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자기 벡터 포텐셜 해석을 이용한 금속 검출기 코일 헤드의 검출 성능 최적화

(Optimization of the Coil Head of Metal Detectors Using a Magnetic Vector Potential Approach)

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요 약

본 논문에서는 자기 벡터 포텐셜 해석을 통해 3- 코일 구조를 갖는 금속 검출기 헤드가 수신 코일에 미치는 유도 전압을 구하고, 금속 물체의 특성이 송신 코일의 임피던스에 미치는 영향을 유도하였다. 이러한 해석을 통하여 최적의 검출 성능을 위한 송신 코일과 수신 코일 간의 거리를 구하였고, 최대 유도 전압을 출력하는 금속의 위치를 유도하였다. 또한 기존의 수식과 비교하여 본 논문의 자기 벡터 포텐셜 해석에 대한 타당성을 입증하였다.

Abstract

We derive an equation that predicts the induced voltage across the receiving terminals of the three-coil head of a metal detector using a magnetic vector potential approach. We also derive an equation that relates the change of the impedance of the transmitting coil to the properties of the metal. We utilize the results to obtain the optimum spacing between the driving and the receiving coils at which the maximum induced voltage is attained. Further, we determine the position of the metallic object where the voltage reaches its peak. We verify our work by comparing the results with those of a previous work.

Keywords : metal detector, head coils, three-coil head, eddy current, optimum spacing, impedance change.

Ⅰ. Introduction

Metal detectors are widely used for quality control of food, medicine and for detecting weapons and mines, etc. There are several types of metal detectors: solenoid type, separated-loop type, and closed-loop type, and so forth. Most of the metal detectors make use of the eddy current induced on the metal surface. Among the various types, the closed-loop type with a three-coil head is most widely used for industrial applications.

The three-coil head detects metal through the imbalance of the magnetic field in the receiving coils caused by the eddy current on the metallic object. The three-coil head has advantages that the path for the material under test is of tunnel type, that the magnetic field in the aperture is uniform and that there are two chances for detection because of the two receiving coils. The sensitivity of the metal detector depends on the structure, dimensions, and spacing of the head coils.

In this paper, we derive an equation that predicts

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the induced voltage across the terminals of the closed-loop receiving coils through analysis of the vector magnetic potential which gives us good insight into the principle of the metal detection. During the derivation of the equation, we also derive an equation of the input impedance variance of the transmitting coil due to the fields generated from the eddy current on the surface of the metallic object. The eddy current is induced by the fields from the transmitting coil. The impedance variation is dependent upon the dimension and properties of the metallic object.

In section II, we analyze the change in vector magnetic potential due to the eddy current on the surface of the metallic object. From the change in vector magnetic potential, we derive the voltage across the terminals of the receiving coils. In the midst of the derivation, we obtain the relation between the variation of the input impedance of the transmitting coil and the properties of the metallic object. In section III, we use a numerical method to obtain the optimum spacing between the transmitting and receiving coils. In addition, we obtain the position of the metallic object where the induced voltage is maximum. In section IV, we conclude our work.

II. Derivation

Fig.1 shows the basic structure of the closed-loop type three-coil head of the metal detector. It consists of a single-turn transmitting coil and two single-turn connected receiving coils. The material under test passes through the aperture formed by the coils. The simplified diagram of the three-coil head is shown in Fig.2, where a metallic sphere to be detected is shown in-between the (inner) transmitting and the (right) receiving coils. For the brevity of analysis, the shape of the coils are assumed to be circular. The three identical coils are arranged such that their centers are aligned coaxially and the spacings between them are equal.

In the figure, s denotes the spacing between the coils, z the distance from the center of the inner coil to the metallic object, r the radius of the coils and a the radius of the metallic sphere (not marked on the figure). The inner coil is the transmitter and is driven with an alternating current. We assume that the dimension of the head is very small compared with the wavelength of the driving alternating current, which means we can neglect the displacement current term in the Maxwell equation. The outer coils are the receiver and are used to detect the imbalance caused by a metallic object put through the coils via a conveyer belt or any other means.

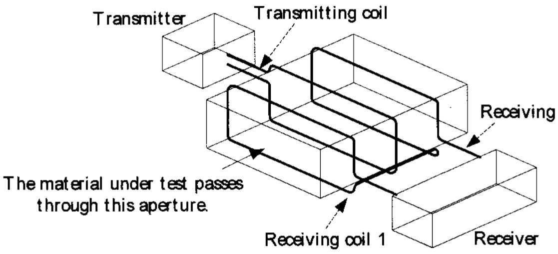

그림 1. 3-코일 헤드 방식 금속 검출기의 폐루프 형 구조

Fig. 1. The structure of the closed-loop type three-coil head of the metal detector.

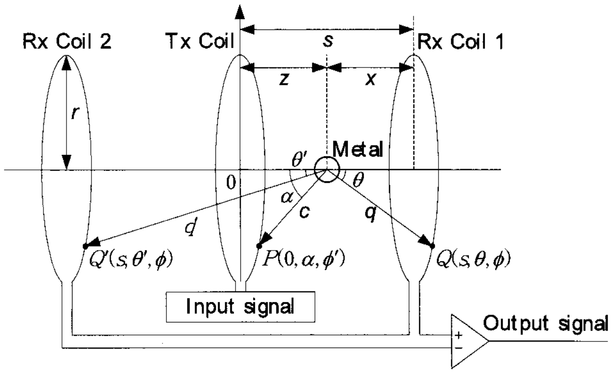

그림 2. 금속 검출기 3-코일 헤드의 구조

Fig. 2. The configuration of the three-coil head of the metal detector.

When there is no electromagnetic material in the head that disturbs the symmetry, the voltages induced in the two receiver coils cancel since the magnetic flux density variations inside both coils are identical. We use magnetic vector potential to calculate the induced voltages in the receiver coils. We first obtain the magnetic vector potential in the space where current I flows in the transmitter coil and a metallic sphere with a conductivity σ and a relative permeability μ_r resides on the co-axis and off from the center of the (inner) transmitter coil with a distance z. Due to the spherical metallic object, the magnitude variation ΔA of the vector magnetic potential becomes[1]

$$\Delta A = \frac{\mu_0 I}{2} \sum_{n=1}^{\infty} \frac{\sin\alpha}{n(n+1)} P_n^1(\cos\alpha) \times P_n^1(\cos\theta) D(p) c^{-n} q^{-n-1} \tag{1}$$

where $D(p)$ is given by

$$D(p) = a^{2n+1} \times \left(1 - \frac{\mu_r (2n+1) I_{n+\frac{1}{2}}(a\sqrt{jp})}{a\sqrt{jp}\, I_{n-\frac{1}{2}}(a\sqrt{jp}) + n(\mu_r - 1) I_{n+\frac{1}{2}}(a\sqrt{jp})} \right) \tag{2}$$

I is the current in the transmitter coil, $P_n^1(\bullet)$ is the associated Legendre function of degree n and of order 1 of the first kind, $I_n(\bullet)$ is the modified Bessel function of order n, and $p = \omega\sigma\mu_0\mu_r$[2~3]. In the expression of p, ω is angular frequency, which is $2\pi f$, σ is conductivity, μ_0 is the permeability of vacuum, and μ_r is relative permeability. For brevity, we approximate Eq.(1) as

$$\Delta A = \frac{\mu_0 I}{2} \frac{\sin\alpha}{2} P_1^1(\cos\alpha) \times P_1^1(\cos\theta) D(p) c^{-1} q^{-2} \tag{3}$$

by taking only the first term. This approximation is valid since the contributions from the other terms are negligible. By Stokes' theorem, magnetic flux can be obtained from the magnetic vector potential as

$$\Phi = \oiint_S \mathrm{B} \cdot d\mathrm{S} = \oiint_S \nabla \times \mathrm{A} \cdot d\mathrm{S} = \oint_\ell \mathrm{A} \cdot d\mathrm{l} \tag{4}$$

where the subscripts S and ℓ in the integral sign denote surface and line, respectively, and the line integral is to be taken along each receiver coil. Therefore, the increase in magnetic flux in coil 1 due to the metallic sphere is obtained as

$$\Delta\Phi = \frac{\pi\mu_0 I r \sin\alpha}{2} P_1^1(\cos\alpha) \times P_1^1(\cos\theta) D(p) c^{-1} q^{-2} \tag{5}$$

by substituting Eq.(3) into Eq.(4) and performing the integration along coil 1. This is the increase in the magnetic flux that passes through coil 1 due to the eddy current on the metallic sphere. Since the electromotive force emf generated by the varying magnetic flux is given by

$$\mathrm{emf} = -\frac{d\Phi}{dt} = -j\omega\Phi,$$

the increases in emf in coil 1 and coil 2 are

$$\mathrm{emf}_1 = -j\omega \frac{\pi\mu_0 I r \sin\alpha}{2} P_1^1(\cos\alpha) \times P_1^1(\cos\theta) D(p) c^{-1} q^{-2} \tag{6}$$

and

$$\mathrm{emf}_2 = -j\omega \frac{\pi\mu_0 I r \sin\alpha}{2} P_1^1(\cos\alpha) \times P_1^1(\cos\theta') D(p) c^{-1} q'^{-2} \tag{7}$$

respectively. Since we use, as the measured voltage, the difference of the two emf's, i.e., $\mathrm{emf}_1 - \mathrm{emf}_2$, we have

$$V = -j\omega \frac{\pi\mu_0 I r \sin\alpha}{2} \times P_1^1(\cos\alpha) D(p) c^{-1} \times \left[P_1^1(\cos\theta) q^{-2} - P_1^1(\cos\theta') q'^{-2} \right]. \tag{8}$$

Now, we investigate the impact of the metallic object on the input impedance of the transmitting coil. We may conjecture that the input impedance of the transmitting coil may vary due to the change of the mutual inductance between the transmitting and receiving coils. The change of impedance due to the eddy current on the metallic sphere may be obtained as

$$\Delta Z = -j\omega \frac{\Delta\Phi}{I}. \tag{9}$$

By substituting Eq.(5) into Eq.(9), we have

$$\Delta Z = -\frac{j\omega\pi\mu_0 r\sin\alpha}{2} P_1^1(\cos\alpha) \times P_1^1(\cos\theta) D(p) c^{-1} q^{-2}. \quad (10)$$

We can obtain the change of the input impedance of the transmitting coil by substituting $\theta = \alpha$, $q = c$, and $x = z$.

Then, as the change of the input impedance of the transmitting coil, we have

$$\Delta Z = -\frac{j\omega\pi\mu_0 r\sin\alpha}{2} \left(P_1^1(\cos\alpha)\right)^2 D(p) c^{-3}. \quad (11)$$

The term $D(p)$ in Eq.(11) is expressed in terms of the dimension and the properties of the metallic object such as the radius, permeability, and conductivity as can be seen from Eq.(2).

III. Result

We utilize the obtained equation to find the optimum spacing s_{opt} between the transmitter and the receiver coils that maximizes the induced voltage. To plot the voltage variation with respect to the spacing s and the position z of the metallic object, we normalize s and z with the radius r of the coils. The voltage is normalized with the transmitter coil current I. The optimum spacing and the position are independent of the properties and dimension of the material. The frequency is chosen for ease of implementation and to meet the assumption that the dimensions of the coils are negligibly small compared to the wavelength of the operating signal. In this paper, we predict the voltage when the metallic object is of iron, i.e., $\mu_r = 110$, $\sigma = 1.06 \times 10^7$ S/m, and the radius is $a/r = 0.03$ and the operating frequency is $f = 100$ kHz. Fig.3 shows the voltage variation vs. z/r and s/r. The voltage attains its maximum when $s_{\text{opt}}/r = 0.65$ and $z/r = 0.5$, which means the induced voltage is maximum when the spacing between the coils is 0.65 of the radius of the coils and when the position of the metallic sphere is at $0.5r$ off the center of the transmitting coil. The

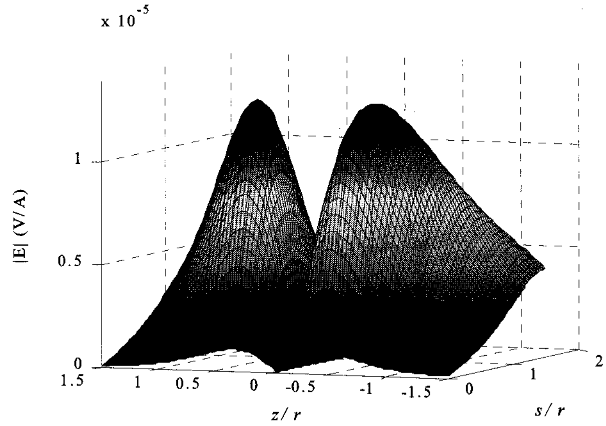

그림 3. 금속의 위치 z/r와 코일 헤드 간의 거리 s/r에 따른 검출 전압의 변화량. 금속의 특성은 $\mu_r = 110$, $\sigma = 1.06 \times 10^7$ S/m, 금속의 크기 $a/r =$ 0.03, 동작 주파수 $f = 100$ kHz

Fig. 3. The voltage variation vs. z/r and s/r. The properties and dimension of the metallic object are $\mu_r = 110$, $\sigma = 1.06 \times 10^7$ S/m, and $a/r =$ 0.03. The operating frequency is $f = 100$ kHz.

results agree well with the result obtained with that of the mutual inductance approach[2].

Further, we investigate the variation of the input impedance of the transmitting coil due to the metallic object given by Eq.(11). The impedance variation is of complex form, consisting of real and imaginary parts as in

$$\Delta Z = \Delta R + j\Delta X \quad (12)$$

Fig. 4 shows the impedance variation vs. the variation of conductivity and relative permeability when the coil dimensions are $r = 0.15$ m, $z/r = 0.5$ and $a = 4$ mm, and the frequency is $f = 100$ kHz. The conductivity σ of the metallic object varies from 0.0001×10^7 to 5.0×10^7 S/m and the relative permeability μ_r from 1 to 100. We find that the relative permeabilities over 50 do not make much difference on the variation of the impedance. Considering that most of the metals have a relative permeability over 50, we may ignore the effect of the relative permeability in metal detection. In Fig.5, we show the variation of the impedance variation vs. the variation of the dimension and the conductivity of the metal. The coil dimension is $r = 0.15$ m, $z/r = 0.5$

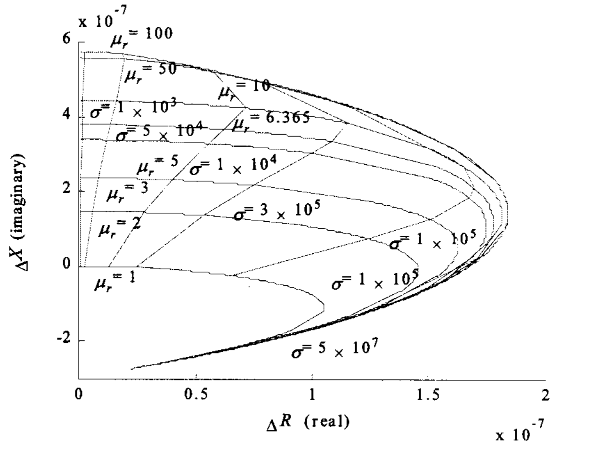

그림 4. 금속의 투자율과 전도도에 따른 송신 코일의 임피던스 변화량 ($\mu_r = 1-100$, $\sigma = 0.0001-5.0\times10^7$ S/m). $a=4$ mm, $r=0.15$ m, $z/r=0.5$, $f=100$ kHz

Fig. 4. The impedance variation vs. $\mu_r = 1-100$ and $\sigma = 0.0001-5.0\times10^7$ S/m. $a=4$ mm and $r=0.15$ m, $z/r=0.5$, $f=100$ kHz.

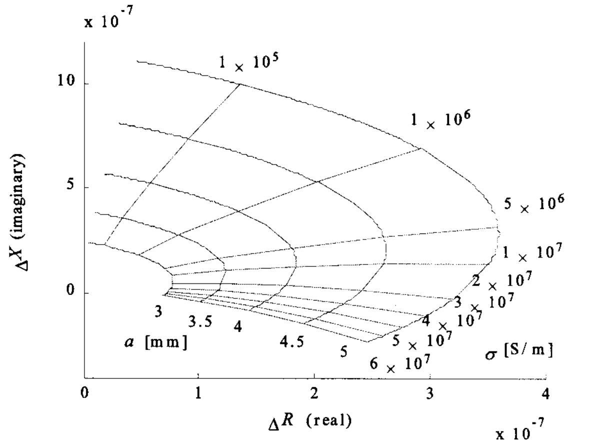

그림 5. 금속의 크기와 전도도에 따른 송신 코일의 임피던스 변화량 ($a=3-5$ mm 와 $\sigma=0.6-6.0\times10^7$S/m.), $r=0.15$ m $z/r=0.5$, $\mu_r=110$, $f=100$ kHz

Fig. 5. The impedance variation vs. $a=3-5$ mm and $\sigma=0.6-6.0\times10^7$ S/m. $r=0.15$ m, $z/r=0.5$, $\mu_r=110$, $f=100$ kHz.

and the frequency is $f=100$ kHz. The dimension and conductivity of the metallic object are radius $a=3-5$ mm, conductivity $\sigma=0.6-6.0\times10^7$ S/m which are typical values for metallic objects.

The impedance variation due to the dimension of the metallic objects is approximately linear, but that due to the conductivity is quite nonlinear.

We can see that the radius and the conductivity of a metallic object can be identified as a point on the plot. However, since the magnitude of the variation of the impedance is so small that, for effective identification, an elaborate scheme should be devised to amplify the variation.

Ⅳ. Conclusion

We derived an equation for the induced voltage in the three single-turn coil head of the metal detector using a magnetic vector potential approach which gives us good insight into the principle. The equation was utilized to find the optimum spacing between the coils with which the induced voltage difference at the receiver coils attains its maximum. It was shown that the optimum spacing between the coils is $0.65r$ (r is the radius of the coil) and the peak voltage is obtained when the metallic sphere is at $0.5r$ off the center of the transmitter coil, which agrees well with the previous results, justifying our work. We further derived an equation of the impedance variation of the transmitting coil due to the metallic object in-between the coils. The impedance variation is dependent upon the dimension and the properties (i.e., conductivity and permeability) of the metallic object. The result may be used to identify the dimension and type of the metal, which we leave as further study.

Preference

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