Inverse Problem Methodology for Parameter Identification of a Separately Excited DC Motor

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Abstract – Identification is considered to be among the main applications of inverse theory and its objective for a given physical system is to use data which is easily observable, to infer some of the geometric parameters which are not directly observable. In this paper, a parameter identification method using inverse problem methodology is proposed. The minimisation of the objective function with respect to the desired vector of design parameters is the most important procedure in solving the inverse problem. The conjugate gradient method is used to determine the unknown parameters, and Tikhonov's regularization method is then used to replace the original ill-posed problem with a well-posed problem. The simulation and experimental results are presented and compared.

Keywords: Identification, inverse problem, optimization, separately excited dc motor, conjugate gradient method

1. Introduction

Separately excited dc motors are very often used as actuators in industrial applications. These actuators have low friction, small size, high speed, low construction cost, no gear back-lash, operate safly without the use of limit switches and generate moderate torque at a high torque to weight ratio. DC motors are preferred over ac motors because of their lower cost and ease of controller implementation, because their mathematical model is simpler [1].

Dynamic model identification has been a major topic of interest in control engineering, motivated by the new achievements in control systems theory and requirements of new industrial and military applications [1-3]. System identification of dc motors is a topic of great importance, because for almost every servo control design a mathematical model is needed [3]. There are situations when identification model is available. For example, the motor parameters might be subject to some time variations [4]. In these cases, a mathematical model that is accurate at the time of the design may not be accurate at a later time. Moreover, a mathematical model is never a complete description of a given system; this is because a model that represents a system well over a range of frequencies, may not represents the system as well as over a different range of frequencies. Therefore, accuracy and adequacy are too major modelling issues that always have to be dealt with. On broader sense, system identification is often the only means of obtaining mathematical models of most physical systems [5-6]; this is because most systems are usually so complex that, unlike dc motors, there is no easy way to derive their models based on the physical laws.

Identification is considered to be among the main applications of inverse theory and its objective for a given physical system is to use data which is easily observable to infer some of the geometric parameters which are not directly observable [6-7]. A general approach in identification seeks to define an objective function that would reach its minimum when the measured data in the physical system under test matches the assumed one. Firstly, the physical system to be investigated is described in terms of parameters, and then, the objective function is minimised with respect to the parameters by an iterative procedure. At the minimum of the objective function, the values of the parameters describe the real structure of the physical system.

The main difficulty with inverse problems is due to their ill-posed character, in the sense of Hadamard [8]. That is, they have no solution or if a solution exists, it might not be unique or not continuous with respect to the given data. Therefore many techniques were proposed to regularize these problems. For example, Beck introduces the "future time" method, while Murio developed the "mollification technique" to obtain smooth solutions of various inverse problems. Tikhonov, Alifanov and others from the Russian school proposed to cast the ill-posed inverse problem into an optimization problem with a regularized objective functional, and/or a self regularizing algorithm of solution.

The crucial point in inverse problem is the efficiency of the process by which the solution is arrived at. The objective functions are said to have multiple minima, and much research effort is expended on global optimization methods, such as zeroth-order probabilistic methods-Monte-Carlo iteration. Recently, though the great number of iterations required, they are mostly developed, since the "pseudo"-deterministic methods, (e.g., steepest descent, conjugate gradient; quasi Newton, etc), that has relatively high speed of convergence, reach only local minima [9-11]. They depend on the initial guess and, usually the number of iterations required cannot be predicted in advance. Ill-conditioned

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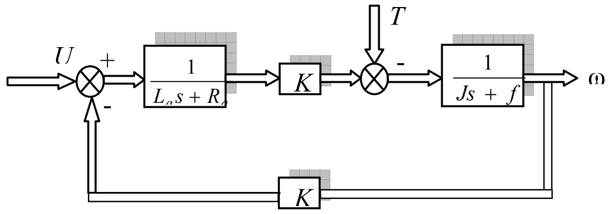

Fig. 1. the block diagram of the dc motor

inverse problems require regularization to prevent the solutions from being excessively sensitive to noise in the data [6]. While efficient algorithms exist for computing inverses, the role of regularization in increasing the cost of the computations has not been well considered. The regularization techniques that are most widely employed are Tikhonov's, Levenberg's and Levenberg-Marquard's, and truncated singular value decomposition (TSVD) [6], [12].

This paper is structured as follows. Section 2, describes the dynamic of the separately excited dc motor. Section 3, describes the inverse problem methodology applied to the parameter identification of a dc motor. In section 4, experimental results are illustrated and compared to the simulation results. Finally, conclusions of the paper are summarized in section 5.

2. DC motor direct model

The block diagram of the dc motor used in this study is shown in Fig. 1. The dynamic of the separately excited dc motor may be expressed by the following equations

$$K\omega(t) = -R_a i_a(t) - L_a \frac{di_a(t)}{dt} + U_a(t) \quad (1)$$

$$Ki_a(t) = J\frac{d\omega(t)}{dt} + f\omega(t) + T_L(t) \quad (2)$$

Where K, R_a, L_a, J and f are respectively, the torque and back-EMF, the armature resistance, the armature inductance, the rotor mass moment of inertia and the viscous friction coefficient. $\omega(t)$, $i_a(t)$, $U_a(t)$ and $T_L(t)$ Respectively denote the rotor angular speed, the armature current, the terminal voltage and the load torque.

3. Inverse problem methodology for parameter identification

The problem that we solve classically by many of our sophisticated numerical analysis techniques is the direct problem-given the device, we predict its performance. But in engineering design it is the inverse problem that has to be solved-given the desired performance, we are asked to predict or synthesize the device [6-11]. Identification inverse problems arise in the search of defects in materials and structures. The identification can be based on propagation phenomena, radiation, and measurement of static response, steady state, transient response and eigen-frequencies. A general approach in identification seeks to define an objective function that would reach its minimum. For our cases the objective function for the inverse problem can be written as the squared sum of errors between measured and calculated values of the rotor angular speed of the separately excited dc motor at 24 testing point (observation data)i. e.,

$$F(X) = \frac{1}{2}\sum_{i=1}^{24}(\omega_i^C - \omega_i^0)^2 \quad (3)$$

Where ω_i^C are the values of ω, calculated using the direct model and ω_i^0 are the measured values of ω at testing point i.

$X = \begin{bmatrix} R_a & L_a & K & J & f & T_{st} \end{bmatrix}^T$ Defines a vector of design parameters.

The minimisation of the objective function with respect to the desired vector is the most important procedure in solving the inverse problem. In this paper, we use the conjugate gradient method to determine the unknown parameters. Iterations are built in the following manner:

$$X^{k+1} = X^k - \alpha^k.d^k \quad (4)$$

Where α^k is the step size, d^k is the director vector of descent given by

$$d^k = \nabla F^T(X^k) + \beta^k d^{k-1} \quad (5)$$

And the conjugate coefficient β^k is determined from

$$\beta^k = \frac{\nabla F(X^k).\nabla F^T(X^k)}{\nabla F(X^{k-1}).\nabla F^T(X^{k-1})} \quad (6)$$

Here, the row vector defined by

$$\nabla F = \begin{pmatrix} \frac{\partial F}{\partial R_a} & \frac{\partial F}{\partial L_a} & \frac{\partial F}{\partial K} & \frac{\partial F}{\partial J} & \frac{\partial F}{\partial f} & \frac{\partial F}{\partial T_{st}} \end{pmatrix} \quad (7)$$

is the gradient of the objective function.

Sensitivity analysis is performed using the direct model with perturbations of each parameter. The sensitivity analysis is of great importance since it gives information on the identification feasibility. It is also of prime importance to evaluate the ratio of sensitivity of one parameter with respect to all other parameters during the process.

The sensitivity of the objective function F as to the design variable X can be written as follows

$$\frac{\partial F}{\partial X} = \sum_{i=1}^{24}\frac{\partial F}{\partial \omega_i^C}.\frac{\partial \omega_i^C}{\partial X} = \sum_{i=1}^{24}(\omega_i^C - \omega_i^0).\frac{\partial \omega_i^C}{\partial X} \quad (8)$$

The finite difference method is employed to approximate the gradient of the objective function.

$$\frac{\partial \omega_i^C}{\partial X} = \frac{\omega_i^C(X + \delta X) - \omega_i^C(X)}{\partial X} \quad (9)$$

where δX represents a small perturbation of the corresponding parameter X.

The main disadvantage of the finite-difference method resides in its resolution cost. In order to determine n first-order derivatives, the direct model has to be solved at least (n+1) times. Nevertheless, for such complex systems, the finite-difference method seems to be the only way to calculate the sensitivity components.

The step size of the K^{th} iteration, α^k, can be determined by minimizing the function $F(X^k - \alpha^k d^k)$ for the given X^k and d^k.

Where the optimization problem is ill-posed that is when existence or unicity of the solution with respect to experimental data is not verified, it is common to use a regularization methods, in order to limit the space parameter [6]. The most commonly used methods are Tikhonov's, Lenvenberg's and Levenberg-Marquardt's. All of these introduce a regularization term F_r representing more or less, the least-squared difference between the calculated parameter vector X (Lenvenberg) and the initial guessed one X^0 (Tikhonov)or the previous calculated one.

$$F^* = (1-\lambda)F + \lambda F_r \tag{10}$$

Where $\lambda \in [0\ ,\ 1]$ is the regularization parameter.

The regularization term for Tikhonov method is of the form (11), where X_i^0 is the initial set of parameters and X_i^k is the current set of parameters.

$$F_r = \sum_i (X_i^k - X_i^0)^2 \tag{11}$$

Levenberg's method is of the same kind, but $X_i^0 = X_i^{k-1}$ is the set of parameters solved by the Gauss-Newton algorithm at previous iteration (k-1).

The convergence criterion in our design is based on the variation of the objective function value. If the differences of the objective function value in two subsequent iterations is less than a specified positive number ε (Eq.12).

$$\left|F(X^{k+1}) - F(X^k)\right| \prec \varepsilon \tag{12}$$

The optimization process will stop and the final optimization is achieved. The computational algorithm for the solution of the inverse problem in this paper can be summarized as follows:

Step 1: pick an initial guess X^0. Set k=0.
Step 2: solve the direct problem given by Eqs (1-2).
Step 3: calculate the objective function $F(X^k)$ given by Eq (3). Terminate the iteration process if the specified stopping criterion is satisfied. Otherwise, go to step 4.
Step 4: solve the equation of sensitivity given by Eqs (8-9), and compute the gradient of the objective function $\nabla F(X^k)$.
Step 5: knowing $\nabla F(X^k)$, compute the conjugate coefficient β^k from Eq (6). Then compute the direction vector of descent d^k from equation (5).
Step 6: knowing α^k and d^k, compute the new estimated vector X^{k+1} from equation (4).
Step 7: set k=k+1 and go back to step 2.

4. Results and discussion

The separately excited dc motor used for experimental tests has the nominal characteristics shown in Table 2.

To evaluate the performance of the proposed identification method using inverse problem, the first thing to do is to properly express the design goal of identifying the parameters of a dc motor into a mathematical equation. The goal is achieved through minimizing an objective function of the summed squared error evaluated at 24 measurements points, as mentioned above. The design parameters are armature resistance R_a, armature inductance

L_a, back-EMF constant K, rotor mass moment of inertia J, viscous friction coefficient f and the static torque T_{st}.

The identification algorithm using conjugate gradient method was executed using Tikhonov method, for a regularization parameter $\lambda=10^{-9}$, an optimal solution was obtained after 22 iterations. Table 1 summarizes the identification results. In order to show the validity of the presented technique, we have simulated dynamic test applying step amplitude of terminated voltage, to the armature circuit of dc motor, as well as, deceleration test and mechanical characteristic.

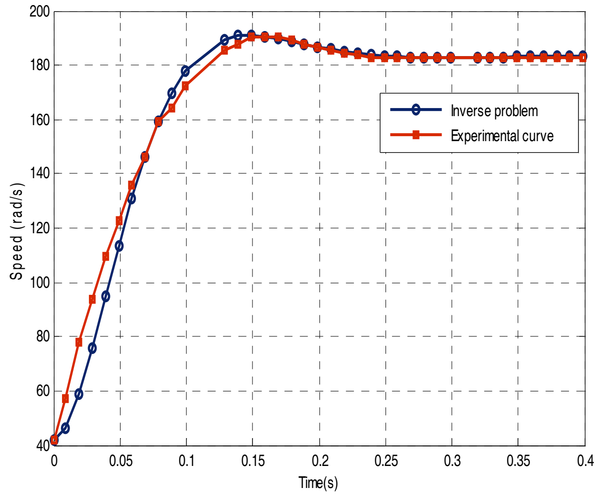

Fig. 2. Rotor speed angular response

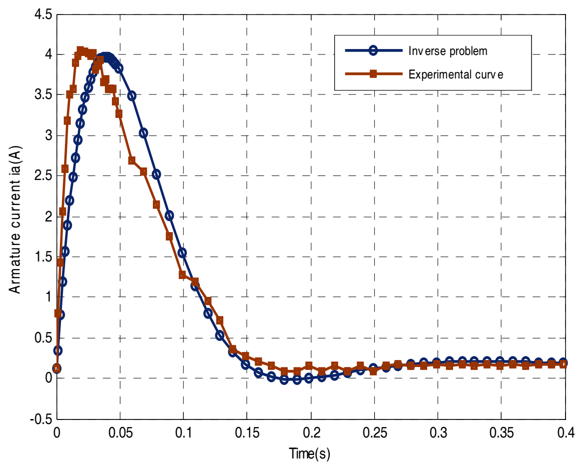

Fig. 3. Armature current response.

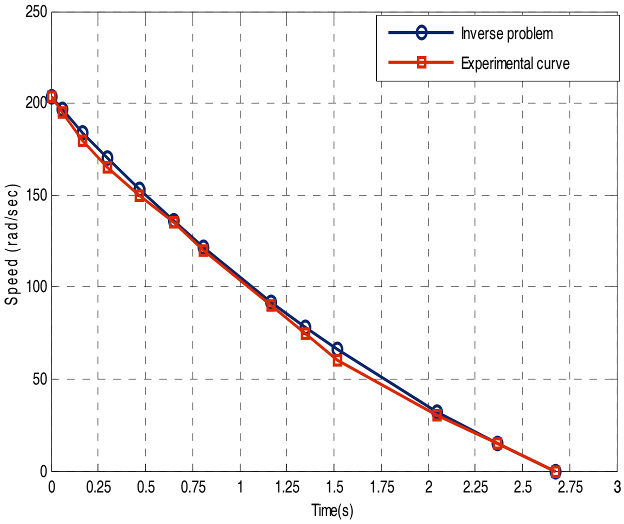

Fig. 4. Deceleration test

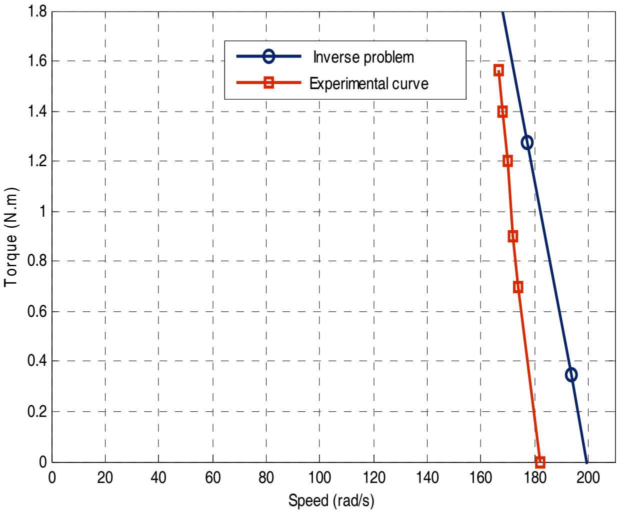

Fig. 5. Mechanical characteristic

According to Figs. 2,4 the curves simulated from the dynamic test parameters with the proposed method are close to the real curve (experimental curve). With regard to Fig. 3 and the steady state Fig. 5, the curves simulated from the dynamic test parameters with the proposed technique are almost identical and close to real measurement (experimental curve) .

Table 1. parameter identification results

Parameter	Initials Values	Optimals Values
$R_a(\Omega)$	28	30.9034
L_a(H)	0.820	0.7954
K (N.m.A^{-1})	1.34	1.3212
J (kg.m^2)	0.0028	0.0022
f (N.m.s/rad)	0.00054	0.0009
T_{st} (N.m)	0.127	0.1230

Table 2. specification of experimental dc motor

Rated power	180 W
Rated speed	1500 rpm
Armature voltage	270 V
Field voltage	220 V
Armature current	1.1 A
Field current	0.4 A

5. Conclusion

The inverse problem of parameter identification of a separately excited dc motor is investigated by employing the conjugate gradient method. The finite difference method is employed to approximate the gradient of the objective function. The Tikhonov's method is then used to cast the ill-posed inverse problem into an optimization problem with a regularized objective functional. The results show a comparison between experiment and the proposed identification technique based on inverse problem, this comparison is made on the basis of real measurements taken in laboratory on a separately excited dc motor, with 180 W of rated power. It shows the advantage of the only dynamic test for identification, coupled to the inverse problem method.

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