiliary variate respectively. Suppose that we are given a set of n paired observations obtained through simple random sampling procedure on two characteristics X and Y. It is assumed that x_i and y_i for the i^{th} sampling unit are recorded instead of true values X_i and Y_i. The observational or measurement errors are defined as

$$u_i = (y_i - Y_i), \tag{2.1}$$

$$v_i = (x_i - X_i) \tag{2.2}$$

which are assumed to be stochastic with mean zero but possibly different variances σ_U^2 and σ_V^2.

For the sake of simplicity in exposition, we assume that $u_i's$ and $v_i's$ are uncorrelated although $X_i's$ and $Y_i's$ are correlated. We further assume that the measurement errors are independent of the true of variables. Such a specification can be, however, relaxed at the cost of some algebraic complexity. We also assume that finite population correction can be ignored.

Let the population means of (X, Y) characteristics be (μ_X, μ_Y) and population variances be (σ_X^2, σ_Y^2) respectively. Further, let ρ be the population correlation coefficient between X and Y. Let $\bar{x} = 1/n \sum_{i=1}^{n} x_i$, $\bar{y} = 1/n \sum_{i=1}^{n} y_i$ be the unbiased estimators of population means μ_X and μ_Y respectively. We note that $s_x^2 = 1/(n-1)\sum_{i=1}^{n}(x_i - \bar{x})^2$ and $s_y^2 = 1/(n-1)\sum_{i=1}^{n}(y_i - \bar{y})^2$ are not unbiased estimators of the population variances σ_X^2 and σ_Y^2. In presence of measurement errors the expected value of s_y^2 is given by

$$E\left(s_y^2\right) = \sigma_Y^2 + \sigma_U^2.$$

When the error variance σ_U^2 associated with study variable Y is known, the conventional estimator of $\theta_{(\alpha,\beta)}$ in presence of measurement errors is defined by

$$\hat{\theta}_{(\alpha,\beta)} = \bar{y}^{\alpha}\hat{\sigma}_y^{2\beta}, \tag{2.3}$$

where $\hat{\sigma}_y^2 = s_y^2 - \sigma_U^2 > 0$ is an unbiased estimators of σ_Y^2 and (α,β) are same as defined in Section 1. It is interesting to observe that

(i) for $(\alpha,\beta) = (-1, 1/2)$, $\hat{\theta}_{(\alpha,\beta)} \to \hat{\theta}_{(-1,1/2)} = \hat{C}_Y = (\hat{\sigma}_y/\bar{y})$ is the conventional estimator of the population coefficient of variation C_Y;

(ii) for $(\alpha,\beta) = (0, 1)$, $\hat{\theta}_{(\alpha,\beta)} \to \hat{\theta}_{(0,1)} = \hat{\sigma}_y^2$, an unbiased estimator of the population variance σ_Y^2;

(iii) for $(\alpha,\beta) = (0, 1/2)$, $\hat{\theta}_{(\alpha,\beta)} \to \hat{\theta}_{(0,1/2)} = \hat{\sigma}_y$, the estimator of the population standard deviation σ_Y;

(iv) for $(\alpha,\beta) = (1, 0)$, $\hat{\theta}_{(\alpha,\beta)} \to \hat{\theta}_{(1,0)} = \bar{y}$, the usual unbiased estimator of the population mean μ_Y.

Further we define

$$\hat{\sigma}_y^2 = \hat{\sigma}_Y^2\left(1 + \delta_{\hat{\sigma}_y^2}\right), \quad \bar{x} = \mu_X\left(1 + \delta_{\bar{x}}\right) \quad \text{and} \quad \bar{y} = \mu_Y\left(1 + \delta_{\bar{y}}\right)$$

such that

$$E\left(\delta_{\hat{\sigma}_y^2}\right) = E\left(\delta_{\bar{x}}\right) = 0, \quad E\left(\delta_{\bar{x}}^2\right) = \frac{C_X^2}{n}\left(1 + \frac{\sigma_V^2}{\sigma_X^2}\right) = \frac{C_X^2}{n\theta_X}, \quad E\left(\delta_{\bar{y}}^2\right) = \frac{C_Y^2}{n}\left(1 + \frac{\sigma_U^2}{\sigma_X^2}\right) = \frac{C_Y^2}{n\theta_Y}$$

and to the first degree of approximation (when finite population correction factor is ignored)

$$E\left(\delta_{\hat{\sigma}_y^2}\right) = \frac{A_Y}{n}, \qquad E\left(\delta_{\bar{x}}\delta_{\bar{y}}\right) = \frac{KC_X^2}{n}, \qquad E\left(\delta_{\hat{\sigma}_y^2}\delta_{\bar{x}}\right) = \frac{\lambda C_X}{n}, \qquad E\left(\delta_{\hat{\sigma}_y^2}\delta_{\bar{y}}\right) = \frac{\gamma_{1Y}^{*}C_Y}{n},$$

where

$$A_Y = \left\{\gamma_{2Y} + \gamma_{2U}\frac{\sigma_U^4}{\sigma_Y^4} + 2\left(1 + \frac{\sigma_U^2}{\sigma_Y^2}\right)^2\right\}, \qquad \lambda = \frac{\mu_{12}(X,Y)}{\sigma_X\sigma_Y^2}, \qquad C_X = \frac{\sigma_X}{\mu_X}, \qquad \theta_X = \frac{\sigma_X^2}{\sigma_X^2 + \sigma_V^2},$$

$$\theta_Y = \frac{\sigma_Y^2}{\sigma_Y^2 + \sigma_U^2}, \qquad \gamma_{1Y}^{*} = \left\{\gamma_{1Y} + \gamma_{1U}\left(\frac{\sigma_U^3}{\sigma_Y^3}\right)\right\}, \qquad \gamma_{1Y} = \sqrt{\beta_1(y)}, \qquad \beta_1(y) = \frac{\mu_3^2(y)}{\mu_2^3(y)},$$

$$\gamma_{2Y} = \beta_2(y) - 3, \qquad \gamma_{2U} = \beta_2(u) - 3, \qquad \gamma_{1Y} = \sqrt{\beta_1(y)}, \qquad \gamma_{1U} = \sqrt{\beta_1(u)} = \frac{\mu_3(u)}{\sigma_U^2},$$

$$\beta_2(u) = \frac{\mu_4(u)}{\mu_2^2(u)}, \qquad \beta_2(y) = \frac{\mu_4(y)}{\mu_2^2(y)}, \qquad \sigma_Y^2 = \mu_2(y) = E(Y_i - \mu_Y)^2, \qquad K = \rho\frac{C_Y}{C_X},$$

$$\sigma_X^2 = \mu_2(x) = E(X_i - \mu_X)^2, \qquad \mu_Y(y) = E(Y_i - \mu_Y)^r, \quad r = 2,3,4, \qquad \mu_4(y) = E(Y_i - \mu_Y)^4,$$

$$\sigma_U^2 = \mu_2(u) = E\left(u_i^2\right), \qquad \mu_4(u) = E\left(u_i^4\right), \qquad \mu_{12}(X,Y) = E\left\{(X_i-\mu_X)(Y_i-\mu_Y)^2\right\}.$$

Note that θ_X and θ_Y are the reliability ratios of X and Y, respectively lying between 0 and 1.

The bias and mean squared error(MSE) of $\hat{\theta}_{(\alpha,\beta)}$ to the first degree of approximation are respectively given by

$$B\left(\hat{\theta}_{(\alpha,\beta)}\right) = \frac{\theta_{(\alpha,\beta)}}{n}\left[\frac{\alpha(\alpha-1)}{2}\frac{C_Y^2}{\theta_Y} + \alpha\beta\gamma_{1Y}^{*}C_Y + \frac{\beta(\beta-1)}{2}A_Y\right], \tag{2.4}$$

$$\text{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right) = \frac{\theta_{(\alpha,\beta)}^2}{n}\left[\alpha^2\frac{C_Y^2}{\theta_Y} + 2\alpha\beta\gamma_{1Y}^{*}C_Y + \beta^2 A_Y\right]. \tag{2.5}$$

See Appendix: A.

3. The Suggested Family of Estimators

Suppose the error variance σ_U^2 associated with study variable is known a priori, see Birch (1964), Schneeweiβ (1976), Srivastava and Shalabh (1997) and Cheng and Van Ness (1991, 1994) *etc.* We can utilize this information in suggesting a family of estimators for estimating the general parameter $\hat{\theta}_{(\alpha,\beta)}$.

Let $a = (\bar{x}/\mu_X)$ and $h(a)$ be a function of 'a' such that $h(1) = 1$ and it satisfies the following conditions:

1. Whatever be the sample chosen, a assumes values in a bounded, closed convex subset, D, of the one-dimensional real space containing the point 'unity'.
2. In D, the function $h(a)$ is continuous and bounded.
3. The first, second and third partial derivatives of $h(a)$ exist and are continuous and bounded in D.

We define a family of estimators for the general parameter $\hat{\theta}_{(\alpha,\beta)}$ as

$$\hat{T}_{(\alpha,\beta)} = \hat{\theta}_{(\alpha,\beta)}h(a). \tag{3.1}$$

Under the conditions (1) and (2) the bias and the mean squared error(MSE) of the estimator $\hat{T}_{(\alpha,\beta)}$ exist, since there are only a finite number of possible samples. The relevant references in this context are Srivastava (1971), Shalabh (1997), Manisha and Singh (2001) and Singh and Karpe (2008a, 2008b, 2009).

The bias and mean squared error(MSE) of the proposed family of estimators $\hat{T}_{(\alpha,\beta)}$ to the first degree of approximation are respectively given by

$$B\left(\hat{T}_{(\alpha,\beta)}\right) = B\left(\hat{\theta}_{(\alpha,\beta)}\right) + \frac{\theta_{(\alpha,\beta)}}{n}\left[(\alpha K C_X + \beta\lambda)C_X h_1(1) + \frac{C_X^2}{\theta_X}h_{11}(1)\right], \tag{3.2}$$

where $B(\hat{\theta}_{(\alpha,\beta)})$ is given by (2.4).

$$\text{MSE}\left(\hat{T}_{(\alpha,\beta)}\right) = \text{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right) + \frac{\theta_{(\alpha,\beta)}^2}{n}\left[2(\alpha K C_X + \beta\lambda)C_X h_1(1) + \frac{C_X^2}{\theta_X}h_1^2(1)\right], \tag{3.3}$$

where MSE($\hat{\theta}_{(\alpha,\beta)}$) is given by (2.5). See Appendix: B.

Minimization of (3.3) with respect to $h_1(1)$ leads to:

$$h_1(1) = -\frac{(\alpha K C_X + \beta\lambda)\theta_X}{C_X}. \tag{3.4}$$

Substitution of (3.4) in (3.3) yields the minimum MSE of $\hat{T}_{(\alpha,\beta)}$ as

$$\min \mathrm{MSE}\left(\hat{T}_{(\alpha,\beta)}\right) = \mathrm{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right) - \frac{\theta^2_{(\alpha,\beta)}}{n}(\alpha K C_X + \beta\lambda)^2\theta_X. \tag{3.5}$$

Thus we established the following theorem.

Theorem 1. *To the first degree of approximation,*

$$MSE\left(\hat{T}_{(\alpha,\beta)}\right) = MSE\left(\hat{\theta}_{(\alpha,\beta)}\right) - \frac{\theta^2_{(\alpha,\beta)}}{n}(\alpha K C_X + \beta\lambda)^2\theta_X$$

with equality holding if

$$h_1(1) = -\frac{(\alpha K C_X + \beta\lambda)\theta_X}{C_X}.$$

From (3.5) it is clear that the proposed class of estimators $\hat{T}_{(\alpha,\beta)}$ is more efficient than conventional estimator $\hat{\theta}_{(\alpha,\beta)}$.

$$\hat{T}^{(1)}_{(\alpha,\beta)} = \hat{\theta}_{(\alpha,\beta)}\left(\frac{\bar{x}}{\mu_X}\right)(\text{product}-\text{type}), \qquad \hat{T}^{(2)}_{(\alpha,\beta)} = \hat{\theta}_{(\alpha,\beta)}\left(\frac{\mu_X}{\bar{x}}\right)(\text{ratio}-\text{type}),$$

$$\hat{T}^{(3)}_{(\alpha,\beta)} = \hat{\theta}_{(\alpha,\beta)}\left(\frac{\bar{x}}{\mu_X}\right)^{\gamma}, \qquad \hat{T}^{(4)}_{(\alpha,\beta)} = \hat{\theta}_{(\alpha,\beta)}\left[1 + \gamma(a-1)\right]^{-1},$$

$$\hat{T}^{(5)}_{(\alpha,\beta)} = \hat{\theta}_{(\alpha,\beta)}\left[1 + \gamma(a-1)\right], \qquad \hat{T}^{(6)}_{(\alpha,\beta)} = \left[\gamma\hat{\theta}_{(\alpha,\beta)} + (1-\gamma)\hat{\theta}_{(\alpha,\beta)}\left(\frac{\bar{x}}{\mu_X}\right)\right],$$

$$\hat{T}^{(7)}_{(\alpha,\beta)} = \left[\gamma\hat{\theta}_{(\alpha,\beta)} + (1-\gamma)\hat{\theta}_{(\alpha,\beta)}\left(\frac{\mu_X}{\bar{x}}\right)\right], \qquad \hat{T}^{(8)}_{(\alpha,\beta)} = \hat{\theta}_{(\alpha,\beta)}\exp[\gamma(a-1)],$$

of the parameter $\theta_{(\alpha,\beta)}$ are the members of the proposed family of estimators $\hat{T}_{(\alpha,\beta)}$ at (3.1) and where γ is a suitably chosen constant. The biases and mean squared errors of the estimators $\hat{T}^{(j)}_{(\alpha,\beta)}$, $j = 1$ to 8 can be obtained easily just by putting the suitable value of the derivatives $h_1(1)$ and $h_{11}(1)$.

Putting $h_1(1) = 1$ and $h_1(1) = -1$ in (3.3) we get the mean squared errors of the estimators $\hat{T}^{(1)}_{(\alpha,\beta)}$ and $\hat{T}^{(2)}_{(\alpha,\beta)}$ respectively as

$$\mathrm{MSE}\left(\hat{T}^{(1)}_{(\alpha,\beta)}\right) = \mathrm{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right) + \frac{\theta^2_{(\alpha,\beta)}}{n}\left[\frac{C_X^2}{\theta_X} + 2(\alpha K C_X + \beta\lambda)C_X\right], \tag{3.6}$$

$$\mathrm{MSE}\left(\hat{T}^{(2)}_{(\alpha,\beta)}\right) = \mathrm{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right) + \frac{\theta^2_{(\alpha,\beta)}}{n}\left[\frac{C_X^2}{\theta_X} - 2(\alpha K C_X + \beta\lambda)C_X\right]. \tag{3.7}$$

It follows from (2.5), (3.6) and (3.7) that:

(i) the product-type estimator $\hat{T}^{(1)}_{(\alpha,\beta)}$ is better than $\hat{\theta}_{(\alpha,\beta)}$ if

$$(\alpha K C_X + \beta\lambda) < -\frac{C_X}{2\theta_X}$$

i.e. if

$$(\alpha K C_X + \beta\lambda)\frac{\theta_X}{C_X} < -\frac{1}{2} \tag{3.8}$$

(ii) the ratio-type estimator $\hat{T}^{(2)}_{(\alpha,\beta)}$ is better than $\hat{\theta}_{(\alpha,\beta)}$ if

$$(\alpha K C_X + \beta\lambda) > \frac{C_X}{2\theta_X}$$

i.e. if

$$(\alpha K C_X + \beta\lambda)\frac{\theta_X}{C_X} > \frac{1}{2}. \tag{3.9}$$

From (2.5), (3.3), (3.6) and (3.7) we note that the proposed family of estimators $\hat{T}_{(\alpha,\beta)}$ is better than:

(i) the conventional estimator $\hat{\theta}_{(\alpha,\beta)}$ if

$$\min\left\{0, \frac{-2(\alpha K C_X + \beta\lambda)\theta_X}{C_X}\right\} < h_1(1) < \max\left\{0, \frac{-2(\alpha K C_X + \beta\lambda)\theta_X}{C_X}\right\} \tag{3.10}$$

(ii) the product-type estimator $\hat{T}^{(1)}_{(\alpha,\beta)}$ if

$$\min\left\{1, -\frac{\theta_X}{C_X}\left(2A + \frac{C_X}{\theta_X}\right)\right\} < h_1(1) < \max\left\{1, -\frac{\theta_X}{C_X}\left(2A + \frac{C_X}{\theta_X}\right)\right\} \tag{3.11}$$

(iii) the ratio-type estimator $\hat{T}^{(2)}_{(\alpha,\beta)}$ if

$$\min\left\{-1, \frac{\theta_X}{C_X}\left(2A + \frac{C_X}{\theta_X}\right)\right\} < h_1(1) < \max\left\{-1, \frac{\theta_X}{C_X}\left(2A + \frac{C_X}{\theta_X}\right)\right\}. \tag{3.12}$$

Following Srivastava (1980), it is easily demonstrated that if we consider a wide class of estimators

$$\hat{T}_H = H\left(\hat{\theta}_{(\alpha,\beta)}, a\right) \tag{3.13}$$

of the parameter $\theta_{(\alpha,\beta)}$, where function $H(\hat{\theta}_{(\alpha,\beta)}, a)$ satisfies $H(\theta_{(\alpha,\beta)}, 1) = \theta_{(\alpha,\beta)}$.
To the first degree of approximation, the MSE of the class of estimators $\hat{T}_H$ is given by

$$\begin{aligned}\mathrm{MSE}\left(\hat{T}_H\right) = \frac{\theta^2_{(\alpha,\beta)}}{n}\bigg[&\alpha^2\frac{C_Y^2}{\theta_Y} + 2\alpha\beta\gamma^*_{1Y}C_Y + \beta^2 A_Y + 2\theta_{(\alpha,\beta)}(\alpha K C_X + \beta\lambda)C_X H_2(\theta_{(\alpha,\beta)}, 1)\\ &+ \frac{C_X^2}{\theta_X}H_2^2(\theta_{(\alpha,\beta)}, 1)\bigg]\end{aligned} \tag{3.14}$$

which is minimized for

$$H_2(\theta_{(\alpha,\beta)}, 1) = \frac{(\alpha K C_X + \beta\lambda)\theta_X \theta_{(\alpha,\beta)}}{C_X}, \tag{3.15}$$

where $H_2(\theta_{(\alpha,\beta)}, 1) = \partial H(\theta_{(\alpha,\beta)}, a)/\partial a|_{(\hat{\theta}_{(\alpha,\beta)},1)}$ is the first-order partial derivative of the function $H(\hat{\theta}_{(\alpha,\beta)}, a)$ about the point $(\hat{\theta}_{(\alpha,\beta)}, a) = (\theta_{(\alpha,\beta)}, a)$.
Substituting (3.15) in (3.14) we get the minimum MSE of the $\hat{T}_H$ as

$$\min \text{MSE}\left(\hat{T}_H\right) = \text{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right) - \frac{\theta^2_{(\alpha,\beta)}}{n}(\alpha K C_X + \beta\lambda)^2 \theta_X, \tag{3.16}$$

which is same as given (3.5). Thus the resulting minimum MSE of $\hat{T}_H$ is equal to the minimum MSE of $\hat{T}_{(\alpha,\beta)}$ given at (3.5) and is not reduced.
It is to be noted that the difference type estimator

$$\hat{T}_d = \hat{\theta}_{(\alpha,\beta)} + d(a-1) \tag{3.17}$$

is a member of the class $\hat{T}_H$ given by (3.13) but not of the class $\hat{T}_{(\alpha,\beta)}$ given by (3.1), where d is a suitably chosen constant.

Remark 1. Suppose that the observations for both the variables X and Y are recorded without error. The MSE of the suggested class of estimator $\hat{T}_{(\alpha,\beta)}$, to the first degree of approximation, is given by

$$\begin{aligned}\text{MSE}\left(\hat{T}_{(\alpha,\beta)}\right)_t &= \frac{\theta^2_{(\alpha,\beta)}}{n}\left[\alpha^2 C_Y^2 + 2\alpha\beta\gamma_{1Y}C_Y + \beta^2(\gamma_{2Y}+2) + 2(\alpha K C_X + \beta\lambda)C_X h_1(1) + C_X^2 h_1^2(1)\right] \\ &= \text{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right)_t + \frac{\theta^2_{(\alpha,\beta)}}{n}\left[2(\alpha K C_X + \beta\lambda)C_X h_1(1) + C_X^2 h_1^2(1)\right],\end{aligned} \tag{3.18}$$

where

$$\text{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right)_t = \frac{\theta^2_{(\alpha,\beta)}}{n}\left[\alpha^2 C_Y^2 + 2\alpha\beta\gamma_{1Y}C_Y + \beta^2(\gamma_{2Y}+2)\right] \tag{3.19}$$

is the mean squared error of the usual estimator $\hat{\theta}_{(\alpha,\beta)}$ to the first degree of approximation when the observations are recorded without error. The $\text{MSE}(\hat{T}_{(\alpha,\beta)})_t$ at (3.18) can be easily obtained from (3.3) by putting $\sigma_U^2 = \sigma_V^2 = 0$.

From (3.3) and (3.18) we have

$$\begin{aligned}\text{MSE}\left(\hat{T}_{(\alpha,\beta)}\right) - \text{MSE}\left(\hat{T}_{(\alpha,\beta)}\right)_t &= \frac{\theta^2_{(\alpha,\beta)}}{n}\left[\alpha^2 C_Y^2\left(\frac{1-\theta_Y}{\theta_Y}\right) + 2\alpha\beta\gamma_{1U}C_Y\left(\frac{1-\theta_Y}{\theta_Y}\right)^{\frac{3}{2}}\right. \\ &\left. + \beta^2\left\{(\gamma_{2Y}+2)\left(\frac{1-\theta_Y}{\theta_Y}\right)^2 + 4\left(\frac{1-\theta_Y}{\theta_Y}\right)\right\} + C_X^2\left(\frac{1-\theta_X}{\theta_X}\right)h_1^2(1)\right],\end{aligned} \tag{3.20}$$

which is always non-negative. Thus, the suggested class of estimators $\hat{T}_{(\alpha,\beta)}$ has larger MSE in presence of measurement errors than in the error free case.

The $\text{MSE}(\hat{T}_{(\alpha,\beta)})_t$ at (3.18) is minimized for

$$h_1(1) = -\frac{(\alpha K C_X + \beta\lambda)}{C_X}. \tag{3.21}$$

Thus the resulting minimum MSE of $\hat{T}_{(\alpha,\beta)}$ is given by

$$\min \text{MSE}\left(\hat{T}_{(\alpha,\beta)}\right)_t = \frac{\theta^2_{(\alpha,\beta)}}{n}\left[\alpha^2 C_Y^2 + 2\alpha\beta\gamma_{1Y}C_Y + \beta^2(\gamma_{2Y}+2) - (\alpha K C_X + \beta\lambda)^2\right]. \tag{3.22}$$

From (3.5) and (3.22) we have

$$\begin{aligned}&\min \text{MSE}\left(\hat{T}_{(\alpha,\beta)}\right) - \min \text{MSE}\left(\hat{T}_{(\alpha,\beta)}\right)_t \\ &= \frac{\theta^2_{(\alpha,\beta)}}{n}\left[\alpha^2 C_Y^2\left(\frac{1-\theta_Y}{\theta_Y}\right) + 2\alpha\beta\gamma_{1U}C_Y\left(\frac{1-\theta_Y}{\theta_Y}\right)^{\frac{3}{2}}\right. \\ &\quad \left. + \beta^2\left\{(\gamma_{2Y}+2)\left(\frac{1-\theta_Y}{\theta_Y}\right)^2 + 4\left(\frac{1-\theta_Y}{\theta_Y}\right)\right\} + (\alpha K C_X + \beta\lambda)^2(1-\theta_X)\right],\end{aligned} \tag{3.23}$$

which is always non-negative. Expression (3.23) clearly indicates that the presence of measurement errors associated with the variables X and Y inflates the MSE of the suggested class of estimators $\hat{T}_{(\alpha,\beta)}$.

4. Estimation of Coefficient of Variation in Presence of Measurement Errors

The problem of estimating population mean μ_Y (or total $(T = N\mu_Y)$ has been considered extensively in the sample survey literature.

The problem of estimating the variance σ_Y^2 has been considered, among others by Singh *et al.* (1973), Liu (1974), Das and Tripathi (1977, 1978), Srivastava and Jhajj (1980), Searls and Interapanich (1990), Singh *et al.* (1988, 1990) and Singh and Karpe (2008b, 2009).

In many practical situations, where the variability and stability among Y values, *e.g.* dispersion per unit mean in the population, is of interest to study, the estimation of C_Y deserves special attention. The problem of estimating of population coefficient of variation C_Y using information on auxiliary variable X has been taken up by Das and Tripathi (1992–1993) and Tripathi *et al.* (2002) when the true observations are recorded.

Here we consider the problem of estimating the population coefficient of variation C_Y using auxiliary information in presence of measurement errors. The conventional estimator of C_Y when the error variance σ_U^2 associated with study variable Y is known in advance is defined by

$$\hat{\theta}_{(-1,\frac{1}{2})} = \hat{C}_y = \left(\frac{\hat{\sigma}_y}{\bar{y}}\right). \tag{4.1}$$

Putting $(\alpha = -1, \beta = 1/2)$ in (2.4) and (2.5) we get the bias and MSE of the conventional estimator $\hat{C}_y$ to the first degree of approximation respectively as

$$B\left(\hat{C}_y\right) = \frac{C_Y}{n}\left[\frac{C_Y^2}{n} - \frac{1}{2}\left\{\gamma_{1Y} + \gamma_{1U}\left(\frac{1-\theta_Y}{\theta_Y}\right)^{\frac{3}{2}}\right\}C_Y - \frac{A_Y}{8}\right] \tag{4.2}$$

and

$$\text{MSE}\left(\hat{C}_y\right) = \frac{C_Y^2}{n}\left[\frac{C_Y^2}{\theta_Y} - \left\{\gamma_{1Y} + \gamma_{1U}\left(\frac{1-\theta_Y}{\theta_Y}\right)^{\frac{3}{2}}\right\}C_Y + \frac{A_Y}{4}\right]. \tag{4.3}$$

Expression (4.2) of bias of $\hat{C}_y$ clearly indicates that the bias of $\hat{C}_y$ is negligible if the sample size n is sufficiently large. In case the population mean μ_X of the auxiliary variable X is known, motivated by Das and Tripathi (1992–1993) we define the following estimators for estimating C_Y in the presence of measurement errors as

$$d_1 = \hat{C}_y\left(\frac{\bar{x}}{\mu_X}\right), \qquad d_2 = \hat{C}_y\left(\frac{\mu_X}{\bar{x}}\right).$$

One can also define the following estimators for C_Y as

$$d_3 = \hat{C}_y\left(\frac{\bar{x}}{\mu_X}\right)^{\delta}, \qquad d_4 = \hat{C}_y[1+\delta(a-1)]^{-1}, \qquad d_5 = \hat{C}_y[1+\delta(a-1)],$$

$$d_6 = \hat{C}_y\exp\left(\frac{a-1}{a+1}\right), \qquad d_7 = \hat{C}_y\exp\left(\frac{1-a}{1+a}\right), \qquad d_8 = \hat{C}_y\exp\left[\frac{\delta(a-1)}{a+1}\right],$$

etc., where δ is a suitable chosen constant.
Putting $(\alpha = -1, \beta = 1/2)$ in (3.1) we get a class of estimators for C_Y as

$$\hat{T}_{(-1,\frac{1}{2})} = \hat{C}_y h(b) = \hat{T}(\text{say}). \tag{4.4}$$

Putting $(\alpha = -1, \beta = 1/2)$ in (3.2) and (3.3) we get the bias and MSE of $\hat{T}$ to the first degree of approximation respectively as

$$B\left(\hat{T}\right) = B\left(\hat{C}_y\right) + \frac{C_Y}{n}\left[\frac{C_X^2}{\theta_X}h_1^2(1) - \frac{(2KC_X - \lambda)C_X}{2}h_1(1)\right], \tag{4.5}$$

$$\text{MSE}\left(\hat{T}\right) = \text{MSE}\left(\hat{C}_y\right) + \frac{C_Y^2}{n}\left[\frac{C_X^2}{\theta_X}h_1^2(1) - (2KC_X - \lambda)C_X h_1(1)\right], \tag{4.6}$$

where $B(\hat{C}_y)$ and $\text{MSE}(\hat{C}_y)$ are respectively defined by (4.2) and (4.3).
The MSE of $\hat{T}$ given by (4.6) is minimized for

$$h_1(1) = \frac{(\lambda - 2KC_X)\theta_X}{2C_X}. \tag{4.7}$$

Thus the resulting minimum $\text{MSE}(\hat{T})$ is given by

$$\begin{aligned}\min\text{MSE}\left(\hat{T}\right) &= \frac{C_Y^2}{n}\left|\frac{C_Y^2}{\theta_Y} - \left\{\gamma_{1Y} + \gamma_{1U}\left(\frac{1-\theta_Y}{\theta_Y}\right)^{\frac{3}{2}}\right\}C_Y + \frac{A_Y}{4} - \frac{(\lambda - 2KC_X)^2\theta_X}{4}\right| \\ &= \text{MSE}\left(\hat{C}_y\right) - \frac{C_Y^2(\lambda - 2KC_X)^2\theta_X}{4n},\end{aligned} \tag{4.8}$$

which clearly indicates that the proposed estimator $\hat{T}$ has smaller MSE than the conventional estimator $\hat{C}_y$.

Thus we state the following theorem:

Theorem 2. *To the first degree of approximation,*

$$\min MSE\left(\hat{T}\right) = MSE\left(\hat{C}_y\right) - \frac{C_Y^2(\lambda - 2KC_X)^2\theta_X}{4n}$$

with equality holding if

$$h_1(1) = \frac{(\lambda - 2KC_X)\theta_X}{2C_X}.$$

It may be easily observed that the estimators d_j, $j = 1$ to 8 are the members of the suggested class of estimators $\hat{T}$ defined at (4.4). The biases and MSEs of the estimators d_j, $j = 1$ to 8 can be easily be obtained from (4.5) and (4.6) respectively just by putting suitable value of the derivatives $h_1(1)$ and $h_{11}(1)$.

To the first degree of approximation the MSEs of the estimators d_j, $j = 1$ to 8 are respectively given by

$$\text{MSE}(d_1) = \text{MSE}\left(\hat{C}_y\right) + \frac{C_Y^2}{n}\left[\frac{C_X^2}{\theta_X} - (2KC_X - \lambda)C_X\right], \tag{4.9}$$

$$\text{MSE}(d_2) = \text{MSE}\left(\hat{C}_y\right) + \frac{C_Y^2}{n}\left[\frac{C_X^2}{\theta_X} + (2KC_X - \lambda)C_X\right], \tag{4.10}$$

$$\text{MSE}(d_3) = \text{MSE}\left(\hat{C}_y\right) + \frac{C_Y^2}{n}\left[\frac{C_X^2}{\theta_X}\delta^2 - (2KC_X - \lambda)\delta C_X\right], \tag{4.11}$$

$$\text{MSE}(d_4) = \text{MSE}\left(\hat{C}_y\right) + \frac{C_Y^2}{n}\left[\frac{C_X^2}{\theta_X}\delta^2 + (2KC_X - \lambda)\delta C_X\right], \tag{4.12}$$

$$\text{MSE}(d_5) = \text{MSE}(d_3), \tag{4.13}$$

$$\text{MSE}(d_6) = \text{MSE}\left(\hat{C}_y\right) + \frac{C_Y^2}{2n}\left[\frac{C_X^2}{2\theta_X} - (2KC_X - \lambda)C_X\right], \tag{4.14}$$

$$\text{MSE}(d_7) = \text{MSE}\left(\hat{C}_y\right) + \frac{C_Y^2}{2n}\left[\frac{C_X^2}{2\theta_X} + (2KC_X - \lambda)C_X\right], \tag{4.15}$$

$$\text{MSE}(d_8) = \text{MSE}\left(\hat{C}_y\right) + \frac{C_Y^2}{2n}\left[\frac{\delta^2 C_X^2}{2\theta_X} - (2KC_X - \lambda)\delta C_X\right]. \tag{4.16}$$

The mean squared errors of d_3, d_4, d_5 and d_8 are respectively minimized for

$$\delta = \frac{(2KC_X - \lambda)\theta_X}{2C_X}, \tag{4.17}$$

$$\delta = \frac{(\lambda - 2KC_X)\theta_X}{2C_X}, \tag{4.18}$$

$$\delta = \frac{(2KC_X - \lambda)\theta_X}{2C_X}, \tag{4.19}$$

$$\delta = \frac{(2KC_X - \lambda)\theta_X}{2C_X}. \tag{4.20}$$

The common minimum MSE of the estimators d_3, d_4, d_5 and d_8 is given by

$$\begin{aligned}\min \mathrm{MSE}(d_j) &= \mathrm{MSE}\left(\hat{C}_y\right) - \frac{C_Y^2(\lambda - 2KC_X)^2\theta_X}{4n} \\ &= \min \mathrm{MSE}\left(\hat{T}\right), \quad j = 3, 4, 5, 8 \end{aligned} \tag{4.21}$$

which is same as the minimum MSE of $\hat{T}$ given by (4.8). Thus the estimators belonging to the proposed class of estimators $\hat{T}$ will not have MSE/min. MSE smaller than that of $\hat{T}$.
Further from (4.8), (4.9),(4.10), (4.14) and (4.15) we have

$$\mathrm{MSE}(d_1) - \min \mathrm{MSE}\left(\hat{T}\right) = \frac{C_Y^2}{4n\theta_X}\left[2C_X - \theta_X(2KC_X - \lambda)\right]^2 \geq 0, \tag{4.22}$$

$$\mathrm{MSE}(d_2) - \min \mathrm{MSE}\left(\hat{T}\right) = \frac{C_Y^2}{4n\theta_X}\left[2C_X + \theta_X(2KC_X - \lambda)\right]^2 \geq 0, \tag{4.23}$$

$$\mathrm{MSE}(d_6) - \min \mathrm{MSE}\left(\hat{T}\right) = \frac{C_Y^2}{4n\theta_X}\left[C_X - \theta_X(2KC_X - \lambda)\right]^2 \geq 0, \tag{4.24}$$

$$\mathrm{MSE}(d_7) - \min \mathrm{MSE}\left(\hat{T}\right) = \frac{C_Y^2}{4n\theta_X}\left[C_X + \theta_X(2KC_X - \lambda)\right]^2 \geq 0. \tag{4.25}$$

It follows from (4.22), (4.23), (4.24) and (4.25) that the proposed class of estimators $\hat{T}$ or (d_j, $j = 3, 4, 5, 8$) is more efficient than d_1, d_2, d_6 and d_7 at its optimum conditions.

In practice it may happen that the value of the constants involved in the estimators may not coincide exactly with its optimum value. In such a situation one has to use some guessed values of the parameters and hence the optimum value. Thus we derive the regions of preference in which the proposed class of estimators $\hat{T}$ is better than conventional estimator $\hat{C}_y$, the product-type estimator d_1 and the ratio-type estimator d_2 and the estimators d_6 and d_7.
From (4.3), (4.6), (4.9), (4.10), (4.14) and (4.15) it follows that the proposed class of estimators $\hat{T}$ is better than:

(i) the conventional estimator $\hat{C}_y$ if

$$\min\left\{0, \frac{(2KC_X - \lambda)\theta_X}{C_X}\right\} < h_1(1) < \max\left\{0, \frac{(2KC_X - \lambda)\theta_X}{C_X}\right\} \tag{4.26}$$

(ii) the product-type estimator d_1 if

$$\min\left\{-1, \left(1 + \frac{(2KC_X - \lambda)\theta_X}{C_X}\right)\right\} < h_1(1) < \max\left\{-1, \left(1 + \frac{(2KC_X - \lambda)\theta_X}{C_X}\right)\right\} \tag{4.27}$$

(iii) the ratio-type estimator d_2 if

$$\min\left\{-1, \left(1 + \frac{(2KC_X - \lambda)\theta_X}{C_X}\right)\right\} < h_1(1) < \max\left\{-1, \left(1 + \frac{(2KC_X - \lambda)\theta_X}{C_X}\right)\right\} \tag{4.28}$$

(iv) the estimator d_6 if

$$\min\left\{\frac{1}{2}, \frac{1}{2}\left(\frac{(2KC_X - \lambda)\theta_X}{C_X} - 1\right)\right\} < h_1(1) < \max\left\{\frac{1}{2}, \frac{1}{2}\left(\frac{(2KC_X - \lambda)\theta_X}{C_X} - 1\right)\right\} \tag{4.29}$$

(v) the estimator d_7 if

$$\min\left\{-\frac{1}{2}, \frac{1}{2}\left(1 + \frac{(2KC_X - \lambda)\theta_X}{C_X}\right)\right\} < h_1(1) < \max\left\{-\frac{1}{2}, \frac{1}{2}\left(1 + \frac{(2KC_X - \lambda)\theta_X}{C_X}\right)\right\}. \quad (4.30)$$

Further from (4.3), (4.9), (4.10), (4.14) and (4.15) we note that

(i) the product-type estimator d_1 is better than the conventional estimator $\hat{C}_y$ if

$$\rho > \frac{1}{2C_Y}\left(\lambda + \frac{C_X}{\theta_X}\right) \quad (4.31)$$

(ii) the ratio-type estimator d_2 is more than the conventional estimator $\hat{C}_y$ if

$$\rho < \frac{1}{2C_Y}\left(\lambda - \frac{C_X}{\theta_X}\right) \quad (4.32)$$

(iii) the modified product-type estimator d_6 is better than the conventional estimator $\hat{C}_y$ and the product-type estimator d_1 respectively if the following inequalities:

$$\rho > \frac{1}{2C_Y}\left(\lambda + \frac{C_X}{2\theta_X}\right), \quad (4.33)$$

$$\rho < \frac{1}{2C_Y}\left(\lambda + \frac{3C_X}{2\theta_X}\right) \quad (4.34)$$

(iv) the modified ratio-type estimator d_7 is better than the conventional estimator $\hat{C}_y$ and the product-type estimator d_2 respectively if the following inequalities:

$$\rho > \frac{1}{2C_Y}\left(\lambda - \frac{C_X}{2\theta_X}\right), \quad (4.35)$$

$$\rho < \frac{1}{2C_Y}\left(\lambda - \frac{3C_X}{2\theta_X}\right). \quad (4.36)$$

Remark 2. Putting ($\alpha = -1, \beta = 1/2$) in (3.13) we get a class of estimators for C_Y wider than the class of estimators defined at (4.1) as

$$\hat{T}^* = H\left(\hat{C}_y, a\right), \quad (4.37)$$

where $H(\hat{C}_y, a)$ is the function of $\hat{C}_y$ and a such that $H(\hat{C}_y, 1) = C_Y$ and also satisfies certain regularity conditions as given in Srivastava (1971).

It can be easily shown to the first degree of approximation that

$$\min \text{MSE}\left(\hat{T}^*\right) = \min \text{MSE}\left(\hat{T}\right), \quad (4.38)$$

where min MSE($\hat{T}$) given by (4.8). It is to be noted that the following estimators:

$$\hat{T}_1^* = \hat{C}_y + d(a-1),$$
$$\hat{T}_2^* = \frac{\hat{\sigma}_y}{\bar{y} + d^*(a-1)}$$

are the members of the class $\hat{T}$ given by (4.37) but not of the class $\hat{T}$ given by (4.4), where d and d^* are suitably chosen constants.

Remark 3. Putting ($\alpha = -1, \beta = 1/2$) in (3.18) we get the mean squared errors of the class of estimators $\hat{T}$ when both the variables X and Y are recorded without error, as

$$\begin{aligned}\min \mathrm{MSE}\left(\hat{T}\right)_t &= \frac{C_Y^2}{n}\left[C_Y^2 - \gamma_{1Y}C_Y + \frac{1}{4}(\gamma_{2Y}+2) - (2KC_X - \lambda)C_X h_1(1) + C_X^2 h_1^2(1)\right] \\ &= \mathrm{MSE}\left(\hat{C}_y\right)_t + \frac{C_Y^2}{n}\left[C_X^2 h_1^2(1) - (2KC_X - \lambda)C_X h_1(1)\right],\end{aligned} \tag{4.39}$$

where

$$\mathrm{MSE}\left(\hat{C}_y\right)_t = \frac{C_Y^2}{n}\left[C_Y^2 - \gamma_{1Y}C_Y + \frac{1}{4}(\gamma_{2Y}+2)\right] \tag{4.40}$$

is the mean squared error of the conventional estimator $\hat{C}_y$ to the first degree of approximation when the observations are error free. The MSE($\hat{T}$)$_t$ at (4.39) can be easily obtained from (4.6) by putting $\sigma_U^2 = \sigma_V^2 = 0$.

Putting ($\alpha = -1, \beta = 1/2$) in (3.20) we get

$$\begin{aligned}\mathrm{MSE}\left(\hat{T}\right) - \mathrm{MSE}\left(\hat{T}\right)_t &= \frac{C_Y^2}{n}\left[C_Y^2\left(\frac{1-\theta_Y}{\theta_Y}\right) - \gamma_{1U}C_Y\left(\frac{1-\theta_Y}{\theta_Y}\right)^{\frac{3}{2}}\right. \\ &\left.+ \frac{1}{4}\left\{(\gamma_{2Y}+2)\left(\frac{1-\theta_Y}{\theta_Y}\right)^2 + 4\left(\frac{1-\theta_Y}{\theta_Y}\right)\right\} + C_X^2\left(\frac{1-\theta_X}{\theta_X}\right)h_1^2(1)\right],\end{aligned} \tag{4.41}$$

which is positive. Hence the proposed class of estimators $\hat{T}$ has larger MSE in the presence of measurement errors than in the error free case. Expression (4.41) clearly indicates that the errors associated with both the variables X and Y are accountable for increasing the MSE of $\hat{T}$.

Further putting ($\alpha = -1, \beta = 1/2$) in (3.23) we get

$$\begin{aligned}\min \mathrm{MSE}\left(\hat{T}\right) - \min \mathrm{MSE}\left(\hat{T}\right)_t &= \frac{C_Y^2}{n}\left[C_Y^2\left(\frac{1-\theta_Y}{\theta_Y}\right) - \gamma_{1U}C_Y\left(\frac{1-\theta_Y}{\theta_Y}\right)^{\frac{3}{2}}\right. \\ &\left.+ \frac{1}{4}\left\{(\gamma_{2Y}+2)\left(\frac{1-\theta_Y}{\theta_Y}\right)^2 + 4\left(\frac{1-\theta_Y}{\theta_Y}\right)\right\} + \frac{1}{4}(2KC_X - \lambda)^2(1-\theta_X)\right],\end{aligned} \tag{4.42}$$

which is positive. It follows from (4.42) that the presence of measurement errors associated with both the variables X and Y increases the MSE of the proposed class of estimators at the optimum condition.

5. Applications of Main Results in Bivariate Normal Population

The main results in Section 3 can easily be employed in bivariate normal populations with obvious modification. Suppose we want to estimate the parameter $\theta_{(\alpha,\beta)}$ when (Y, X) follows bivariate normal distribution $(\mu_X, \mu_Y, \sigma_Y^2, \sigma_X^2, \rho)$, $u_i \sim N(0, \sigma_U^2)$ and $v_i \sim N(0, \sigma_V^2)$. In such a case, $\gamma_{2Y} = \gamma_{2U} = 0$, $\gamma_{1Y} = \gamma_{1U} = 0$ and. Thus the expressions in (2.5), (3.3), (3.5), (3.6), (3.7), (3.18), (3.19), (3.20), (3.23), (4.3), (4.6), (4.7), (4.8), (4.9), (4.10), (4.14), (4.15), (4.39), (4.40), (4.41) and (4.42) respectively reduce to;

(i) $$\mathrm{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right) = \frac{\theta^2_{(\alpha,\beta)}}{n\theta_Y}\left(\alpha^2 C_Y^2 + \frac{2\beta^2}{\theta_Y}\right) \tag{5.1}$$

(ii) $$\mathrm{MSE}\left(\hat{T}_{(\alpha,\beta)}\right) = \mathrm{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right) + \frac{\theta^2_{(\alpha,\beta)} C_X^2}{n\theta_X}\{2\alpha K\theta_X + h_1(1)\} h_1(1) \tag{5.2}$$

(iii) $$\min \mathrm{MSE}\left(\hat{T}_{(\alpha,\beta)}\right) = \mathrm{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right) - \frac{\alpha^2\theta^2_{(\alpha,\beta)} K^2 C_X^2 \theta_X}{n} \tag{5.3}$$

(iv) $$\mathrm{MSE}\left(\hat{T}^{(1)}_{(\alpha,\beta)}\right) = \mathrm{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right) + \frac{\theta^2_{(\alpha,\beta)} C_X^2}{n\theta_X}(1 + 2\alpha K\theta_X) \tag{5.4}$$

(v) $$\mathrm{MSE}\left(\hat{T}^{(2)}_{(\alpha,\beta)}\right) = \mathrm{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right) + \frac{\theta^2_{(\alpha,\beta)} C_X^2}{n\theta_X}(1 - 2\alpha K\theta_X) \tag{5.5}$$

(vi) $$\mathrm{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right)_t = \frac{\theta^2_{(\alpha,\beta)}}{n}\left(\alpha^2 C_Y^2 + 2\beta^2\right) \tag{5.6}$$

(vii) $$\mathrm{MSE}\left(\hat{T}_{(\alpha,\beta)}\right)_t = \mathrm{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right)_t + \frac{\theta^2_{(\alpha,\beta)}}{n}\{2\alpha K + h_1(1)\} h_1(1) \tag{5.7}$$

(viii) $$\mathrm{MSE}\left(\hat{T}_{(\alpha,\beta)}\right) - \mathrm{MSE}\left(\hat{T}_{(\alpha,\beta)}\right)_t = \frac{\theta^2_{(\alpha,\beta)}}{n}\left[\left(\frac{1-\theta_Y}{\theta_Y}\right)\left\{\alpha^2 C_Y^2 + 2\beta^2\left(\frac{1-\theta_Y}{\theta_Y}\right)\right\} + C_X^2\left(\frac{1-\theta_X}{\theta_X}\right) h_1^2(1)\right] \geq 0 \tag{5.8}$$

(ix) $$\min \mathrm{MSE}\left(\hat{T}_{(\alpha,\beta)}\right) - \min \mathrm{MSE}\left(\hat{T}_{(\alpha,\beta)}\right)_t = \frac{\theta^2_{(\alpha,\beta)}}{n}\left[\left(\frac{1-\theta_Y}{\theta_Y}\right)\left\{\alpha^2 C_Y^2 + 2\beta^2\left(\frac{1+\theta_Y}{\theta_Y}\right)\right\} + \alpha^2 K^2 C_X^2 (1-\theta_X)\right] \geq 0 \tag{5.9}$$

(x) $$\mathrm{MSE}\left(\hat{C}_y\right) = \frac{C_Y^2}{n\theta_Y}\left(C_Y^2 + \frac{1}{2\theta_Y}\right) \tag{5.10}$$

(xi) $$\mathrm{MSE}\left(\hat{T}\right) = \mathrm{MSE}\left(\hat{C}_y\right) + \frac{C_Y^2 C_X^2}{n\theta_X}\{h_1(1) - 2K\} h_1(1) \tag{5.11}$$

(xii) $$h_1(1) = -K\theta_X \tag{5.12}$$

(xiii) $$\min \mathrm{MSE}\left(\hat{T}\right) = \mathrm{MSE}\left(\hat{C}_y\right) - \frac{C_Y^2 C_X^2 K^2}{n} = \mathrm{MSE}\left(\hat{C}_y\right) - \frac{C_Y^4\rho^2}{n} \tag{5.13}$$

(xiv) $$\mathrm{MSE}(d_1) = \mathrm{MSE}\left(\hat{C}_y\right) + \frac{C_Y^2 C_X^2}{n\theta_X}(1 - 2K\theta_X) \tag{5.14}$$

(xv) $\text{MSE}(d_2) = \text{MSE}\left(\hat{C}_y\right) + \frac{C_Y^2 C_X^2}{n\theta_X}(1 + 2K\theta_X)$ (5.15)

(xvi) $\text{MSE}(d_6) = \text{MSE}\left(\hat{C}_y\right) + \frac{C_Y^2 C_X^2}{4n\theta_X}(1 - 4K\theta_X)$ (5.16)

(xvii) $\text{MSE}(d_7) = \text{MSE}\left(\hat{C}_y\right) + \frac{C_Y^2 C_X^2}{4n\theta_X}(1 + 4K\theta_X)$ (5.17)

(xviii) $\text{MSE}\left(\hat{C}_y\right)_t = \frac{C_Y^2}{n}\left(C_Y^2 + \frac{1}{2}\right)$ (5.18)

(xix) $\text{MSE}\left(\hat{T}\right)_t = \text{MSE}\left(\hat{C}_y\right)_t + \frac{C_Y^2 C_X^2}{n}\{h_1(1) - 2K\}\, h_1^2(1)$ (5.19)

(xx) $\text{MSE}\left(\hat{T}\right) - \text{MSE}\left(\hat{T}\right)_t = \frac{C_Y^2}{n}\left[\left(\frac{1-\theta_Y}{\theta_Y}\right)\left\{C_Y^2 + \frac{1}{2}\left(\frac{1+\theta_Y}{\theta_Y}\right)\right\} + C_X^2\left(\frac{1-\theta_X}{\theta_X}\right)h_1^2(1)\right]$ (5.20)

(xxi) $\min \text{MSE}\left(\hat{T}\right) - \min \text{MSE}\left(\hat{T}\right)_t = \frac{C_Y^2}{n}\left[\left(\frac{1-\theta_Y}{\theta_Y}\right)\left\{C_Y^2 + \frac{1}{2}\left(\frac{1+\theta_Y}{\theta_Y}\right)\right\} + \rho^2 C_Y^2 (1-\theta_X)\right].$ (5.21)

From (5.1), (5.4) and (5.5) it follows:

(i) $\text{MSE}\left(\hat{T}^{(1)}_{(\alpha,\beta)}\right) < \text{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right)$, if $\rho < -\frac{C_X}{2\alpha C_Y \theta_X}$

(ii) $\text{MSE}\left(\hat{T}^{(2)}_{(\alpha,\beta)}\right) < \text{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right)$, if $\rho > \frac{C_X}{2\alpha C_Y \theta_X}$.

From (5.1), (5.2), (5.4) and (5.5) we note that

(i) $\text{MSE}\left(\hat{T}_{(\alpha,\beta)}\right) < \text{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right)$, if $\min(0, -2\alpha K\theta_X) < \max(0, -2\alpha K\theta_X)$

(ii) $\text{MSE}\left(\hat{T}_{(\alpha,\beta)}\right) < \text{MSE}\left(\hat{T}^{(1)}_{(\alpha,\beta)}\right)$, if $\min\{1, -(1 + 2\alpha K\theta_X)\} < h_1(1) < \max\{1, -(1 + 2\alpha K\theta_X)\}$

(iii) $\text{MSE}\left(\hat{T}_{(\alpha,\beta)}\right) < \text{MSE}\left(\hat{T}^{(2)}_{(\alpha,\beta)}\right)$, if $\min\{-1, (1 - 2\alpha K\theta_X)\} < h_1(1) < \max\{-1, (1 - 2\alpha K\theta_X)\}$.

It is also observed from (5.8) and (5.9) that

(i) the estimator $\hat{T}_{(\alpha,\beta)}$ has larger MSE in presence of measurement errors than error free case even in bivariate normal population, $u_i \sim N(0, \sigma_U^2)$ and $v_i \sim N(0, \sigma_V^2)$; and

(ii) (ii) the presence of measurement errors associated with both the variables X and Y inflates the MSE of the suggested class of estimators $\hat{T}_{(\alpha,\beta)}$ at the optimum conditions. Thus the presence of measurement errors disturbs the optimal properties of the suggested class of estimators $\hat{T}_{(\alpha,\beta)}$.

From (5.10), (5.14), (5.15), (5.16) and (5.17) it is observed that:

(i) $\text{MSE}(d_1) < \text{MSE}\left(\hat{C}_y\right)$, if $\rho > \frac{C_X}{2C_Y\theta_X}$

(ii) $\text{MSE}(d_2) < \text{MSE}\left(\hat{C}_y\right)$, if $\rho < -\frac{C_X}{2C_Y\theta_X}$

(iii) $\text{MSE}(d_6) < \text{MSE}\left(\hat{C}_y\right)$, if $\rho > \frac{C_X}{4C_Y\theta_X}$

(iv) $\mathrm{MSE}(d_7) < \mathrm{MSE}\left(\hat{C}_y\right)$, if $\rho < -\dfrac{C_X}{4C_Y\theta_X}$.

Further from (5.20) and (5.21) it is observed that the presence of measurement errors associated with two variables X and Y enhances the MSE/minimum MSE of the estimator $\hat{T}$ in bivariate normal populations, $u_i \sim N(0, \sigma_U^2)$ and $v_i \sim N(0, \sigma_V^2)$.

For simplicity we assume that $C_X = C_Y = C(\text{say})$, (X, Y) follows a bivariate normal populations with parameters $(\mu_X, \mu_Y, \sigma_Y^2, \sigma_X^2, \rho)$, $u_i \sim N(0, \sigma_U^2)$, $v_i \sim N(0, \sigma_V^2)$ and the ratio of measurement error variance and the true variance are same *i.e.*

$$\frac{\sigma_U^2}{\sigma_Y^2} = \frac{\sigma_V^2}{\sigma_X^2} = \phi(\text{say}).$$

The percent relative efficiencies of the different estimators of population coefficient of variation C_Y over usual estimators are defined by:

$$\mathrm{PRE}\left(d_1, \hat{C}_y\right) = \frac{(1+\phi)(2C^2+\phi+1)}{[(1+\phi)(2C^2+\phi+1)+2C^2(\phi-2\rho+1)]} \times 100, \tag{5.22}$$

$$\mathrm{PRE}\left(d_6, \hat{C}_y\right) = \frac{2(1+\phi)(2C^2+\phi+1)}{[2(1+\phi)(2C^2+\phi+1)+C^2(\phi-4\rho+1)]} \times 100, \tag{5.23}$$

$$\mathrm{PRE}\left(\hat{T}, \hat{C}_y\right) = \frac{(1+\phi)(2C^2+\phi+1)}{[(1+\phi)(2C^2+\phi+1)-2\rho^2C^2]} \times 100. \tag{5.24}$$

It follows from (5.22), (5.23) and (5.24) that for the case of:

(i) d_1, if $(\phi - 2\rho + 1) < 1$, $\mathrm{PRE}(d_1, \hat{C}_y)$ is bigger than 100

(ii) d_6, if $(\phi - 4\rho + 1) < 1$, $\mathrm{PRE}(d_6, \hat{C}_y)$ is bigger than 100 and

(iii) $\hat{T}$, $\mathrm{PRE}(\hat{T}, \hat{C}_y)$ is always bigger than 100.

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Appendix: A

To the first degree of approximation, the bias and mean squared error of $\hat{\theta}_{(\alpha,\beta)}$ are obtained as follows:

$$\begin{aligned}
\hat{\theta}_{(\alpha,\beta)} &= \bar{y}^{\alpha}\hat{\sigma}_y^{2\beta} \\
&= \mu_Y^{\alpha}\left(1+\delta_{\bar{y}}\right)^{\alpha}\left(1+\delta_{\hat{\sigma}_y^2}\right)^{\beta}\sigma_Y^{2\beta} \\
&= \mu_Y^{\alpha}\sigma_Y^{2\beta}\left(1+\delta_{\bar{y}}\right)^{\alpha}\left(1+\delta_{\hat{\sigma}_y^2}\right)^{\beta} \\
&= \theta_{(\alpha,\beta)}\left(1+\delta_{\bar{y}}\right)^{\alpha}\left(1+\delta_{\hat{\sigma}_y^2}\right)^{\beta} \\
&= \theta_{(\alpha,\beta)}\left[1+\alpha\delta_{\bar{y}}+\frac{\alpha(\alpha-1)}{2}\delta_{\bar{y}}^2+\cdots\right]\left[1+\beta\delta_{\hat{\sigma}_y^2}+\frac{\beta(\beta-1)}{2}\delta_{\hat{\sigma}_y^2}^2+\cdots\right] \\
&= \theta_{(\alpha,\beta)}\left[1+\alpha\delta_{\bar{y}}+\beta\delta_{\hat{\sigma}_y}+\frac{\alpha(\alpha-1)}{2}\delta_{\bar{y}}+\frac{\beta(\beta-1)}{2}\delta_{\hat{\sigma}_y}^2+\alpha\beta\delta_{\bar{y}}\delta_{\hat{\sigma}_y}+\cdots\right]
\end{aligned}$$

or

$$\left(\hat{\theta}_{(\alpha,\beta)} - \theta_{(\alpha,\beta)}\right) = \theta_{(\alpha,\beta)}\left[\alpha\delta_{\bar{y}} + \beta\delta_{\hat{\sigma}_y^2} + \alpha\beta\delta_{\bar{y}}\delta_{\hat{\sigma}_y^2} + \frac{\alpha(\alpha-1)}{2}\delta_{\bar{y}}^2 + \frac{\beta(\beta-1)}{2}\delta_{\hat{\sigma}_y}^2\right]. \tag{A.1}$$

Taking expectations of both sides in (A.1) we get the bias of $\hat{\theta}_{(\alpha,\beta)}$ to the first degree of approximation, as

$$B\left(\hat{\theta}_{(\alpha,\beta)}\right) = \frac{\theta_{(\alpha,\beta)}}{n}\left[\frac{\alpha(\alpha-1)}{2}\frac{C_Y^2}{\theta_Y} + \alpha\beta\gamma_{1Y}^*C_Y + \frac{\beta(\beta-1)}{2}A_Y\right]. \tag{A.2}$$

Squaring both sides of (A.1) and neglecting terms of $\delta's$ having power greater than two we have

$$\left(\hat{\theta}_{(\alpha,\beta)} - \theta_{(\alpha,\beta)}\right)^2 = \theta_{(\alpha,\beta)}^2\left(\alpha\delta_{\bar{y}} + \beta\delta_{\hat{\sigma}_y^2}\right)^2$$

or

$$\left(\hat{\theta}_{(\alpha,\beta)} - \theta_{(\alpha,\beta)}\right)^2 = \theta_{(\alpha,\beta)}^2\left(\alpha^2\delta_{\bar{y}}^2 + 2\alpha\beta\delta_{\bar{y}}\delta_{\hat{\sigma}_y^2} + \beta^2\delta_{\hat{\sigma}_y^2}^2\right)^2. \tag{A.3}$$

Taking expectations of both sides in (A.3) we get the mean squared error of $\hat{\theta}_{(\alpha,\beta)}$ to the first degree of approximation, as

$$\text{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right) = \frac{\theta_{(\alpha,\beta)}^2}{n}\left[\alpha^2\frac{C_Y^2}{\theta_Y} + 2\alpha\beta\gamma_{1Y}^*C_Y + \beta^2A_Y\right] \tag{A.4}$$

Appendix: B

The bias and mean squared error(MSE) of $\hat{T}_{(\alpha,\beta)}$ to the first degree of approximation are obtained as follows:
Expanding $h(b)$ about the point $a = 1$ in third order Taylor's series, we obtain

$$\hat{T}_{(\alpha,\beta)} = \hat{\theta}_{(\alpha,\beta)}\left[h(1) + (a-1)h_1(1) + \frac{(a-1)^2}{2}h_{11}(1) + \frac{(a-1)^3}{6}h_{111}(a^*)\right], \tag{B.1}$$

where $a^* = 1 + \psi(u-1)$, $0 < \psi < 1$ and ψ may depend on a and

$$h_1(1) = \left.\frac{\partial h(a)}{\partial a}\right|_{a=1}, \qquad h_{11}(1) = \left.\frac{\partial^2 h(a)}{\partial a^2}\right|_{a=1}, \qquad h_{111}(1) = \left.\frac{\partial^3 h(a)}{\partial a^3}\right|_{a=a^*}.$$

Noting that $h(1) = 1$ in (B.1) we have

$$\hat{T}_{(\alpha,\beta)} = \hat{\theta}_{(\alpha,\beta)}\left[1 + (a-1)h_1(1) + \frac{(a-1)^2}{2}h_{11}(1) + \frac{(a-1)^3}{6}h_{111}(a^*)\right].$$

Further

$$(a-1) = \left(\frac{\bar{x}}{\mu_X} - 1\right) = \left\{\frac{\mu_X(1+\delta_{\bar{x}})}{\mu_X} - 1\right\} = \delta_{\bar{x}}.$$

Expressing (B.1) in terms of $\delta' s$ we have

$$\hat{T}_{(\alpha,\beta)} = \theta_{(\alpha,\beta)}\left[1 + \alpha\delta_{\bar{y}} + \frac{\alpha(\alpha-1)}{2}\delta_{\bar{y}}^2 + \beta\delta_{\hat{\sigma}_y^2} + \alpha\beta\delta_{\bar{y}}\delta_{\hat{\sigma}_y^2} + \frac{\beta(\beta-1)}{2}\delta_{\hat{\sigma}_y^2}^2\right]\left[1 + \delta_{\bar{x}}h_1(1) + \frac{\delta_{\bar{x}}^2}{2}h_{11}(1) + \frac{\delta_{\bar{x}}^3}{6}h_{111}(a^*)\right]$$

$$= \theta_{(\alpha,\beta)}\left[1 + \alpha\delta_{\bar{y}} + \frac{\alpha(\alpha-1)}{2}\delta_{\bar{y}}^2 + \beta\delta_{\hat{\sigma}_y^2} + \alpha\beta\delta_{\bar{y}}\delta_{\hat{\sigma}_y^2} + \frac{\beta(\beta-1)}{2}\delta_{\hat{\sigma}_y^2}^2 + \delta_{\bar{x}}h_1(1) + \alpha\delta_{\bar{x}}\delta_{\bar{y}}h_1(1) + \beta\delta_{\hat{\sigma}_y^2}\delta_{\bar{x}}h_1(1) + \frac{\delta_{\bar{x}}^2}{2}h_{11}(1) + \frac{\delta_{\bar{x}}^3}{6}h_{111}(a^*) + \cdots\right]$$

or

$$\left(\hat{T}_{(\alpha,\beta)} - \theta_{(\alpha,\beta)}\right) = \theta_{(\alpha,\beta)}\left[\alpha\delta_{\bar{y}} + \frac{\alpha(\alpha-1)}{2}\delta_{\bar{y}}^2 + \beta\delta_{\hat{\sigma}_y^2} + \alpha\beta\delta_{\bar{y}}\delta_{\hat{\sigma}_y^2} + \frac{\beta(\beta-1)}{2}\delta_{\hat{\sigma}_y^2}^2 + \delta_{\bar{x}}h_1(1) + \alpha\delta_{\bar{x}}\delta_{\bar{y}}h_1(1) + \beta\delta_{\hat{\sigma}_y^2}\delta_{\bar{x}}h_1(1) + \frac{\delta_{\bar{x}}^2}{2}h_{11}(1) + \frac{\delta_{\bar{x}}^3}{6}h_{111}(a^*) + \cdots\right]. \tag{B.2}$$

Neglecting terms of $\delta' s$ having power greater two we have

$$\left(\hat{T}_{(\alpha,\beta)} - \theta_{(\alpha,\beta)}\right) = \theta_{(\alpha,\beta)}\left[\alpha\delta_{\bar{y}} + \frac{\alpha(\alpha-1)}{2}\delta_{\bar{y}}^2 + \beta\delta_{\hat{\sigma}_y^2} + \alpha\beta\delta_{\bar{y}}\delta_{\hat{\sigma}_y^2} + \frac{\beta(\beta-1)}{2}\delta_{\hat{\sigma}_y^2}^2 + \delta_{\bar{x}}h_1(1) + \alpha\delta_{\bar{x}}\delta_{\bar{y}}h_1(1) + \beta\delta_{\hat{\sigma}_y^2}\delta_{\bar{x}}h_1(1) + \frac{\delta_{\bar{x}}^2}{2}h_{11}(1)\right]. \tag{B.3}$$

Taking expectations of both sides in (B.2) we get the bias of $T_{(\alpha,\beta)}$ to the first degree of approximation as

$$B\left(\hat{T}_{(\alpha,\beta)}\right) = \theta_{(\alpha,\beta)}\left[\alpha E\left(\delta_{\bar{y}}\right) + \frac{\alpha(\alpha-1)}{2}E\left(\delta_{\bar{y}}^2\right) + \beta E\left(\delta_{\hat{\sigma}_y^2}\right) + \alpha\beta E\left(\delta_{\bar{y}}\delta_{\hat{\sigma}_y^2}\right) + \frac{\beta(\beta-1)}{2}E\left(\delta_{\hat{\sigma}_y^2}^2\right) + E\left(\delta_{\bar{x}}\right)h_1(1) + \alpha E\left(\delta_{\bar{x}}\delta_{\bar{y}}\right)h_1(1) + \beta E\left(\delta_{\hat{\sigma}_y^2}\delta_{\bar{x}}\right)h_1(1) + E\left(\frac{\delta_{\bar{x}}^2}{2}\right)h_{11}(1)\right]$$

$$= \frac{\theta_{(\alpha,\beta)}}{n}\left[\frac{\alpha(\alpha-1)}{2}\frac{C_Y^2}{\theta_y} + \alpha\beta\gamma_{1Y}^*C_Y + \frac{\beta(\beta-1)}{2}A_Y + (\alpha KC_X + \beta\lambda)C_Xh_1(1) + \frac{C_X^2}{\theta_X}h_{11}(1)\right]$$

$$= B\left(\hat{\theta}_{(\alpha,\beta)}\right) + \frac{\theta_{(\alpha,\beta)}}{n}\left[(\alpha KC_X + \beta\lambda)C_Xh_1(1) + \frac{C_X^2}{\theta_X}h_{11}(1)\right], \tag{B.4}$$

where $B(\hat{\theta}_{(\alpha,\beta)})$ is given by (A.2).
is the bias of $\hat{\theta}_{(\alpha,\beta)}$ to the first degree of approximation.
Squaring both sides of (B.3) and neglecting terms of $\varepsilon' s$ having power greater than two we have

$$\left(\hat{T}_{(\alpha,\beta)} - \theta_{(\alpha,\beta)}\right)^2 = \theta_{(\alpha,\beta)}^2\left[\alpha\delta_{\bar{y}} + \beta\delta_{\hat{\sigma}_y^2} + \delta_{\bar{x}}h_1(1)\right]^2$$

$$= \theta_{(\alpha,\beta)}^2\left[\alpha^2\delta_{\bar{y}}^2 + \beta^2\delta_{\hat{\sigma}_y^2}^2 + \delta_{\bar{x}}^2h_1^2(1) + 2\alpha\beta\delta_{\bar{y}}\delta_{\hat{\sigma}_y^2} + 2\beta\delta_{\hat{\sigma}_y^2}\delta_{\bar{x}}h_1(1) + 2\alpha\delta_{\bar{x}}\delta_{\bar{y}}h_1(1)\right]. \tag{B.5}$$

Taking expectations of both sides in (B.6) we get the MSE of $(\hat{T}_{(\alpha,\beta)})$ to the first degree of approximation as

$$\mathrm{MSE}\left(\hat{T}_{(\alpha,\beta)}\right) = \mathrm{MSE}\left(\hat{\theta}_{(\alpha,\beta)}\right) + \frac{\theta^2_{(\alpha,\beta)}}{n}\left[2(\alpha K C_X + \beta\lambda)C_X h_1(1) + \frac{C_X^2}{\theta_X}h_1^2(1)\right], \qquad (B.6)$$

where $\mathrm{MSE}(\hat{\theta}_{(\alpha,\beta)})$, which is given by (A.4), is the MSE of $\hat{\theta}_{(\alpha,\beta)}$ to the first degree of approximation.

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