Modeling the Calculation of Lateral Accelerations in Railway Vehicles as a Tool of Alignment Design

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Abstract

Railway track alignment Standards set a minimum lenght value for straight and circular alignments (art. 5.2.9.), in order to ensure passenger ride comfort in railway vehicles of which dynamic oscillations will thus have to be limited. The transitions between alignments can cause abrupt changes (usually called discontinuities or singular points of the alignment) of curvature, of rate of change of curvature or of rate of change of cant. A passenger is likely to experience effects due to the excitation of the elastic suspension of the vehicle which generates oscillations that are damped as the vehicle moves away from the singularity. The amplitude of these oscillations should be adequately attenuated by the damping of the suspension system within the interval between two successive singular points, especially to avoid resonances. Therefore minimum lengths between two successive singular points are stated in alignment standards. Nevertheless, these normative values can be overly conservative in some cases. As an alternative, track alignment designers could try to assess how much the excitation has been attenuated between two successive singular points and thus assess at which point a new singularity may be present without affecting ride comfort. Although such assessment can be made with commercial SW packages which simulate the dynamic behavior of a vehicle considered as a set of rigid bodies interconnected with elastic elements simulating the suspension systems (such as SIMPACK, ADAMS or VAMPIRE), a simplified and user-friendly computation method (based upon the analytical solution of differential equations governing the phenomenon) is made available in this paper to track design engineers, not always used to working with full dynamic models.

Keywords : *Lateral Acceleration, Track Alignment*

1. Introduction

The parameters used to assess the level of discomfort of passengers are the non-compensated lateral acceleration and its first time derivative. The alignment standards restrict the values of these parameters to guarantee that ride comfort thresholds are not exceeded.

At all points of the alignment, the value of these parameters will be the sum of a quasi-static component depending on vehicle speed and alignment characteristics, and a transient component due to the oscillations generated when running over singular points.

The alignment standard [1] states that the transient component depends on the rolling movement alone. However, there

are actually other movements that can increase the non-compensated lateral acceleration and its first time derivative.

In this research the effects of both the lateral displacement and the vertical axis rotation due to the presence of lateral suspensions have been considered, which means an improvement with respect to the guidelines of the standard for computing accelerations and their first derivative. However, the relative displacement between wheel and rail, as well as the vertical displacements and pitching movement of the vehicle have not been included in the computation as their contribution is considered irrelevant.

The values of the lateral acceleration and its rate of change depend on the speed, the curvature of the track, the cant and the characteristics of the vertical and lateral suspension of the vehicle.

Let us assume that:
• The track is composed of straight, circular and clothoid (or cubic parabolas) alignments, tangent and with the

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corresponding cant.

- The vehicle (suspended mass) is considered as a rigid solid and is connected to each bogie by means of a vertical suspension, a lateral suspension and a longitudinal suspension made up of elastic springs associated with viscous dampers.
- The accelerations felt within the vehicle are the inertia accelerations.
- All angles are considered small so that for all angles we have: $\alpha \approx \sin\alpha \approx tg\alpha$, $\cos\alpha \approx 1$, $\alpha^2 \approx 0$.

The non compensated lateral acceleration (a_g) of the vehicle body at each point of the track comes both from the centrifugal acceleration (V^2/R) and the component of the acceleration of the gravity (g) working parallel to the vehicle floor[1].

Whenever the vehicle passes over a point where is an abrupt change of curvature, of rate of change of curvature or cant, a damped transient acceleration (a_{tr}) appears, that shall be added to the permanent acceleration (a_i):

$$a_g = a_i + a_{tr} \tag{1}$$

The permanent part depends on the cant deficiency as follows:

$$a_i = (1+s) \cdot a_q \tag{2}$$

where: s= rolling flexibility coefficient of vehicle
$\quad\quad a_q = gI/b$
$\quad\quad I$ = cant deficiency
$\quad\quad b$ = distance between running threads of axle wheels

As said earlier, the European Standard EN 13803-1 [1] states that the transient acceleration comes from the rolling movement (θ):

$$a_g = a_i + a_{tr\theta} = (1+s) \cdot a_q + a_{tr\theta} \tag{3}$$

The transient acceleration $a_{tr\theta}$ may thus be computed as follows:

$$a_{tr\theta} = g \, e^{-\zeta\omega t}(C\sin\omega t + S\cos\omega t) \tag{4}$$

where: ζ: relative damping of the rolling movement
$\quad\quad \omega$: angular frequency of the rolling movement:
$\quad\quad\quad \omega = \sqrt{K_\theta/J_0} = 2\pi f_\theta$
$\quad\quad f_\theta$: own frequency of the rolling movement
$\quad\quad K_\theta$: stiffness to rolling rotation of vertical suspension
$\quad\quad J_0$: moment of inertia of the suspended mass with respect to the axis of rolling rotation
$\quad\quad C$ and S are the values shown in Table 1, where D is the cant:

Table 1. Constants C and S for different discontinuities

Discontinuity	$\Delta(D-s \cdot I)$	$\Delta(D-s \cdot I)$
C	0	$\Delta(D-s \cdot I)/b\omega$
S	$\Delta(D-s \cdot I)/b$	0

The expression of a_g is a sum of linear expressions a_i and trigonometric expressions $a_{tr\theta}$ (sines and cosines) of which all coefficients are defined, therefore it is easy to obtain its values at each time.

The rate of change of acceleration is simply the first time derivative.

The above equations may be easily programmed by track alignment designers and applied to track alignment design.

However, the rolling movement is not the only one contributing to the transient part of the non compensated lateral acceleration. As said before, both the lateral and vertical axis rotation movements, due to the presence of the lateral suspension, shall be taken into account.

Let $u = u(t)$ be the time variation of one of these two movements, either the lateral movement of the vehicle body (y) with respect to the position of equilibrium, or the rotation with respect to a vertical axis (α), considered as independent one another and independent of the rolling rotation (i.e. uncoupled movements), we have:

$$\ddot{u} + 2\varsigma\omega\dot{u} + \omega^2 u = 0 \tag{5}$$

where: ζ: relative damping of lateral suspensions for the lateral movement or vertical axis rotation
$\quad\quad \omega = \sqrt{K/M}$: angular frequency of the lateral movement or vertical axis rotation
$\quad\quad K$: lateral or α rotational stiffness
$\quad\quad M$: suspended mass or moment of inertia of the suspended mass with respect to the vertical axis

The values of $\ddot{u}$ corresponding to a discontinuity in curvature or in rate of change of curvature can be obtained similarly as they were for the rolling movement, using the following equation:

$$\ddot{u} = e^{-\zeta\omega t}(C\sin\omega t + S\cos\omega t) \tag{6}$$

The inertial non compensated acceleration (transient) parallel to the track plane due to each one of these movements will be $a_{tr} = \ddot{u}$.

Therefore, the overall lateral acceleration perceived by a passenger (inertial acceleration of the suspended mass) would come from the rolling rotation (θ), lateral movement (y) and vertical axis rotation (α). Therefore, the over-

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[1]The accelerations coming from the dynamic interaction of the vehicle with track irregularities (that depend on the level of conservation of the track and increase with the running speed) shall be added to the former.

all acceleration, assuming that these movements are uncoupled, may be found with:

$$a_J = a_i + a_{tr\theta} + a_{try} + a_{tr\alpha} \qquad (7)$$

The rate of change of such acceleration is also given by the first time derivative of (7).

2. Purpose of the Study

Its main purpose will be the comparison of results of non compensated lateral acceleration and rate of change of acceleration in the vehicle body, when running over track alignment singularities, by calculating the values of accelerations and first derivatives of accelerations in three different ways:

i. Considering only the transient component due to the rolling movement, according to equations (3) and (4).

ii. Considering the rolling movement as well as the lateral movement and vertical axis rotation, assuming that these movements are uncoupled, according to the equations (6) and (7).

iii. Using a full dynamic model including these three coupled movements.

As seen below, the analysis of the results shall allow to assess how well the simplified cases (i and ii) may compare with case (iii), and a possible application as a track alignment design tool.

3. Dynamic Calculations

Five case studies have been examined, calculating the movements of a vehicle passing over track alignment singularities, by means of a numerical simulation with a dynamic Finite Element Model.

3.1 Alignments Analyzed

The calculations of lateral excitations have been made for five typical alignments, of which the relevant features

are described in Table 2:

The first three alignments include a transition curve and are strictly compliant with the limits set for passenger trains (See [1]).

Alignments 4 and 5 correspond to UIC track devices for $V=100$ km/h and 160 km/h respectively for diverging track (See [4]).

Therefore the values obtained for the accelerations and the rate of change of accelerations will correspond to normative limits.

3.2 Characteristics of the Dynamic Model and Vehicle Parameters

The characteristics of the vehicle used for the dynamic model are those corresponding to the UIC vehicle, described in [3], and summarized as follows:

Mass of vehicle body (m)	32000 kg
Roll Inertia	56800 kgm^2
Vertical axis rotation Inertia	1970000 kgm^2
Distance between running threads of axle wheels	
	1500 mm
Distance between bogie centers (E)	20 m
Height of vehicle body center of gravity with respect to rolling rotation axis (h)	1.21 m
Stiffness of lateral suspension (Ky) *(per bogie)*:	
	305 kN/m

Damping of lateral suspension *(per bogie)*: 59 kN/(m/s)

The characteristics of the vertical suspension have been simplified by using an elastic spring with stiffness to rotation and damping adapted to the usual values, i.e., to obtain a rolling coefficient of $s = 0.4$, an angular frequency of the rolling movement of $\omega = 3.03$ rad/s, and a relative damping of $\varsigma = 0.1$. Fig. 1 shows a schematic diagram of the dynamic model used.

Assuming that the positive axis x of the figure indicates the direction of the advancing vehicle, the curves of the tracks are clockwise.

Table 2. Alignments analysed

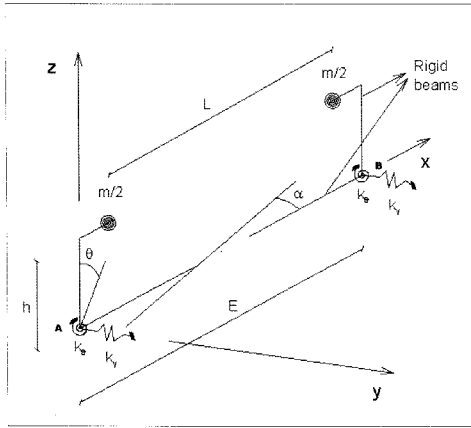|	ALIGNMENT 1	ALIGNMENT 2	ALIGNMENT 3	ALIGNMENT 4	ALIGNMENT 5
R (m)	593	1646	6621	1200	3000
L_{curve} (m)	40	67	200	65	101
$L_{clothoid}$ (m)	91	152	134	0	0
V (km/h)	120	200	300	100	160
D_{max} (mm)	137	137	80	0	0
I_{max} (mm)	150	150	80	98	101
$(dI/dt)_{max}$ (mm/s)	55	55	50	-	-
$(dD/dt)_{max}$ (mm/s)	50	50	50	-	-

Fig. 1 Dynamic model

4. Results of Dynamic Calculations

The values obtained for the movements of the dynamic model when passing over the different alignments considered are used to compute the non compensated lateral acceleration and its rate of change in the vehicle.

These accelerations have been calculated at a point located at the height of the axis of rolling of the vehicle (points A and B in Fig. 1). We assume that this axis is approximately located on the vehicle floor plane.

The calculations have been made for four cases:

1. Three uncoupled models where there is alternatively a restriction on two of the three degrees of freedom of the vehicle, i.e.:
   • Rolling rotation (θ)
   • Lateral movement (y)
   • Vertical axis rotation (α)
2. Coupled model, where the rigid vehicle has three degrees of freedom

The following Figs. 2 to 11 show the values of the non compensated lateral acceleration and its rate of change. There are three curves for each track, describing the time evolution of the acceleration and rate of change of acceleration:
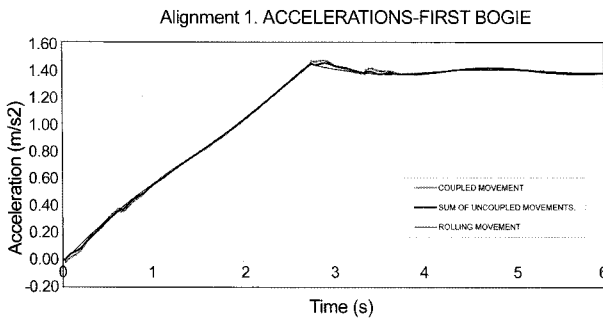
a. Coming only from the calculation with rolling movement.
b. Coming from the sum of the accelerations obtained with the calculations of the uncoupled models (uncoupled calculation).
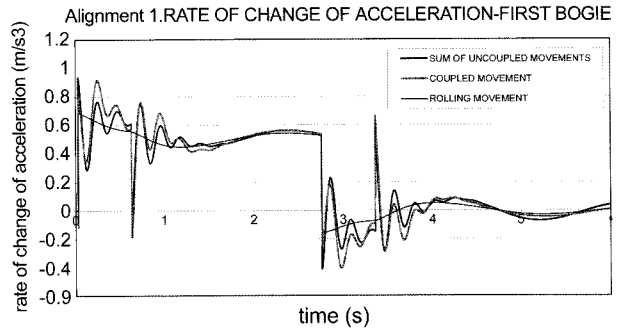

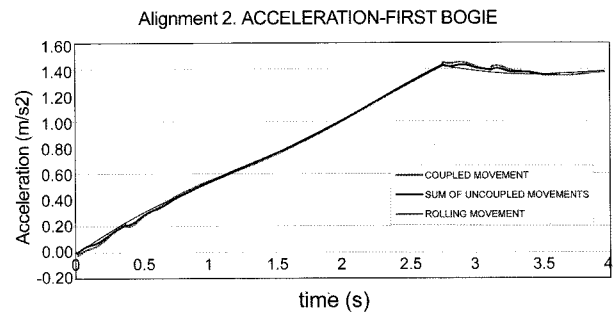**Fig. 3** Rate of change of acceleration. Alignment 1
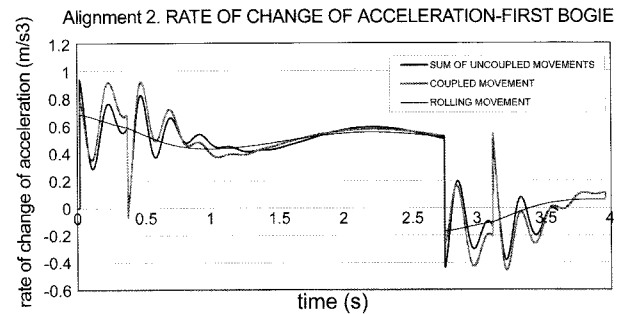

Fig. 4 Accelerations. Alignment 2

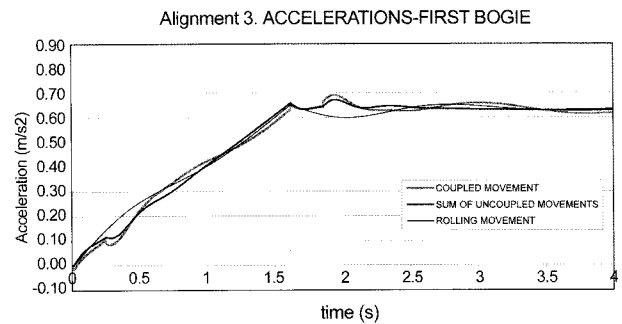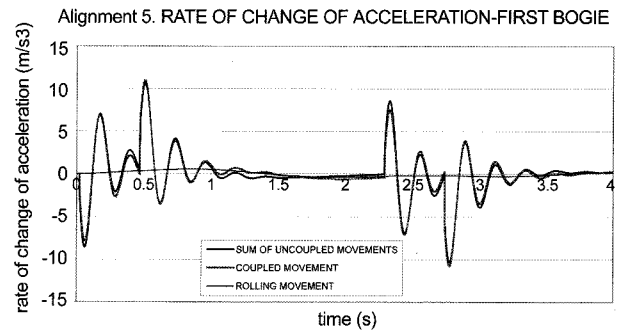
Fig. 5 Rate of change of Acceleration. Alignment 2

Fig. 2 Accelerations. Alignment 1

Fig. 6 Accelerations. Alignment 3
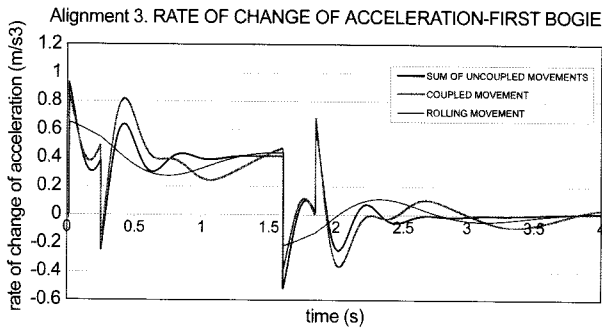
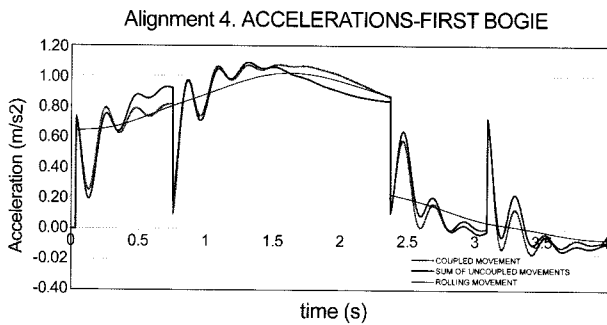**Fig. 7** Rate of change of Acceleration. Alignment 3
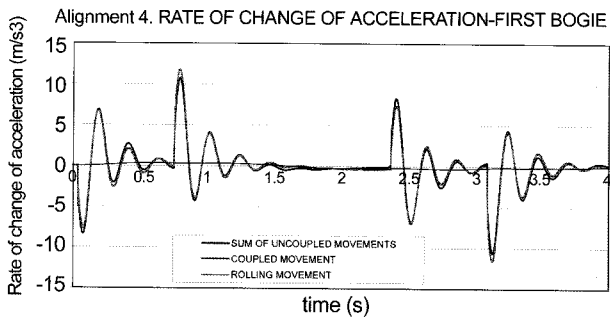

Fig. 8 Accelerations. Alignment 4
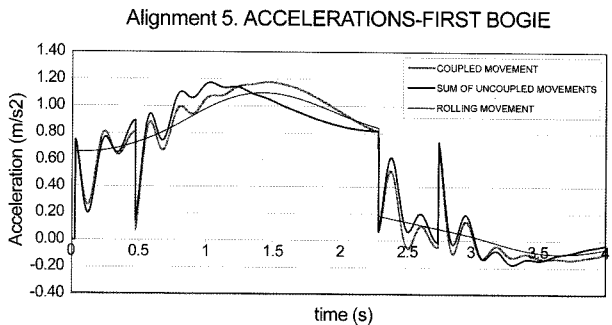

Fig. 9 Rate of change of Acceleration. Alignment 4

Fig. 10 Acceleration. Alignment 5

c. Coming from the calculation with the full model (coupled calculation).

Fig. 11 Rate of change of Acceleration. Alignment 5

5. Analysis of Results

The analysis of the above results show the following:

For osculating alignments (Alignments 1 to 3)

As regards alignments 1 to 3, that may be considered representative for circular curves with transition and cant, the values of transient accelerations are not significant against the permanent values.

However, the transient values of the rate of change of acceleration are significant and may be up to three times the permanent values. Therefore, a dynamic calculation is needed to assess the maximum values of the rate of change of acceleration.

On the other hand, the dynamic study where only the rolling movement has been considered is not representative enough: there is no coincidence with the coupled calculation as regards frequency and the maximum values drift from those of the coupled calculation (the coupled calculation may produce values almost double as those of the rolling movement).

Finally, the sum of the calculations for each uncoupled degree of freedom shows a close proximity to the values obtained from the coupled calculation, as frequency is almost the same and the maximum values are slightly different (less than 5%).

For non-osculating alignments (Alignments 4 and 5)

As regards alignments 4 and 5, that may be considered representative for circular curves without transition and without cant, transient values of both acceleration and rate of change of acceleration are greater than the permanent values and thus a dynamic calculation is needed to assess the maximum values of both variables.

The dynamic study where only the rolling movement has been considered provides quite different results with respect to the coupled calculation and would thus be incomplete.

The sum of the calculations for each uncoupled degree of freedom shows a close proximity to those of the coupled calculation, as frequency is quite similar and the maximum values are slightly separated (less than 5%).

6. Conclusions

For osculating alignments the transient values of accelerations are not significant against the permanent values, but a dynamic analysis has to be performed in order to assess the maximum values of the rate of change of acceleration.

A simplified analysis of each uncoupled degree of freedom and a further addition of the corresponding accelerations for each degree of freedom may be deemed representative enough as regards the cases of study.

For non osculating alignments the transient values of both the accelerations and the rate of change of accelerations are significant against the permanent values, therefore a dynamic analysis has to be performed.

As for the osculating alignments, a simplified analysis of each uncoupled degree of freedom and a further addition of the corresponding accelerations for each degree of freedom may be deemed representative enough as regards the cases of study.

The uncoupled analysis for each degree of freedom may be easily approached and programmed by track alignment designers and thus be used as a practical tool for track alignment design optimization, allowing in particular the analysis of cases not considered in Track Alignment Standards.

Acknowledgments

Reference

1. European Standard EN 13803-1, "Railway applications - Track alignment design parameters -Track gauges 1435 mm and wider - Part 1: Plain line", January 2006.

2. European Preliminary Standard prEN 13803-2, "Railway applications - Track alignment design parameters -Track gauges 1435 mm and wider - Part 2: Switches and crossings and comparable alignment design situations with abrupt changes of curvature", December 2005.

3. Rapport ERRI D 181/DT 329 "Lateral forces on railway bridges. Parametric study", July 1995.

4. Specification UIC 711, "Geometry of points and crossings with UIC rails permitting speeds of 100 km/h or more on the diverging track", January 1981.

5. Y.-G. Kim, H.-B. Kwon, S.-W. Kim, C.-K. Kim and T.-W. Kim, "Correlation of ride comfort evaluation methods for railway vehicles", Journal of RAIL AND RAPID TRANSIT. Vol. 217, No. 2, pp. 73-88, 2003.

List of Symbols

a_i : permanent acceleration
a_{tr} : transient acceleration
$a_{tr\theta}$: transient acceleration due to the rolling movement
a_J : non compensated lateral acceleration
a_q : gI/b
b : distance between running threads of axle wheels
D : cant
E : distance between bogie centers
f_θ : own frequency of the rolling movement
g : gravity acceleration
h : height of vehicle body center of gravity with respect to rolling rotation axis
I : cant deficiency
J_0 : moment of inertia of the suspended mass with respect to the axis of rolling rotation
K : lateral or α rotational stiffness
K_θ : stiffness to rolling rotation of vertical suspension
K_y : stiffness of lateral suspension
m : mass of vehicle body
M : suspended mass or moment of inertia of the suspended mass with respect to the vertical axis
s : rolling flexibility coefficient of vehicle
y : lateral movement
α : yaw angle or nosing
θ : rolling angle
ς : relative damping of the rolling movement
ω_q : angular frequency of the rolling movement:
$$\omega_\theta = \sqrt{K_\theta/J_0} = 2\pi f_\theta$$