

An Analytical Approach for Design of Nth-band FIR Digital Filters with Equi-Ripple Passband

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Abstract – In FIR (Finite Impulse Response) filter applications, Nth-band FIR digital filters are known to be important due to their reduced computational requirements. The conventional methods for designing FIR filters use iterative approaches such as the well-known Parks-McClellan algorithm. The Parks-McClellan algorithm is also used to design Nth-band FIR digital filters after Mintzer's research. However, a disadvantage of the Parks-McClellan algorithm is that it needs a large amount of design time. This paper describes a direct design method for Nth-band FIR Filters using Chebyshev polynomials, which provides a reduced design time over indirect methods such as the Parks-McClellan algorithm. The response of the resulting filter is equi-ripple in passband. Our proposed method produces a passband response that is equi-ripple to within a minuscule error, comparable to that of Mintzer's design method which uses the Parks-McClellan algorithm.

Keywords: Chebyshev polynomials, digital filter design, FIR filter, multi-rate filters, Nth-band filters

1. Introduction

FIR digital filters have important roles in many areas of signal processing. FIR filters are used widely because they have a characteristic of linear-phase, which IIR (Infinite Impulse Responses) filters do not have [1][2]. In addition, FIR filters are necessary in constructing optimal interpolation and decimation filters with a characteristic of linear-phase during over-sampling and down-sampling process in multi-rate digital systems [3] [4].

General methods in designing linear-phase FIR filters are the window function method based on Discrete Fourier Transform, and the optimal-FIR filter design program method which is also called the Parks-McClellan algorithm. Generally speaking, the window function method is used in designing FIR filters when equi-ripple property is not required. Otherwise, the Parks-McClellan algorithm is used in other cases [4]-[7].

One other interesting topic of FIR filter design is about constructing Nth-band FIR filters. One benefit of using Nth-band filters is that the number of coefficients of the filter is reduced by imposing some conditions on coefficients, and so the memory requirements and the number of arithmetic operations used in realization of the FIR filters are reduced [8]. For example, using half-band FIR filters in hardware implementations has many advantages because only half of the coefficients are used [4][8][9]. For this reason, Nth-band filters have been studied widely, and there exists many literatures on this subject: Mintzer has specified requirements of passband and stopband to design

Nth-band low-pass FIR filters. He also showed that the Parks-McClellan algorithm is useful in designing Nth-band FIR filters [8]. Vaidyanathan and Nguyen have found a method to improve Parks-McClellan algorithm so that computing time to design half-band FIR filters is reduced [4]. Nohrden and Nguyen have specified the critical frequency and bandwidth of Nth-band FIR filters [10]. Oraintara and Nguyen constructed a function, based on cosine modulation, between two different Nth-band FIR filters, and the function gives a simple relations between coefficients of two filters [11] [12].

Even though the Parks-McClellan algorithm is widely used in designing Nth-band filters, it has a disadvantage that considerable amount of computing time is needed to obtain coefficients of filters because the Parks-McClellan algorithm uses an iterative algorithm to compute coefficients. Although computing time has decreased by Vaidyanathan and Nguyen's research, indirect methods based on the Parks-McClellan algorithm still make designers hard. So such indirect methods are not desirable in a real-time environment [9].

On the other hand, direct methods such as analytical design methods have an advantage that they are ten or hundred times faster than indirect methods [9]. For this reason, direct design methods based on analytical approaches have been studied for a long time [7]. One of the noticeable direct methods is proposed by Willson and Orchard in designing half-band FIR filters.

Willson and Orchard have contributed a design method for half-band FIR filters with equi-ripple property using Chebyshev polynomial approximation via an analytical approach [9]. However, the question still remains whether there exists a direct design technique for Nth-band FIR filters, by extending Willson and Orchard's studies. Our study has started with this question and gets a partial answer to the question in this paper.

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Received 21 January 2009; Accepted 29 July 2009

We apply an analytical approach to obtain relations of the coefficients of Nth-band FIR filters. In addition, we are able to derive solutions for Nth-band FIR filters with equi-ripple passband. In this paper, we describe a design method for Nth-band FIR low-pass filters via an analytical approach. We describe the analysis of Nth-band FIR filters in section 2 and a design method for Nth-band FIR filters in section 3. In section 4, we compare the results of our design method with that of Mintzer using the Parks-McClellan algorithm, and verify the applicability of our proposed method.

2. Analysis of Nth-band FIR filters

2.1 Analysis of third-band FIR filters

We can define the FIR filter which is positive symmetric of length $L = 6M - 1$ as

$$H(z) = h_0 + \sum_{n=1}^{3M-1} h_n (z^n + z^{-n}). \quad (1)$$

To satisfy the property of third-band filters, the coefficients h_n should be defined as

$$h_{3p} = 0, \quad p \neq 0. \quad (2)$$

Let $z = e^{j\omega}$ and $z + z^{-1} = 2\cos(\omega)$. Then, equation (1) can be written as

$$\begin{aligned} H(\omega) &= h_0 + 2 \times \sum_{n=1}^{3M-1} h_n \cos(n\omega) \\ &= h_0 + 2h_1 \cos(\omega) + 2h_2 \cos(2\omega) + 2h_4 \cos(4\omega) \\ &\quad + 2h_5 \cos(5\omega) + \cdots + 2h_{3M-2} \cos((3M-2)\omega) \\ &\quad + 2h_{3M-1} \cos((3M-1)\omega), \end{aligned} \quad (3)$$

with the condition of (2). We can rewrite (3) as

$$\begin{aligned} H(\omega) &= h_0 + 2 \cos(2\omega) \times \sum_{k=0}^{M-1} p_{1,k} \cos(3k\omega) \\ &\quad + 2 \cos(\omega) \times \sum_{k=0}^{M-1} p_{2,k} \cos(3k\omega) \end{aligned} \quad (4)$$

because of a property of cosine function.

Note that we have the following relations:

$$\begin{aligned} 2h_{3M-1} &= p_{1,M-1} \\ 2h_{3M-2} &= p_{2,M-1} \\ 2h_{3M-4} &= p_{1,M-2} + p_{2,M-1} \\ 2h_{3M-5} &= p_{1,M-1} + p_{2,M-2} \\ &\vdots \\ 2h_s &= p_{1,1} + p_{2,2} \\ 2h_4 &= p_{1,2} + p_{2,1} \\ 2h_2 &= 2p_{1,0} + p_{2,1} \\ 2h_1 &= p_{1,1} + 2p_{2,0}. \end{aligned} \quad (5)$$

Now we rewrite equation (4) as

$$H(\omega) = h_0 + 2 \cos(2\omega) P_1(\omega) + 2 \cos(\omega) P_2(\omega), \quad (6)$$

$$P_i(\omega) = \sum_{k=0}^{M-1} p_{i,k} \cos(3k\omega), \quad (7)$$

In matrix forms, (6) and (7) are rewritten as

$$H(\omega) = h_0 + 2 \cdot [\cos(2\omega) \quad \cos(\omega)] \cdot \begin{bmatrix} P_1(\omega) \\ P_2(\omega) \end{bmatrix} \quad (8)$$

where

$$\mathbf{P}(\omega) = \begin{bmatrix} P_1(\omega) \\ P_2(\omega) \end{bmatrix}. \quad (9)$$

Note that $P_i(\omega)$ has a period of $(2/3)\pi$ and is symmetric about $\omega = (2/3)\pi$ because of a property of cosine function. In other words, $P_i(\omega)$ has the following property:

$$P_i(\omega) = P_i\left(\frac{2}{3}\pi - \omega\right) = P_i\left(\frac{2}{3}\pi + \omega\right). \quad (10)$$

Let's assume the ideal low-pass filter, which has infinitely many coefficients and cut-off frequency at $\omega_c = \frac{\pi}{3}$.

We can define $\mathbf{P}_{id}(\omega)$ for $-\frac{\pi}{3} \leq \omega \leq \frac{\pi}{3}$ which is corresponding to (9). Then, we consider the following condition at the passband.

$$H(\omega) = h_0 + 2 \cdot [\cos(2\omega) \quad \cos(\omega)] \cdot \mathbf{P}_{id}(\omega) = 1 \quad (11)$$

Also, we use the symmetric property obtained above to consider the following property at the stopband.

$$\begin{aligned} &H\left(\frac{2}{3}\pi - \omega\right) \\ &= h_0 + 2 \cdot \left[\cos\left(\frac{4}{3}\pi - 2\omega\right) \quad \cos\left(\frac{2}{3}\pi - \omega\right) \right] \cdot \mathbf{P}_{id}(\omega) \end{aligned} \quad (12)$$

$$\begin{aligned} &= 0 \\ &H\left(\frac{2}{3}\pi + \omega\right) \\ &= h_0 + 2 \cdot \left[\cos\left(\frac{4}{3}\pi + 2\omega\right) \quad \cos\left(\frac{2}{3}\pi + \omega\right) \right] \cdot \mathbf{P}_{id}(\omega) \end{aligned} \quad (13)$$

$$= 0$$

Equations (11)-(13) can be written as

$$\begin{bmatrix} 2 \cos(2\omega) & 2 \cos(\omega) & 1 \\ 2 \cos\left(\frac{4}{3}\pi - 2\omega\right) & 2 \cos\left(\frac{2}{3}\pi - 2\omega\right) & 1 \\ 2 \cos\left(\frac{4}{3}\pi + 2\omega\right) & 2 \cos\left(\frac{2}{3}\pi + 2\omega\right) & 1 \end{bmatrix} \begin{bmatrix} \mathbf{P}_{id}(\omega) \\ h_0 \\ 1 \end{bmatrix} \quad (14)$$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

Now use (14) to obtain

$$\mathbf{P}_{\text{id}}(\omega) = \begin{bmatrix} \frac{\sin(\omega)}{3\sin(3\omega)} \\ \frac{\sin(2\omega)}{3\sin(3\omega)} \end{bmatrix} \quad (15)$$

and

$$h_0 = \frac{1}{3}. \quad (16)$$

Therefore we can approximate (9) with (15).

2.2 Analysis of Nth-band filters

We extend the above procedure to Nth-band FIR filters.

Let's define the Nth-band FIR filter which is positive symmetric of length $L = 2NM - 1$ as

$$H(z) = h_0 + \sum_{n=1}^{NM-1} h_n (z^n + z^{-n}). \quad (17)$$

$$h_{Np} = 0, \quad p \neq 0. \quad (18)$$

Because of a property of cosine function and (18), we can express $H(z)$ as $H(\omega)$ like (19) and (20).

$$\begin{aligned} H(\omega) &= h_0 + 2 \times \sum_{n=1}^{NM-1} h_n \cos(n\omega) \\ &= h_0 + 2 \times \sum_{i=1}^{N-1} \cos((N-i)\omega) \cdot P_i(\omega), \end{aligned} \quad (19)$$

$$\begin{aligned} P_i(\omega) &= p_{i,0} + p_{i,1} \cos(N\omega) + p_{i,2} \cos(2N\omega) \\ &\quad + \cdots + p_{i,M-1} \cos((M-1)N\omega) \\ &= \sum_{j=0}^{M-1} p_{i,j} \cos(jN\omega), \quad (1 \leq i \leq N-1). \end{aligned} \quad (20)$$

Then using $(N-1) \times 1$ row matrix $\mathbf{C}(\omega)$ and $1 \times (N-1)$ column matrix $\mathbf{P}(\omega)$, we write (19) as

$$H(\omega) = h_0 + 2 \cdot \mathbf{C}(\omega) \cdot \mathbf{P}(\omega), \quad (21)$$

where

$$\begin{aligned} \mathbf{C}(\omega) &= [\cos((N-1)\omega) \quad \cos((N-2)\omega) \quad \cdots \quad \cos(\omega)] \\ &= [C_1 \quad C_2 \quad \cdots \quad C_{N-1}], \end{aligned} \quad (22)$$

and

$$\mathbf{P}(\omega) = \begin{bmatrix} P_1(\omega) \\ P_2(\omega) \\ \vdots \\ P_{N-1}(\omega) \end{bmatrix}. \quad (23)$$

$P_i(\omega)$ has a period of $(2/N)\pi$ and it is symmetric about $\omega = (2/N)\pi$. It follows that $P_i(\omega)$ has the following property:

$$P_i(\omega) = P_i\left(\frac{2}{N}\pi - \omega\right) = P_i\left(\frac{2}{N}\pi + \omega\right). \quad (24)$$

Assuming that the LP is ideal with cut-off frequency at $\omega_c = \frac{\pi}{N}$, let $\mathbf{P}_{\text{id}}(\omega)$ defined for $-\frac{\pi}{N} \leq \omega \leq \frac{\pi}{N}$ which is corresponding to (23). Then, we consider the following conditions at the passband and the stopband:

$$\begin{aligned} H(\omega) &= h_0 + 2 \cdot \mathbf{C}(\omega) \cdot \mathbf{P}_{\text{id}}(\omega) = 1 \\ H\left(\frac{2k}{N}\pi \mp \omega\right) &= h_0 + 2 \cdot \mathbf{C}\left(\frac{2k}{N}\pi \mp \omega\right) \cdot \mathbf{P}_{\text{id}}(\omega) = 0 \\ H(\pi - \omega) &= h_0 + 2 \cdot \mathbf{C}(\pi - \omega) \cdot \mathbf{P}_{\text{id}}(\omega) = 0, \end{aligned} \quad (25)$$

where $k = 1, 2, \dots, (N/2)$, and the last condition is needed only if N is even.

Equations (25) can be written as

$$\begin{bmatrix} 2 \cdot \mathbf{C}(\omega) & 1 \\ 2 \cdot \mathbf{C}\left(\frac{2}{N}\pi - \omega\right) & 1 \\ 2 \cdot \mathbf{C}\left(\frac{2}{N}\pi + \omega\right) & 1 \\ \vdots & \vdots \\ 2 \cdot \mathbf{C}\left(\frac{N-1}{N}\pi + \omega\right) & 1 \end{bmatrix} \times \begin{bmatrix} \mathbf{P}_{\text{id}}(\omega) \\ h_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad (26)$$

for N is odd, and

$$\begin{bmatrix} 2 \cdot \mathbf{C}(\omega) & 1 \\ 2 \cdot \mathbf{C}\left(\frac{2}{N}\pi - \omega\right) & 1 \\ 2 \cdot \mathbf{C}\left(\frac{2}{N}\pi + \omega\right) & 1 \\ \vdots & \vdots \\ 2 \cdot \mathbf{C}\left(\frac{N-1}{N}\pi + \omega\right) & 1 \\ 2 \cdot \mathbf{C}(\pi - \omega) & 1 \end{bmatrix} \times \begin{bmatrix} \mathbf{P}_{\text{id}}(\omega) \\ h_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix},$$

for N is even.

Now use (26) to obtain

$$\mathbf{P}_{\text{id}}(\omega) = \begin{bmatrix} \frac{\sin(\omega)}{N\sin(N\omega)} \\ \frac{\sin(2\omega)}{N\sin(N\omega)} \\ \vdots \\ \frac{\sin((N-1)\omega)}{N\sin(N\omega)} \end{bmatrix}. \quad (27)$$

Therefore we can approximate (23) with (27). If we express (20) using Chebyshev polynomials, we can obtain $p_{i,k}$ by approximating (23) with (27) according to the specific requirement. Also we can obtain h_n from $p_{i,k}$ using suitable relations of h_n and $p_{i,k}$.

3. A Design Method with Equi-Ripple Passband

3.1 Approximation

We consider an Nth-band filter which has the equi-ripple property at $0 \leq \omega \leq \omega_p$, where $\omega_p = 2\pi f_p$. If we substitute $x = \cos \omega$ and $y = \sin \omega$ in (20), we have the following expression using Chebyshev polynomials $T_k(x)$.

$$P_i^x(x) = \sum_{j=0}^{M-1} p_{i,j} T_{2j}(x) \quad (28)$$

In this case, the constraint $-1 \leq x \leq 1$ subjects to $0 \leq \omega \leq \frac{N}{2}\pi$, and so it is not suitable for (25). However, if (28) is expressed using y , where $-1 \leq y \leq 1$ subjects to $-\frac{\pi}{2} \leq \omega \leq \frac{\pi}{2}$, which is used in (25), we get the following expression:

$$P_i^y(y) = \sum_{j=0}^{M-1} (-1)^j p_{i,j} T_{2j}(y) \quad (29)$$

Note that we use the following property of Chebyshev polynomials to obtain (29):

$$T_{2k}(x) = (-1)^k T_{2k}(y). \quad (30)$$

To obtain the equi-ripple response, (29) must be approximated to (27) at the passband. If we substitute $y = \alpha \cdot t$ for $\alpha = \sin(\frac{N}{2}\omega_p)$ in (29), $-1 \leq t \leq 1$ corresponds to the range of passband. So we obtain the expression as (31) with $t = \frac{y}{\alpha}$.

$$P_i^t(t) = \sum_{j=0}^{M-1} (-1)^j a_{i,2j} T_{2j}(t) \quad (31)$$

To approximate (31) to (27), ω must be expressed as

$$\omega = \frac{2}{N} \sin^{-1}(\alpha \cdot t). \quad (32)$$

We can obtain coefficients a_k in (31) using standard numerical methods [13]. Chebyshev polynomial expansion of $f(x)$, which is defined in $-1 \leq x \leq 1$, is generally formulated as

$$a_k = \frac{2}{m} \sum_{i=1}^m f(\cos \theta_i) \cos k\theta_i, \quad \theta_i = \frac{(2i-1)\pi}{2m}. \quad (33)$$

Equation (27) is an even function of t , so we can reduce the number of terms in the summation from m to $(m/2)$ or $(m+1)/2$, depending on either m is even or odd. Also $a_k = 0$ for k is odd, which is expected from (31).

To obtain an appropriate approximation, we need to carefully select m , the number of sample points, and so it is important to determine which evaluation function we use. From this point, we use the method similar to that of Willson and Orchard to design half-band FIR filters. In Chebyshev polynomial approximations, the ripple size tends to increase normally near the band edges. So we can compare

ripple size of Chebyshev polynomial at $t = 0$ with that at $t = 1$. The values of $P_i^t(t)$ at $t = 0$ and $t = 1$ are obtained as

$$P_i^t(1) = \sum_{j=0}^{N-1} a_{i,2j}, \quad (34)$$

and

$$P_i^t(0) = \sum_{j=0}^{N-1} (-1)^j a_{i,2j}. \quad (35)$$

If we assume that $f_i(0)$ and $f_i(1)$ are the value of the elements of (27) at $t = 0$ and $t = 1$ respectively, then we can obtain the evaluation function as

$$J_{p_i} = \left| \frac{P_i^t(1)/f_i(1)-1}{P_i^t(0)/f_i(0)-1} \right|. \quad (36)$$

When one increases the number of sample points within appropriate range, it is not difficult to get optimal value for m which makes evaluation function near to 1.

And using the method for half-band FIR filters by Willson and Orchard, we can obtain the coefficients $p_{i,j}$ from $a_{i,j}$ which are obtained from (33) [9].

3.2 Introducing weight matrices

We observe that the ripple property in passband is not regular as N increases, or f_p approaches to the critical frequency. We conclude that using the Chebyshev approximation of $P(\omega)$ is not efficient in such cases.

The reason for such phenomenon is that P_{id} increases to infinite as ω goes close to $\frac{\pi}{N}$, so the errors, which come from the approximation of P_{id} , are accumulative, and hence the ripple property is not regular.

However, by introducing weight matrices \mathbf{W}_c and \mathbf{W}_p of $(N-1) \times (N-1)$ such that $\mathbf{W}_c \times \mathbf{W}_p = 2 \times \mathbf{I}$, where \mathbf{I} is an identity matrix, we can rewrite (21)-(23) as

$$\begin{aligned} H(\omega) &= h_0 + 2 \cdot \mathbf{C}(\omega) \cdot \mathbf{P}(\omega) \\ &= h_0 + 2 \cdot \mathbf{C}(\omega) \cdot \mathbf{W}_c \cdot \mathbf{W}_p \cdot \mathbf{P}(\omega) \\ &= h_0 + 2 \cdot \mathbf{C}^w(\omega) \cdot \mathbf{P}^w(\omega) \end{aligned} \quad (37)$$

and

$$\mathbf{C}^w(\omega) = \mathbf{C}(\omega) \cdot \mathbf{W}_c \quad (38)$$

$$\mathbf{P}^w(\omega) = \mathbf{W}_p \cdot \mathbf{P}(\omega). \quad (39)$$

We expect these are more efficient. Now we should approximate $\mathbf{P}^w(\omega)$ with $\mathbf{P}_{id}^w(\omega)$ as

$$\mathbf{P}_{id}^w(\omega) = \mathbf{W}_p \cdot \mathbf{P}_{id}(\omega). \quad (40)$$

If one selects \mathbf{W}_p so that all elements of \mathbf{P}_{id}^w , except the first element, do not increase to infinite, then the errors, coming from the element of \mathbf{P}_{id}^w , except the first element,

do not affect $H(\omega)$. In other words, only the first element dominates the property of $H(\omega)$. If we assume that $W_{i,j}^p$ are the elements of \mathbf{W}_p , then, the elements $P_{i,j}^w$ of \mathbf{P}_{id}^w are written as

$$P_{i,j}^w = \frac{\sum_{j=1}^{N-1} W_{i,j}^p \sin(j\omega)}{\sin(N\omega)}, \quad (1 \leq i \leq N-1). \quad (41)$$

To select appropriate weight matrices, we decompose $\mathbf{W}_c \times \mathbf{W}_p = 2 \times \mathbf{I}$, such that the elements $W_{i,j}^p$ of \mathbf{W}_p satisfy the following conditions:

$$\sum_{j=1}^{N-1} W_{i,j}^p \sin(j\omega) \Big|_{\omega=\frac{\pi}{N}} = 0, \quad (2 \leq i \leq N-1) \quad (42)$$

Note that there are many solutions to (41), and their choices remain with the designer.

4. Design Example and Discussions

To evaluate our design method proposed in this paper, we re-design the example that can be found in Mintzer's [8]. A low-pass filter, which has a filter length of 23, requires a passband of from 0.00000 to 0.10000 and a stopband of from 0.23333 to 0.43333. A filter is designed to meet these specifications. We choose \mathbf{W}_c and \mathbf{W}_p as

$$\mathbf{W}_c = \mathbf{W}_p = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}. \quad (43)$$

The coefficients of the filter by our proposed method are given in Table 1 together with those by Mintzer [8]. The frequency response for the filter is shown in Fig. 1 and the frequency response in passband is shown in Fig. 2. Also those of Mintzer's are shown in the same figures. To evaluate the differences between the results of the two filters, we define the ripple as

$$R(\omega) = 20 \log H(\omega) \text{ (dB)}. \quad (44)$$

Table 1. Comparison of coefficients for the example

Parks-McClellan Method	The Proposed Method
$h_0 = +0.33333333$	$h_0 = +0.33333333$
$h_1 = +0.26474821$	$h_1 = +0.26735214$
$h_2 = +0.13677322$	$h_2 = +0.13413176$
$h_4 = -0.04856929$	$h_4 = -0.05068677$
$h_5 = -0.04297682$	$h_5 = -0.04097800$
$h_7 = +0.01466941$	$h_7 = +0.01589007$
$h_8 = +0.01538212$	$h_8 = +0.01416732$
$h_{10} = -0.00319225$	$h_{10} = -0.00374572$
$h_{11} = -0.00397174$	$h_{11} = -0.00351493$

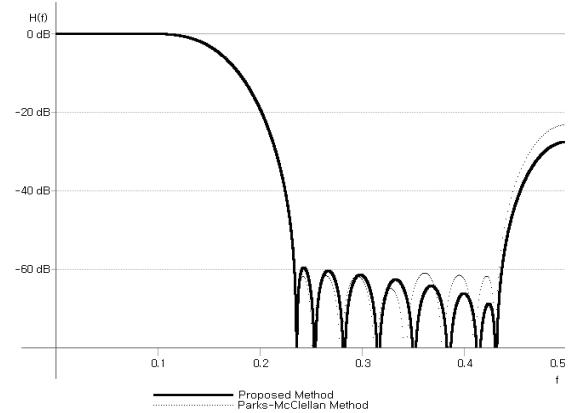


Fig. 1. Frequency responses of the third-band FIR filter designed for the example

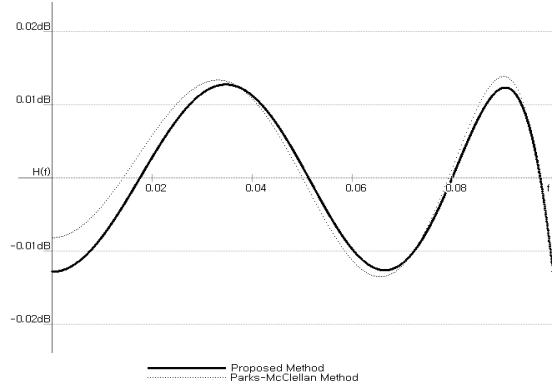


Fig. 2. Frequency responses in the passband of the third-band FIR filter designed for the example

The filter that is designed by the proposed method has maximum passband ripples of $+0.001446/-0.001458$ or $(+0.0126dB/-0.0127dB)$, and a maximum stopband ripple of 0.001099 or $(-59.181dB)$. The filter that is designed by Mintzer's has maximum passband ripples of $+0.001598/-0.001555$ or $(+0.0139dB/-0.0135dB)$ and a maximum stopband ripple of 0.0008965 or $(-60.949dB)$.

If we compare the result of the example with that of Mintzer's, our method has better equi-ripple property in the passband even though it has a loss in the stopband. Mintzer used the Parks-McClellan algorithm, and he aimed to obtain equi-ripple property in both the passband and the stopband. So the behavior of Mintzer method is well in the stopband and the passband.

In our method, we substituted $t = \frac{y}{\alpha} = \frac{\sin \omega}{\alpha}$ for $\alpha = \sin(\frac{N}{2}\omega_p)$ in (3.4), so $-1 \leq t \leq 1$ corresponds to the passband of $-\omega_p \leq \omega \leq \omega_p$. Therefore, the approximation with the range of t had equi-ripple property in the passband. In other word, the proposed method aims for the quiet-ripple property in the passband, rather than the stopband. Consequently, our method cannot have better property in the stopband compared with Mintzer method.

To compare computing time with Mintzer's, we programmed our algorithm in Matlab using a PC with 2GHz CPU. Our method required 0.31ms to calculate coefficients while Mintzer's method needed 5.5ms. So our method is considerably faster(approximately 18 times) than Mintzer's.

5. Conclusion

The contribution of the proposed method in this paper is that presents a closed-form solution to the design of Nth-band FIR filters. We presented an equation and an approximation related to the coefficients of Nth-band FIR filters via an analytical approach. In addition, we demonstrated a design method for Nth-band FIR filters with equi-ripple passband and applicability of our proposed method in the example.

In the example of third-band FIR filter design, we reduced the passband ripple from +0.0139dB/-0.0135dB to +0.0126 dB /-0.0127dB, compared with Mintzer's design method which uses the Parks-McClellan algorithm. As well as improved passband property, we achieved 18 times faster in computing time, compared with Mintzer's method. However, the stopband ripple increased from -60.949dB to -59.181dB.

In interpolation filters which are used in application such as sample-rate conversion, the characteristic in the passband is more important than the characteristic in the stopband. So, the method that has the equi-ripple property in passband is useful to sample-rate conversion, and real-time processing. .

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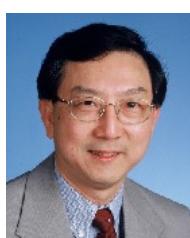
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