A Deterministic Inventory Model with an Inventory-Level-Dependent Demand Rate under Day-terms Supplier Credit in a Supply Chain

Seong Whan Shinn*

*Department of Industrial Engineering, Halla University

신용거래가 허용되는 공급체인에서 재고종속형 제품 수요를 고려한 확정적 재고모형

신 성 환*

*한라대학교 산업경영공학과

Abstract

본 연구는 공급자(supplier), 중간분배자(retailer/distributor) 그리고 고객(customer)으로 구성된 2 단계 공급사슬을 대상으로 공급자가 수요 증대를 목적으로 중간분배자에게 일정기간 동안 제품대금에 대한 지불 연기를 허용한다는 가정 하에 중간분배자의 최적 재고정책 결정을 위한 재고 모형을 다루었다. 소비성 상품의 경우, 고객의 수요는 일반적으로 상품 진열대에 진열되어있는 상품의 재고량에 많은 영향을 받는 것으로 알려져 있다. 따라서 본 연구에서는 이와 같은 상황을 고려하여 고객의 수요가 중간분배자의 재고량에 영향을 받는다는 가정 하에 재고 모형을 설계하였고, 모형 분석을 통하여 이익을 최대화하는 경제적 주문량 결정 방법을 제시하였다. 또한 예제를 통하여 제시된 해법을 적용하고, 그 타당성을 보였다.

Keywords : Inventory, Day-terms Credit, Inventory-Level-Dependent Demand Rate, Supply Chain

1. Introduction

A significant part of the manufacturing goods is usually kept in a supply chain, especially either in a manufacturer's/wholesaler's inventory or in a retailer's storage. And it induces the excessive cost for carrying inventory in a supply chain. So, an effective supply chain network requires a cooperative relationship between the supplier and the distributor/retailer. In today's business transactions, it is more and more common to see that the buyers are allowed some grace period before they settle accounts with the supplier. Trade credit would play an important role in the conduct of business for many reasons. For a supplier who offers trade credit, it is an effective means of price discrimination which circumvents anti-trust measures and is also an efficient method to stimulate the demand of product. For a retailer, it is an efficient method of bonding a supplier when the retailer is at the risk of receiving inferior quality goods or service and is also an effective means of reducing the cost of holding stocks.

In this regard, Haley and Higgins[5], Kingsman[6], Chapman et al.[2], Goyal[4], and Ward and Chapman[13] examined the effects of trade credit on the optimal inventory policy. All the research works mentioned above held the assumption that the demand is a known constant.

† 교신저자: 신성환, 강원도 원주시 흥업면 한라대1길 32 한라대학교 공과대학 산업경영공학과
M · P : 010-9214-5434, E-mail : swshinn@halla.ac.krr
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However, for certain commodities, such as consumer goods, food grains, stationery items, consumable etc., the customer's demand rate may depend on the size of the quantityon hand. According to Levin et al.[7], one of the function of inventories is that of a motivator; they indicated: "At times, the presence of inventory has a motivational effect on the people around it. It is a common belief that large piles of goods displayed in a supermarket will lead the customer to buy more".

Such a situation generally arises for a consumer-goods type of inventory and the demand rate may go up or down if the on-hand inventory level increases or decreases. It is, therefore, likely to have an effect of increasing the size of each order and the availability of opportunity to delay the payments have an important effect on the retailer's lot size.

Some research papers evaluated an inventory system where the demand rate has been assumed to be dependent on the on-hand inventory. Among the important research papers published so far with inventory-level-dependent demand rate, mention should be made of works by Baker and Urban[1], Mandal and Phaujdar[8], Datta and Pal[3], Padmanabhan and Vrat[9], and Vrat and Padmanabhan[12], etc. Baker and Urban[1] evaluated an inventory system assuming the demand rate to be a polynomial functional form of the on-hand inventory level at that time. Datta and Pal[3] discussed a similar situation where the demand rate declines along with inventory level down to a certain level of the inventory, and then the demand rate becomes constant for the rest of cycle. Mandal and Phaujdar[8], and Padmanabhan and Vrat[9] analyzed an inventory model for deteriorating products where the demand rate has been assumed to be dependent on the on-hand inventory. Vrat and Padmanabhan[12] also discussed an inventory model for inventory-level-dependent demand rate products under inflation. With this type of product, the probability of making a sale would increases as the amount of the product in inventory increases. It is, therefore, likely to have an effect of increasing the size of each order. Also, the availability of opportunity to delay the payments effectively reduces the retailer's cost of holding inventories, and thus is likely to result in larger order quantity.

In this regard, Song and Shinn[11], and Shinn[10] evaluated an inventory system under day-terms supplier credit where the customer's demand rate has been assumed to be dependent on the on-hand inventory. They examined the effects of trade credit on the optimal inventory policy assuming that the demand rate is to be a linear function of the on-hand inventory level at that time.

In this paper, we evaluate the problem of determining the retailer's optimal ordering policy for an inventory-level-dependent demand rate items when the supplier offers a fixed credit period. For the analysis, we assume that the customer's demand rate is of a polynomial function form. In the next section, we formulate a relevant mathematical model.

The properties of an optimal solution are discussed and solution algorithm is given in Section 3. Numerical example is provided in Section 4, which is followed by concluding remarks.

2. Supply Chain Model Formulation

The model presented is the continuous, deterministic case of an inventory system in which the demand rate is dependent on the inventory level. The assumptions of this model are as follows:

(1) Replenishments are instantaneous with a known and constant lead time.
(2) No back orders are allowed.
(3) The inventory system involves only one item.
(4) The demand rate is deterministic and is a known function of the level of inventory.
(5) The supplier proposes a certain credit period and sales revenue generated during the credit period is deposited in an interest bearing account with rate I.

At the end of the period, the credit is settled and the retailer starts paying the capital opportunity cost for the items in stock with rate R($R \geq I$).

And in deriving the model, the following notations are used:

P : unit retail price.
C : unit purchase cost.
S : fixed ordering cost.

tc : credit period set by the supplier.

H : inventory carrying cost, excluding the capital opportunity cost.

R : capital opportunity cost(as a percentage).

I : earned interest rate(as a percentage).

Q : order size.

T : replenishment cycle time.

$q(t)$: inventory level at time t.

$D(q(t))$: annual demand rate, as a function of the on-hand inventory(q(t)).

This analysis will concentrate on the situation in which the customer's demand rate is of a polynomial function form. That is, the demand rate will take the form

$$D(q(t)) = \alpha q(t)^{\beta},\ \alpha > 0,\ 0 < \beta < 1, \qquad (1)$$

where D(q(t)) is the demand rate of the product, q(t) is the inventory level at time t, α is the scale parameter, and β is the shape parameter. Given this demand function, the inventory level as a function of time will decrease rapidly initially, since the quantity demanded will be greater at a high level of inventory. As the inventory is depleted, the quantity demanded will decrease, resulting in the inventory level decreasing more slowly. Figure 1 illustrates the time behavior of the inventory level. As stated by Baker and Urban[1], the mathematical expression of the inventory function over time can be determined by equating the rate of change of inventory level per unit time with minus the demand rate, and solving the resulting differential equation as follows :

$$\frac{dq(t)}{dt} = -D(q(t)) \qquad (2)$$

$$\frac{dq(t)}{dt} = -\alpha q(t)^{\beta} \qquad (3)$$

$$\int q(t)^{-\beta} dq(t) = \int -\alpha dt \qquad (4)$$

$$\frac{q(t)^{1-\beta}}{1-\beta} = -\alpha t + k \qquad (5)$$

$$q(t)^{1-\beta} = -(\alpha(1-\beta)t) + k, \qquad (6)$$

when $t=0$, $q(t)=Q$, so $k = Q^{1-\beta}$ and $q(t)^{1-\beta} = -(\alpha(1-\beta)t) + Q^{1-\beta}$. Solving for q(t) yields

$$q(t) = \begin{cases} (Q^{1-\beta} - \alpha(1-\beta)t)^{\frac{1}{1-\beta}} & ,t \le \dfrac{Q^{1-\beta}}{\alpha(1-\beta)}, \\ 0 & ,t > \dfrac{Q^{1-\beta}}{\alpha(1-\beta)}, \end{cases} \qquad (7)$$

where

Q = the order level(the quantity to order up to);

t = time from the start of a cycle.

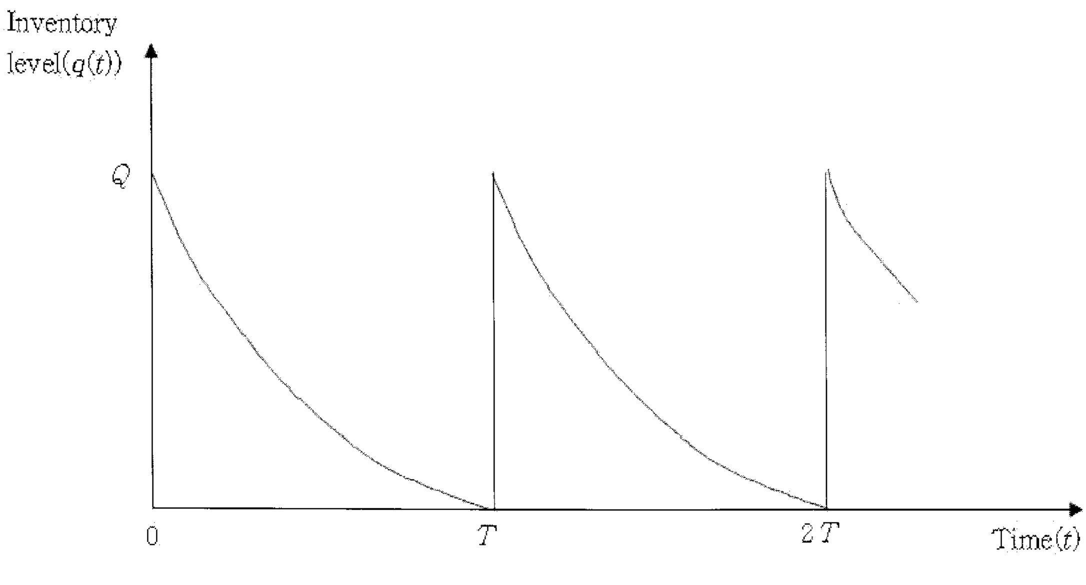


Figure 1.　Inventory level(q) vs. Time(t).

Note that due to the inventory carrying costs, it is clearly optimal to let the inventory level reach zero before reordering, i.e., q(T) = 0. So, given that q(T) = 0,

$$Q^{1-\beta} - \alpha(1-\beta)T = 0 \qquad (8)$$

and so,

$$T = \frac{Q^{1-\beta}}{\alpha(1-\beta)}\ . \qquad (9)$$

The objective of this model will be to maximize the annual net profit for retailer. The annual net profit, $\Pi(Q)$ would be as follows:

$$\Pi(Q) = \begin{matrix}Annual\\ Sales\\ Revenue\end{matrix} - \begin{matrix}Annual\\ Purchasing\\ Cost\end{matrix} - \begin{matrix}Annual\\ Ordering\\ Cost\end{matrix}$$

$$- \begin{matrix}Annual\\ Inventory\ Carrying\\ Cost\end{matrix} - \begin{matrix}Annual\\ Capital\ Opportunity.\\ Cost\end{matrix}$$

And they consist of the following five elements.

(1) Annual sales revenue = $\frac{PQ}{T}$.

(2) Annual purchasing cost = $\frac{CQ}{T}$.

(3) Annual ordering cost = $\frac{S}{T}$.

(4) Annual inventory carrying cost:
From the equation (7), the inventory level per order is

$$q_i = \int_0^T (Q^{1-\beta} - \alpha(1-\beta)t)^{\frac{1}{1-\beta}} dt,\ T = \frac{Q^{1-\beta}}{\alpha(1-\beta)} \qquad (10)$$

$$= -\frac{(Q^{1-\beta} - \alpha(1-\beta)T)^{\frac{2-\beta}{1-\beta}} - Q^{2-\beta}}{\alpha(2-\beta)} \qquad (11)$$

$$= \frac{Q^{2-\beta}}{\alpha(2-\beta)}\ . \qquad (12)$$

So, the inventory carrying cost per order becomes Hq_i and then

Annual inventory carrying cost = $\frac{Hq_i}{T}$.

(5) Annual capital opportunity cost:
(i) Case 1($tc \le T$): (see Figure 2. (a)) The number of products in stock earning interest during time $(0, tc)$ is

$$q_e = Qtc - \int_0^{tc} (Q^{1-\beta} - \alpha(1-\beta)t)^{\frac{1}{1-\beta}} dt \qquad (13)$$

and the interest earned per order becomes CIq_e.
Also, the number of products in stock paying interest, during time (tc, T) becomes

$$q_f = \int_{tc}^T (Q^{1-\beta} - \alpha(1-\beta)t)^{\frac{1}{1-\beta}} dt \qquad (14)$$

and the interest payable per order can be expressed as CRq_f. Therefore,

Annual capital opportunity cost = $\frac{CRq_f - CIq_e}{T}$.

(ii) Case 2($tc > T$): (see Figure 2. (b)) The number of products in stock earning interest during time $(0, tc)$ is

$$q_e = Qtc - \int_0^T (Q^{1-\beta} - \alpha(1-\beta)t)^{\frac{1}{1-\beta}} dt. \qquad (15)$$

And in this case, the number of products in stock paying interest, $q_f = 0$. Therefore,

Annual capital opportunity cost = $-\frac{CIq_e}{T}$.

So, depending on the relative size of tc to T, $\Pi(Q)$ has two different expressions as follows:

1. Case 1 ($tc \le T$)

$$\Pi_1(Q) = \frac{(P-C)Q - S - Hq_i - C(Rq_f - Iq_e)}{T}, \qquad (16)$$

$$= \alpha(1-\beta)(P-C)Q^{\beta} - \alpha(1-\beta)SQ^{-(1-\beta)} + \alpha(1-\beta)CItcQ^{\beta}$$

$$- \frac{1-\beta}{2-\beta}QH - CR\left(\frac{1-\beta}{2-\beta}\right)(Q^{1-\beta} - \alpha(1-\beta)tc)^{\frac{2-\beta}{1-\beta}} Q^{-(1-\beta)}$$

$$+ CI\left(\frac{1-\beta}{2-\beta}\right)(Q^{1-\beta}-\alpha(1-\beta)tc)^{\frac{2-\beta}{1-\beta}}Q^{-(1-\beta)}-\frac{1-\beta}{2-\beta}QCI, \quad (17)$$

2. Case 2 ($tc > T$)

$$\Pi_2(Q)=\frac{(P-C)Q-S-Hq_i+CIq_e}{T}, \quad (18)$$

$$=\alpha(1-\beta)(P-C)Q^{\beta}-\alpha(1-\beta)SQ^{-(1-\beta)}-\frac{1-\beta}{2-\beta}QH$$

$$+\alpha(1-\beta)CItcQ^{\beta}-\frac{1-\beta}{2-\beta}QCI, \quad (19)$$

3. Determination of Optimal Policy for Retailer

The problem is to find an economic ordering quantity(EOQ) for retailer which maximizes $\Pi(Q)$.

Then, we can consider the necessary and sufficient conditions for maximizing $\Pi(Q)$ with respect to Q.

Note that the annual net profit, $\Pi(Q)$, at T = tc is obtained on substituting tc $=\frac{Q^{1-\beta}}{\alpha(1-\beta)}$ in equations (17) and (19), respectively. And we have the following relationship between $\Pi_i(Q)$, $i=1,2$,

$$\Pi_1(Q) = \Pi_2(Q),\ \text{tc} = \frac{Q^{1-\beta}}{\alpha(1-\beta)}. \quad (20)$$

And this implies that the annual net profit function $\Pi_i(Q)$, $i=1,2$, is continuous at T = tc. Now, we are going to investigate the characteristics of $\Pi_i(Q)$, $i=1,2$. The first order condition with respect to Q is

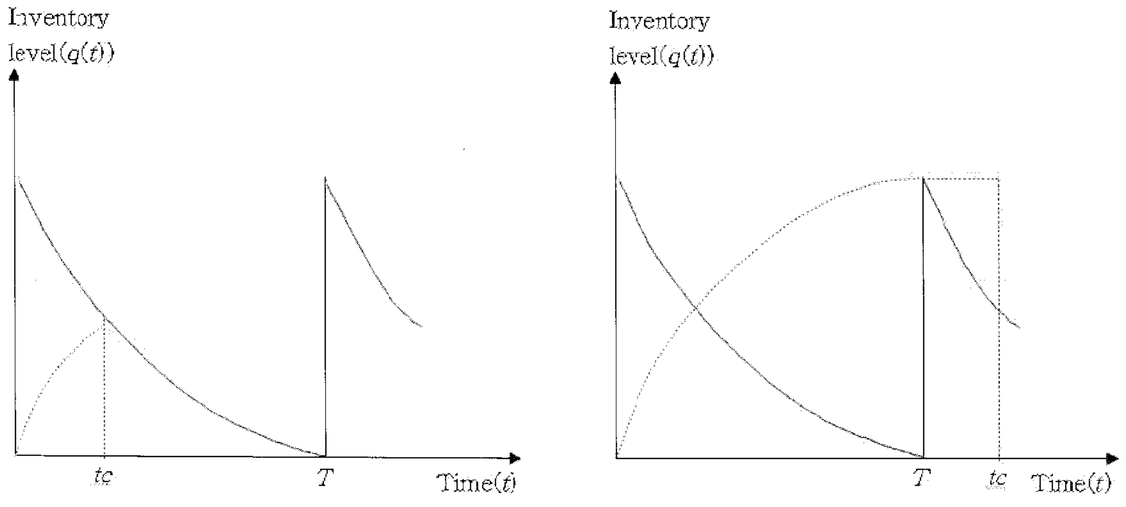
$$\Pi_1(Q)' = -\frac{1-\beta}{2-\beta}(H+CI)+\alpha(1-\beta)^2SQ^{-(2-\beta)}$$
$$+\alpha\beta(1-\beta)(P-C(1-Itc))Q^{-(1-\beta)}$$
$$-\frac{(1-\beta)C(R-I)(Q^{1-\beta}-\alpha(1-\beta)tc)^{\frac{1}{1-\beta}}}{Q}$$
$$+\frac{(1-\beta)^2C(R-I)(Q^{1-\beta}-\alpha(1-\beta)tc)^{\frac{2-\beta}{1-\beta}}}{(2-\beta)Q^{2-\beta}}, \quad (21)$$

$$\Pi_2(Q)' = -\frac{1-\beta}{2-\beta}(H+CI)+\alpha(1-\beta)^2SQ^{-(2-\beta)}$$
$$+\alpha\beta(1-\beta)(P-C(1-Itc))Q^{-(1-\beta)}. \quad (22)$$

Similarly the second order condition with respect to Q is

$$\Pi_1(Q)'' = -\alpha\beta(1-\beta)^2(P-C(1-Itc))Q^{-(2-\beta)}$$
$$-\alpha(1-\beta)^2(2-\beta)SQ^{-(3-\beta)} -$$
$$\frac{(1-\beta)C(R-I)(Q^{1-\beta}-\alpha(1-\beta)tc)^{\frac{1}{1-\beta}}}{Q^2}\left(\frac{Q^{1-\beta}}{Q^{1-\beta}-\alpha(1-\beta)tc}-1\right)-$$
$$\frac{(1-\beta)^2C(R-I)(Q^{1-\beta}-\alpha(1-\beta)tc)^{\frac{1}{1-\beta}}}{Q^2}\left(\frac{Q^{1-\beta}-\alpha(1-\beta)tc}{Q^{1-\beta}}-1\right), \quad (23)$$

$$\Pi_2(Q)'' = -\alpha\beta(1-\beta)^2(P-C(1-Itc))Q^{-(2-\beta)}$$
$$-\alpha(1-\beta)^2(2-\beta)SQ^{-(3-\beta)}. \quad (24)$$



(a) $tc \le T$　　(b) ($tc > T$)

Figure 2. Credit period(tc) vs. Replenishment cycle time(T).

For the normal condition($R \geq I$), it can be shown that $\Pi(Q)$ is a concave function of Q.

Based on the above results, we develop the following solution procedure to determine an EOQ for retailer.

Solution algorithm

Step 1. Determine Q_1 from $\Pi_1(Q_1)' = 0$ by equation (21). If $Q_1 \geq (\alpha(1-\beta)tc)^{\frac{1}{1-\beta}}$, then obtain $\Pi_1(Q_1)$ by equation (17) and go to step 2. Otherwise, go to step 2.

Step 2. Determine Q_2 from $\Pi_2(Q_2)' = 0$ by equation (22). If $Q_2 < (\alpha(1-\beta)tc)^{\frac{1}{1-\beta}}$, then obtain $\Pi_2(Q_2)$ by equation (19) and go to step 3. Otherwise, go to step 3.

Step 3. If $Q_1 < (\alpha(1-\beta)tc)^{\frac{1}{1-\beta}}$ and $Q_2 \geq (\alpha(1-\beta)tc)^{\frac{1}{1-\beta}}$, then obtain $\Pi_1(Q)$ by equation (17) at $Q = (\alpha(1-\beta)tc)^{\frac{1}{1-\beta}}$ and go to step 4. Otherwise, go to step 4.

Step 4. Select the optimal order quantity which gives the maximum annual net profit among those obtained in steps 1, 2 and 3.

4. Numerical Example

As an example, suppose the parameters of the inventory function are as follows:

$\alpha = 1,500$ units per year;
$\beta = 0.3$;
S = $500 per order;
H = $5 per unit per year;
R = 15%;
I = 10%;
P = $65 per unit;
C = $50 per unit;
tc = 0.3 year per order.

In order to solve this problem, a computer program written in QBASIC was developed. The input required for this program includes the demand(scale and shape) parameters, the set-up cost, the inventory(carrying, capital opportunity cost and earned interest rate) parameters, the selling price and cost of the item. With the credit period of 0.3 year per order, the solution algorithm provides a solution to this problem of:

Optimal order quantity, Q = 21,275 units;
Maximum annual net profit $\Pi(Q)$=$246,891.

An optimal solution for the example can be obtained through the following steps.

Step 1. From equation (21), $\Pi_1(Q)' = 0$ at $Q = 21,275$. So, $Q_1 = 21,275$. Since $Q_1 \geq (\alpha(1-\beta)tc)^{\frac{1}{1-\beta}} = 3,707$, compute $\Pi_1(Q_1)$ by equation (17), and go to Step 2.

Step 2. From equation (22), $\Pi_2(Q)' = 0$ at $Q = 27,029$. So, $Q_2 = 27,029$. Since $Q_2 > (\alpha(1-\beta)tc)^{\frac{1}{1-\beta}} = 3,707$, go to Step 3.

Step 3. Since $Q_1 \geq (\alpha(1-\beta)tc)^{\frac{1}{1-\beta}}$, go to Step 4.

Step 4. From the results in Steps 1, 2 and 3, an EOQ becomes 21,275 units with its maximum annual net profit $246,891.

5. Conclusion

In conclusion, the on-hand inventory level is one of the important factors related to the variation of the customer's demand rate and it is, therefore, likely to have an effect on increasing the size of each order. But, no inventory model has been found in the literature that addresses an inventory-level dependent demand rate pattern with a polynomial functional form.

This paper dealt with the retailer's optimal ordering quantity determination problem under an inventory-level-dependent demand rate assuming that the customer's demand function is of a polynomial function form when the supplier offers a fixed credit period. For the system presented, a mathematical model formulated. And we proposed the solution procedure, which leads to a retailer's optimal ordering quantity for the model developed. To illustrate the validity of the solution procedure, an example problem was chosen and solved.

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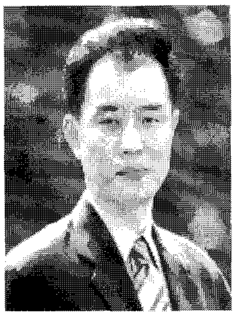
저 자 소 개

신 성 환



현재 한라대학교 공과대학 산업경영공학과 교수로 재직 중이며, 인하대학교 산업공학과를 졸업하고, 한국과학기술원 산업공학과에서 공학석사 및 공학박사학위를 취득하였다. 주요 관심분야는 SCM, 물류관리, 생산관리 등이다.

주소: 강원도 원주시 흥업면 한라대1길 32 한라대학교 공과대학 산업경영공학과