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Estimation for the half triangle distribution based on Type-I hybrid censored samples

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Abstract

A hybrid censoring is a mixture of Type-I and Type-II censoring schemes. This paper deals with estimation based on Type-I hybrid censored samples from the half triangle distribution. We derive some estimators of the scale parameter of the half triangle distribution based on Type-I hybrid censored samples. We compare the proposed estimators in the sense of the mean squared error for various censored samples.

Keywords: Approximate maximum likelihood estimator, half triangle distribution, Type-I hybrid censored sample.

1. Introduction

The half-triangle distribution has the following cumulative distribution function (cdf)

$$F(x) = 1 - \left(1 - \frac{x}{\theta}\right)^2, \quad 0 < x < \theta, \tag{1.1}$$

and the probability density function (pdf)

$$f(x) = \frac{2}{\theta}\left(1 - \frac{x}{\theta}\right), \quad 0 < x < \theta. \tag{1.2}$$

A triangle distribution was applied to a kernel function in nonparametric density estimation. Johnson (1997) studied the possibility of using the more intuitively obvious triangular distribution as a proxy for the beta distribution. Some properties of the triangular distribution was studied by Balakrishnan and Nevzorov (2003). Lee *et al*. (2008) proposed the

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approximate maximum likelihood estimators (AMLEs) of the scale parameter in a triangular distribution based on multiply Type-II censored samples by the approximate maximum likelihood estimation methods. Recently, Kang (2007) proposed some explicit estimators for the half triangle distribution based on multiply Type-II censored samples. Hybrid censoring scheme is a mixture of Type-I ad Type-II censoring schemes.

Let us assume that the ordered lifetimes be denoted by $X_{1:n}, X_{2:n}, ..., X_{n:n}$. Epstein (1954) introduced Type-I hybrid censoring scheme, and considered lifetime experiments assuming that the lifetime of each unit follows an exponential distribution. Type-I hybrid censoring scheme in which the life testing experiment is terminated at a random time $T^* = \min\{X_{r:n}, T\}$, where $1 \le r \le n$ and $T \in (0, \infty)$ are fixed in advance. As with a conventional Type-I censoring scheme, the termination point here is at most T. A schematic illustration is depicted in Figure 1.1, when $x_{1:n} < ... < x_{r:n}$ denote the observed failure times if $x_{r:n} < T$ and $x_{1:n} < ... < x_{d:n}$ denoted the observed failure times if $x_{d:n} < T$, $d < r$ and the $(d+1)^{th}$ failure does not take place before times point T.

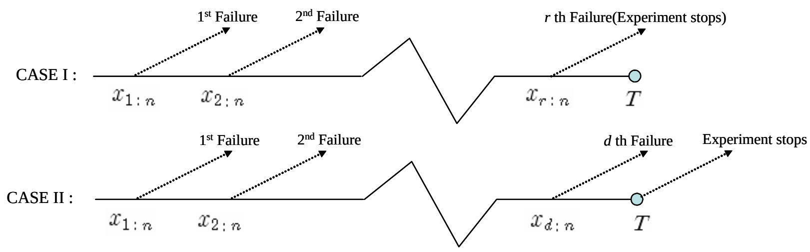

Figure 1.1 The Type-I hybrid censoring scheme

Childs *et al.* (2003) proposed the Type-II hybrid censoring scheme, and it can be described as follows. Put n identical items on test, and then stop the experiment at the random time $T^* = \max\{X_{r:n}, T\}$, where r and T are prefixed numbers. Then, the Type-II hybrid censoring scheme ensures that at least r failures take place.

In this paper, we derive some estimators of the parameter θ in the half triangle distribution based on Type-I hybrid censored samples. We also compare the proposed estimators in the sense of the mean squared error (MSE) for various censored samples.

2. Estimation for the scale parameter

We will discuss the maximum likelihood estimation of the scale parameter based on Type-I hybrid censored samples. The log-likelihood function for two different cases follow. Based on the observed data, the log-likelihood function (without the constant term) for Case I is

$$\ln L_I = (n-r)\ln[1 - F(x_{r:n})] + \sum_{i=1}^{r} \ln f(x_{i:n}), \tag{2.1}$$

and for Case II, it is

$$\ln L_{II} = (n-d)\ln[1 - F(T)] + \sum_{i=1}^{d} \ln f(x_{i:n}). \tag{2.2}$$

The random variable $Z_{i:n} = X_{i:n}/\theta$ then has a standard half triangle distribution with pdf and cdf;

$$f(z_{i:n}) = 2(1 - z_{i:n}) \text{ and } F(z_{i:n}) = 1 - (1 - z_{i:n})^2.$$

On differentiating the log-likelihood function with respect to θ of (2.1) and (2.2) in turn and equation to zero, we obtain the estimating equations as

$$\frac{\partial \ln L_I}{\partial \theta} = -\frac{1}{\theta}\left[r - 2(n-r)\frac{1}{(1 - z_{r:n})} z_{r:n} - \sum_{i=1}^{r} \frac{1}{(1 - z_{i:n})} z_{i:n} \right] = 0, \tag{2.3}$$

and

$$\frac{\partial \ln L_I}{\partial \theta} = -\frac{1}{\theta}\left[d - 2(n-d)\frac{1}{(1 - z_T)} z_T - \sum_{i=1}^{d} \frac{1}{(1 - z_{i:n})} z_{i:n} \right] = 0, \tag{2.4}$$

respectively, where $z_T = T/\theta$.

We can find the maximum likelihood estimator (MLE) of θ as values $\widetilde{\theta}$ that maximize the log-likelihood function in (2.1) and (2.2) by solving the equation $\partial \ln L_I/\partial\theta = 0$ and $\partial \ln L_{II}/\partial\theta = 0$. Since the equation (2.3) and (2.4) cannot be solved explicitly, some numerical method must be employed.

Since the log-likelihood equation do not admit explicit solutions, it will be desirable to consider an approximation to the likelihood equation which provide us with explicit estimator for the scale parameter.

We expand the functions $z_{i:n}/(1 - z_{i:n})$ and $1/(1 - z_{i:n})$ in Taylor series around the points ξ_i, where $\xi_i = F^{-1}(p_i) = 1 - \sqrt{q_i}$ and $q_i = 1 - p_i$.

First, we can approximate this function by

$$\frac{z_{i:n}}{1 - z_{i:n}} \approx -\frac{\xi_i^2}{q_i} + \frac{1}{q_i} z_{i:n}. \tag{2.5}$$

By substituting equation (2.5) into equation (2.3), we may approximate the equation in (2.3) by

$$\frac{\partial \ln L_I}{\partial \theta} = -\frac{1}{\theta}\left[r + 2(n-r)\left\{ \frac{\xi_r^2}{q_r} - \frac{1}{q_r} z_{r:n} \right\} + \sum_{i=1}^{r} \left\{ \frac{\xi_i^2}{q_i} - \frac{1}{q_i} z_{i:n} \right\} \right] = 0. \tag{2.6}$$

We can derive an estimator of θ as follows;

$$\widehat{\theta}_{I,1} = \frac{B_1}{A_1}, \tag{2.7}$$

where

$$A_1 = r + 2(n-r)\frac{\xi_r^2}{q_r} + \sum_{i=1}^{r} \frac{\xi_i^2}{q_i} \quad \text{and} \quad B_1 = 2(n-r)\frac{1}{q_r} X_{r:n} + \sum_{i=1}^{r} \frac{1}{q_i} X_{i:n}.$$

Since $\xi_i^2/q_i > 0$ and $1/q_i > 0$, the estimator $\widehat{\theta}_1$ is always positive.

Second, we can approximate this function by equations

$$\frac{1}{1-z_{i:n}} \approx \frac{(1-2\xi_i)}{q_i} + \frac{1}{q_i} z_{i:n}. \tag{2.8}$$

By substituting equation (2.8) into equation (2.3), we may approximate the likelihood equation in (2.3) by

$$\frac{\partial \ln L_{II}}{\partial \theta} = -\frac{1}{\theta}\left[r - 2(n-r)\left\{ \frac{(1-2\xi_r)}{q_r} + \frac{z_{r:n}}{q_r} \right\} z_{r:n} - \sum_{i=1}^{r} \left\{ \frac{(1-2\xi_i)}{q_i} + \frac{z_{i:n}}{q_i} \right\} z_{i:n} \right] = 0. \tag{2.9}$$

Equation (2.9) is a quadratic equation in θ, with its roots given by

$$\widehat{\theta}_{I,2} = \frac{-A_2 \pm \sqrt{A_2^2 - 4rB_2}}{2r}, \tag{2.10}$$

where

$$A_2 = -2(n-r)\frac{1-2\xi_r}{q_r} X_{r:n} - \sum_{i=1}^{r} \frac{1-2\xi_i}{q_i} X_{i:n}$$

and

$$B_2 = -2(n-r)\frac{1}{q_r} X_{r:n}^2 - \sum_{i=1}^{r} \frac{1}{q_i} X_{i:n}^2.$$

Since $B_2 < 0$, only one root is admissible, and hence the AMLE of θ is given by

$$\widehat{\theta}_{I,2} = \frac{-A_2 + \sqrt{A_2^2 - 4rB_2}}{2r}. \tag{2.11}$$

Similarly for Case II, the AMLE of θ can be obtained by solving

$$\frac{\partial \ln L_{II}}{\partial \theta} = -\frac{1}{\theta}\left[d + 2(n-d)\left\{ \frac{\xi_{d^*}^2}{q_{d^*}} - \frac{1}{q_{d^*}} z_T \right\} + \sum_{i=1}^{d} \left\{ \frac{\xi_i^2}{q_i} - \frac{1}{q_i} z_{i:n} \right\} \right] = 0, \tag{2.12}$$

and

$$\frac{\partial \ln L_{II}}{\partial \theta} = -\frac{1}{\theta}\left[d - 2(n-d)\left\{ \frac{(1-2\xi_{d^*})}{q_{d^*}} + \frac{1}{q_{d^*}} z_T \right\} z_T - \sum_{i=1}^{d} \left\{ \frac{(1-2\xi_i)}{q_i} + \frac{1}{q_i} z_{i:n} \right\} z_{i:n} \right] = 0, \tag{2.13}$$

respectively, where $\xi_{d^*} = F^{-1}(p_{d^*}) = 1 - \sqrt{q_{d^*}}$, $\quad p_{d^*} = (p_d + p_{d+1})/2$ and $q_{d^*} = 1 - p_{d^*}$.

Therefore, in this case following the same steps as before, we obtain the AMLE of θ as follows;

$$\widehat{\theta}_{II,1} = \frac{D_1}{C_1} \quad \text{and} \quad \widehat{\theta}_{II,2} = \frac{-C_2 + \sqrt{C_2^2 - 4rD_2}}{2r}, \tag{2.14}$$

where

$$C_1 = d + 2(n-d)\frac{\xi_{d^*}^2}{q_{d^*}} + \sum_{i=1}^{d}\frac{\xi_i^2}{q_i}, \quad D_1 = 2(n-d)\frac{1}{q_{d^*}}T + \sum_{i=1}^{d}\frac{1}{q_i}X_{i:n},$$

$$C_2 = -2(n-d)\frac{1-2\xi_{d^*}}{q_{d^*}}T - \sum_{i=1}^{d}\frac{1-2\xi_i}{q_i}X_{i:n}, \text{ and } D_2 = -2(n-d)\frac{1}{q_{d^*}}T^2 - \sum_{i=1}^{d}\frac{1}{q_i}X_{i:n}^2.$$

Cheng and Amin (1983) proposed maximum product of spacings estimator (MPSE) method. The new suggested method can be applied in principle to the estimation of parameters in any continuous univariate distribution. The MPSE of θ is the value of this parameter, which maximize the geometric mean of spacings and expressed below after taking logarithm;

$$G = \frac{1}{n+1}\sum_{i=1}^{n+1}\ln(G_i), \tag{2.15}$$

where $G_i = F(x_i) - F(x_{i-1})$, $i = 1, 2, ..., n+1$, $x_0 = 0$ and $x_{n+1} = \infty$.

We can modify equation (2.15) based on Type-I hybrid censored samples as follows; For Case I, it is

$$G_I = \frac{1}{r+1}\sum_{i=1}^{r}\ln(G_i), \tag{2.16}$$

and for Case II, it is

$$G_{II} = \frac{1}{d+1}\sum_{i=1}^{d}\ln(G_i). \tag{2.17}$$

On differentiating the equations (2.16) and (2.17) with respect to θ in turn and equation to zero. The resulting equations are given below.

$$\sum_{i=1}^{r}\frac{2X_{i:n} - 2X_{i:n}^2 - 2X_{i-1:n} + 2X_{i-1:n}^2}{2X_{i:n}\theta - X_{i:n}^2 - 2X_{i-1:n}\theta + X_{i-1:n}^2} = 0, \tag{2.18}$$

and

$$\sum_{i=1}^{d}\frac{2X_{i:n} - 2X_{i:n}^2 - 2X_{i-1:n} + 2X_{i-1:n}^2}{2X_{i:n}\theta - X_{i:n}^2 - 2X_{i-1:n}\theta + X_{i-1:n}^2} = 0, \tag{2.19}$$

respectively.

Since the equation (2.18) and (2.19) cannot be solved explicitly, solution of (2.18) and (2.19) are obtain by using numerical method. We can find the MPSE of θ as values $\bar{\theta}$.

Woo (2007) considered the inference on the reliability, and derived the k th moment of the ratio of two independent half triangle distributions with different supports. He also proposed unbiased estimators of the parameter θ in the half triangle distribution based on complete samples as follows;

$$\theta^* = \frac{X_{n:n}}{c}, \tag{2.20}$$

where

$$c = \frac{n \sum_{i=0}^{n-1} (-1)^i 2^{n-i} \binom{n-1}{i}}{(n+1+i)(n+i+2)}.$$

Finally, for Type-I hybrid censored samples, we can modify the estimator (2.20) as follows; For Case I is

$$\theta_I^* = \frac{X_{r:n}}{c_1}, \tag{2.21}$$

where

$$c_1 = \frac{r \sum_{i=0}^{r-1} (-1)^i 2^{r-i} \binom{r-1}{i}}{(r+1+i)(r+i+2)},$$

and for Case II, it is

$$\theta_{II}^* = \frac{T}{c_2}, \tag{2.22}$$

where

$$c_2 = \frac{d \sum_{i=0}^{d-1} (-1)^i 2^{d-i} \binom{d-1}{i}}{(d+1+i)(d+i+2)}.$$

3. Simulated results

From the above formula, the mean squared errors of the estimators are simulated by Monte Carlo method (based on 10,000 Monte Carlo runs) for sample size n =20, 40, and different r and T values. We mainly compare the performances of the proposed estimators of the scale parameter θ, in terms of their biases and mean squared errors (MSEs) for different censoring scheme. The convergence of Newton-Raphson method depended on the choice of the initial values. For this reason, the AMLE $\widehat{\theta}_1$ ($\widehat{\theta}_{I,1}$ and $\widehat{\theta}_{II,1}$) was used as starting values for the iterations, the MLE $\widetilde{\theta}$ and MPSE $\bar{\theta}$ are obtained using Newton-Raphson method.

From Table 3.1 the following general observations can be made. For all the method, for fixed n and r the MSEs decrease as T increases, for fixed r and T the MSEs decrease as n increases from 20 to 40, for fixed n and T the MSEs decrease as r increases.

The performances of the AMLEs and MLE are very similar in all aspects. The AMLE $\widehat{\theta}_1$ is a linear function of available order statistic and the performance is good. The AMLE $\widehat{\theta}_2$ is generally more efficient than $\widehat{\theta}_2$ in the sense of the MSE when $n > r$.

The MSEs of the estimator θ^* (θ^*_I and θ^*_{II}) and $\bar{\theta}$ are larger than the AMLEs or MLE for all scheme. For $T \geq 0.7$ and $n = r$ =20, the estimator θ^* is more efficient than the others.

As note earlier, as $T \to \infty$ the results for Type-I hybrid censoring reduce to results for conventional Type-II censoring. Therefore, when T is large, or equivalently when there is a high probability that $X_{r:n} < T$, we would expect the results for Type-I hybrid censoring to agree closely with the results for conventional Type-II censoring. By taking $\theta = 1$, $r = 19$, $n = 20$, $T = 0.5$, we get $P(X_{12:20} < 0.5) \approx 95.91\%$ (see, equation 3.1 and Table 3.2).

Since this probability is high, it is reasonable that the results agree quite closely with conventional Type-II censoring in this case. But when $r = 18$, $P(X_{18:20} < 0.5) \approx 9.13\%$. As expected, for fixed n and r the $P(X_{r;n} < T)$ increases as T increases, for fixed n and T the $P(X_{r;n} < T)$ increases as r decrease (see, Table 3.2).

$$P(X_{12:20} < 0.5) = \sum_{i=12}^{20} \binom{20}{i} \left\{1 - \left(1 - \frac{0.5}{1}\right)^2\right\}^i \left(1 - \frac{0.5}{1}\right)^{2(20-i)} = 0.9591. \qquad (3.1)$$

Table 3.1 The relative mean squared errors and biases for the estimators of the scale parameter θ

T	n	r	MSE(Bias)				
			$\widehat{\theta}_1$	$\widehat{\theta}_2$	$\widehat{\theta}$	θ^*	$\bar{\theta}$
0.5	20	20	0.0435(0.0176)	0.0459(-0.0108)	0.0477(0.0590)	0.1262(-0.3551)	0.1958(-0.4396)
		18	0.0448(0.0155)	0.0446(-0.0086)	0.0505(0.0539)	0.1306(-0.3603)	0.1964(-0.4403)
		16	0.0492(0.0046)	0.0456(-0.0081)	0.0573(0.0320)	0.1552(-0.3885)	0.2027(-0.4474)
		14	0.0527(-0.0133)	0.0498(-0.0155)	0.0591(-0.0005)	0.2136(-0.4522)	0.2197(-0.4665)
		12	0.0573(-0.0240)	0.0564(-0.0226)	0.0598(-0.0191)	0.2965(-0.5341)	0.2380(-0.4869)
	40	40	0.0202(0.0076)	0.0211(-0.0081)	0.0212(0.0280)	0.1633(-0.4041)	0.1844(-0.4276)
		35	0.0205(0.0070)	0.0208(-0.0075)	0.0218(0.0267)	0.1646(-0.4055)	0.1847(-0.4280)
		30	0.0234(-0.0038)	0.0219(-0.0082)	0.0258(0.0062)	0.1995(-0.4435)	0.2005(-0.4457)
		25	0.0258(-0.0126)	0.0256(-0.0118)	0.0264(-0.0105)	0.2983(-0.5414)	0.2377(-0.4868)
		20	0.0342(-0.0133)	0.0343(-0.0126)	0.0343(-0.0124)	0.4190(-0.6439)	0.2493(-0.4993)
0.6	20	20	0.0329(0.0043)	0.0364(-0.0373)	0.0360(0.0525)	0.0579(-0.2399)	0.1272(-0.3497)
		18	0.0363(-0.0040)	0.0329(-0.0268)	0.0429(0.0309)	0.0760(-0.2682)	0.1327(-0.3570)
		16	0.0394(-0.0210)	0.0360(-0.0243)	0.0447(-0.0056)	0.1295(-0.3450)	0.1618(-0.3944)
		14	0.0425(-0.0285)	0.0417(-0.0265)	0.0442(-0.0226)	0.2074(-0.4416)	0.2058(-0.4487)
		12	0.0523(-0.0283)	0.0523(-0.0261)	0.0527(-0.0251)	0.2958(-0.5328)	0.2355(-0.4837)
	40	40	0.0150(0.0008)	0.0171(-0.0252)	0.0155(0.0249)	0.0832(-0.2872)	0.1099(-0.3279)
		35	0.0167(-0.0049)	0.0152(-0.0181)	0.0188(0.0116)	0.0975(-0.3099)	0.1164(-0.3370)
		30	0.0188(-0.0165)	0.0184(-0.0158)	0.0194(-0.0131)	0.1847(-0.4229)	0.1775(-0.4161)
		25	0.0246(-0.0141)	0.0246(-0.0130)	0.0247(-0.0125)	0.2978(-0.5407)	0.2365(-0.4852)
		20	0.0342(-0.0134)	0.0343(-0.0126)	0.0343(-0.0124)	0.4190(-0.6439)	0.2493(-0.4993)
0.7	20	20	0.0268(-0.0148)	0.0287(-0.0645)	0.0301(0.0393)	0.0192(-0.1324)	0.0823(-0.2712)
		18	0.0295(-0.0283)	0.0250(-0.0373)	0.0342(-0.0050)	0.0573(-0.2163)	0.0991(-0.2996)
		16	0.0317(-0.0347)	0.0308(-0.0318)	0.0328(-0.0266)	0.1258(-0.3348)	0.1500(-0.3751)
		14	0.0394(-0.0316)	0.0393(-0.0287)	0.0396(-0.0270)	0.2071(-0.4407)	0.2039(-0.4458)
		12	0.0516(-0.0287)	0.0517(-0.0265)	0.0518(-0.0257)	0.2958(-0.5327)	0.2353(-0.4834)
	40	40	0.0118(-0.0052)	0.0157(-0.0468)	0.0121(0.0246)	0.0356(-0.1642)	0.0720(-0.2545)
		35	0.0140(-0.0184)	0.0126(-0.0207)	0.0152(-0.0105)	0.0743(-0.2624)	0.0962(-0.2985)
		30	0.0177(-0.0183)	0.0177(-0.0167)	0.0177(-0.0158)	0.1840(-0.4216)	0.1763(-0.4139)
		25	0.0246(-0.0141)	0.0246(-0.0130)	0.0246(-0.0126)	0.2978(-0.5407)	0.2365(-0.4852)
		20	0.0342(-0.0134)	0.0343(-0.0126)	0.0343(-0.0124)	0.4190(-0.6439)	0.2493(-0.4993)

Table 3.1 Continued

T	n	r	MSE(Bias)				
			$\widehat{\theta}_1$	$\widehat{\theta}_2$	θ	θ^*	$\bar{\theta}$
0.8	20	20	0.0245(-0.0405)	0.0219(-0.0795)	0.0293(0.0118)	0.0087(-0.0484)	0.0662(-0.2269)
		18	0.0242(-0.0429)	0.0226(-0.0398)	0.0251(-0.0286)	0.0553(-0.2015)	0.0904(-0.2787)
		16	0.0300(-0.0367)	0.0299(-0.0328)	0.0300(-0.0298)	0.1257(-0.3338)	0.1490(-0.3729)
		14	0.0391(-0.0318)	0.0391(-0.0288)	0.0391(-0.0274)	0.2071(-0.4406)	0.2038(-0.4456)
		12	0.0516(-0.0287)	0.0517(-0.0265)	0.0518(-0.0257)	0.2958(-0.5327)	0.2353(-0.4834)
	40	40	0.0101(-0.0169)	0.0131(-0.0673)	0.0108(0.0183)	0.0615(-0.1314)	0.0657(-0.2248)
		35	0.0125(-0.0229)	0.0123(-0.0208)	0.0125(-0.0181)	0.0722(-0.2546)	0.0924(-0.2898)
		30	0.0177(-0.0183)	0.0177(-0.0168)	0.0177(-0.0159)	0.1840(-0.4216)	0.1763(-0.4139)
		25	0.0246(-0.0141)	0.0246(-0.0130)	0.0246(-0.0126)	0.2978(-0.5407)	0.2365(-0.4852)
		20	0.0342(-0.0134)	0.0343(-0.0126)	0.0343(-0.0124)	0.4190(-0.6439)	0.2493(-0.4993)
0.9	20	20	0.0226(-0.0720)	0.0183(-0.0798)	0.0268(-0.0341)	0.0132(-0.0062)	0.0747(-0.2212)
		18	0.0226(-0.0460)	0.0222(-0.0403)	0.0220(-0.0337)	0.0555(-0.2002)	0.0911(-0.2783)
		16	0.0300(-0.0368)	0.0299(-0.0328)	0.0299(-0.0299)	0.1257(-0.3338)	0.1490(-0.3730)
		14	0.0391(-0.0318)	0.0391(-0.0288)	0.0391(-0.0274)	0.2071(-0.4406)	0.2038(-0.4456)
		12	0.0516(-0.0287)	0.0517(-0.0265)	0.0518(-0.0257)	0.2958(-0.5327)	0.2353(-0.4834)
	40	40	0.0098(-0.0389)	0.0089(-0.0624)	0.0114(-0.0101)	0.1358(-0.3012)	0.0780(-0.2236)
		35	0.0124(-0.0231)	0.0123(-0.0207)	0.0123(-0.0184)	0.0722(-0.2544)	0.0926(-0.2900)
		30	0.0177(-0.0183)	0.0177(-0.0168)	0.0177(-0.0159)	0.1840(-0.4216)	0.1763(-0.4139)
		25	0.0246(-0.0141)	0.0246(-0.0130)	0.0246(-0.0126)	0.2978(-0.5407)	0.2365(-0.4852)
		20	0.0342(-0.0134)	0.0343(-0.0126)	0.0343(-0.0124)	0.4190(-0.6439)	0.2493(-0.4993)

Table 3.2 $P(X_{r;n} < T)$

T	n	r	$P(X_{r;n} < T)$	T	n	r	$P(X_{r;n} < T)$	T	n	r	$P(X_{r;n} < T)$
0.5	20	20	0.0032	0.6	20	20	0.0306	0.7	20	20	0.1516
		18	0.0913			18	0.3580			18	0.7334
		16	0.4148			16	0.7941			16	0.9710
		14	0.7858			14	0.9696			14	0.9987
		12	0.9591			12	0.9979			12	1.0000
	40	40	0.0000		40	40	0.0009		40	40	0.0230
		35	0.0433			35	0.3654			35	0.8535
		30	0.5839			30	0.9547			30	0.9994
		25	0.9738			25	0.9998			25	1.0000
		20	0.9998			20	1.0000			20	1.0000
0.8	20	20	0.4420	0.9	20	20	0.8179				
		18	0.9561			18	0.9990				
		16	0.9990			16	1.0000				
		14	1.0000			14	1.0000				
		12	1.0000			12	1.0000				
	40	40	0.1954		40	40	0.6690				
		35	0.9951			35	1.0000				
		30	1.0000			30	1.0000				
		25	1.0000			25	1.0000				
		20	1.0000			20	1.0000				

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