Signal Space Representation of Half-Symbol-Rate-Carrier PSK Modulations

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Abstract—This paper proposes a new concept of a signal constellation of the recently introduced half-symbol-rate-carrier phase-shift keying (HSRC-PSK) modulations for bandwidth-efficient high speed data communications. Since the HSRC-PSK modulations contain different symbol energies representing the same bit sequences due to the loss of orthogonality of their HSRC signals, it is very hard to represent the symbol using the conventional signal constellation. To resolve the problem, two different energies are assigned to represent one symbol for the HSRC offset quadrature phase shift keying (OQPSK) modulation. Similarly, the different energies exist to display the different symbol for HSRC minimum shift keying (MSK) modulation. With the proposed signal space representation, HSRC-PSK symbol can easily be shown with a two-dimensional scatter plot which provides helpful information of evaluating HSRC-PSK signal's quality.

Index Terms—half-symbol-rate-carrier, phase-shift-keying, signal space diagram, symbol energy

I. INTRODUCTION

Recently, HSRC-PSK modulations for high speed data communication have been introduced and their performances are analyzed [1][2]. The proposed modulations can be used to increase data transmission rate with reasonable degradation of bit-error-rate (BER) performances. The different symbol's energies, which are caused from the loss of othogonality of the carrier signals of the HSRC-PSK modulations, result in the different signal's spectra and the BER performances than those of the conventional ones [1]. These properties are analyzed theoretically and simulated to prove the theory [1]. However, so far, it is very hard to show signal space representation using a two-dimensional scatter diagram because of the different signal energies for the same symbol caused from the loss of othogonality of the carrier signals.

In this paper, a new concept of signal representation of HSRC-PSK signals using a two-dimensional scatter diagram called signal constellation is proposed to display the HSRC-PSK signals. An assumption that the orthogonality of the carrier signals in the HSRC-PSK are still valid and the different signal's energies for one symbol that come out from the result of this assumption is applied to make use of the conventional two-dimensional scatter diagram. Unlike a conventional PSK's symbol representation using a signal constellation, two different symbol states, which are determined by the former symbol states or sequences of data bits, are assigned to represent one symbol, which will be discussed in this paper.

Although the properties of the HSRC-PSK modulations are not identical to those of the conventional PSK modulations [1], similar forms of the conventional signal constellation are proposed in this paper. With the proposed signal constellation for HSRC-PSK modulations, it is possible to analyze a signal's quality for HSRC-PSK modulations with a similar form of the convenient two-dimensional scatter diagram.

II. HSRC-PSK SIGNALING & SPECTRA

Recently, HSRC-PSK modulations such as HSRC-QPSK, HSRC-OQPSK, and HSRC-MSK have been introduced and analyzed their spectra and BER performances [1]. Using two orthogonal half-symbol-rate-carrier components, $\sin(\pi/T_s)t$, $\cos(\pi/T_s)t$, where T_s=$2T_b$ in QPSK. The HSRC-QPSK signal can be described as (1).

$$s(t)=\sqrt{\frac{2E_b}{T_b}}m_I(t)\cos\left(\frac{\pi}{2T_b}\right)t+\sqrt{\frac{2E_b}{T_b}}m_Q(t)\sin\left(\frac{\pi}{2T_b}\right)t \quad (1)$$

where m_I and m_Q are data for odd bits and even bits, in the I channel and the Q channel respectively.

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Fig.3 shows quadrature modulation scheme and time domain waveforms of half-symbol-rate-carrier signals, which are called HSRC-QPSK and HSRC-OQPSK, respectively [1]. HSRC-OQPSK signal can be obtained by staggering the I and Q data by a bit period, T_b. This modulation can be implemented simply by delaying the quadrature data by a bit period, T_b, as shown in Fig. 4.

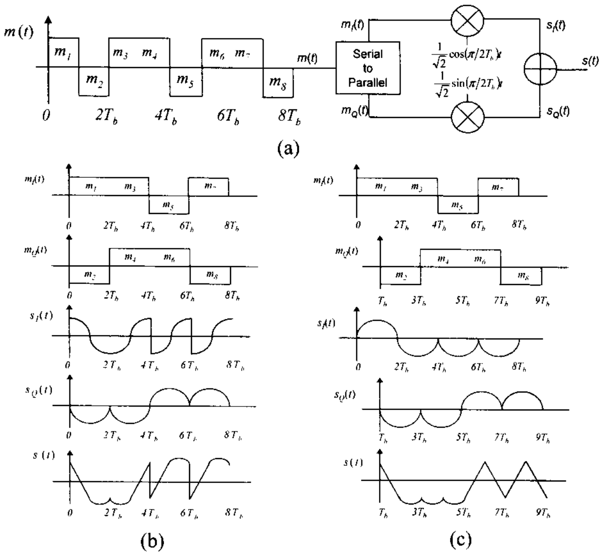

Fig. 1 HSRC-(O)QPSK (a) modulation scheme, time-domain waveforms of (b) HSRC-QPSK (c) HSRC-OQPSK

The cross product of the two carrier signals should be zero over the symbol period in order to keep the property of orthogonality of the two carrier signal. For the HSRC-QPSK, the orthogonality of the two HSRC carrier signals are valid over the symbol period of $2T_b$, calculated in (3). Note that, if the carrier frequency is below half symbol rate, $f_c<1/2T_s$, those two quadrature carriers are not orthogonal anymore over the period of T_s [1].

$$\int_0^{T_s=2T_b} \cos\left(\frac{\pi}{2T_b}\right)t \cdot \sin\left(\frac{\pi}{2T_b}\right)t \; dt = 0 \qquad (2)$$

The I and Q channel data are staggered by 1 bit time which is equal to T_b, hence, the symbol is changing every T_b instead of $2T_b$. Therefore, the integration period of the equation (2) is changed to T_b for the HSRC-OQPSK and the result of the integration is not zero, which means the orthogonality of the HSRC-OQPSK is no longer valid anymore. The modulated signal is essentially a MSK signal with zero carrier frequency and looks more like a pulse-shaped baseband waveform. Since the phase of carrier and the data have changed simultaneously over the symbol period of T_s, there are no discontinuities in time domain signal, as shown in Fig. 1(c). At the beginning of every bit period, in-phase and quadrature-phase data bits are mixed with two carriers respectively, at the level of 0 or $\pm A_c$. The in-phase carrier signal is at a level of either A_c or $-A_c$ and the quadrature-phase carrier signal is 0, at the beginning of every symbol time. The modulated signal level always starts with $\pm A_c$ and ends at $\pm A_c$, and there is no signal discontinuity in the modulated signal, which improves signal spectral efficiency. This is different from conventional OQPSK where there is still discontinuity in the modulated waveforms. Therefore, the spectrum of HSRC-OQPSK is similar to MSK that also has no discontinuity in its waveform, as shown in Fig. 3.

Furthermore, the HSRC-MSK modulation can be obtained [1]. In this case, the shaping signals for MSK are the same as half-symbol-rate-carrier signals. It can be obtained by applying the HSRC signal to the conventional MSK modulation format shown in Fig. 2. Therefore, the HSRC-MSK modulation signal is represented as (3). As shown in Fig.2, the modulated I and Q channel signals of the HSRC-MSK are similar to those of original binary data except the waveforms are shaped with $cos^2(\pi/2T_b)t$, $sin^2(\pi/2T_b)t$.

$$s(t) = m_I(t)\cos^2\left(\frac{\pi}{2T_b}\right)t + m_Q(t)\sin^2\left(\frac{\pi}{2T_b}\right)t \qquad (3)$$

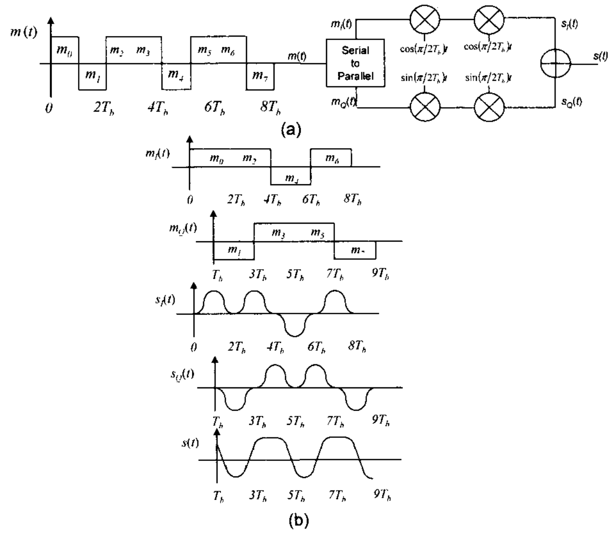

Fig. 2 HSRC-MSK (a) modulation scheme (b) time domain waveforms

The spectra and the BER performances of the HSRC-PSK modulations, named as HSRC-QPSK, HSRC-OQPSK, and HSRC-MSK, are different from those of the conventional ones. These properties of the

HSRC-PSK were analyzed theoretically and the simulations were compared to those of the conventional modulations [1].

Fig. 3 shows the comparison of the spectra of non-return-to-zero(NRZ), HSRC-QPSK, HSRC-OQPSK, and HSRC-MSK. Although the signal spectrum of conventional OQPSK is identical to that of QPSK, the spectrum of HSRC-OQPSK signal is similar to the spectrum of minimum-shift keying (MSK), not QPSK, shown in Fig 3. The properties of the HSRC-PSK modulations are discussed in [1].

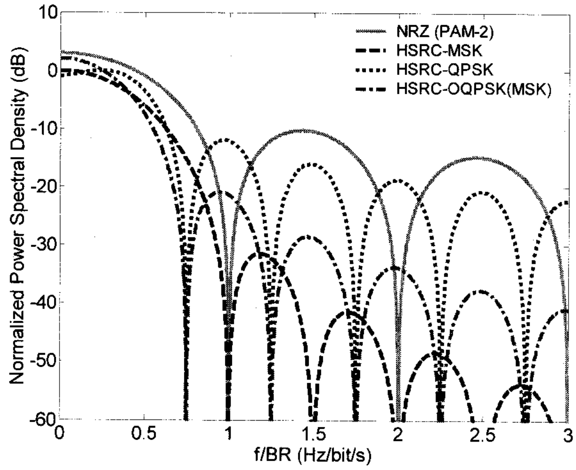

Fig. 3 Normalized power spectral density of the proposed HSRC-PSK modulations

III. SIGNAL SPACE REPRESENTATION

Two-dimensional symbol diagram is usually used to show the signal sets of the M-PSK modulations [3]. It represents the signal modulated with quadrature sinusoidal signals as a two-dimensional diagram on a complex plane. Fig. 4 shows a signal constellation and its decision threshold for M-ary PSK signal. The decision thresholds spaced equivalently by 2π/M [3]. The axes of the complex plane with cosine and sine carriers are often called the in-phase and the quadrature-phase. Each point, a symbol, represents a complex number and is equivalent to $\log_2(M)$-bits.

However, since the HSRC-PSK modulations have different symbol energies to represent the same data bits due to the loss of orthogonality of the carrier signals, it is hard to represent signal sets of using two-dimensional symbol representation. In this section, a new concept of a symbol representation for HSRC-PSK modulations will be proposed and discussed.

A. HSRC-QPSK

Fig. 5 shows the two-dimensional symbol representation called signal constellation of HSRC-QPSK modulations. Since the orthogonality of the HSRC-QPSK is still valid even though the HSRC signals are used as carriers, the signal constellation of HSRC-QPSK can be displayed as the conventional QPSK by plotting the signal using two orthogonal carrier signals, $\phi_1(t)=\cos(\pi/2T_b)t$, $\phi_2(t)=\sin(\pi/2T_b)t$ as in-phase (real) and quadrature-phase (imaginary) axes.

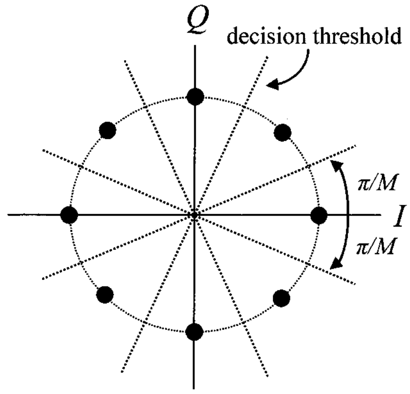

Fig. 4 Two-dimensional symbol representation and decision thresholds for M-ary PSK modulation signal

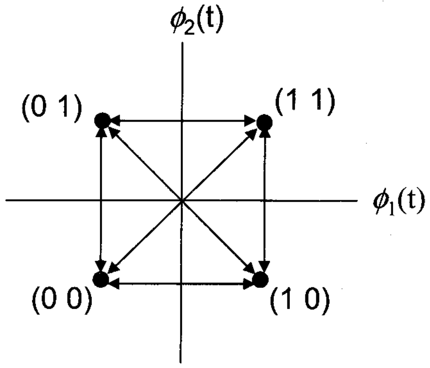

Fig. 5 HSRC-QPSK signal constellation

B. HSRC-OQPSK

The signal constellation of the HSRC-OQPSK cannot be easily represented by conventional OQPSK signal constellation because of the loss of orthogonality of the carrier signals which produces the different signal energies for the same symbol [1]. Therefore, it is necessary to analyze the energies of the modulated signal to propose a signal constellation for the HSRC-OQPSK. In HSRC-OQPSK signal, there are two different signal energies which make three different symbol energies named as $E_{s,t}$, $E_{s,t}$, $E_{s,p}$, respectively. The symbol energies are calculated using (3) and the results are represented as (4)~(6) [1]

$$E_s = \int_0^{T_s=2T_b} |s(t)|^2 dt \quad (3)$$

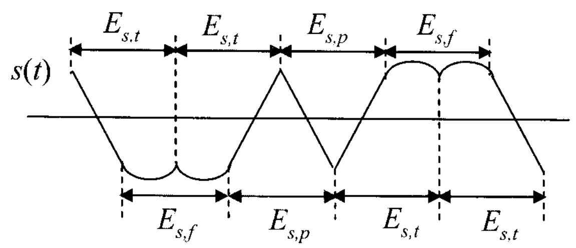

Fig. 6 The time domain signal of the HSRC-OQPSK modulation and its symbol energies.

$$E_{s,t} = A_c^2 \int_0^{2T_b} \left| \sin\left(\frac{\pi t}{2T_b} + \frac{\pi}{4}\right) \right|^2 dt \quad (4)$$

$$E_{s,f} = 2 \cdot A_c^2 \int_0^{T_b} \left| \sin\left(\frac{\pi t}{2T_b} + \frac{\pi}{4}\right) \right|^2 dt = k_f E_{s,t} \ ; \ k_f \equiv \left(\frac{\pi+2}{\pi}\right) \quad (5)$$

$$E_{s,p} = 2 \cdot A_c^2 \int_{T_b}^{2T_b} \left| \sin\left(\frac{\pi t}{2T_b} + \frac{\pi}{4}\right) \right|^2 dt = k_p \cdot E_{s,t} \ ; k_p \equiv \left(\frac{\pi-2}{\pi}\right) \quad (6)$$

The violation of the orthogonality of the HSRC-OQPSK modulation causes the three different symbol energies as calculated in (4)~(6). And two times of $E_{s,t}$ equals to the sum of $E_{s,f}$ and $E_{s,p}$. Now, with two signal states defined as $E_{s,f}$ and $E_{s,p}$, the three different symbol energies of the HSRC-OQPSK can be represented using a proposed signal constellation shown in Fig. 7.

There are two different energy states for each symbol defined $E_{s,f}$ and $E_{s,p}$, where the symbol transition sequence determines which energy state the signal should be in. Every energy state can only be occupied for a half symbol period (one bit period, T_b) and cannot not be occupied more than a half symbol period even if the data of the second bit has the same value. Then, the symbol can be defined from the bit sequences.

The clockwise transition, shown in the inner square of the signal constellation in Fig. 7, represents the low energy($E_{s,p}$) symbol transition. On the other hand, the counter-clockwise transition, shown in the outer square of the signal constellation in Fig. 6, represents the high energy symbol($E_{s,f}$) transition. The transition from the inner symbol to the outer symbol represents a normal energy($E_{s,t}$). That is, the low energy data transition traces the inner square in a clockwise direction and the high energy transition traces the outer square in a counter clockwise direction.

For example, the data transition from (0 1) to (0 0) with the energy of $E_{s,t}$ can start from the high energy state (outer symbol) of (0 1) to the high energy state of (0 0), stay for a half symbol period, and then move to and stay in the low energy state (inner symbol) of (0 0) for the second half symbol period. Also, the data transition can also start from the low energy state of (0 1) to the high energy state of (0 1) stay for half symbol period, and move to and stay in the high energy state of (0 0) for the second half symbol period. Note that, each symbol points can be occupied for a half symbol period which is equal to 1-bit time (T_b).

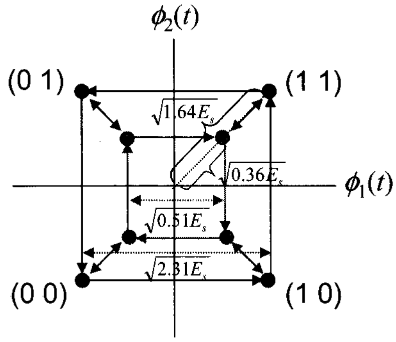

Fig. 7 HSRC-OQPSK signal constellation

C. HSRC-MSK

The signal of HSRC-MSK modulation is a pulse-shaped waveform of the original signal with a raised cosine filter, as shown in Fig. 8. With a similar approach to the HSRC-OQPSK, the signal constellation of HSRC-MSK can be obtained, shown in Fig. 8. There are two signal energies of the HSRC-MSK which are given by (7), (8) [1].

$$E_{s,t,m} = A_c^2 \int_0^{2T_b} \left| \sin^2\left(\frac{\pi t}{2T_b}\right) - \cos^2\left(\frac{\pi t}{2T_b}\right) \right|^2 dt = A_c^2 T_b \quad (7)$$

$$E_{s,f,m} = A_c^2 \int_0^{2T_b} \left| \sin^2\left(\frac{\pi t}{2T_b}\right) + \cos^2\left(\frac{\pi t}{2T_b}\right) \right|^2 dt = 2A_c^2 T_b \quad (8)$$

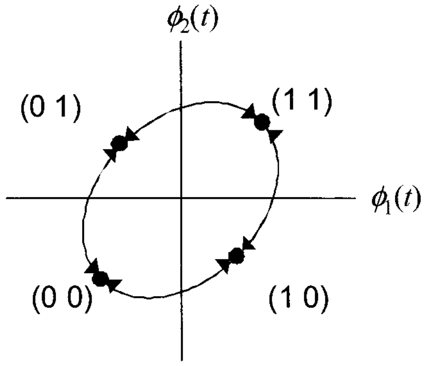

Fig. 8 HSRC-MSK Signal constellation

Fig. 8 shows the signal constellation of the HSRC-

MSK. From (7), it is analyzed that transition data bit sequences, such as (0 1) or (1 0), makes the symbol energy $E_{s,t,m}$. Similarly, the two same data bits sequences, such as (0 0) and (1 1), makes the symbol energy $E_{s,f,m}$ and $E_{s,t,m}$ and they are equal to the half energy of $E_{s,f,m}$. However, being different from the HSRC-OQPSK, only one symbol energy exists for the one symbol, shown in Fig. 8. Consequently, the signal constellation is an oval shape and not a circle shape like that of the conventional MSK's signal constellation.

V. CONCLUSIONS

Recently, HSRC-PSK modulation techniques were proposed to increase the data rate over band-limited channels [1]. A new concept of signal representation with a two-dimensional diagram has been proposed to show HSRC-PSK signals, such as HSRC-QPSK, HSRC-OQPSK, and HSRC-MSK. With the assumption that the orthogonality of the quadrature modulation system of the HSRC-PSK is effective and the different signal's energies can exist to represent one symbol, the modified conventional two-dimensional scatter diagram has been proposed as a signal constellation for the HSRC-PSK signals.

The two-dimensional signal space diagram of the HSRC-QPSK is the same as that of QPSK modulation due to the validation of the orthogonality of the carrier signals. On the other hand, a new concept of signal constellation has been proposed to show two different symbols and their energies. There are two –clockwise and counter-clockwise– transitions exit for the signal constellation. Similarly, a new signal constellation of the HSRC-MSK modulation has two different energy symbols. As a result, an oval type symbol transition occurs.

New concepts of symbol representation for HSRC-PSK modulations proposed in this paper offer useful information on representing the modulated signals and evaluating the signal's quality as the conventional signal constellation does.

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