Method of Study Modules in Higher Mathematics Studies¹

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Being aware of the present situation in Latvia and in whole Europe, Mathematics curriculum development is a topical issue. One of the ways how to deal with it is the application of study modules in the study process. The division of Mathematics studies in the Forms and Content Modules allows students to understand better the organization of study process of mathematics, and creates conceptual awareness of Mathematics logics and its practical application helping students to understand causal relationships and to develop cognitive skills. In the article will be present same theoretical aspects and practical experience in Latvia University of Agriculture.

Keywords: curriculum, study modules, mathematics studies, forms modules, content

modules

ZDM Classification: B45, D35

MSC2000 Classification: 97B45, 97D35, 51-01

1. INTRODUCTION

Science knowledge is a basic element in modern education. It is indispensable in all areas of life — everyday social and professional life. In various professions the science knowledge is an essential element. Examples are doctors, farmers, engineers, environmental protection specialists as well as science subject teachers and scientists dealing with corresponding areas of research. That's right! But, in our opinion, the most

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important role of science education in forming developed cognitive skills, such as interdisciplinary critical thinking, problem solving, decision making, is not estimated enough. In this context, everybody needs science education, but the development of Science education needs international collaboration. The 7th IOSTE Symposium for Central and Eastern Europe marked that added to the greater integration of EU, there is need for the exploration, discussion and exchange of education ideas, analysis of common problems, implementations of European dimension in initial science education (Nezvalova *et al.*, 2009). Moreover there were stressed that science and technology education should stimulate international cooperation in research and development and promote cooperation with other international organizations (Lyons, 2009).

Of late years the main assignment of mathematics was reviewed to give knowledge for the best learning of technical subjects. But nowadays the objectives of mathematics appear to be the following: students must be prepared for practical use of the subject and the basis of theoretical knowledge must be established as well for students will need it, in their future, work in order to understand the available literature and to use it creatively. Therefore the improving of mathematics program is based on two categories of description of the experience of mathematics: mathematics as the basis of other subject and as a tool for analyzing problems that occur in the world (Vintere & Aboltins, 2007).

There are three most important factors that affect on the development of mathematics education in Latvia:

- 1) The demand for highly qualified engineering employees;
- 2) The fixed volume of the study content that has not changed for a few years in comparison with the time that, on the contrary, has been reduced for mathematic studies;
- 3) Students' lack of background knowledge and cognitive skills. Having never achieved success in their studies, students often lose any interest about natural sciences.

Basically all students spend only a year to acquire the basic course in mathematics. Unfortunately the frequent poor results cause expulsion from the university. From this point of view universities should work on making the mathematics studies more attractive, as well as prevent lack of knowledge caused by insufficient work at school or college. The main problem is to find and investigate methods of the formation of scientific and technological literacy.

INVESTIGATION METHODOLIGY

Research methodology used in this article is scientific analysis and assessment of a number of information sources and reports, taking into consideration the authors' reflection experience and observations as well as analysis of the results of participation in research and development activities in connection with the development of the study process of mathematics in Latvia University of Agriculture (LUA).

MATERIALS AND METHODS

The reasons and conditions mentioned above require seeking for effective teaching methods, approaches and resources that can be used in the process of studying to attain the aims and objectives fixed in the study program. It means certain changes and improvements in the mathematic education processes are necessary which would increase achievements measured with definite criteria.

Mathematics is a hierarchical subject and the idea that it can be learned in a piecemeal fashion is not sensible. There is a need to construct a coherent knowledge base with connections between topics rather than a collection of seemingly isolated methods. The aim should be the development of an overview of the coherence, power and general applicability of mathematics as a means of analysis, a problem-solving tool and a language of exact scientific communication. It means that it is necessary to find and investigate methods of the formation of scientific and technological literacy

Curriculum should therefore be based on the corresponding competencies and teaching should promote the building of the appropriate cognitive schemes.

One way:

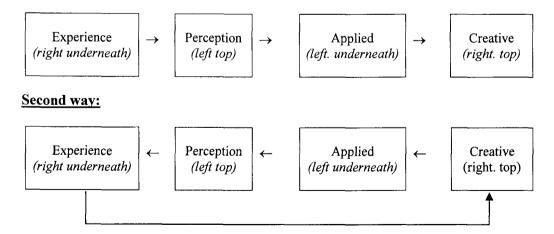


Figure 1. Two ways of organize teaching process.

Thinking about cognitive development it is necessary to understand the process of thinking of each student. Teaching any subject, but especially science subjects and mathematics, the teachers' main goal is to work on the students' strong hemisphere, not underlining and lessening the weak one (Zeidmane, 2007).

Human brain hemispheres' anatomical structural asymmetry is closely connected with its functional asymmetry. It is important to know each hemisphere's function and use this knowledge in the teaching process. The teacher must differentiate the teaching process, thinking in four ways Figure 1:

- 1) How experience develops students personality (right underneath),
- 2) How to help to develop concepts' and laws' perception (left top),
- 3) How to help to apply the adopted (left underneath),
- 4) How to create new ideas, new meaning (right top).

In terms of cognition and learning theory, students continually build up new cognitive schemes. Beginners rely on every-day schemes. Very soon, they realize that those are not adequate for solving technical problems. They start to learn new formal schemes, but in doing so, these schemes get confounded with the old every-day schemes. The result is the well-known type of errors occurring at this stage: students may find nonsense answers that normal, non-scientifically trained people would never accept. The reason is found in the difficulty of combining formal and intuitive thinking. Only after having gained some expertise, the new, formal concepts are stable enough to lead to answers without confusion by everyday schemes and students become able to solve problems in a scientifically correct way. Gaining more and more experience, a new sort of scientifically based intuition appears in a last step which is typical for experts in the field. In authors' opinion mathematics curricula should help engineers to acquire the right cognitive schemes making them able to solve new problems in a correct and creative way.

In order to satisfy the above mentioned requirements in mathematics, in accordance with the requirements of the Bologna process, the mathematics curriculum must be developed in compliance with the credit-module system. Modularization is as known the summary of analogue fields to a thematic and temporal rounded in closed itself and with performance points provided in testable units (Schlattmann, 2007). ECTS (ETCS-Points = European Transfer Credit System-Points) give an average workload for the complete course of studies. It is necessary to develop the study modules.

Term "module" in the Longman Dictionary of Contemporary English (1995) is explained as one of the units that a course of study has been divided into, each of which can be studied separately.

Study modules objective is to provide definite knowledge and skills which are realized integrating separately differentiated study modules.

Study modules:

- contain certain information;
- are mutually coordinate (have entrance and exit);
- are coordinated with professional and didactic tasks.

International practice shows that there are various approaches to forming modules (Artjukh, 2007):

- 1. Modules can include various parts, united according to subject (lectures, exercises, practice).
- 2. Modules can include disciplines/courses of several semesters and have the exam after completing the module.
- 3. Every module requires knowledge, skills and abilities acquired in previous modules.
- 4. Modules are defined by the number of credits and hours in semester week.

In authors' opinion it is necessary to form the modules for separate study subjects by elements all approaches. Therefore the course of mathematics was designed according to **study module** principle (Zeidmane, 2003).

For realizing separate subject syllabuses development we can divide the study process into STUDY FORMS modules, for realizing cognitive development we can divide the study process into STUDY CONTENT modules. The content module must form knowledge in the concrete field. That's why the course should be divided into separate themes, the sequence of which is appropriate to the logic of the given science. Because mathematic is the basic subject for many faculties, but every specialty have different amount of mathematics (credit points), it is necessary divide each Study Content module into levels (Figure 2).

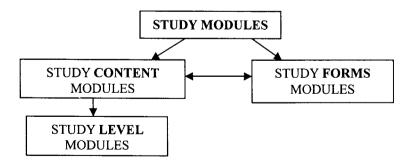


Figure 2. Division of study modules

The authors have worked out a structural scheme of the study modules method, where the process is divided into **study form modules** (see Figure 3).

The most important in the theme acquisition is supposed THE PROVISION OF

INFORMATION which includes previously learned material ("from the Background knowledge"), the Material from other subject, necessary for the given theme and new material. For the processing of the given information integration of different form of study modules (lectures, tasks, using ICT, students individual works) can be used. The most important task is TO PROVIDE RECOMMENDATIONS FOR PRACTICAL WORK, which could be used in annual project work, specials subjects or in the next module (the next theme). The above mentioned corresponds to the division of study process into forms of study modules.

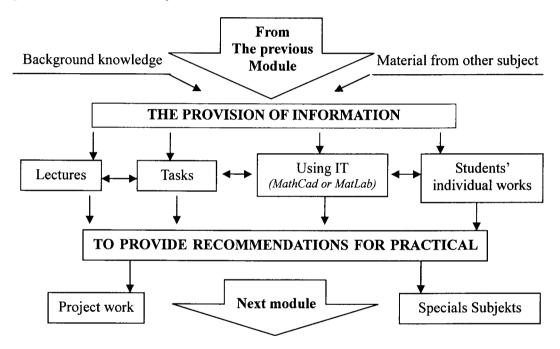


Figure 3. Structural scheme of study form modules

In order to optimize the study process in mathematics the following should be observed:

- 1. The course should be divided into separate themes ("The **TITLE**"), the sequence of which is appropriate to the logic of the given science;
- The goal of this module BACKGROUND KNOWLEDGE is to indicate, which
 questions should be known in order to follow the solution of the scheme successfully.
 Thus this module is structured into methods, concepts, and regularities;
- 3. In the module **PROBLEM** it is important to formulate the problem, including:
 - 1) Motivation, why this problem exists or why it is not sufficient with background knowledge to acquire the given theme;

- 2) Concise and clear formulation of the problem, so that the module theme solution could solve this problem;
- 4. The module ADDITIONAL INFORMATION is considered as the continuation of the module BACKGROUND KNOWLEDGE. If the questions included in BACKGROUND KNOWLEDGE can not comply with the solution, then there is necessity to study new methods or come to an agreement about new concepts. This module contains also regularities, which are not learned before, but which will be necessary for the solution.

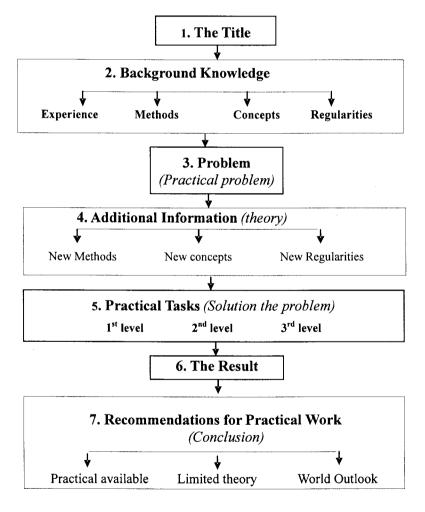


Figure 4. Structural scheme of study matters modules.

5. The module **PRACTICAL TASKS** as a logic solution of regularities is presented which is based on background knowledge and additional information and that leads to the solution of the problem. The module PRACTICAL TASKS depends on volume of content.

- That's why it is necessary all mathematics course to divide in levels in accordance with ECTS which determine an average workload for the each level.
- 6. The module **THE RESULT** is necessary to review a theme solution once more shortly formulating concepts and laws, and classifying them according to causal relationship or analogy, thus encouraging the process of memorizing.
- 7. The perception of an individual is always selective and everyone in his/ her lifetime has created the evaluation criteria, according to which all new information is classified into significant, less significant and negligible. Consequently, the module RECOMMENDATIONS FOR PRACTICAL WORK or conclusion is considered to be of great importance.

RESULTS

Improvements in the syllabus of mathematics studies include evaluation of the existing volume of studies, contents as well as competences to be acquired. The volume of the program of mathematics studies is regulated by the normative documentation of the Republic of Latvia and Latvia University of Agriculture.

In order to inspect the contents of studies, experience of similar profile universities, the guidelines for the 21st century Mathematics for European Engineers and Professions Standards were explored. The latter is also important in terms of **competence** — it determines both the competence in mathematics learned at university and the general competences necessary for certain specialization that can be acquired during the studies of mathematics.

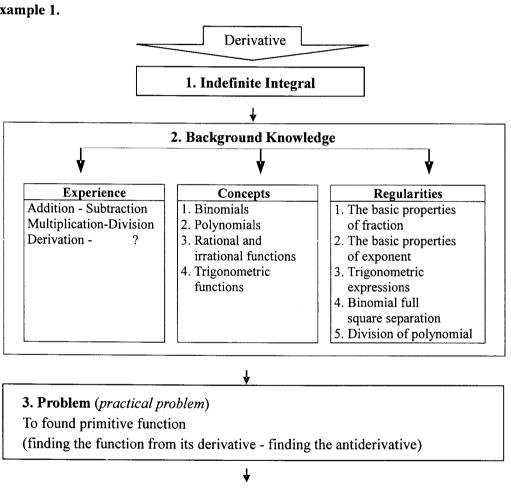
For this reason the programs of mathematics studies' and its didactics development at LUA consists of two directions: to make the mathematics studies more attractive and to improve the study program based on students' competence by activating practical use of mathematics in their specialization area and latest achievements in pedagogy and elaboration of materials based on modern technologies. Syllabus development includes focus on the use of ICT as well. The innovative part of the basic mathematics syllabus development appears to be the substitution of practical classes with laboratory practice in the ICT class, which mainly will be focused on the use of the *MathCAD* software program.

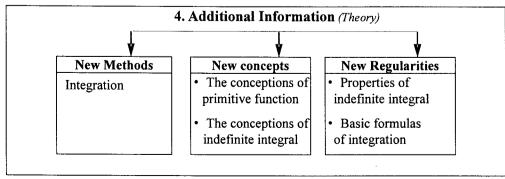
According the **Study Forms** modules Courses of Mathematics is divided into Lectures, Tasks, where student training in solution mathematical problems and receive the Individual tasks. After that students can control their individual tasks which had done using only head (brain) in Computer classrooms. So we can provide cognitive development relevance with using ICT. In Latvia University of Agriculture we include

MathCAD in given Credit points, but we are going to obtain the additional Credit points for using Math Lab in Mathematic Study course.

All Mathematic courses were divided into separate themes according to Study Concept modules. For example we illustrate the module "INDEFINITE INTEGRAL" which is establishing combine Forms and Content modules (see Example 1).







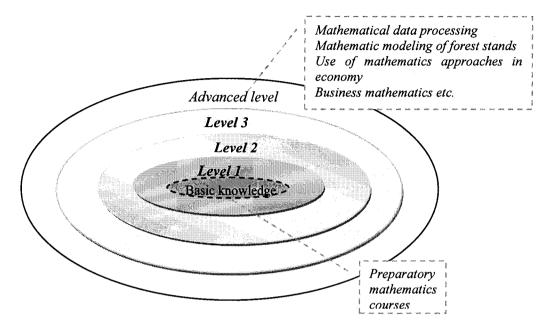


Figure 5. Schematic diagram of the mathematics structure in LUA

In order to create module "5, PRACTICAL TASKS" the basic higher mathematics course in LUA is divided into three levels. A schematic diagram of the mathematics structure in LUA is shown in Figure 5. There is a central core (Level 1) of material essential for all graduates (2–3 ECTS). Level 2 (7–8 ECTS) comprises the knowledge and skills which are necessary in order to underpin the general Engineering Science that is assumed to be essential for most engineering graduates. Level 3 (12–14 ECTS) comprises specialist or advanced knowledge and skills which are considered essential for individual engineering disciplines. Advanced level comprises highly specialist knowledge and skills which are associated with advanced levels of study and incorporates synoptic mathematical theory and its integration with real-life engineering examples.

Researches show that almost like in the Latvia University of Agriculture the problem persists in other universities as well — unsatisfactory preparation of students and high differentiation of the student's level of knowledge and skills. Unfortunately the frequent poor results cause expulsion from the university. That is why the LLU lecturers at present involved in creating, designing and testing the preparatory mathematics courses on MLS Moodle. It could be called as a zero module.

As a result of learning materials at 1st level students should be able (Example 2) to:

- reverse the process of differentiation to obtain an indefinite integral for simple functions
- understand the role of the arbitrary constant
- use a table of indefinite integrals of simple functions

- understand and use the notation for indefinite integrals
- use the constant multiple rule and the sum rule
- use indefinite integration to solve practical problems such as obtaining velocity from a formula for acceleration or displacement from a formula for velocity.

The material at the 2nd level is regarded as basic to all engineering disciplines in that it provides the fundamental understanding of many mathematical principles. The material in level 2 can be used by engineers in the understanding and the development of theory and in the sensible selection of tools for analysis of engineering problems. As a result of learning this material students should be able to apply the methods (the method of integration by parts, of substitution etc.) of integration, to obtain indefinite integrals of rational functions in partial fraction form as well as solve practical problems which require the evaluation of an integral.

The material at the 3rd level is advanced enough for simple real engineering problems to be addressed. As a result of learning this material students should be able to use integrals in the solution of engineering problems.

Example 2.

5. PRACTICAL TASKS (Solution the problem) 1. Direct integration method 2. Variable substitution method 3. Method of integration by parts 4. Integration of similar trigonometric function 5. Integration of irrational expressions 6. Integration of irrational functions (linear) 3rd level 7. Integration of trigonometric function with substitution method 8. Integration of irrational expressions (quadratic)

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

$$\int a^x dx = \frac{a^x}{\ln a} + C$$

$$\int e^x dx = e^x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \frac{dx}{\cos^2 x} = tgx + C$$

$$\int \frac{dx}{\sin^2 x} = -ctgx + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \frac{1}{a} arctg \frac{x}{a} + C$$

THE RESULT Torm module "Lecture"

$$\int u^n du = \frac{u^{n+1}}{n+1} + C, n \neq -1$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int a^u du = \frac{a^u}{\ln a} + C$$

$$\int e^u dx = e^u + C$$

$$\int \cos u du = \sin u + C$$

$$\int \sin u du = -\cos u + C$$

$$\int \frac{du}{\cos^2 u} = tgu + C$$

$$\int \frac{du}{\sin^2 u} = -ctgu + C$$

$$\int \frac{du}{\sin^2 u} = \arcsin \frac{u}{a} + C$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$$



6.2. THE RESULT – 1st level

Form module "Tasks"

- 1. Direct integration method using
 - the table of basic integrals
 - properties of indefinite integral
 - identical transformation of under integral function

2. Substitution method

- Simpler substitutions u = kx + a
- trigonometric's substitutions

3. Method of integration by parts

 $\int u dv = uv - \int v du$

Important multiplier u and dv choise

4. Integration of similar trigonometric expression

- Transformation multiplication of function to addition of function

$$\cos\alpha\cdot\cos\beta = \frac{1}{2}(\cos(\alpha-\beta)+\cos(\alpha+\beta))$$

$$\sin^2\alpha = \frac{1-\cos 2x}{2}$$

$$\sin \alpha \cdot \cos \beta = \frac{1}{2} (\sin(\alpha - \beta) + \sin(\alpha + \beta))$$

$$\sin \alpha \cdot \sin \beta = \frac{1}{2} (\cos(\alpha - \beta) - \cos(\alpha + \beta))$$

$$\cos^2\alpha = \frac{1+\cos 2x}{2}$$

6.3. THE RESULT - 2nd LEVEL

Form module "Tasks"

5. Integration of rational expressions with substitution method

$$\boxed{m < n} \leftarrow \left[\frac{Q_m(x)}{P_n(x)} dx \right] \rightarrow \boxed{m > n}$$

- to express as addition of particle fractions to divide $Q_m(x)$ by $P_n(x)$ acquire $q_{m-n}(x)$
- to integrate acquire particle fractions

and residual value $\frac{r_k(x)}{r_k(x)}$ -to divide the fraction $\frac{P_n(x)}{P_n(x)}$ into addition of particle fractions -to integrate polynomial $q_{m-n}(x)$ and

6. 6. Integration of linear irrational functions with substitution method

- if

$$\int R\left(x; \sqrt[n_1]{ax+b}; \sqrt[n_2]{ax+b}; \dots; \sqrt[n_k]{ax+b}\right) dx = ax+b = t^m$$

m – lowest common dividend of numbers $n_1, n_2, ..., n_k$

-if

$$\int R\left(x; \sqrt[n_1]{\frac{ax+b}{cx+e}}; \sqrt[n_2]{\frac{ax+b}{cx+e}}; ...; \sqrt[n_k]{\frac{ax+b}{cx+e}}\right) dx = \frac{ax+b}{cx+e} = t^m$$



6.4. THE RESULT - 3rd LEVEL

Form module "Tasks"

7. Integration of trigonometric expression with substitution method

- universal substitution
$$\int R(\sin x; \cos x) dx \rightarrow \left(t = tg \frac{x}{2}\right)$$

- if $\Rightarrow \int R(\sin x; \cos x) dx \rightarrow (t = tgx)$
- if $R(-\sin x; -\cos x) = R(\sin x; \cos x)$ $\Rightarrow \int R(\sin x; \cos x) dx \rightarrow (t = \cos x)$
- if $R(-\sin x; \cos x) = -R(\sin x; \cos x)$ $\Rightarrow \int R(\sin x; \cos x) dx \rightarrow (t = \sin x)$ $R(\sin x; -\cos x) = -R(\sin x; \cos x)$

8. Integration of quadratic irrational functions with substitution method

$$\int R\left(x; \sqrt{a^2 - x^2}\right) dx \to \left[x = a \cdot \sin t\right]$$

$$\int R\left(x; \sqrt{a^2 + x^2}\right) dx \to \left[x = \operatorname{arctgt}\right]$$

$$\int R\left(x; \sqrt{x^2 - a^2}\right) dx \to \left[x = \frac{a}{\cos t}\right]$$



7. RECOMMENDATIONS FOR PRACTICAL WORK

(Conclusion)

Practical available - in Kinematic

The direct task of kinematic $s \Rightarrow v \Rightarrow a$

Velosity is the derivative Acceleration is the of distance with respect to time derivative of velosity

with respect to time

The inverse task of kinematic

 $a \Rightarrow v \Rightarrow s \qquad v = \int a \cdot dt$ \Rightarrow $s = \{v \cdot dt\}$

integrals of the acceleration integrals of the velosity

Special case – motion with constant acceleration (a = const)

 $v = \int a dt = a \int dt = a \cdot t + C$ $\Rightarrow C = v_0$ - constant of integration - initial velosity

 $s = \int vdt = \int (v_0 + at)dt = v \cdot t + \frac{a \cdot t^2}{2} + C \implies C = s_0 \quad \text{Constant of integration - initial}$

"distance" (position) $s = s_0 + v \cdot t + \frac{a \cdot t^2}{2}$

Limited theory

Integrals which cannot formulate with elementary functions,

 $\int \frac{Q_m(x)}{\sqrt{P_n(x)}} dx \qquad \text{when } n > 2 \qquad \int e^{-x^2} dx$ integrals of probability - Frenel's integrals

 $\int \frac{\sin x}{x} dx \int \frac{\cos x}{x} dx$

World Outlook

In order to identify something at first it could be divided (derivative), to definite connection between them and then to unite (integration)

DEFINITE INTEGRAL

Figure 6. Example of study module in Mathematics

In order to activate practical use of mathematical knowledge in certain specialization and mathematical competence in the aspect of professional application several programs of advanced mathematics studies have been produced at LUA – Mathematical data processing, Applying mathematical methods, Use of mathematics approaches, Mathematic modeling of forest stands, Use of mathematics approaches in economy, Business mathematics etc.

The syllabus development is connected with the proportion of compulsory and elective courses. Rather all advanced courses are entered into studies' programs LUA as subjects of free choice. The electives should not be only some supplement to the compulsory study program. Under such circumstances the students seldom choose electives. Disadvantage of great number of electives is in greater demand on number of teachers, lecture-rooms, laboratories etc. Among advantages may be included competition of different courses and teachers. Unfortunately students sometimes do not always prefer better courses. They often search for courses that are easier to pass.

ICT might be a tool to solve the problem in education - to eliminate something from the curriculum, develop interconnections among units and courses, and link the formal curriculum to the informal curriculum. In the age of the Internet all students have access to an enormous amount of information from countless sources outside schools (Zeidmane & Vintere, 2008a; 2008b). That is why the advanced mathematics studies' (as separate modules) at LUA have focus on designing e-courses on MLS *Moodle*. Therefore is solved problem of the proportion of compulsory and elective courses

CONCLUSION

The advantage of the study process **module method** in comparison with other study process models is:

- Systematic approach to course design and its contents;
- Coordination of all study forms in every module and among modules;
- Structure flexibility of study process:
- Opportunity to find solution of the theme, to be able to see the essence and to be aware of application of the results;
- Effective assessment system.

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