Korean Mathematics Adds Value to Teachers' Conceptual Understanding in the United States¹

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Researchers at Truman State University in Missouri, located in the heartland of the United States, have been using materials adapted from the English translations of the sixth national primary mathematics curriculum from Korea for professional development and assessment with groups of Missouri teachers for the purpose of enhancing teachers' understanding of the fundamentals of mathematics since 2002 [gecKo Mathematics (2008). Korean Mathematics in American Classrooms. Edited by J. Grow-Maienza. Adapted from Korean Mathematics (2001). Kirksville, MO: Truman State University. http://kmath.truman.edu/]. A professional development initiative for 50 teachers conducted in Missouri this past year is reported here. Significant gains in teacher understanding of fundamental mathematics concepts and pedagogy necessary for student achievement in primary mathematics were found as a result of the initiative.

Keywords: teachers' conceptual understanding, Korean mathematics curriculum

ZDM Classification: B50, C33, C39, C70 MSC2000 Classification: 97C30, 97C70

1. INTEREST IN ASIAN MATHEMATICS CURRICULA IN THE WEST

Asian elementary and secondary students have been surpassing aggregated groups of students in the United States on international tests of mathematics achievement for years

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(U. S. Dept. of Ed., 1996; Travers et al., 1985; Husen, 1967). When looking for causes of those differences in student achievement both Asian and American researchers acknowledge the many cultural differences between Asia and the United States, including differences in family values and educational practices (Stevenson et al., 1985; 1987; Stevenson & Stigler, 1992; Fuson & Kwon, 1992a; 1992b; Miura et al., 1994). Some international comparison studies have focused on classroom practices in mathematics classrooms (Stevenson & Lee, 1995; Stigler & Hiebert, 1998; Hahn & Grow-Maienza, 1998; Stigler et al., 1999; Grow-Maienza, Hahn & Joo, 2001; Cai & Wang, 2006; Wang & Cai, 2007a; 2007b). Important insights have come from those studies. In addition, in the last decade researchers and educators in the United States have begun to explore Asian mathematics curricula and textbooks for at least partial explanation for the higher mathematics achievement of students in Asian countries compared to students in the United States (Mayer, Sims & Tajika, 1995; Watanabe, 2001; 2002; 2003; 2006; Li, 2002; Grow-Maienza, 2002; Grow-Maienza & Beal, 2004). Mathematics curricula in Asia are seen as narrowly focused and in-depth (Schmidt et al., 1996; Stigler & Hiebert, 1998). American curricula are often characterized as "a mile wide and an inch deep (Schmidt et al., 1996)." As a result there is considerable interest among many researchers and educators in the United States in curricula from Asia which is more narrowly focused and designed to be taught in-depth. Some American school districts are adopting Asian curricula (e.g., Prystay, 2004).

A widely read book in the United States is Liping Ma's Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States (1999). Ma reports in detail the results of her in-depth interview study of the mathematical understanding of Chinese elementary teachers. Ma demonstrates in her study the superior "knowledge packages" that Chinese teachers reveal when they discuss their pedagogy for teaching primary mathematics concepts. Chinese teachers focus on developing the conceptual underpinnings of mathematics. American teachers, by contrast, generally focus on procedures. Others have reported the concentration on developing conceptual understanding that is observed in Asian classrooms. (Stevenson & Lee, 1995; Stigler & Hiebert, 1998; Hahn & Grow-Maienza, 1998; Grow-Maienza, Hahn & Joo, 2001; Cai, 2005; Wang & Cai, 2007).

Ma suggests that American teachers focus on procedures because they themselves lack deep conceptual understanding of the underpinnings of fundamental mathematics (Ma, 1999). In his forward to Ma's book, Lee Shulman suggests that Ma's findings should be very relevant to university professors and teacher educators in the United States. Future teachers in the United States are coming to teacher preparation programs without the fundamental mathematics knowledge needed to teach primary mathematics. American teachers must get that fundamental knowledge in their university teacher preparation

programs. In China, says Ma, teachers have received the fundamentals of mathematics from their own elementary and middle school teachers (Shulman in Ma, 1999). Grow-Maienza, in her analysis of the Korean Sixth National Primary Mathematics Curriculum for the National Science Foundation, found many of those fundamentals embedded in the Korean curriculum and very explicitly developed (Grow-Maienza, 2002; accessible at http://eisenhowermathematics.truman.edu.)

2. THE 2008–2009 MISSOURI PROFESSIONAL DEVELOPMENT INITIATIVE

In 2008 researchers at Truman State University introduced *gecKo* mathematics, materials adapted from the English translations of the 6th national primary mathematics curriculum of Korea to 50 teachers in two settings in Missouri. Purpose of the project was to enhance the conceptual understanding of teachers for the fundamental mathematics and pedagogy typically taught in grades 1 through 6. The initiative was delivered in two institutes of different duration. In the first setting twenty-seven teachers in 10 small rural districts in Northeast Missouri attended an eight-day institute on the university campus in August, 2008. These teachers attended two half-day follow-up meetings, one in October, 2008, and one in May, 2009. In the second setting 21 teachers in the large urban Kansas City Missouri School District attended two two-day weekend institutes in Kansas City, one in October, one three weeks later in November, 2008, and two full-day follow-up meetings, one in February, and one in May, 2009.

In both settings, teacher participants were introduced to a 23-chapter pre-algebra module from *gecKo* mathematics, material adapted from the English translations of the sixth national curriculum of Korea. The module consisted of chapters on multiplication, division, fractions, and decimals, grades 2 through 5, which are prerequisite to the learning of algebra. In both settings focus was on understanding and building pedagogy around the fundamental principles underlying the procedures being taught. Emphasis was on important content, such as the inverse relationship of operations, the meaning of place value, the meaning of the unit in measurement and multiple ways of solving problems—all strengths in *gecKo* mathematics, as in the traditional Korean curriculum. Another emphasis was on the coherence seen in the curriculum, illustrated in one way by the progressively complex demonstration of operations on the number line consistent in grades 1 through 6.

Teachers were given three hours graduate credit for the course and monetary stipends for attending the institutes and for developing modules of instruction from gecKo mathematics materials. Participants also developed pretests and a post tests for the

modules they constructed. Teachers were expected to implement the modules in their classrooms, integrating the modules into their own curricula. Participants also assessed student performance on the modules and reported results to the project at Truman State University. The first five days in the August institute and the first four days of the Kansas City institute were devoted to understanding how *gecKo* mathematics develops the underlying concepts of the important content of curriculum. The second week of the August institute was given to hands-on development of the modules and student assessment instruments. Participants in the Kansas City institute developed modules in teams at their schools during the academic year. All modules and classroom results were reported to the project at follow-up meetings in April and May, 2009, or submitted to the researchers electronically.

Pre and post internal assessment on teacher understanding of mathematical and pedagogical content instruments were developed by the project team prior to the August institute based on questions from three sources:

- 1) Liping Ma's Knowing and Teaching Elementary Mathematics,
- 2) the TIMSS released items, and
- 3) end-of-chapter practice lessons in *gecKo* Mathematics, adapted from the Korean Sixth National Primary Mathematics Curriculum.

The instruments were designed to measure participants' understanding of key concepts in the *gecKo* mathematics curriculum. Participants were presented with problems elementary students might see, and asked to select the key concept students who responded incorrectly on the item were likely to be missing. The instruments had been piloted and revised in spring, 2008, based on a pilot conducted in the pre-service Master of Arts in Education program at Truman State University. Data on participant attitudes and changes in pedagogical behavior were also collected internally, as well as by an external evaluation team from the University of Missouri. Results of the analysis of the pre- and post-test gain scores obtained by the internal evaluation team is below, followed by the analysis of survey data gathered by both the external evaluation team and the internal evaluation team.

3. RESULTS OF PROFESSIONAL DEVELOPMENT USING gecKo MATHEMATICS MATERIALS BASED ON KOREAN 6TH NATIONAL MATHEMATICS CURRICULUM

Internal Assessment of Teacher Knowledge

Scott Alberts, Associate Professor of Mathematics and Statistics at Truman State

University performed part of the internal evaluation on the effectiveness of *gecKo* mathematics institutes for the MDHE Improving Teacher Quality Cycle VI project at Truman State University and first examined the Pre-test/Post-test instrument pair designed by the *gecKo* team.

The design consisted of two tests of mathematical concepts. Both instruments contained 10 multiple-choice items, with each item posing a student problem and asking teachers to choose from eight choices the concept or broad topic most important to successful mastery of the item. A scoring key was developed by a consensus of the evaluation team, giving 5 points for a correct answer; several items were given fewer points for a secondary or alternate answer. A perfect score was 50 points, although no particular exam scored higher than 42 points on any administration. Higher scores were generally given for reflecting the key concepts of the Korean tradition such as place value, multiple representations of number, etc. Few or no points were generally given for answers reflecting mathematics as a set of memorized facts or recipes. Each test had two open-ended questions that will be analyzed presently. As described above, two different workshops were held, one in Northeast Missouri in the summer, and another in Kansas City during the school year.

Northeast Missouri Summer Institute:

The summer institute was an eight-day intensive workshop on Truman's campus in August, 2008 with follow-ups in October, 2008 and May, 2009. The tests were given at the beginning and end of the intensive session. An initial analysis of the pre-test and post-test of the participating teachers at the summer workshop shows a significant change in the way they analyze problems and consider key concepts.

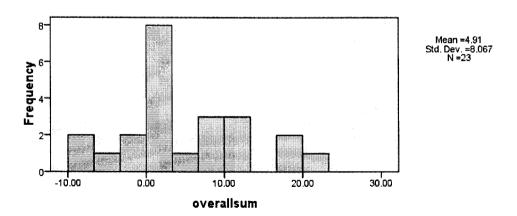


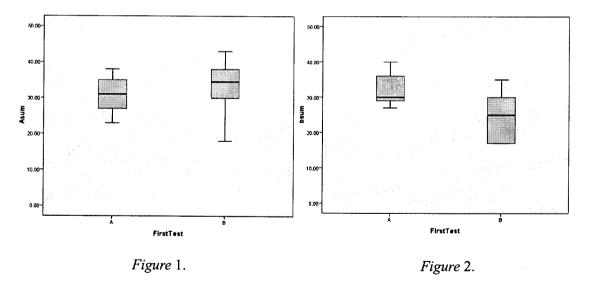
Figure 0.

Here, 23 participants are analyzed out of 26 pairs of forms. One participant was removed because she had not completed a pre-test, one had not completed a post-test, and a third was removed because the post-test was only half-completed. The remaining 23 had completed pre-tests and post-tests. Of the two forms of the exam (arbitrarily named "A" and "B"), 13 had completed the A form as the pre-test and 10 had completed the B form as the pretest with all 23 completing the opposite form as the corresponding post-test.

Overall, respondents scored five points higher on their post-test than their pre-test, with three respondents showing extraordinary improvement (15 points or more improvement), while two respondents showed a decrease of five to ten points.

Overall, the mean improvement of 4.91 points was significant (two-sided p-value = 0.008). Given the small sample size and lack of normality, two nonparametric tests on the median were performed as well and both found significance; the Wilcoxon signed-rank test showed a p-value of 0.010, and the weaker Sign test showed a p-value of 0.026.

Looking at the two tests, A and B, similar differences were seen regardless of which test was taken first. The box plots on the left, Figure 1, shows that those who took form A as a pretest scored about five points worse on test A than those who took form B as the pretest. The box plots on the right, Figure 2, show the same difference for those who took the B test as the pretest.



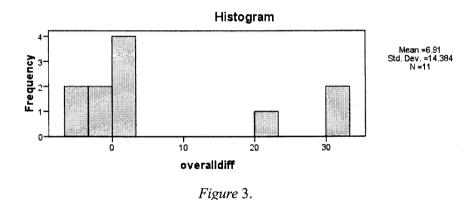
Although the mean and median differences were significant, some items showed much more improvement than others. Items A3, B4, B5, and B6 were found to show significant improvement (*p*-value < 0.05), while items A2, A5, and B10 displayed almost no change. Five Items: A2, A6, A8, B1 and B7 actually got slightly worse.

Urban Kansas City Institute:

The Kansas City institute was a total of six days of sessions, a two-day session in October, 2008, another two-day session in November, and single-day sessions in February and May, 2009. Tests were given at the October and May sessions. The May session was not well attended; of twenty-two participants in October and November, only half attended the May session. The less-intensive experience and reduced sample size reduces the statistical significance of our analysis, but does support the results found in the more intensive session.

Here, 11 participants are analyzed out of 22 pairs of forms. One participant was removed because she had not completed a pre-test and ten had not completed a post-test. The remaining 11 had completed pre-tests and post-tests. Of the two forms of the exam, 8 had completed the A form as the pre-test and 3 had completed the B form as the pretest with all completing the opposite form as the post-test. This unbalance may also skew results, as the forms have received only cursory work to equalize difficulty.

Overall, respondents scored almost seven points higher on their post-test than their pre-test, with three respondents showing extraordinary improvement (15 points or more improvement), while two respondents showed a decrease of six points; six of the eleven respondents scored virtually the same, with a net change between -1 and +2 points on the exam. This bimodal distribution is shown in Figure 3:



This mean improvement of 6.91 points was larger than the summer session, but due to the smaller sample size, was not significant (two-sided p-value = 0.142).

In the same way, differences were found across both tests, but the small sample size of this administration, particularly of those who took test B as the pre-test, precludes significant findings.

A few items showed significant improvement (p-value < 0.10) despite the small sample size, A3, A5, A6, B9, and B10. Several items showed a decrease in score, A7, A8,

A10, B3, B5, B8 none significantly.

Combined results:

In some ways, combining the data from the two sessions is not wise because of the differences of the workshops. The first was an intensive experience, while the second occurred over a longer span of time. Given the small sample size of the second session, comparing the trends of the two sessions will at least allow comparison of the results to suggest consistencies and variation that could imply long term trends versus one-time random effects.

Overall pre-test/post-test improvement was stable between the two sessions. The August session had an outlier which accounts for a mean difference of 3.2 points. This difference was not significant from a t-test comparison with a p-value > 0.50. Non-parametric comparison via the Mann-Whitney test was even less significant.

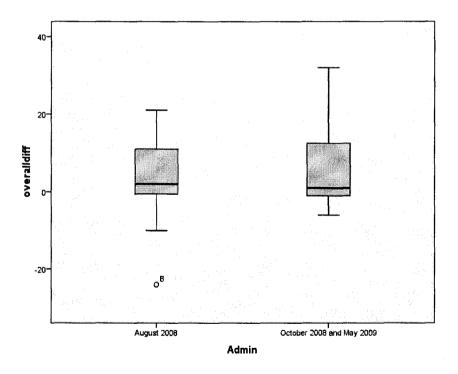


Figure 4.

Looking at items, A3 showed up as improvement in both administrations, but no other items were significant in both. Looking at all items in the combined data set, three items register as significant using a *t*-test, A3, B6, and B9, all with a two-tailed *p*-value below 0.02. All of these items gave half or full credit to concepts (B), place value, and (C),

composition of numbers, which are the key concepts in the *gecKo* mathematics system and the concepts most focused on in the workshop sessions.

By May, 2009, all the participants had followed through in developing units of instruction and assessment instruments from *gecKo* mathematics, incorporated the units into their mathematics curricula, and reported results to the initiative. Qualitative evaluations of the follow-up meetings in May when participants demonstrated units they developed for their classroom and reported results of implementing units in their classrooms reflect satisfaction and enthusiasm for integrating *gecKo* mathematics into their own curricula. The most successful results were exhibited by teachers who participated as part of a team from one school. The more members from one school present at the institute, the more successful the initiative seems to be. We attribute this to the in-school support teachers received from each other in implementing *gecKo* mathematics in their classrooms.

External and Internal Assessment of Change in Teacher Attitudes and Behavior

External evaluators, a team from the University of Missouri, reported that participants reported that the extent of emphasis on number and operations and on inquiry-based instruction in the project had been high. Participants also reported their increase in knowledge was high. Northeast participants' mean perceived level of project-focused content knowledge was at 5.3 (on a scale of 1 to 10) prior to the summer institute, and was at 7.3 after the summer institute. Kansas City participants reported a perceived level of project-focused content knowledge at the end of the professional development program at 7.0 on a scale of 0 to 10. On a scale of 0 to 5, Northeast participants rated improvement in content knowledge relevant to teaching assignment at 4.3. Northeast participants rated their confidence in content knowledge at the end of the summer institute at 2.7 on a scale of 0 to 3. Kansas City participants rated their confidence in content knowledge at the end of the professional development program at 2.3.

Responses to questions on inquiry based instruction in the external evaluators' teacher survey indicated that teacher participants had received input on inquiry learning and felt more comfortable teaching using inquiry learning. In addition, responses to questions on perceived improvement in teaching practice yielded mean scores of 2.6 to 2.7 (on a scale of 0 to 3) on using inquiry based teaching, implementing activities and developing materials for the classroom for Northeast participants. Kansas City participants had a mean of 2.4 to 2.6 on implementing activities and developing materials for use in the classroom.

An indirect goal of this initiative was to impact the mathematical understanding and achievement of children in Missouri, students of the teacher participants. Though standardized test scores on Missouri children were not available to us at this time,

qualitative and anecdotal data collected by our internal evaluation team from participating teachers indicate a perception that their students' mathematical understanding increased and their mathematics performance went up because of increased knowledge of teachers and changes in teacher behavior. One teacher, for instance, wrote:

"We've looked at number lines, graphs, word and number sentences, pictures and any other way we could think of to examine and work a math problem. My students have loved being able to share their own ideas with their peers. They love being able to show a new way to solve an old problem. And YES, it did help to strengthen the students' knowledge and understanding, as well. I don't know how many times this year I've heard students exclaim, "THAT'S how that works!" or "NOW I UNDERSTAND WHY WE DO THAT!"

Another teacher wrote: "I have seen great gains in my students' understanding of mathematical topics." And another wrote: "When I first started this process with gecKo mathematics, I did not dream my students would make the gains that they did. My 6th graders enjoyed doing the Korean mathematics and used many of the new strategies throughout the entire year. The hands on lessons were so enjoyable and made a lasting impression on my students. A teacher from another school wrote: "I know that my kiddos have gained confidence and knowledge. It's like sneaking up on them and teaching, as they learn, without their ever knowing they're learning. Pow!" Another teacher wrote: "My students remained engaged and productive throughout the lessons, which significantly cut down on behavior problems. My class has been excited about our math time and really look forward to it! I am amazed at their ability to explain their thoughts and help each other with the learning process." And another wrote: "...after implementing the gecKo methods into my core curriculum, I have found an overwhelming difference in students' performance. It was beneficial for students to use manipulatives, to develop number sense and basic operations of addition and subtractions problems.

A Kansas City teacher with a large Hispanic population credited the project with helping her English Language Learners, "The mathematical language is strong in *gecKo* because each lesson has a teacher explanation page. Using this language specifically in direct mini-lesson teaching has been very beneficial to my ELL students. Another Kansas City teacher claimed he, "... taught in three weeks what it took me two months to teach last year, and this year they all got it!" Another Kansas City teacher claimed her students made significant gains when she incorporated *gecKo* math into her regular Investigation in number data and space curriculum.

Responses to the project's internal evaluation instrument indicated considerable changes in teacher behavior over the course of the project. Wrote one teacher:

"I've changed my teaching style significantly as a result of *gecKo* mathematics. I am no longer one of those teachers who demonstrate to the class one way to solve a problem, expects the class to copy my example, and then assigns dozens of practice problems. Now, my students are actively involved in the learning process. They examine a problem from many angles and are invited to share their discoveries and methods with me and the other students. I've seen improvement from that alone." Wrote another: "GecKo has given me many tools to use with my students such as lessons that involve number lines,

pictures, manipulatives, vertical solving, horizontal solving, and word problems. I have incorporated these tools into my curriculum for all of my classes and have seen the benefits."

Another teacher wrote: "The 2 week institute was a true eye opener for me as an educator. The hands-on demonstrations and in-depth mathematical conversations were all valuable resources I took back to my own classroom. *gecKo* math has changed, not only the way I teach math, but how I think about mathematics.

A teacher from LaPlata wrote: "I did not feel this would have an impact on my classroom! I didn't have a connection to the *gecKo* math at first. Once I understood that it had underlying principles and wasn't just another curriculum, I began to see the benefits. I was pleasantly surprised to see my positive results. I have changed the way I teach in a way that seems to work better for me in teaching and assessing my students. Making students think was enjoyable and very successful in reaching the desired results. My students became thinkers and problem solvers rather than students who compute answers. Students had a deep understanding of the concept which enabled them to attack any problem rather than memorize facts. Students were always engaged, challenged and excited to learn. Student journals were great tools for assessments. I got much more for their explanations rather than graded papers.

And another LaPlata teacher wrote: "When I first started this process with gecKo mathematics, I did not dream my students would make the gains that they did. My 6th graders enjoyed doing the Korean mathematics and used many of the new strategies throughout the entire year. The hands on lessons were so enjoyable and made a lasting impression on my students. My teaching style has changed in a way that now I ask myself, "How else can I teach this? What hands-on approach can I use? How does this relate to Base 10?"

A third LaPlata teacher wrote: "The module I taught was geared toward all learning styles and levels. I was able to help my students find success in a "foreign mathematical system" with the learning goals we have. My students were engaged and anxious to see what we would be doing each day. My approach to teaching mathematics is now more hands-on and in-depth questioning where getting the answer is not more important than the approach students use to get the problem solving process going." And another: "The gecKo theory has changed the way I have introduced and taught math lessons. I was able to reach all learners by using a variety of activities to support mathematical concepts."

Participants did use assessment data they gathered to monitor the effectiveness of their instruction. For instance, more than one teacher reported she had used assessment data much more effectively than she had ever used it before.

One teacher wrote: "I'd like to comment on my use of pre-tests and post-tests. The only subject I regularly administered pre and post tests for was spelling! However, I have found that not only does the testing give me a greater grasp on what my students are accomplishing in class, but it also has taught them to use the pre-tests as a sort of introduction to our new unit. The first pretest I gave my class greatly frustrated them. I was faced with questions like, "Why are you testing me before you even teach this to me?" and "How are we supposed to know this? We haven't even learned it yet." and comments such as, "I can't do this." and "I know I am going to flunk this because I don't know what any of this means." Yet, they quickly learned that not only did the pretests

tell me where they were beginning, but it also told them a little about themselves, too. They looked forward to taking the post-tests so they could show me (and themselves!) how much they had learned. It ended up being a great motivator to learn and perform well.

At the last follow-up meetings in May, 2009, after the summer, 2008, institute and after the October/November Kansas City weekend institutes, we collected narrative responses to the question, "What did utilizing gecKo mathematics do for your students' understanding of the fundamentals of mathematics and your students' mathematics achievement?" Responses were overwhelmingly positive. All participants reported gains and increased understanding in their students. Teachers also reported they had taught the given concept more effectively than they had taught it before.

4. IMPLICATIONS FOR ADOPTION OF KOREAN-BASED MATERIALS FOR PROFESSIONAL DEVELOPMENT OF AMERICAN TEACHERS

Those who study what makes good professional development have found these elements to be important: a focus on content knowledge, an emphasis on active learning, promotion of coherence and encouragement of collaboration among teachers (Garet *et al.*, 1999; Iris, 2006). Clearly data from this project would indicate that these phenomena noted in the professional development literature were present and working effectively in this project.

One of the formative recommendations the external evaluators offered in October, 2008, was that we concentrate on a few rural districts, and concentrate our professional development. That advice makes sense. Data demonstrated that teachers who were present at the Institute in teams from their schools perceived themselves to be getting more out of the institute than those who attended alone. This was very clear in the institute. Where there were four or more teachers from the school or district attending the summer institute the principal and teachers expressed enthusiasm and reported changes in teacher behavior. Their participation and assessment results showed real changes. Two of the six teachers who attended the summer 2008 institute from the LaPlata district returned to earn the graduate credit in the project extension granted by the MDHE in summer 2009, and helped to teach the content to the new participants by presenting the modules they had developed the past year. Two more teachers from Green City attended the 2009 institute funded by the project extension on the recommendations of the four Green City teachers that had attended the 2008 institute. A third Green City teacher returned to present the extensive module she had developed for 7th and 8th graders, material she had presented at a state conference earlier in the year

One surprising finding we have from the internal anecdotal data in which participants

report they used formative evaluation more effectively than they ever had before shows the value of committing to pre- and post testing for formative purposes as well as for summative purposes. Being required to construct pre- and post-assessment instruments on the concept modules they developed to integrate into their classroom mathematics instruction seemed to make teachers much more aware of the benefits of formative and summative assessment.

What is important here is that our focus was on adding value to the mathematical content and pedagogical knowledge for participant teachers, and adding value to student mathematical learning for participants' students. An oft-cited 1998 analysis of evaluative studies of professional-development programs in mathematics and science found that programs focusing contextually "on subject knowledge and on student learning of particular subject matter" had a greater effect on student learning than those prescribing generic sets of "teaching behaviors (Kennedy, 1998)."

An important implication of the findings of this project is that engaging teachers in developing content knowledge from an Asian curriculum and creating and integrating units into their own curriculum, pre- and post-testing students and reporting results is an effective professional development model that can be easily adapted to use with other curricula. The *gecKo* mathematics institute in which this model was developed has all the characteristics of effective professional development that have been generalized from recent empirical evidence (*e.g.*, Kennedy, 1998; Garet *et al.*, 1999; 2001; Weiss *et al.*, 2003; 2006).

gecKo mathematics is a work in progress. Six chapters on fractions and decimals, both student texts and teacher manuals, are currently online at a Truman State University website http://gecKomath.truman.edu/lessons, as well as on a website in the public domain called www.gecKomath.com. As developer and editor of gecKo mathematics, Grow-Maienza hopes to see the entire series, now in manuscript form, online, available to teachers, parents, and students all over the English-speaking world. The Korean teacher manuals, from which gecKo mathematics was adapted, are rich in mathematical background for teachers. The Singapore and Japanese text series in English have no such teacher manuals. Korea has made a unique and important contribution to the western world in the development of consistent written materials for teachers to accompany the systematic coherent primary mathematics curriculum. The English-speaking world owes gratitude to Korean educators.

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