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The Status of *Scientiae Mediae* in the History of Mathematics: Biancani's Case* †

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【Abstract】 We can witness the recent surge of interest in the controversy over the scientific status of mathematics among Jesuit Aristotelians around 1600. Following the lead of Wallace, Dear, and Mancosu, I propose to look into this controversy in more detail. For this purpose, I shall focus on Biancani's discussion of *scientiae mediae* in his dissertation on the nature of mathematics. From Dear's and Wallace's discussions, we can gather a relatively nice overview of the debate between those who championed the scientific status of mathematics and those who denied it. But it is one thing to fathom the general motivation of the disputation, quite another to appreciate the subtleties of dialectical strategies and tactics involved in it. It is exactly at this stage when we have to face some difficulties in understanding the point of Biancani's views on *scientiae mediae*. Though silent on the problem of *scientiae mediae*, Mancosu's discussions of the Jesuit Aristotelians' views on *potissima* demonstrations, mathematical explanations, and the problem of cause are of utmost importance in this regard, both historically and philosophically. I will carefully examine and criticize some of Mancosu's interpretations of Piccolomini's and Biancani's views in order to approach more closely what was really at stake in the controversy.

【Key Words】 Scientiae Mediae, Biancani, Mancosu, demonstration, Aristotelian mathematics

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1. Introduction

We can witness the recent surge of interest in the controversy over the scientific status of mathematics among Jesuit Aristotelians around 1600. Both historians and philosophers of science (and mathematics) have good reason to be enthusiastic about scrutinizing this controversy. For, it must not only deepen our understanding of the role of mathematics in the scientific revolution, but also present a nice case study for Aristotelian scientific methodology and philosophy of mathematics in action. Following the lead of Wallace, Dear, and Mancosu, I propose to look into this controversy in more detail. For this purpose, I shall focus on Biancani's discussion of *scientiae mediae* (subordinate, subalternated, intermediate, middle, or mixed sciences; applied mathematics or mixed mathematics) in his dissertation on the nature of mathematics. In the discussion of the perennial problem of unity or disunity of sciences, Aristotle's general prohibition of *metabasis* has always played a key role by securing the autonomy of individual sciences. But some disciplines like astronomy or music provide us with obvious counterexamples to this rule by applying the results of pure mathematics. According to Dear(1995), Aristotle's *ad hoc* solution of this problem by classifying them as *scientiae mediae* caused a battle for later Jesuit Aristotelians on the problem of scientific status of *scientiae mediae*, and thereby the problem of relationship among pure mathematics, applied mathematics, and physics. From Dear's and

Wallace's discussions, we can gather a relatively nice overview of the debate between those who championed the scientific status of mathematics and those who denied it. But it is one thing to fathom the general motivation of the disputation, quite another to appreciate the subtleties of dialectical strategies and tactics involved in it. It is exactly at this stage when we have to face some difficulties in understanding the point of Biancani's views on *scientiae mediae*. It is in part due to the strange structure of his dissertation, which concludes with surprisingly brief discussion of *scientiae mediae*. More seriously, some of Biancani's views seem to be conflicting each other. For example, it is not clear why he emphasized the difference in kind between pure and applied mathematics if he would have *scientiae mediae* in the realm of the ideal model of science. Though silent on the problem of *scientiae mediae*, Mancosu's discussions of the Jesuit Aristotelians' views on *potissima* demonstrations, mathematical explanations, and the problem of cause are of utmost importance in this regard, both historically and philosophically. I will carefully examine and criticize some of Mancosu's interpretations of Piccolomini's and Biancani's views in order to approach more closely what was really at stake in the controversy.

2. Wallace and Dear on the Motivations of Jesuit Debates on *Scientiae Mediae*

In both Dear and Wallace we can find very useful perspective for understanding Biancani in more general context of the period around 1600. For they tried to understand Biancani against the background of their detailed discussion of Clavius, the reformer of mathematical education among the Jesuits that promoted scientific knowledge in the period of Scientific Revolution.

Wallace starts his discussion of Biancani immediately after the following remark: "The full effect of Clavius's program, was not seen until students whom he prepared himself under the new Ratio Studiorum completed their studies and began to write on the nature of the mathematical disciplines." (Wallace(1984), p. 141) Dear also points out that even though the Jesuits uplifted the status of mathematics and the results were reflected in the educational program, the exact relation between mathematics and philosophy was a matter of controversy. And he pins down Clavius's role as a defender of the philosophical status of mathematics against the opponents in the controversy around the curriculum in Rome in 1580's. He also made it explicit that Clavius was not attacking the straw men but the real objectors like Piccolomini, Pereyra, and Coimbra commentators. In Dear's discussion we can also find many suggestive comments on the strategies and tactics of Clavius in the controversy. For example, according to Dear, Clavius suggested to let mathematicians attend the regular disputation as well as philosophers. Clavius argued for the necessity of mathematics in order to study natural philosophy by using the point that mathematical astronomy is needed for cosmological studies. (Dear(1995), p. 35-36)

Even more interesting is the fact that, according to Dear, arguments given by Clavius for his claim were merely arguments from authorities. By resorting to Aristotle's authority, for example, Clavius emphasized the fact that mathematics, physics, and metaphysics had been the three constituents of speculative philosophy from the antiquity. Also he claimed the excellence of mathematics in terms of Ptolemy's authority. Moreover, according to Dear, Clavius evaded the crucial objections that mathematical disciplines are not scientific. In other words, Clavius's strategy in the controversy was not launching any positive argument but simply subsuming mathematical sciences under the Aristotelian model of ideal science. (Dear(1995), pp. 37-39)

In order to appreciate the import of Dear's comments we need to grant a fair hearing to his explanation of how a crucial problem was raised in Aristotelian methodology, which was destined to bring about the controversy among the Jesuit Aristotelians before and after 1600. According to Dear's description, individual sciences in Aristotle's model must be based on unique and proper principles that would function as the major premise in syllogistic demonstrations. As a consequence, individual sciences are separated sharply from each other. And that is a logical necessity expressed in the methodological principle of homogeneity. That homogeneity requires that the principle of an individual science must be related to the genus as its object in order to secure deductive link. But some disciplines such as astronomy and music clearly violate this rule by applying the results of pure mathematics to celestial motions and sounds. Aristotle made special arrangement for these sciences by classifying them as subordinated to higher scientific disciplines. According to Dear, such an Aristotelian solution was more or less an *ad hoc* device for the problem of classifying sciences. And, as a consequence, it was destined to bring about the controversy among the Jesuit Aristotelians as to "whether demonstrations in a subject such as optics yielded true scientific knowledge if the presupposed theorems of geometry were not proved at the same time". (Dear(1995), pp. 38-39; Here Dear refers to Wallace(1984), p. 134.)

So far, Dear's description of the problem situation is quite persuasive and illuminating. But, much more than that, Dear presents a subtle and ingenious interpretation of Clavius's strategy in the situation. According to Dear, since the attempt itself to assimilate applied disciplines to the general model of science made it clear that Aristotle granted the scientific status to all mathematical disciplines, such an Aristotelian approach was in full service for Clavius's purposes. Finally, Dear's ingenious interpretation arrives at the pinnacle in his claim that unlike Clavius, who was able to claim the scientific status of mathematics without involvement with the thorny problem of cause, Biancani directly tackled the problem of cause. (Dear(1995), p. 40)

3. The Chapter on *Scientiae Mediae*

Happily beginning to understand the general motivation of the disputation, one might desire to appreciate the subtleties of dialectical strategies and tactics involved in it. For this reason, it is truly timely that Mancosu publisheded Klima's English translation of Biancani's dissertation on the nature of mathematics as an appendix to his book *Philosophy of Mathematics and Mathematical Practice in the Seventeenth Century* (Mancosu(1996)). Furthermore, Mancosu discussed Biancani's philosophical views on mathematical sciences rather extensively in several writings. (Mancosu(1991), (1992), (1996))

It is exactly at this stage when we have to face some difficulties in understanding the point of Biancani's views on *scientiae mediae*. It is in part due to the strange structure of his dissertation, which concludes with surprisingly brief discussion of *scientiae mediae*. Thanks to Mancosu and Klima, now more people have come to know that Biancani's *De Mathematicarum Natura Dissertatio*(1615) was published as an appendix to his *Aristotelis Loca Mathematica*, which was an excerpt of passages on mathematics in Aristotelian corpus. Biancani's disseration on the nature of mathematics consists of five chapters. The first chapter deals with the subject of mathematics. Middle term in geometrical demonstration is discussed in the second chapter, where the crucial issue is whether geometrical demonstrations are perfect (*potissima*). In chapter three he criticizes opponents' errors. The fourth chapter is an attempt to argue for the excellence of mathematics. At the end of the chapter four Biancani enumerates all the conclusions he secured from his discussion of pure mathematics in the first four chapters. And, finally in chapter five, he discusses *scientiae mediae* such as astronomy, optics, mechanics, and music. Why does he discuss all these themes in this particular order? In particular, why does he discuss *scientiae mediae* only at the end? Is it due to their prime importance, or their relative unimportance? Since Biancani's dissertation on the nature of mathematics is itself an appendix to his *Aristotelis Loca Mathematica*, he might be excused for his all too frequent cross-references in the former to the latter. What is interesting is the fact that his final chapter follows the same pattern of referring back to his previous discussions. Be that as it may, his chapter on *scientiae mediae* is all too brief to allow sound understanding of his views about them.

Roughly speaking, what we can read in this chapter on *scientiae mediae* is this. In the opening and the closing paragraph, we can confirm his conclusion that *scientiae mediae* present us with perfect demonstrations. He resorts to Aristotle's authority to show that *scientiae mediae* do have demonstrations of causes: "For here the 'what' is to be known by those who perceive, but the 'why' by the mathematicians, for they have the demonstrations of the causes."[1] Then, he provides us with a few examples of alleged perfect demonstrations in applied mathematical sciences. What is troublesome is that his discussion of them is too succinct, if not cryptic. Our expectation to find some further hints as to the implications of his views on *scientia mediae* is almost completely betrayed.

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1) Hic enim ipsum quidem quod sensitiuorum est scire, ipsum vero propter quid Mathematicorum, hi namque. (M 206, B 313; Anal. Post. 79a 3-6)

Let me quote his discussion of some examples: one from astronomy and another from optics.

> "To begin with astronomy, isn't the demonstration of the eclipse of the moon (even by the testimony of Aristotle and his commentators, especially Zabarella) a perfect demonstration? For it renders evident the proper and adequate cause of the property [*affectio*] in question, i.e., the eclipse, namely, the interposition of the Earth. But we should say the same of the solar eclipse, the cause of which is shown to be the interposition of the Moon. And that these demonstrations were discovered by the astronomers is known from their books, as well as the fact that they use geometrical media, namely, circle, diameter, and diametric opposition, and that therefore how certain they are is obvious from the infallible prediction of eclipses."[2)]

> "Again, in optics we do not lack perfect demonstrations either. For example, why is the eye spherical? So that perpendicular lines can fall on it from every direction. But why perpendicular lines? In order to produce distinct sight. Here you have the final cause." [3)]

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2) Et, ut ab Astronomia initum faciamus, demonstratio eclypsis Lunae (etiam Arist. Eiusque; interpretibus praecipue Zabarella testibus) non ne potissima? Nam affectionis illius, seu defectus propriam, & adaequatum causam euidentem facit, interpositionem, videlicet terre. Idem de solari defectu dicendum, cuius causam ostendunt esse Lunae obiectionem. Quas demonstrationes ab Astronomis inuentas esse ex ipsorum libris constat. & quod medio utantur Geometrico, nimirum circulo, & diametris, & diametrali oppositione. Quam deinde certae sint, patet ex eclypsium infallibili praedictione. (M. 206, B. 314)

3) In perspectiua etiam neque desunt perfectae demonstrationes, v.g. cur oculus sphaericus? Ut illi undique lineae perpendiculares possint accidere. Sed cur lineae perpendiculares? Ut distincta fieret visio; ecce causae dinales. (M. 207, B. 314)

Could we understand what a perfect demonstration should be like from such brief discussions as these? After all, wherein lies the novelty of Biancani's own contribution?

Laird's assessment that Biancani defended mathematics more with rhetoric and examples than with detailed exposition of Aristotle seems not entirely unfair. Also, even after having considered what Biancani did in *Aristotelis loca mathematica*, he points out quite justifiably that Biancani failed to solve the difficulties raised in the commentaries on the *Posterior Analytics*..(Laird(1997), p. 266)

4. Biancani's Target

But we have to note that there is one potentially revealing remark at the closing part of chapter five. For, there he claims:

> "From these it manifestly appears that mathematical sciences have perfect demonstrations, whose causes are so distinct from their effects that no calumnies can do any harm to them. Therefore, even if our opponent could prove, which they never can, that geometry and arithmetic lacks them, they would have to admit this concerning the other [disciplines] mentioned above that they reason by all genera of causes, and that they excel with such clarity that they leave nothing in ambiguity or controversy."[4]

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[4] Ex quibus liquido constant Mathematicas habere perfectissimas demonstrationes, quarum cause ita ab effectu distinguntur, ut nullis calumniis sint obnoxiae: quare etiam si aduersarii conuincant, quod neutiquam faciunt, Geometriam, & Arithmeticam illis carere; reliquis tamen praedictis concedere

This exceptional remark is interesting in view of the fact that for the last three decades Quine-Putnam indispensability argument has been counted as the only persuasive argument for realism in philosophy of mathematics. (See Colyvan(2001), Maddy(1997), Putnam(1979), and Quine(1980).) For, in both Biancani and twentieth century realists, applied mathematics seems to play the role of the last bulwark for realism in mathematics. Be that as it may, Biancani's remark seems to give us a definite hint for identifying the prime target of his dissertation.

In chapter three of his dissertation, Biancani criticizes seventeen errors of his opponents. Fortunately, at least some of them have definite relevance for *scientiae mediae*: i.e, the third, the fourteenth, and the sixteenth. The third errors stems from Plato's remark in Book 7 of the *Republic* that mathematicians dream about quantity. The fourteenth error has something to do with the alleged ignobility of the subject matter of mathematics. And, the sixteenth error is related to an inconsistency involved in the opponents of Biancani. For my present purpose, what does matter is the sixteenth error:

> "The sixteenth is [that] which they put forward by asking [first] in general whether mathematics has perfect demonstrations, then later in the discussion they bring up several points against mathematics, and at the end of the treatise they claim that these concern only geometry and arithmetic. Wherefore, unless the

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coguntur: easque; per omne causarum genus excurrere, quod tanta praeterea euidentia praestant, ut nihil ambiguum, nihl controuersum relinquatur. (M. 208, B. 315)

> reader peruses everything to the end, which rarely happens, he will be deceived, for he will think that all mathematical sciences were concerned, while the authors themselves acknowledge that they have never spoken about applied mathematics, i.e., astronomy, music, optics, and mechanics, which they readily admit to be true demonstrative sciences."[5]

Together with Biancani's revealing remark discussed above, this point seems to indicate how important it is for him that his opponents grant the status of demonstrative science to *scientiae mediae*. For, now it becomes clear that we have to sharply distinguish between (1) those adversaries of mathematics who concedes the existence of perfect demonstrations in pure mathematics but denying it for *scientia mediae*, (2) those who concede it for *scientiae mediae* but denying it for pure mathematics, and (3) those who deny it for both pure mathematics and *scientiae mediae*. It is obvious that he has to respond somewhat differently for strategic purposes depending upon the different type of his opponents. As a consequence, now we can see that the prime target of Biancani's dissertation must be of the second type, i.e., those who concede the existence of perfect demonstrations in *scientiae mediae* but denying it for pure mathematics.

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[5] Decimasexta, qua in universum proponunt hoc modo, utrum Mathematicae habeant perfectas demonstrationes, postea in discursu multa contra cas adducunt, quae tandem in fine tratationis contra solas Geometriam, & Arithmeticam valere fatentur. Quare nisi lector ad finem usque omnia per legerit, quod raro accidit, decipitur, putat enim in omnes Mathematicas illa quadrare, cum tamen ipsi fateantur, se nunquam loquutos esse de mediis Astronomia, Musica, Optica, Mecanica, quibus inesse veram scientiae demonstratiuae rationem libenter concedunt. (M 203, B. 310).

5. Biancani on Perfect Demonstrations

In chapter three of his dissertation, Biancani starts his discussion by his claim that the mathematicians of his age are "compelled to guard by every effort what was so far their safe, ancient, and rightful possession from some recent thinkers who strive to take it away". (B294, M184) And, in the next sentence, he raises a rhetorical question as to whether there "was ever a philosopher of stature before Alessandro Piccolomini who attempted to rob geometers of perfect demonstrations". By exploiting Piccolomini's claim for his own originality, Biancani declares assuredly that there has been none. Biancani's portrayal of the situation is apparently confirmed by Mancosu. According to Mancosu, in his work *Commentarium de Certitudine Mathematicarum Disciplinarium*(1547),

> "Piccolini challenged the traditional argument that mathematical sciences possess the highest degree of certainty because they make use of the highest type of demonstration, the *potissima* demonstration, defined by him as that which gives at once the cause and the effect (*simul et quia et propter quid*). (Mancosu(1996), p. 12)

On the other hand, in Laird's rather extensive comparisons and assessments of medieval and renaissance philosophers' views on the demonstrative power of mathematical sciences, we seem to find an entirely different picture. According to him, most of them except for Aquinas and Zabarella, denied the existence of *potissima* proofs in *scientiae mediae*.[6] Furthermore, according to Laird, despite their citations of Aquinas and Zabarella, none of the Jesuit commentators associated with the Collegio Romano allowed *propter quid* demonstrations for mixed sciences. (Laird(1997), p. 260) Finally, according to Laird, Biancani was a rare exception to the general tradition for his "rare confidence in the demonstrative power of the mixed sciences." (Laird(1997), p. 266)

Of course, we must be able to resolve many apparent inconsistencies involved in these reports by sharply distinguishing between the problem of *potissima* proof in pure mathematics and that of *scientiae mediae*, as Biancani did. Still we need to be alert to some potentially misguiding emphases and perspectives as well as incompatible reports. For, even if we restrict our concern to the problem of *potissima* proof in *scientiae mediae*, we find a contradiction between Biancani's and Laird's perceptions of the situation. If Biancani is right, then there was no disagreement about the existence of *potissima* proofs in *scientiae mediae*. On the other hand, if Laird is right, then there was even more

6) "In accordance with their understanding of Aristotle, the commentators all sought to keep the terms of premises and conclusions within the same subject genus. For most of them this meant that a mathematical proof in a mixed science is *quia*, not *propter quid*, since it is not made through the proximate, necessarily physical cause of the composite predicate's adhering in the composite subject. And when a mathematical middle term proved a mathematical predicate of a physical subject, the proof was usually considered *quia* through the remote cause. Only Zabarella and, in a more limited way, Aquinas, allowed for *propter quid* mathematical demonstrations in the mixed sciences." (Laird(1997), pp. 259-260.

serious disagreement about the existence of *potissima* proofs in *scientiae mediae* than in pure mathematics. Furthermore, it is not a minor problem whether to appreciate Biancani as championing the majorities or minorities in the disputes.

It seems to me that there is room for suspicion whether Laird and Mancosu are a bit unfair to Biancani in their description of the controversy about the scientific status of mathematical sciences. Above all, both tend to call or characterize the controversy in terms of the certainty of mathematical sciences. As we saw above, Macosu presents Piccolomini as challenging the traditional argument that mathematical sciences possess the highest degree of certainty because they use *potissima* demonstrations. Laird too, again as we saw above, testifies "the widespread doubt over the certainty of mathematics in general expressed by several prominent Jesuit professors at the Collegio Romano." (Laird(1997), p. 260) Mancosu's evidence for the existence of the allegedly traditional argument is Piccolomini's list of authorities including Aristotle, Averroes, Albert, Aquinas, and Nifo. (Mancosu(1992), 244; See also M 187, B.297) In order to prove the existence of a tradition, however, it may not be good enough to enumerate a list of authorities. Nor do I find in Biancani's dissertation any emphasis on the certainty of mathematical sciences.

If we assume Mancosu's perspective, it appears that the alleged traditional argument that mathematical sciences possess the highest degree of certainty might be another expression of the Platonic position according to which mathematics is superior to physics. According to the more widely accepted view in history of science, however, possibly more Aristotelian position according to which physics is superior to mathematics was more dominant in both medieval and renaissance period.. For example, Westman wrote: "If, however, an astronomer were determined to reconcile physical and mathematical issues, it would be customary within the Aristotelian tradition (which prevailed within the universities) to defer to the physicist, for in the generally accepted medieval hierarchy of the sciences, physics or natural philosophy was superior to mathematics." (Westman(1986), p. 78) In the footnote, he identifies that tradition as going back to Albertus Magnus, and contrasts it with the opposing tradition at Oxford. (Westman(1986), p. 105) In view of the existence of these two rival traditions, I think, Mancosu's presentation of Piccolomini as challenging the only extant tradition could be a distortion of history.

The reason why Piccolomini grants the certainty of mathematics might be understood as a subtle strategy for not allowing *potissima* proof for pure mathematics. According to Mancosu's exposition of Piccolomini's position, Piccolomini thinks that mathematics possesses the highest degree of certainty because mathematical objects are created by human mind. (Mancosu(1992), 244) Since such a position must deny the existence of mathematical objects independent of human mind, mathematical proofs apparently cannot be the *potissima* proof as Piccolomini defines. In other words, it could be a trap to formulate the issue in terms of the certainty of mathematics, and Biancani was wise enough to avoid the issue of mathematical certainty itself.

Laird provides us with a hint as to why some Jesuit commentators wanted to deny the certainty of mathematical sciences. For he writes: "they often doubted whether demonstrations within the mixed sciences could even be demonstratively certain, let alone *propter quid*." (Laird(1997), p. 260) Citing Toledo and Pereyra as his examples, Laird goes on to point out that "demonstrations in physics answer to Aristotle's ideal of the most powerful demonstrations." (Laird(1997), p. 261) One interesting consequence of such a view for *scientiae mediae*, according to Laird, is that "they have all the disadvantages of being mathematical—they do not demonstrate through cause—and all the uncertainty of physics—their subjects involve physical things." (Laird(1997), p. 261)

Now we can see that all the possible types of denying *potissima* proof to mathematical sciences indeed have actual instances in history. First, in Pereyra, we have the most radical position of denying *potissima* proof in both pure mathematics and in *scientiae mediae*. Secondly, as was pointed out, Biancani's primary targets were those who denied *potissima* proofs for pure mathematics but allowing them in *scientiae mediae*. Finally, more common opinion must have been allowing *potissima* proofs for pure mathematics but denying them for *scientiae mediae*. Since Biancani grants *potissima* proofs for both pure and *scientiae mediae*, it is curious why he does not bother with the first and the third type of adversaries in his dissertation.

Biancani explicitly mentions Pereyra as one of the two who are following Piccolomini's footsteps. (M 187, B. 297) Also, there are passages indicating that Biancani takes Pereyra's view quite seriously. For example, we may cite Biancani's interpretation of 32d proposition of the first book of Euclid's *Elements* as evidence. Mancosu discussed how Pereyra raised the issue of causality by using the same proposition as a clear counterexample to the causal theory of mathematics. According to Mancosu's exposition, Pereyra's criticism is that (1) the middle term in the proof of the 32d proposition is the appeal to auxiliary segments and to the external angles, but (2) the auxiliary segments and the external angles cannot be the true formal cause of the equality. (Mancosu(1996), pp. 14-15) Interestingly, Biancani uses the same proposition from Euclid's *Elements* as an example of the material cause in pure mathematics. Also, he notes that "the parallel line by which the [external] angle is divided is drawn in order to find the medium of the demonstration, but it is by no means the medium itself." (M 191-192, B 300); See also Piccolomini, p. 97) If so, it needs some explanation why Biancani simply ignores Pereyra's denial *of potissima* proof both for pure mathematics and *scientiae mediae*.

Even more strange is that Biancani does not discuss in detail the relationship between the subalternating science and the subalternated science. As in the discussion of Wallace and Dear, when we first hear about Aristotle's prohibition of *metabasis* and his *ad hoc* solution of allowing subalternated sciences, we tend to think that in subalternated science we can merely have *quia* demonstrations and that only in subalternating science we can have *propter quid* explanations. Indeed, we get the same impression from the 13th chapter of the first book of Aristotle's *Posteior Analytics*. According to recent studies on *scientia mediae* in medieval and renaissance commentaries on Aristotle's *Posterior Analytics*, however, there were several ingenious interpretations regarding the relationship between demonstrations in subalternating science and subalternated science. (Laird(1997), (1983); Livesey(1982), (1989))

Laird's discussion of Robert Grosseteste's case can be a nice sample for showing what kind of issues are involved. (Laird(1983), p. 36f.) Suppose that we have a proposition to be proved in optics:

> "that every two angles of which one constitutes the ray incident upon a mirror and the other the reflected ray are two equal radiant angles."

According to Grosseteste, we need to appropriate a proposition from geometry in order to prove it. In this case, the proposition from geometry is this:

> "Of any pair of triangles of which one angle of one is equal to one angle of the other and the sides containing the equal angles are proportional, the corresponding remaining angles are equal."

Grosseteste appropriates this geometrical proposition to optics by adding the condition "radiant" to its terms:

> "of any two radiant triangles of which one radiant angle of one is equal to one radiant angle of the other and the radiant sides containing the equal radiant angles are proportional, the

corresponding remaining radiant angles are equal".

Ultimately, as Laird represents the situation, we will have a syllogism of the following pattern:

<rA> rS rM
rM rP
rS rP

Within geometry the purely geometrical proposition MP could be used in a demonstration as follows:

<A> S M
M P
S P

Now, according to Laird, Grosseteste counts <A> as a demonstration *propter quid*, while <rA> as a demonstration *quia*: "For the cause of the equality of radiant angles is not in the major premise borrowed from geometry and appropriated to *perspectiva*, but it is rather in the nature of light and the regularity of nature". (Laird(1983), p. 40; cf. Robert Grosseteste, *Comm. Post. Anal.* I. 8, 93-101)

Though interesting, I believe that Grosseteste might be misrepresenting Aristotle's intention here. But I would not discuss this matter any further. For my present purpose, what is needed is to note simply that Latin commentators after Robert Grossesteste had to take stance in this matter on one way or another. For, as Laird points out, Grosseteste's commentary was not only the first systematic exposition of Aristotle's *Posterior Analytics* in the Latin West but also "the starting point for all subsequent discussions of the logical problems of demonstration within the intermediate sciences". (Laird(1983), p. 53) Further, insofar as the problem of the existence of *potissima* demonstrations in *scientiae mediae* does matter, the key lies in answering whether <rA> presents us a *propter quid* demonstration.

Apparently, Biancani does not discuss this issue extensively. But again he leaves us a revealing remark in an intriguing context:

> "Finally, in the fourth place, we confirm the same point by the common authority of all ancient authors, who always call geometrical proofs by appropriation *[per antonomasiam]*, and not reasons, or opinions, or tenets [*sententiae*], as it happens in other parts of philosophy. But let us turn from authority to reason." [7]

In chapter two of his dissertation on the nature of mathematics, where he discusses *potissima* demonstrations, Biancani provides us with four arguments based on authority and three arguments based on reason. In his arguments based on authorities, Binacani resorts to Aristotle, Plato, and Proclus, and devotes large space for each of them. Unlike these authorities, Biancani writes just one sentence for the fourth argument based on authority, as we have

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[7] Quarto tandem loco, communi authoritate omnium antiquorum idem comprobatur, apud quos semper demonstraationes Geometrice appellatae sunt per antonomasiam demonstrationes, non rationes, non opiniones, non sententiae, quemadmodum in reliquis philosophiae partibus fieri solet. Sed iam ab authoritatibus ad rationes. (M. 188, B. 297)

just seen. Why? Nor does he identify the authoritative philosopher either. Why? Even though the key word, i.e., "appropriation" indicates the opportunity to discuss Grosseteste, Biancani is referring to "the common authority of all ancient authors". Why?

6. Biancani's Strategy

Mancosu is almost silent on the problem of *scientiae mediae* in his book, for he declares that he will not discuss the relationship between mathematics and physics. (Macosu(1996), p. 3, p. 213) In fact, even when confined to pure mathematics, Mancosu's discussions of the Renaissance Jesuit Aristotelians' views on *potissima* demonstrations, mathematical explanations, and the problem of cause are interesting and of utmost importance both historically and philosophically. Further, Mancosu does not investigate the problem of mathematical explanation merely as of historical curiosity but as of utmost philosophical value. (Mancosu(2000), (2001)) I believe that Mancosu could have been indebted greatly to Biancani's dissertation in this regard, for the problem of explanation has been rarely discussed in contemporary philosophy of mathematics.[8] From this point of view, the first chapter of Biancani's dissertation on the nature of mathematics, which deals with the problem of the subject matter of pure mathematics, the problem of mathematical abstraction, and the

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[8] Mark Steiner must be a notable exception in this regard. See Steiner(1975) and Steiner(1998).

problem of definition in mathematics, seems to be a treasure house for fascinating issues for philosophers of mathematics.

Though tempting, it must be beyond the scope of this article to delve into some such fundamental philosophical issues in pure mathematics. Let me just draw your attention to one point that has definite relevance for understanding Biancani's strategy to handle his adversaries. Biancani begins his discussion of the first chapter with a revealing claim that pure mathematics differs in kind from applied mathematics. More interestingly, he writes:

> "Quantity abstracted from sensible matter is usually considered in two ways. For it is considered by the natural scientist and the metaphysician in itself, that is absolutely, insofar as it is quantity, whether it is delimited [terminata] or not; and in this way its properties are divisibility, locatability, figurability, etc. But the geometer and the arithmetician consider [quantity] not absolutely, but insofar as it is delimited, as are the finite straight or curved lines in continuous quantity…" [9]

What I find interesting and revealing here might be expressed by raising the following two questions: (1) Why does Biancani contrast pure mathematics with natural science and metaphysics, when we expect to hear about exactly wherein lies the difference between pure and applied mathematics?; (2) Why does he not compare mathematics, natural science, and metaphysics[10], thereby

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9) Quantitas igitur abstracta a materia sensibili dupliciter considerari solet. Consideratur enim a Physico, & Mathematico secundum se, idest, absolute, quatenus Quantitas est, siue terminata sit, siue non; qua ratione affectiones ipsisus sunt, diuisibilitas, locabilitas, figurabilitas, &c. a Gepmetra vero, & Aritthmetico consideratur non absolute, sed quatenus est terminata, ut sunt in quantitate continua lineae finitae rectae, aut curuae, ··· (M 179, B 289)

highlighting the peculiarity of each, but comparing mathematics with "natural science and metaphysics lumped together"? As for the first question, the only possible explanation seems to this. Bicancani thinks that the difference in kind between pure and applied mathematics cannot be understood without contrasting pure mathematics with "natural science and metaphysics". As for the second question, we might speculate that Biancani is worried about the possibility of losing sight of the differentiating characteristic of mathematics from other speculative sciences (i.e., natural science and metaphysics) in case he adopts the traditional way of comparing the three speculative sciences. But what is the ultimate reason for his ways of thinking and worries? I cannot figure out any other than strategic, rhetorical, and political reasons.

The reason why Biancani depends on ancient and more recent authorities, thereby being parsimonious to cite medieval sources might be explained if we incorporate the general tendency of

[10] Clavius provides us with a clear example of this way of comparing the three disciplines: "Because the mathematical disciplines discuss things that are considered apart from any sensible matter—although they are themselves immersed in matter—it is evident that they hold a place intermediate between metaphysics and natural sciences, if we consider their subjects, as is rightly shown by Proclus. For the subject of metaphysics is separated from all matter, both in the thing and in reason; the subject of physics is in truth conjoined to sensible matter, both in the thing and in reason; when, since the subject of the mathematical disciplines is considered free from all matter—although it[i.e., matter] is found in the thing itself—clearly it is established intermediate between the other two." (Clavius, "*In disciplinas mathematicas prolegomena*" in *Opera mathematica*, Vol. 1, p. 5; requoted from Dear(1995), p. 37).

Renaissance scholars to recover the purity of ancient philosophers eliminating all Arabic and Latin medieval accretions. (cf. Lohr(1999)) Also, the reason why Biancani simply ignores Pereyra's radical denial of potissima proof both for pure and applied mathematics might be explained if we remember that Biancani, as a contemporary of Galileo. In other words, Biancani was in a position to exploit fully the external support from the remarkable success of applied mathematical sciences of his time.

But why does Biancani emphasize the difference in kind between pure and applied mathematics? If Dear is right in his interpretation, Biancani as well as Clavius would include mathematical sciences within the realm of the Aristotelian ideal model of science. But, if so, would they include (1) pure mathematics only in that realm, or (2) applied mathematics only, or (3) both pure and applied mathematics? (1) cannot be the case, for that seems contradicting the overall project of securing the scientific status of applied mathematics. Nor (2) can be the case, for that would make Biancani's entire discussion of the excellence of pure mathematics futile and useless. If (3) is Biancani's intention, why does he emphasize the difference in kind between pure and applied mathematics?

7. Concluding Remarks

Following the lead of Wallace, Dear, and Mancosu, I tried to improve our understanding of the controversy among Renaissance

Jesuit Aristotelians around the scientific status of mathematical sciences. Largely due to rhetorical nature of Biancani's work on the nature of mathematics, however, we are left with more puzzles and problems. At best, we might fathom Biancani's hidden agenda as follows:

> If we show that applied mathematics belongs to the same camp with physics and metaphysics, and if we loosen the alertness of our adversaries by emphasizing the difference in kind between pure and applied mathematics, (since applied mathematics is nothing but the result of applying pure mathematics after all) they themselves might have accepted inadvertently the point that the entirety of physics (and possibly even metaphysics) is the application of pure mathematics.

Now, in view of all this, what do we have to learn from Biancani's discussion of *scientiae mediae*? I think that there are at least two lessons if we compare our current situation with that of Biancani. One has to do with the practice of philosophy of mathematics, the other with redrawing the map of human knowledge.

Philosophy of mathematics in the twentieth century largely ignored the problems of applying mathematics. Only quite recently, we began to realize the significance of the philosophical issues involved in the application of mathematics in all the different individual sciences. On the other hand, virtually all mathematicians from Biancani's time to 1900 had to struggle with the philosophical problems of applying mathematics. That means, we cannot afford to ignore Aristotelian philosophies of mathematics in 17^{th}, 18^{th}, and 19^{th} centuries as antecedent cases in philosophy of applied mathematics.

In the age of ever growing science, engineering, and technology, philosophers have no voice in organizing human knowledge. On the other hand, in the medieval and renaissance Latin West, philosophers were not only equipped with a general model of Aristotelian science but also able to detail it by probing questions as to the relationships among individual sciences. As was the case in Aristotle's discussion of geometry, optics, and the study of rainbow, the issue of subordination between mathematical sciences tends to be extended to other scientific disciplines as well. As was hinted at above, some such debates are found in contesting the scientific status of theology in the middle ages. We also noted that, according to some Renaissance Jesuit Aristotelians, physics provides us with the best examples of perfect demonstrations. No matter what variations, complications, and changes were there, however, the dream of drafting the blueprint for the entire edifice of human knowledge modeling after the relationship between pure and applied mathematics was always there.

Biancani was so confident about the success of *scientiae mediae* as to ignore adversaries who denied *potissima* proofs for them. Mancosu was so sure about the promise of his project of rewriting the history of philosophy of mathematics as to be silent about the relation between mathematics and physics. As for the *fortuna* of philosophy and science in the 21^{st} century, could we happily adopt their strategy?

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The Status of *Scientiae Mediae* in the History of Mathematics: Biancani's Case

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최근 1600년 경 예수회 아리스토텔레스주의자들 사이에서 벌어졌던 수학의 과학으로서의 지위에 관한 논쟁에 대한 관심이 급증하고 있다. 필자는 월러스, 디어, 그리고 만코수를 좇아 이 논쟁을 조금 더 자세히 살펴보고자 한다. 이를 위해 필자는 비앙카니가 수학의 본성에 관한 논고에서 중간과학을 논의한 바에 초점을 맞출 것이다. 디어와 월러스의 논의로부터 우리는 수학의 과학적 지위를 옹호한 이들과 부인한 이들 사이의 논쟁에 관한 비교적 훌륭한 조감도를 얻을 수 있다. 그러나 그 논쟁의 일반적 동기를 이해하는 일과 그 안에 내포된 변증적 전략과 전술의 정교함을 감식하는 일은 전혀 별개의 문제이다. 바로 이 단계에서 우리는 중간과학에 관한 비앙카니의 견해의 요점을 이해하는 데에서 어려움에 봉착한다. 비록 중간과학의 문제에 관해서는 침묵을 지켰지만, 만코수가 완벽한 증명, 수학적 설명, 그리고 원인의 문제에 관한 예수회 아리스토텔레스주의자들의 입장을 논의한 바는 역사적으로나 철학적으로나 모두 최고의 중요성을 지닌다. 필자는 그 논쟁에서 진실로 무엇이 쟁점이었는지를 보다 심도 있게 이해하기 위하여 피콜로미니와 비앙카니의 견해에 대한 만코수의 해석을 주의 깊게 검토하고 비판할 것이다.

[주요어] 중간과학, 비앙카니(블랑카누스), 만코수, 증명, 수학적 설명, 아리스토텔레스주의 수학철학