

## Reconsidering the Formal Accounts of Continuity in the Theory-Change from Newtonian to Einsteinian Physics\*

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**【Abstract】** This essay will consider evolutionary views that attempt to capture the continuity of theory-change from Newtonian to Einsteinian physics via the formal aspects of these theories. Although it cannot be denied that the formal aspects such as ‘correspondence principles’ and ‘covariance principles’ provide important information concerning this theory-change, these formal properties are *not sufficient* to capture the essential elements of any evolutionary account of the development of Einstein’s special and general theories of relativity from Newtonian mechanics.

**【Key Words】** Theory-Change from Newtonian to Einsteinian Physics, Evolutionary View of Theory-change, Correspondence Principle, Covariance Principle, Newtonian Limit

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## 1. Introduction

The theory-change from Newtonian to Einsteinian physics has been cited as a classic example of a great scientific revolution by both scientists and philosophers. For example, the physicist Max Born identified the development of the theories of relativity as “the Einsteinian revolution” which opened “the beginning of a new era.” (Born 1965, p. 2) And the philosopher of science Karl Popper wrote that “Einstein revolutionized physics.” (Withrow 1967, p. 25)

Einstein himself, however, rarely employed the term “revolution” in order to characterize his theories of relativity. (Cohen 1985) He instead warned that the term “revolution” mischaracterizes the way that the special and the general theories were developed. Their development is considered as one which “slowly leads to a deeper conception of the laws of nature” based on results of “the best brains of successive generations.” (Klein 1975, p. 113) According to Einstein, the special theory is claimed as “simply a systematic development of the electromagnetics of Maxwell and Lorentz”. (Einstein 1954, p. 230) As for the general theory it was “the last step in the development of the program of field theory, ... [and] it modified Newton’s theory only slightly” (ibid., p. 260).

Two opposing points of view have characterized theory change in science as either evolutionary or revolutionary. The dispute between the two views is concerned with whether or not the

development of scientific knowledge has an accumulative (or quasi-accumulative) nature, and is closely related to other epistemological issues, such as those of scientific realism and scientific rationality. The evolutionary view, supported by Duhem (1914/1954) and logical empiricists such as Hempel (1960), maintains that scientific change is an essentially continuous and cumulative progress. On the other hand, Kuhn (1962) explicitly articulated and defended a revolutionary view involving “paradigm changes.” According to Kuhn, two different paradigms are incommensurable in their assertions about the world, aims, criteria of appraisal, conceptual frameworks, and even observational basis.

Advocates of both the evolutionary and the revolutionary views have employed the case of theory-change from Newtonian to Einsteinian physics in order to support their position. While Zahar (1973) and Friedman (1983) point out the commonality of mathematical formalisms in both theories as evidence of the accumulative nature of the theory-change, Kuhn considers the conceptual discontinuities concerning notions such as ‘mass’ and ‘space-time’ as evidence of the occurrence of a revolution brought about by Einsteinian physics.

This essay will consider evolutionary views that attempt to capture the continuity of theory-change from Newtonian to Einsteinian physics via the formal aspects of these theories. This view seems to reflect well the accounts in physics textbooks, in which the formal heuristics, such as ‘the Newtonian limit’ and ‘the covariance principle,’ connect Newtonian physics to Einsteinian physics. According to Rohrlich (1988), for example, a

newer theory is related with an older one in a way that “[t]he mathematical framework of [the latter] is rigorously derived from that of [the former] (a derivation which involves limiting procedures)”. (Rohrlich 1988, p. 303) Rohrlich emphasizes that physicists regards this formal aspect as the essential part of the relations between the two theories.

Physicists, however, typically deduce *only the mathematical structure of S* from that of *T* and pay little attention to whether the concepts resulting from the physical interpretations of the symbols involved in those structures permit such a functional relation. They work largely intuitively. The mathematical structure or framework of the theory is considered to be primary, and the central terms (the meaning of certain central symbols) are later derived from the applications of that framework to actual situations. (ibid., my italics)

Batterman (1995) basically reiterates Rohrlich’s view: “*only the mathematical structures of the two theories can be related by this limiting derivational procedure*” (Batterman 1995, p. 173), whereas “the interpretation and the ensuing ontologies [of the two theories] are in general not so related.” (Rohrlich 1988, p. 303) This view maintains that the essential aspect of the continuity between the two theories occurs in the formal aspect of the mathematical equations of the two theories. We will see that this attitude can be found within various accounts which emphasize the continuity between Newtonian and Einsteinian physics, such as that of Hempel (1960), Zahar (1973) and Friedman (1983).

In this essay, I will attempt to clarify what is in fact involved in this formal continuity. We will see that although it cannot be

denied that the formal aspects provide important information concerning this theory-change, these formal properties are *not sufficient* to capture the essential elements of any evolutionary account of the development of special and general relativity from Newtonian mechanics. There are four separate ways in which it has been claimed that formal relationships exist between the equations in Newtonian and in Einsteinian physics (i.e. 'Newtonian limits' and 'covariance principles' within special and general relativity). I will argue that none of these four ways succeeds in capturing the essential aspects of continuity within the theory-change.

## 2. The Correspondence Limit as A Formal Condition

Logical positivists and their advocates emphasize the formal continuity that the correspondence relations exist between Newtonian and Einsteinian physics within the limit. Hempel writes his *Philosophy of Natural Science*:

[The new] theory [in a scientific revolution] does not simply refute the earlier empirical generalizations in its field; rather, it shows that within a certain limited range defined by qualifying conditions, the generalizations hold true in fairly close approximation. (Hempel 1960, p. 76)

Despite his differences with the positivists on the generalization of theory-change, Zahar makes essentially the same claim (1973):

[A] new relativistic law should yield the corresponding classical theory as a limiting case. In the most general case laws will involve the speed of light  $[c]$ , the velocities  $v_1, \dots, v_n$  of a finite number of particles or processes ... If  $R = 0$  and  $K = 0$  are the relativistic and classical laws respectively, we require that:

$R \rightarrow K$  as  $(v_1/c, v_2/c, \dots, v_n/c) \rightarrow (0, 0, \dots, 0)$ . (Zahar 1973, p. 244)

A successor theory is more comprehensive than the old one in that a limiting case of the former approximates the latter. The old theory is then, in this sense, a special case of the more comprehensive new theory. So, our case of theory-change seems to be an accumulative process.

According to Nickles too, the correspondence in the limit is the key aspect of “the reduction of the Einsteinian formula for momentum,  $p = m_0 v / \sqrt{1 - v^2/c^2}$ , where  $m_0$  is the rest mass, to the classical formula  $p = m_0 v$  in the limit as  $v \rightarrow 0$ .” (Nickles 1973, p. 182) We can see the relationship from the expression of the Lorentz factor, i.e.  $1 / \sqrt{1 - v^2/c^2}$ , in the Lorentz transformation. As a Taylor series, the Lorentz factor can be expanded as  $1 / \sqrt{1 - v^2/c^2} = 1 - 1/2 (v/c)^2 - 1/8 (v/c)^4 - 1/16 (v/c)^6 - \dots$ . From this mathematical framework, we can consider the key expressions of the special theory of relativity as “Newtonian or classical quantities plus an expansion of corrections in powers of  $(v/c)^2$ .” (Batterman 1995, p. 173) Consequently, Rohrlich claims that the mathematical framework of Newtonian mechanics is “rigorously derived” from that of the special theory of relativity in a “derivation which involves limiting procedures.”<sup>1)</sup> (Rohrlich 1988, p. 303)

Recall, however, that the mathematical formalisms themselves do not carry any physical meaning. The conceptual schemes which underlie the formalisms are surely necessary to comprehend the way these formalisms work within a specific physical theory.<sup>2)</sup> A mathematical term such as the Lorentz factor is provided with empirical significance only within the theoretical framework of a specific theory. Lorentzian ether theory interprets the Lorentz factor as the effect of the contraction of matter moving through the ether, whereas special relativity interprets it as the modification of the spatio-temporal relations between events.

Furthermore, the limiting process imposes an empirical condition. The operation of neglecting higher powers of  $(v/c)^2$  is based on the empirical consideration that the speed  $v$  of moving bodies is generally small with respect to the speed of light  $c$ . In this way, the limiting operation refers to the physical situation of a moving body. Hence, it seems that the limiting process also is

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1) This relationships between the two theories are characterized as a formal one in that “the mathematical framework  $M(T$  [the special theory]) implies  $M(S$ [Newtonian mechanics]) when the domain  $D(T)$  is restricted to  $D(S)$ ” (Rohrlich 1988, p. 303) where “ $D$  is given by the characteristic parameter  $[p]$  which provides the error estimate; if the error is negligible, one is within  $[D(S)]$ .” (ibid., 305) This restriction then involves a limiting process:  $(p \rightarrow 0) \lim M(T) \rightarrow M(S)$ .

2) We can read this in Duhem: “If a physicist is given only an equation, he is not taught anything. To this equation must be joined rules by which the letters that the equation bears upon are made to correspond to the physical magnitudes they represent. And that which allows us to know these rules is the set of hypotheses and reasonings by which one has arrived at the equations in question. [This set of rules] is the theory that the equations summarize in a symbolic form: in physics, an equation, detached from the theory that leads to it, has no meaning.” (Duhem 1902, p. 223)

not devoid of empirical content.

At this point, defenders of the formal continuity can claim:

The mathematical framework of [a older theory] is rigorously derived from that of [a newer theory] (a derivation which involves limiting procedures); but the interpretations and the ensuing ontologies [of two theories] are in general not so related. They involve qualitative differences and for this reason demand independent recognition. In this way, one comes close to Feyerabend's theoretical pluralism and at the same time one ensures a well-defined logical-mathematical linkage between [two theories]. (Rohrlich 1988, p. 303)

This view maintains that although we can admit that the mathematical structure involves more than the formal aspects, the essential aspect of the continuity between the two theories occurs within the formal aspect of the mathematical equations, whereas its conceptual aspects experience a radical shift.

But this view is not tenable. A central aim behind the limiting procedure is in fact to provide an empirical meaning for mathematical formalisms. Hence, the formal aspect is intricately interrelated with its empirical one. Before we examine this claim in the limiting process within the special theory, we first look at a similar case in the general theory since the moral there is manifest.

The empirical aspect involved in the correspondence principle can be shown through investigation of the so-called 'Newtonian limit' procedure within the general theory of relativity. Einstein's general theory is related to Newtonian gravitation theory by the formal procedure limiting the physical quantities to the empirical



region where the effects specific to general relativity are small. According to the latter theory, the field equation is the Poisson equation  $\nabla^2\Phi = 4\pi G\rho$ , that represents the distribution of a gravitational potential  $\Phi$  due to the distribution of mass, when mass, the source of the potential, is distributed in continuous manner. And the equation of motion, which describes the trajectory of a test body influenced by gravitational field, can be written as  $m d^2x_i/dt^2 = -m\partial\Phi/\partial x_i$  (1). In contrast, the field equations in the general theory are Einstein's field equations  $R_{ij} - 1/2g_{ij}R = 8G\pi/c^4T_{ij}$ , and the equation of motion is the geodesic equation  $d^2x_i/d\tau^2 + \Gamma^i_{jk}(dx_j/d\tau)(dx_k/d\tau) = 0$  (2). In order to derive (1) from (2), we need the following physical hypotheses: a) the body moves at a speed that is negligible compared to the speed of light; b) the gravitational field that effects the body is very weak; c) the gravitational field is stationary, that is, the metric field is not changing with respect to time.

Under condition a), the equation of motion  $m d^2x_i/d\tau^2 + \Gamma^i_{jk}(dx_j/d\tau)(dx_k/d\tau) = 0$  (2) can be approximated to  $d^2x_i/dt^2 + \Gamma^i_{jk}(dt/d\tau)^2 = 0$  (3), since  $dt/d\tau$  is small compared to  $dx/d\tau$ . Under the condition c), the connection  $\Gamma^i_{jk}$  can be rewritten as  $\Gamma^i_{00} = -1/2 g_{ij}\partial g_{00}/\partial x_i$  (i) because the derivative of  $g_{ij}$  with respect to  $t$  can be neglected. In addition, given the condition b), we can choose the coordinate where the metric  $g_{ij}$  is slightly different from the Minkowski metric  $\eta_{ij}$ , that is,  $g_{ij} = \eta_{ij} + h_{ij}$ . Since only  $\eta_{ij}$  contributes and  $h_{ij}$  can be neglected under the condition b), (i) becomes  $\Gamma^i_{00} = -1/2 \eta_{ij}\partial\eta_{00}/\partial x_i$  (ii). By inserting (ii) into (3), the equation of motion then become  $d^2x_i/dt^2 = -1/2 \nabla h_{00}$  (5).

At this point, (5) is associated with empirical content that is imbedded in Newtonian theory of gravitation. The comparison between (1) and (5) then gives  $h_{00} = -2\Phi + C$ , where  $C$  is an integration constant, and  $C = 0$  in a boundary condition that both  $h_{00}$  and  $\Phi$  approach 0 when  $r$  goes to infinity. In other words, (1) is approximated to (2) only when  $g_{00}$  is equal to  $-(1 + 2\Phi)$ . In this way, the unknown quantity  $g_{00}$  within the equation of general relativity obtains empirical meaning by referring to its classical counterpart, i.e. a variable specified by Newtonian gravity. So, in addition to the fact that each symbol has specific physical meaning<sup>3)</sup>, an apparently formal limiting procedure is

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<sup>3)</sup> It is apparent that the Newtonian limit procedure cannot be characterized as only a formal one. Instead we can see that various physical concepts and empirical conditions are also involved along with the mathematical formulae. First of all, the mathematical quantities and frameworks are used in order to represent physical quantities and models. In the above case, the mathematical quantities  $x$  and  $t$  represent kinematical concepts, i.e. the spatial and the temporal coordinates. Then the combination of the quantities constitutes the geodesic equation (2), which represents the equation of motion by means of the physical interpretation of the mathematical framework. Physical significance is given to the geodesic equation by interpreting the formula, that is, specifying which aspect of the world the formula represents. In this way, the mathematical formulae are employed in order to realize the physical concepts.

Secondly, the mathematical operations relate the mathematical frameworks to empirical conditions. In the above case, the limiting procedures that realize various physical hypotheses such as *a)*, *b)*, and *c)* provide the mathematical formulae with specific empirical conditions. The condition *a)* imposes an empirical condition justifies the neglect of the "relativistic effect" of the physical system, whereas the conditions *b)* and *c)* specify the empirical properties of the gravitational field which can be neglected in a non-relativistic system. Along these lines, the mathematical procedures implement empirical conditions by the physical hypotheses.

intended as relating the symbols with empirical quantities which are well-confirmed by Newtonian mechanics.<sup>4)</sup>

The above account provides a clue to understanding the case of limiting procedures within the special theory. Just as the above procedures in the general theory provide mathematical formalism with empirical significance, so physics textbooks refer to the mathematical similarity between Newtonian mechanics and the special theory in order to show that mathematical terms, such as the Lorentz factor, are connected with empirical consequences that are well confirmed and already entailed by the predecessor theory.

The Einsteinian formula for momentum  $p = m_0 v / \sqrt{1 - v^2/c^2}$  is developed from the two principles of the special theory. And the fact that this formula converges to its counterpart within Newtonian mechanics secures indirect empirical support from the empirical confirmation of Newtonian mechanics. In this way, the formal aspect is intricately related with the conceptual aspect of a given theory. Accordingly, in order to assess the continuity between Newtonian mechanics and the special theory, one must take into account not only the formal aspects common to the two theories, but also how the evolving formalisms are given physical significance within them. In the following sections we will see cases showing that even though one insists on the shared formal

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<sup>4)</sup> Einstein, according to Renn and Sauer (1999), employed the heuristics of the correspondence principle, i.e. the limiting procedure, to provide mathematical formalism with the empirical significance. So, they characterize the heuristics of the correspondence as “the physical strategy,” in that the formalism in the general theory starts from the well-known limiting case of the predecessor theory.

structure being tied to observable phenomena in the same way, two quite different interpretations remain open within these particular cases. If this is true, the formal aspect in the correspondence principle cannot completely capture the continuity between Newtonian and Einsteinian physics.

### 3. The Covariance Principle as a Formal Condition

A more convincing case emphasizing the formal aspect of the continuity between Newtonian and Einsteinian physics seems to be the extension of the “covariance principle” involved in the theory-change. This principle has the formal feature that the equations expressing the laws of nature should be invariant under a given class of coordinate transformations. Friedman makes this clearly:

Covariance ... is really *a property of formulations* of space-time theories rather than space-time theories themselves; it characterizes systems of differential equations ... representing the intrinsic laws of a space-time theory relative to some particular coordinatization in  $R^4$ . (Friedman 1983, p. 213, my italics)

Along the same line, Zahar also claims:

The new [Lorentz covariant] laws were mathematically derived from assumptions like the Relativity Principle which seem so *'formal' and innocuous as to be devoid of empirical content*. (Zahar 1973, p. 249, my italics)

Einstein in his landmark 1905 paper “On the Electrodynamics of Moving Bodies” viewed the extension of the covariance requirement as an essential heuristic: Maxwell’s equations governing electro-magnetic phenomena are to be invariant in their form under a group of coordinate transformations specified by the principle of special relativity. Furthermore, Einstein considered the extension of the requirement of covariance to all coordinate systems (i.e. not just to the inertial frames) to be essential in the development of the general theory of relativity. It seems, then, that the extensions of the requirement of covariance are a central characteristic of the evolutionary development from Newtonian to Einsteinian physics:

Einstein decided to treat all coordinate systems on a par and to impose a condition of general covariance on all physical laws. This condition, which is a strengthening of the requirement of Lorentz covariance (General Covariance of course implies Lorentz covariance), is an important element of continuity between the special and the general theories of Relativity. (Zahar 1973, p. 252)

The aim of this section is the same as one of the previous section – that is, to show that the formal aspect of covariance also falls short of giving an adequate characterization of the continuity involved in the theory-change. Janssen maintains that formal covariance by itself cannot distinguish Einstein’s special relativity from Lorentzian ether theory. And in the case of general relativity, Friedman claims that despite the fact that the formal covariance of the general theory is same as that of its

counterparts in Newtonian and the special theory, their physical significances are different. Hence, it is difficult to defend the view that the extension of the covariance requirement as simply a formal condition captures the essential aspects of the evolution from Newtonian to Einsteinian physics.

**3.1. The Covariance Principle within Special Relativity:** An emphasis on the formal property of covariance as the essential part of the development of the special and the general theory of relativity can be found in Einstein's writings:

The content of the restricted relativity theory can accordingly be summarized in one sentence: all natural laws must be so conditioned that they are covariant with respect to Lorentz transformations. (Einstein 1940, p. 329)

A specific principle of covariance requires the invariance of the laws of mechanics under coordinate transformations from one inertial coordinate system to another. In the case of Newtonian physics, the laws of mechanics hold with respect to a set of inertial coordinates, which are related through a group of Galilean transformations that implement the principle of Galilean relativity. But, Maxwell's equations are not Galileo-invariant. So, assuming that Maxwell's equations are correct, Einstein modified the coordinate transformations relating two inertial coordinate systems. With the introduction of the Lorentz transformations, the laws of electrodynamics take the same form with respect to all inertial coordinate systems, and hence the covariance of Maxwell's equations is established. In this way, Einstein's special theory

re-establishes the principle of relativity in electrodynamics, and hence eliminates the preferred frame of reference, i.e. the absolute rest frame. In section two of "On the Electrodynamics of Moving Bodies," under the postulate that all physical laws take the same form under this coordinate transformation and the speed of light is always the constant  $c$ , Einstein derives the coordinate transformation implementing the principle of relativity of inertial motions. And sections six and nine of this paper show that the covariance of Maxwell's equations holds under the group of Lorentz transformations. It seems, then, that the extension of covariance to Maxwell's equations characterizes a sort of generalization from Newtonian mechanics to Einstein's special relativity.

But, it can be argued that the covariance principle has in fact empirical contents. We have seen in the last section that the procedure of requiring the 'Newtonian limit' is intended to relate mathematical quantities in relativistic theories with empirical quantities well-confirmed in Newtonian mechanics. In a similar manner, the apparently formal procedure of imposing covariance in fact starts from the corresponding laws in Newtonian mechanics. Accordingly, the covariance principle by no means involves only formal properties, since it provides a constraint over the entities involved in a physical law. We can see this in Zahar:

Einstein based his heuristic on the requirement that all physical laws should be Lorentz-covariant; i.e. all theories should assume the same form, whether they are expressed in terms of  $x, y, z, t$  or in terms of  $x', y', z', t'$ . *But it would be practically impossible to discover new laws simply by looking out for all*

*the equations which are covariant under the Lorentz transformation. A good method is to start from well-tested laws whose past success would anyway have to be explained by any new theory.* Thus the heuristic of Einstein's programme is based on two distinctive requirements: (1) a new law should be Lorentz-covariant and (2) it should yield some classical law as a limiting case. (Zahar 1989, p. 243, my italics)

Despite the aforementioned accounts of Friedman and Zahar emphasizing the formal aspect of covariance, it seems that both authors are well aware that the covariance principles in the development of the special and the general theory involves physical content. (We will see a paragraph where Friedman points out this in the next section.) Zahar writes:

[O]n the face of it, the most distinctive requirement of Einstein's heuristic is empty ... However, the requirement is trivialised only if one is allowed complete freedom in reformulating the law. If one is restricted to a given number of entities:  $a_1, a_2, \dots, a_n$ , then the covariant requirement, far from being empty, becomes a very stringent condition. ... [I]n each particular case in which the heuristic is applied, the entities occurring in the covariant law are precisely those involved in the corresponding [and empirically well-confirmed] classical law. (Zahar 1989, p. 110, my italics)

Based on Janssen's recent work on Lorentzian ether theory, there is an additional argument that the evolutionary process from Newtonian to Einsteinian physics cannot be completely captured by means of the formal aspect of covariance alone. In fact, the theory-change from Newtonian to Einsteinian physics is more than the formal extension of covariance requirement. This is because



the formal aspect of covariance fails to capture the physical interpretation of Einstein's theory, given that it may have many different physical realizations. A supporting case is provided by Janssen, who points out that Lorentzian ether theory employs an identical mathematical structure which, however, has an interpretation quite different from that of Einstein's special relativity. (Janssen 2002)

Lorentzian ether theory was proposed as an attempt to eliminate the inconsistency between the concept of a stationary ether and electrodynamics, both of which were then accepted physics. Electrodynamics predicts that light, as an electromagnetic wave, propagates at the constant speed of  $c$ . In the middle of the 19th century, it was widely assumed that light was propagated through a stationary medium known as the ether, with respect to which Maxwell's equations were assumed to hold. Given this assumption together with Newtonian kinematics, it can be inferred that the speed of light is not constant in a frame at rest with respect to the Earth. The Earth is moving with respect to the ether, and so the velocity of light with respect to the Earth is the vector sum of the velocity of the light with respect to the ether and the velocity of the Earth with respect to the ether. But the null results of a whole series of optical experiments attempting to detect the change of velocity of light seems to establish the constancy of the speed of light regardless of the motion of the light source.

Lorentzian ether theory attempted to resolve this inconsistency by assuming that the laws governing matter are Lorentz-covariant.

Janssen characterizes this attempt by Lorentz as a combination of Newtonian mechanics and electrodynamics summarized in “the theorem of the corresponding states”:

[Lorentz] replaced the real space-time coordinates of an arbitrary inertial frame in Newtonian space-time by cleverly chosen fictive space-time coordinates that depend on the frame’s velocity with respect to the ether. ... He likewise replaced the real electric and magnetic fields by fictive fields. In terms of these fictive fields as functions of the fictive space-time coordinates, Maxwell’s equations hold in every frame in which the ether is either at rest or in uniform motion. ... This configuration of fictive fields translates into a configuration of real fields in the moving frame that is different from the configuration in the initial frame at rest in the ether. Lorentz referred to two such configurations of real fields as corresponding states, and to the mathematical result that if one is allowed the other also is as *the theorem of the corresponding states*. (Janssen 2002, p. 424)

Lorentzian ether theory maintains a Newtonian view of *space-time*, but assumes that the laws governing *matter* are Lorentz-invariant. Furthermore, in order to eliminate *any* difference of interference patterns associated with the corresponding states (the so-called second-order effect), that might detect the Earth’s motions with respect to the ether, Lorentzian ether theory interprets the theorem of the corresponding states as amounting to the contraction hypothesis:

[A] matter configuration producing a certain field configuration in a frame at rest in the ether will, when the system is set in motion, change into the matter configuration producing the corresponding state of that field configuration in the frame moving with the system. (*ibid.*, p. 425)

In other words, material bodies such as the interferometer arms employed in the Michelson-Morley experiment experience contractions by a factor of  $\sqrt{1 - v^2/c^2}$  in the direction of their motion through the ether. As a consequence, this physical system modifies its shape and mass depending on the velocity of the system with respect to the ether. Lorentzian ether theory is thus able to explain by means of the contraction hypothesis why no effect of the ether can be detected.

According to Janssen, both Lorentzian ether theory and Einstein's special relativity employ an *identical* mathematical formalism despite their essentially different physical interpretations. (Janssen 2004) What the theorem of the corresponding states involves is essentially the Lorentz invariance of the laws governing matter. In other words, the fictive space-time coordinates and the fictive fields are introduced in order that Maxwell equations remain invariant under the group of Lorentz transformations. Hence, if we capture the essence of special relativity as a theory all of whose laws are Lorentz-invariant, we cannot distinguish Einstein's special relativity from Lorentzian ether theory.

These two theories are however importantly distinguished by their different interpretations of how the Lorentz invariance of laws is physically realized. Lorentz retained the kinematical properties of Newtonian space-time, and interpreted Lorentz invariance as stemming from the property of laws governing matter: the Lorentz-transformed quantities represent the modified configurations of matter in motion, which result in Lorentz

invariance of the laws of nature. In other words, Lorentz viewed Maxwell's equations as holding with respect to the ether frame, and the configuration of the fictive fields as representing the modified configurations of matter in motion, which are different from its configuration at rest in the ether.

Einstein's interpretation, as Janssen sees it, is completely different in that it is founded on "a new concept of space-time." Lorentzian ether theory was supposed to reveal the motion of a body relative to the ether. Yet Einstein viewed the concept of the ether, which provides a preferred frame, as at odds with his belief that all inertial frames are equivalent. As a result, Einstein interpreted the very same formalism as a result of the kinematical properties of a new space-time, which stem from the principle of special relativity; the fictive fields in the theorem of the corresponding states are interpreted as the fields measured by inertially moving observers. In contrast with Lorentz's interpretation, Einstein interpreted Lorentz transformations as relating the space-time coordinates of one moving observer to those of another.<sup>5)</sup> Accordingly, Lorentz-transformed quantities reflect different space-time coordinates measured by two observers in uniform relative motion. While Lorentzian ether theory is based on Newtonian space-time and Lorentz invariant laws governing matter, Einstein's special theory views both as founded on the

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<sup>5)</sup> It seems that Einstein wrote in this spirit as follows: "the new feature of [the 1905 relativity theory] was the realization of the fact that the bearing of the Lorentz transformation transcended its connection with Maxwell equations and was connected with the nature of space-time in general." (Einstein 1955, a letter to C. Seeling)

kinematical properties of a new concept of space-time:

*From a purely mathematical point of view, Lorentz had thereby arrived at special relativity. To meet the demands of special relativity, all that needs to be done is to make sure that any proposed law is Lorentz invariant. Conceptually, however, Lorentz's theory is very different from Einstein's. In Einstein's theory, the Lorentz invariance of all physical laws reflects a new space-time structure. [On the other hand,] Lorentz retained Newton's conception of space and time, the structure of which is reflected in the invariance of the laws of Newtonian physics under what are now called Galilean transformations. (Janssen 2004)*

Accordingly, from the perspective of the formal aspect of Lorentz invariance, Lorentzian ether theory cannot be distinguished from special relativity. However, Lorentzian ether theory and special relativity are essentially different theories. Accordingly, Einstein's special relativity amounts to more than just the theory that claims that all laws must satisfy Lorentz invariance. It follows that we cannot capture the essential feature of special relativity through the covariance principle alone. So, it can be concluded that the formal aspect of the covariance cannot completely substantiate any account of the development from Newtonian to Einsteinian physics as evolutionary.

**3.2. The Covariance Principle within General Relativity:** A similar moral can be drawn in the case of the general theory of relativity. By means of a formal principle of general covariance that requires the coordinate independence of the laws of physics, Einstein aimed to generalize the principle of relativity to apply to

arbitrary coordinate systems including non-inertial frames. Einstein viewed the requirement of general covariance as a way to construct a theory realizing the generalized principle of relativity:

The general laws of nature are to be expressed by equations which hold good for all systems of co-ordinates, that is, are covariant with respect to any substitutions whatever (generally co-variant).

It is clear that a physical theory which satisfies this postulate will also be suitable for the general postulate of relativity. For the sum of all substitutions in any case includes those which correspond to all relative motions of three-dimensional systems of coordinates. (Einstein 1916, p. 117)

Under the influence of Mach's empiricism, Einstein considered the concept of a preferred inertial frame (which enables us to define the concept of absolute motion) as epistemologically unsatisfactory, since such a frame could not be identified through any measurable observation. Accordingly, Einstein thought that a new theory of motion needed to be based on what was observable, i.e., relative motions. Given that both Newtonian mechanics and the special theory of relativity have this "epistemological defect" of admitting a set of preferred inertial frames, Einstein intended to develop a theory that did not refer to any preferred coordinate systems:

Of all imaginable spaces  $R_1, R_2$ , etc., in any kind of motion relatively to one another, there is none which we may look upon as privileged *a priori* ... . *The laws of physics must be of such a nature that they apply to systems of reference in any kind of motion.* Along this road we arrive at an extension of

the postulate of relativity. (Einstein 1923, p. 113)

Just as the principle of special relativity eliminated the preferred inertial frames of electrodynamics, Einstein, by means of general covariance, intended to eliminate *any* preferred frame. Just as the relativity between inertial coordinate systems was achieved through the special theory, Einstein intended to generalize relativity to be applied to all arbitrary coordinate systems. In this way, acceleration could be viewed as being an artifact of the choice of the coordinate system. According to this account, it seems that the extension of covariance to non-inertial coordinates could be identified as the generalization of the principle of relativity to all frames of reference. The evolution from the special to the general theories can be characterized as the generalization of a relativity principle through extending covariance.

The above account of the development of general relativity, however, was not in fact fully realized as Einstein originally intended. Given that acceleration is still a physically significant concept in the general theory, general covariance by no means implements the generalized principle of relativity. While the concept of velocity is relativitized through the principles of relativity in Newtonian mechanics and special relativity, accelerating motions in the general theory remain as absolute motions. With reference to the local inertial frames i.e. a privileged subclass of frames, the concept of acceleration of a given body can be defined as the deviation from a geodesic trajectory of a free-falling body. None of the geodesic trajectories

can be transformed into non-geodesic trajectories by means of coordinate change, and the distinction between the former and the latter is absolute with respect to coordinate change.

Furthermore, although the formal role of covariance within the general theory of relativity is the same as the corresponding one within Newtonian mechanics and special relativity, the physical significance of covariance is interpreted as being distinct within the contexts of the different theories. The status of covariance in the general theory of relativity is complicated because the space-time it postulates is curved. To appreciate this, we need to separate the concept of an 'indistinguishability group' from a 'covariance group.' The latter characterizes the range of coordinate systems where the equations representing physical laws of a given space-time theory hold. On the other hand, the former selects those "reference frames (states of motion) [that] are distinguishable (by a "mechanical experiment") relative to those laws." (Friedman 1983, p. 213) Hence, the latter is concerned with the formulation of a given theory, and the former is about the laws of that theory. Friedman claims that:

"Sameness of form" (covariance) is much too weak to guarantee physical equivalence (indistinguishability) and therefore much too weak to express a relativity principle. The notions of "sameness of form" and covariance correspond to the notions of physical equivalence and relativity only in the context of flat space-time theories in which there exists a privileged class of inertial coordinate systems. ... But in non-flat space-time theories like general relativity these "nice" connections between indistinguishability and covariance break down. (Friedman 1983, p. 208)



While in Newtonian mechanics and special relativity, the covariance and the indistinguishability groups coincide, this is not the case in general relativity. The covariance group of the general theory is identical to the group of all coordinate transformations, whereas the indistinguishability group is the restricted group of transformations from one local inertial frame to another. The discrepancy between the two groups shows that the same mathematical requirement of covariance turns out to play different roles within the different theories. In the case of Newtonian mechanics and special relativity, where the flat space-time formalisms are available, their covariance implements the physical equivalence of inertial frames. Yet, in general relativity, without the existence of a privileged reference frame defined globally, covariance does not implement a relativity principle.

Hence, the continuity between the development of the special and the general theory cannot be captured by the formal requirement of the extension of covariance. It follows that we cannot capture the essential feature of the development of the special and the general theory through the covariance principle alone. So, it can be concluded that the formal aspect of the covariance cannot completely substantiate any account of the development from Newtonian to Einsteinian physics as evolutionary.

#### 4. Conclusion

We have, then, considered four ways in which formal aspects

of the relationships between Newtonian and Einstein physics by themselves do not supply physical interpretations that give information about the way the formalism represents the world. Accordingly, we cannot rule out the possibility that in spite of the continuity of mathematical formalisms, their conceptual interpretations could experience radical changes. The identical mathematical framework can have radically different interpretations. So, it seems that the mathematical continuity in theory-change as characterized so far is not inconsistent with conceptual discontinuity within the process.

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Reconsidering the Formal Accounts of Continuity in the  
Theory-Change from Newtonian to Einsteinian Physics

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본 논문은 형식적인 조건을 통하여 뉴턴 물리학에서 아인슈타인 물리학으로의 이론변화의 연속성을 이해하려는 과학이론에 대한 진보적 견해를 비판적으로 고찰한다. ‘상응 원리’이나 ‘공변 원리’와 같은 형식적인 측면이 이 이론변화에서 중요한 정보를 담고 있는 것은 부정할 수 없지만, 이 형식적 속성은 뉴턴역학에서 아인슈타인의 특수 그리고 일반 상대성 이론으로의 발전에 대한 진화적 견해를 위한 본질적 요소를 제공하기에는 충분치 않다.

[주요어] 뉴턴 물리학에서 아인슈타인 물리학으로의 이론변화,  
이론발전에 대한 진화적 견해, 상응 원리, 공변 원리,  
뉴턴 극한