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INTUITIONISTIC FUZZY MINIMAL SPACES

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Abstract. We introduce the concept of intuitionistic fuzzy minimal structure which is an extension of the intuitionistic fuzzy topological space. And we introduce and study the concepts of intuitionistic fuzzy M-continuity, intuitionistic fuzzy M-open mappings and several types of intuitionistic fuzzy minimal compactness on intuitionistic fuzzy minimal spaces.

1. Introduction

The concept of fuzzy set was introduced by Zadeh [6]. As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [2]. Chang [3] defined fuzzy topological spaces using fuzzy sets. In [1], Alimohammady and Roohi introduced the notion of fuzzy minimal space which is a generalization of fuzzy topological space [5] in Lowen's sense.

Çoker [4] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. The intuitionistic fuzzy topological spaces are a generalization of fuzzy topological spaces in Chang's sense.

In this paper, we introduce the concept of intuitionistic fuzzy minimal space which is an extension of the intuitionistic fuzzy topological space. We introduce and study the concepts of intuitionistic fuzzy M-continuity, intuitionistic fuzzy M-open maps and intuitionistic fuzzy M-closed maps. Finally, we introduce the concepts of intuitionistic fuzzy minimal compactness, almost intuitionistic fuzzy minimal compactness and nearly intuitionistic fuzzy minimal compactness on intuitionistic fuzzy minimal spaces and study properties of such intuitionistic fuzzy minimal compactness.

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2. PRELIMINARIES

Let I be the unit interval $[0,1]$ of the real line. A member μ of I^X is called a fuzzy set of X. For any $\mu \in I^X$, μ^c denotes the complement $1-\mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and 1, respectively. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A(x)+\gamma_A(x) \leq 1$ for all $x \in X$.

An *intuitionistic fuzzy point* $x_{(\alpha,\beta)}$ in X is an intuitionistic fuzzy set

$$x_{(\alpha,\beta)} = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \to I$ and $\gamma_A : X \to I$ are defined as follows:

$$(\mu_A(y), \gamma_A(y)) = \begin{cases} (\alpha,\beta) \text{ if } y = x \\ (0,1) \text{ if } y \neq x \end{cases}$$

and $\alpha + \beta \leq 1$.

An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to belong to an intuitionistic fuzzy set $A = (\mu_A, \gamma_A)$ in X, denoted by $x_{(\alpha,\beta)} \in A$, if $\mu_A(x) \geq \alpha$ and $\gamma_A(x) \leq \beta$.

An intuitionistic fuzzy set A in X is the union of all intuitionistic fuzzy points which belong to A.

Obviously every fuzzy set μ on X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1}-\mu)$.

Definition 2.1 ([2]). Let $A = (\mu_A, \gamma_A)$ and $B = (\mu_B, \gamma_B)$ be intuitionistic fuzzy sets on X. Then

(1) $A \subseteq B$ iff $\mu_A \leq \mu_B$ and $\gamma_A \geq \gamma_B$.
(2) $A = B$ iff $A \subseteq B$ and $B \subseteq A$.
(3) $A^c = (\gamma_A, \mu_A)$.
(4) $A \cap B = (\mu_A \wedge \mu_B, \gamma_A \vee \gamma_B)$.
(5) $A \cup B = (\mu_A \vee \mu_B, \gamma_A \wedge \gamma_B)$.
(6) $\mathbf{0} = (\tilde{0}, \tilde{1})$ and $\mathbf{1} = (\tilde{1}, \tilde{0})$.

Let f be a map from a set X to a set Y. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy set of X and $B = (\mu_B, \gamma_B)$ an intuitionistic fuzzy set of Y. Then:

(1) The image of A under f, denoted by $f(A)$ is an intuitionistic fuzzy set in Y defined by

$$f(A) = (f(\mu_A), \tilde{1} - f(\tilde{1} - \gamma_A)).$$

(2) The inverse image of B under f, denoted by $f^{-1}(B)$ is an intuitionistic fuzzy set in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).$$

An *intuitionistic fuzzy topology* [4] on X is a family $\mathcal{T}$ of intuitionistic fuzzy sets in X which satisfies the following properties:

(1) $\mathbf{0}, \mathbf{1} \in \mathcal{T}$.
(2) If $A_1, A_2 \in \mathcal{T}$, then $A_1 \cap A_2 \in \mathcal{T}$.
(3) If $A_i \in T$ for all i, then $\cup A_i \in \mathcal{T}$.

The pair $(X, \mathcal{T})$ is called an *intuitionistic fuzzy topological space.*

3. Intuitionistic Fuzzy Minimal Spaces

Definition 3.1. Let X be a nonempty set. An *intuitionistic fuzzy minimal structure* $\mathcal{M}$ on X is a map $\mathcal{M} : I(X) \to I$ if the family $\mathcal{M}$ satisfies

$$\mathbf{0}, \mathbf{1} \in \mathcal{M}.$$

Then the $(X, \mathcal{M})$ is called an *intuitionistic fuzzy minimal space* (simply IFMS). Every member of $\mathcal{M}$ is called an *intuitionistic fuzzy minimal open* set. An intuitionistic fuzzy set A is called a *intuitionistic fuzzy minimal closed* set if the complement of A (simply, A^c) is an intuitionistic fuzzy minimal open set.

Example 3.2. Let $(X, \mathcal{T})$ be an IFTS. Then $\mathcal{T}$ is obvious an intuitionistic fuzzy minimal structure on X.

Definition 3.3. Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in an IFMS $(X, \mathcal{M})$. Then an intuitionistic fuzzy set A is called an *intuitionistic fuzzy minimal neighborhood* of $x_{(\alpha,\beta)}$ if there is an intuitionistic fuzzy minimal open set B in X such that $x_{(\alpha,\beta)} \in B \subseteq A$.

Definition 3.4. Let $(X, \mathcal{M})$ be an IFMS. The intuitionistic fuzzy minimal closure and the intuitionistic fuzzy minimal interior of A, denoted by $mC(A)$ and $mI(A)$, respectively, are defined as

$$mI(A) = \cup\{B \in I(X) : B \in \mathcal{M} \text{ and } B \subseteq A\},$$

$$mC(A) = \cap\{B \in I(X) : B^c \in \mathcal{M} \text{ and } A \subseteq B\},$$

respectively.

Theorem 3.5. *Let $(X, \mathcal{M})$ be an IFMS and A, B in $I(X)$.*

(1) *$mI(A) \subseteq A$ and if A is an intuitionistic fuzzy minimal open set, then $mI(A) = A$.*

(2) *$A \subseteq mC(A)$ and if A is an intuitionistic fuzzy minimal closed set, then $mC(A) = A$.*

(3) *If $A \subseteq B$, then $mI(A) \subseteq mI(B)$ and $mC(A) \subseteq mC(B)$.*

(4) *$mI(A \cap B) \subseteq mI(A) \cap mI(B)$ and $mC(A) \cup mC(B) \subseteq mC(A \cup B)$.*

(5) *$mI(mI(A)) = mI(A)$ and $mC(mC(A)) = mC(A)$.*

(6) *$\mathbf{1} - mC(A) = mI(\mathbf{1} - A)$ and $\mathbf{1} - mI(A) = mC(\mathbf{1} - A)$.*

Proof. (1), (2) and (3) are obvious. (4) Let A, B in $I(X)$. Since $mC(A) \subseteq mC(A \cup B)$ and $mC(B) \subseteq mC(A \cup B)$, it is $mC(A) \cup mC(B) \subseteq mC(A \cup B)$. Similarly, $mI(A) \cap mI(B) \supseteq mI(A \cap B)$.

(5) It follows from (1) and (2).

(6) It is obvious. □

Example 3.6. Let $X = \{a, b\}$, let A and B be intuitionistic fuzzy sets defined as follows:

$$A(a) = (0.1, 0.5), A(b) = (0.2, 0.4)$$

and

$$B(a) = (0.2, 0.6), B(b) = (0.3, 0.5).$$

Then $\mathcal{M} = \{\mathbf{0}, \mathbf{1}, A, B\}$ is an intuitionistic fuzzy minimal structure on X. Let C be an intuitionistic fuzzy set defined as follows: $C(a) = (0.2, 0.5)$ and $C(b) = (0.3, 0.4)$; then since $C = A \cup B$, $mInt(C) = C$ but C is not intuitionistic fuzzy minimal open.

Definition 3.7. Let $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ be two IFMS's. Then $f : X \to Y$ is said to be an *intuitionistic fuzzy M-continuous* function if for every $A \in \mathcal{N}$, $f^{-1}(A) \in \mathcal{M}$.

Theorem 3.8. *Let $f : X \to Y$ be a function on two IFMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$.*

(1) *f is intuitionistic fuzzy M-continuous.*

(2) *$f^{-1}(B)$ is an intuitionistic fuzzy minimal closed set in X for each intuitionistic fuzzy minimal closed set B in Y.*

(3) *$f(mC(A)) \subseteq mC(f(A))$ for $A \in I(X)$.*

(4) $mC(f^{-1}(B)) \subseteq f^{-1}(mC(B))$ *for* $B \in I(Y)$.

(5) $f^{-1}(mI(B)) \subseteq mI(f^{-1}(B))$ *for* $B \in I(Y)$.

Then (1) $\Leftrightarrow$ (2) $\Rightarrow$ (3) $\Leftrightarrow$ (4) $\Leftrightarrow$ (5).

Proof. (1) $\Leftrightarrow$ (2) Obvious.

(2) $\Rightarrow$ (3) For $A \in I(X)$,

$$\begin{aligned} f^{-1}(mC(f(A))) &= f^{-1}(\cap\{F \in I(Y) : f(A) \subseteq F \text{ and } F^c \in \mathcal{N}\}) \\ &= \cap\{f^{-1}(F) \in I(X) : A \subseteq f^{-1}(F) \text{ and } F^c \in \mathcal{N}\} \\ &\supseteq \cap\{K \in I(X) : A \subseteq K \text{ and } K^c \in \mathcal{M}\} \\ &= mC(A) \end{aligned}$$

Hence $f(mC(A)) \subseteq mC(f(A))$.

(3) $\Rightarrow$ (4) Let $B \in I(Y)$. From (3), it follows

$$f(mC(f^{-1}(B))) \subseteq mC(f(f^{-1}(B))) \subseteq mC(B).$$

Hence we have (4). Similarly, we have (4) $\Rightarrow$ (3).

(4) $\Leftrightarrow$ (5) It follows from Theorem 3.5(6). □

Example 3.9. Let $X = \{a, b\}$. Let A, B and C be intuitionstic fuzzy sets defined as in Example 3.6. Consider $\mathcal{M} = \{\mathbf{0}, \mathbf{1}, A, B\}$ and $\mathcal{N} = \{\mathbf{0}, \mathbf{1}, A, B, C\}$ as IFMS's on X. Let $f : (X, \mathcal{M}) \to (X, \mathcal{N})$ be a function defined as follows: $f(x) = x$ for each $x \in X$. Then f satisfies the condition (5) in Theorem 3.8. But it is not intuitionistic fuzzy M-continuous because $f^{-1}(C)$ is not intuitionistic fuzzy minimal open for an intuitionistic fuzzy minimal open set C.

Corollary 3.10. *Let* $f : X \to Y$ *be a function on two IFMS's* $(X, \mathcal{M})$ *and* $(Y, \mathcal{N})$. *Then the following equivalent:*

(1) $f(A) \subseteq mC(f(A))$ *for each intuitionistic fuzzy minimal closed set* A *in* X.

(2) $f^{-1}(B) = mC(f^{-1}(B))$ *for each intuitionistic fuzzy minimal closed set* B *in* Y.

(3) $f^{-1}(B) = mI(f^{-1}(B))$ *for each intuitionistic fuzzy minimal open set* B *in* Y.

Theorem 3.11. *Let* $f : X \to Y$ *be a function on two IFMS's* $(X, \mathcal{M})$ *and* $(Y, \mathcal{N})$. *Then the statements are equivalent:*

(1) *If for every intuitionistic fuzzy point* $x_{(\alpha,\beta)}$ *and each intuitionistic fuzzy minimal open set* V *of* $f(x_{(\alpha,\beta)})$, *there exists an intuitionistic minimal open*

neighborhood U *of* $x_{(\alpha,\beta)}$ *such that* $f(U) \subseteq V$ *where* $\alpha, \beta \in I$.

(2) $f^{-1}(mI(B)) \subseteq mI(f^{-1}(B))$ *for* $B \in I(Y)$.

Proof. (1) $\Rightarrow$ (2) Let $B \in I(Y)$ and $x_{(\alpha,\beta)} \in f^{-1}(mI(B))$. Then since $f(x_{(\alpha,\beta)}) \in mI(B)$, there exists an intuitionistic fuzzy minimal open set V such that $f(x_{(\alpha,\beta)}) \in V \subseteq B$ By hypothesis, there exists an intuitionistic fuzzy minimal open neighborhood $U_{x_{(\alpha,\beta)}}$ containing $x_{(\alpha,\beta)}$ such that $f(U_{x_{(\alpha,\beta)}}) \subseteq V \subseteq B$. Now we can say for $x_{(\alpha,\beta)} \in f^{-1}(B)$, there exists an intuitionistic fuzzy minimal open set $U_{x_{(\alpha,\beta)}}$ such that

$$x_{(\alpha,\beta)} \in U_{x_{(\alpha,\beta)}} \subseteq f^{-1}(B).$$

By definition of intuitionistic fuzzy minimal interior operator, it is obtained $x_{(\alpha,\beta)} \in mI(f^{-1}(B))$.

Hence, we have $f^{-1}(mI(B)) \subseteq mI(f^{-1}(B))$.

(2) $\Rightarrow$ (1) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and V an intuitionistic fuzzy minimal open set of $f(x_{(\alpha,\beta)})$; then from (2) and Corollary 3.10, it follows $x_{(\alpha,\beta)} \in f^{-1}(V) = mI(f^{-1}(V))$. Thus there exists an intuitionistic fuzzy minimal open neighborhood U such that $x_{(\alpha,\beta)} \in U \subseteq f^{-1}(V)$. Hence (1) is obtained. □

Definition 3.12. Let X be a nonempty set and $\mathcal{M}$ an intuitionistic fuzzy minimal structure on X. The intuitionistic fuzzy minimal structure $\mathcal{M}$ is said to have the property $(\mathcal{U})$ if for $A_i \in \mathcal{M}$ $(i \in J)$,

$$\mathcal{M}(\cup A_i) = \cup \mathcal{M}(A_i).$$

Theorem 3.13. *Let* $(X, \mathcal{M})$ *be an IFMS with the property* $(\mathcal{U})$. *Then*

(1) $mI(A) = A$ *if and only if* $A \in \mathcal{M}$ *for* $A \in I(X)$.

(2) $mC(A) = A$ *if and only if* $A^c \in \mathcal{M}$ *for* $A \in I(X)$.

Proof. (1) From Theorem 3.5, it is sufficient to show that if $mI(A) = A$, then $A \in \mathcal{M}$. Let $mI(A) = A$. Then since

$$\mathcal{M}(mI(A)) = M(\cup\{B \in I(X) : B \in \mathcal{M} \text{ and } B \subseteq A\}) = \cup \mathcal{M}(B),$$

thus $A \in \mathcal{M}$.

(2) Let $mC(A) = A$; then from Theorem 3.5, it follows $\mathbf{1} - A = \mathbf{1} - mC(A) = mI(\mathbf{1} - A)$. Hence by (1), we have $\mathbf{1} - A \in \mathcal{M}$.

The converse follows from (1) and Theorem 3.5. □

From Theorem 3.8, Theorem 3.11 and Theorem 3.13, we have the following:

Corollary 3.14. *Let $f : X \to Y$ be a function on two IFMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$. If $\mathcal{M}$ has the property $(\mathcal{U})$, the following are equivalent:*

(1) *f is intuitionistic fuzzy M-continuous.*

(2) *$f^{-1}(B)$ is an intuitionistic fuzzy minimal closed set, for each intuitionistic fuzzy minimal closed set B in Y.*

(3) *$f(mC(A)) \subseteq mC(f(A))$ for $A \in I(X)$.*

(4) *$mC(f^{-1}(B)) \subseteq f^{-1}(mC(B))$ for $B \in I(Y)$.*

(5) *$f^{-1}(mI(B)) \subseteq mI(f^{-1}(B))$ for $B \in I(Y)$.*

(6) *If for every intuitionistic fuzzy point $x_{(\alpha,\beta)}$ and each intuitionistic fuzzy minimal open neighborhood V of $f(x_{(\alpha,\beta)})$, there exists an intuitionistic minimal open neighborhood U of $x_{(\alpha,\beta)}$ such that $f(U) \subseteq V$.*

Definition 3.15. Let $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ be two IFMS's. Then a function $f : X \to Y$ is called *intuitionistic fuzzy M-open* if for every $A \in \mathcal{M}$, $f(A)$ is in $\mathcal{N}$.

Theorem 3.16. *Let $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ be two IFMS's.*

(1) *f is intuitionistic fuzzy M-open.*

(2) *$f(mI(A)) \subseteq mI(f(A))$ for $A \in I(X)$.*

(3) *$mI(f^{-1}(B)) \subseteq f^{-1}(mI(B))$ for $B \in I(Y)$.*

Then (1) $\Rightarrow$ (2) $\Leftrightarrow$ (3).

Proof. (1) $\Rightarrow$ (2) For $A \in I(X)$,

$$\begin{aligned} f(mI(A)) &= f(\cup\{B \in I(X) : B \subseteq A, B \in \mathcal{M}\}) \\ &= \cup\{f(B) \in I(Y) : f(B) \subseteq f(A), f(B) \in \mathcal{N}\} \\ &\subseteq \cup\{U \in I(Y) : U \subseteq f(A), U \in \mathcal{N}\} \\ &= mI(f(A)) \end{aligned}$$

Hence $f(mI(A)) \subseteq mI(f(A))$.

(2) $\Rightarrow$ (3) For $B \in I(Y)$, from (3) it follows that

$$f(mI(f^{-1}(B))) \subseteq mI(f(f^{-1}(B))) \subseteq mI(B).$$

Hence we get (3). Similarly, we get (3) $\Rightarrow$ (2). □

Corollary 3.17. Let $f : X \to Y$ be a function on two IFMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$. If f is intuitionistic fuzzy M-open, then $f(A) = mI(f(A))$ for every intuitionistic fuzzy minimal open set A in X.

Proof. From Theorem 3.5 and Theorem 3.16, it is obvious. □

From Theorem 3.13 and Theorem 3.16, we have the following:

Corollary 3.18. *Let* $(X, \mathcal{M})$ *and* $(Y, \mathcal{N})$ *be two IFMS's. If* $\mathcal{N}$ *has the property* $(\mathcal{U})$, *the following are equivalent:*

(1) f *is intuitionistic fuzzy M-open.*
(2) $f(mI(A)) \subseteq mI(f(A))$ *for* $A \in I(X)$.
(3) $mI(f^{-1}(B)) \subseteq f^{-1}(mI(B))$ *for* $B \in I(Y)$.

Definition 3.19. Let $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ be two IFMS's. Then a function $f : X \to Y$ is called *intuitionistic fuzzy M-closed* if for every intuitionistic fuzzy minimal closed set A in X, $f(A)$ is an intuitionistic fuzzy minimal closed set in Y.

Theorem 3.20. *Let* $f : X \to Y$ *be a function on two IFMS's* $(X, \mathcal{M})$ *and* $(Y, \mathcal{N})$.

(1) f *is intuitionistic fuzzy M-closed.*
(2) $mC(f(A)) \subseteq f(mC(A))$ *for* $A \in I(X)$.
(3) $f^{-1}(mC(B)) \subseteq mC(f^{-1}(B))$ *for* $B \in I(Y)$.

Then $(1) \Rightarrow (2) \Leftrightarrow (3)$.

Proof. It is similar to Corollary 3.18. □

Corollary 3.21. *Let* $f : X \to Y$ *be a function on two IFMS's* $(X, \mathcal{M})$ *and* $(Y, \mathcal{N})$. *If* f *is intuitionistic fuzzy M-closed, then* $f(A) = mC(f(A))$ *for every intuitionistic fuzzy minimal closed set* A *in* X.

Proof. Obvious. □

From Theorem 3.13 and Theorem 3.20, it follows:

Corollary 3.22. *Let* $f : X \to Y$ *be a function on two IFMS's* $(X, \mathcal{M})$ *and* $(Y, \mathcal{N})$. *If* $\mathcal{N}$ *has the property* $(\mathcal{U})$, *the following are equivalent:*

(1) f *is intuitionistic fuzzy M-closed.*
(2) $mC(f(A)) \subseteq f(mC(A))$ *for* $A \in I(X)$.
(3) $f^{-1}(mC(B)) \subseteq mC(f^{-1}(B))$ *for* $B \in I(Y)$.

4. SEVERAL TYPES OF FUZZY MINIMAL COMPACTNESS

Definition 4.1. Let $(X, \mathcal{M})$ be an IFMS and $\mathcal{A} = \{A_i \in I(X) : i \in J\}$. $\mathcal{A}$ is called an *intuitionistic fuzzy minimal cover* if $\cup\{A_i : i \in J\} = \mathbf{1}$. It is an *intuitionistic fuzzy minimal open cover* if each A_i is an intuitionistic fuzzy minimal open set. A subcover of an intuitionistic fuzzy minimal cover $\mathcal{A}$ is a subfamily of it which also is an intuitionistic fuzzy minimal cover.

Definition 4.2. Let $(X, \mathcal{M})$ be an IFMS. An intuitionistic fuzzy set A in $I(X)$ is said to be *intuitionistic fuzzy minimal compact* if every intuitionistic fuzzy minimal open cover $\mathcal{A} = \{A_i \in \mathcal{M} : i \in J\}$ of A has a finite subcover.

Theorem 4.3. *Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be an intuitionistic fuzzy minimal continuous function on two IFMS's. If A is an intuitionistic fuzzy minimal compact set, then $f(A)$ is also an intuitionistic fuzzy minimal compact set.*

Proof. Let $\{B_i \in I(Y) : i \in J\}$ be an intuitionistic fuzzy minimal open cover of $f(A)$ in Y. Then since f is an intuitionistic fuzzy minimal continuous function, $\{f^{-1}(B_i) : i \in J\}$ is an intuitionistic fuzzy minimal open cover of A in X. By Definition 4.2, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{j \in J_0} f^{-1}(B_j)$. Hence $f(A) \subseteq \cup_{j \in J_0} B_j$. It completes the proof. □

Definition 4.4. Let $(X, \mathcal{M})$ be an IFMS. An intuitionistic fuzzy set A in X is said to be *almost intuitionistic fuzzy minimal compact* if for every intuitionistic fuzzy minimal open cover $\mathcal{A} = \{A_i \in I(X) : i \in J\}$ of A, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{j \in J_0} mC(A_j)$.

Theorem 4.5. *Let $(X, \mathcal{M})$ be an IFMS. If an intuitionistic fuzzy set A in X is intuitionistic fuzzy minimal compact, then it is also almost intuitionistic fuzzy minimal compact.*

Proof. Obvious. □

In Theorem 4.5, the converse is not always true as shown in the next example.

Example 4.6. Let $X = I$ and $n \in N - \{1\}$. Let A_n be an intuitionistic fuzzy set defined as follows: $A_n =< x, \mu_{A_n}, \nu_{A_n} >$ and $A_1 =< x, \mu_{A_1}, \nu_{A_1} >$ by

$$\mu_{A_n}(x) = \begin{cases} 0.8, & x = 0, \\ nx, & 0 < x \leq 1/n, \\ 1, & 1/n < x \leq 1, \end{cases}$$

$$\nu_{A_n}(x) = \begin{cases} 0.1, & x = 0, \\ 1 - nx, & 0 < x \leq 1/n, \\ 0, & 1/n < x \leq 1, \end{cases}$$

$$\mu_{A_1}(x) = \begin{cases} 1, & x = 0, \\ 1/2, & \text{otherwise}, \end{cases}$$

$$\nu_{A_1}(x) = \begin{cases} 0, & x = 0, \\ 1/2, & \text{otherwise}. \end{cases}$$

Consider an intuitionistic fuzzy minimal structure $\mathcal{M} : I(X) \to I$ on X as follows:

$$\mathcal{M} = \{A_n : n \in N\} \cup \{\mathbf{0}, \mathbf{1}\}.$$

Let $\mathcal{A} = \{A_n : n \in N\}$ be an intuitionistic fuzzy minimal open cover of X. Then there does not exist a finite subcover of $\mathcal{A}$. Thus X is not intuitionistic fuzzy minimal compact. But X is almost intuitionistic fuzzy minimal compact.

Theorem 4.7. *Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be an intuitionistic fuzzy M-continuous function on two IFMS's. If A is an almost intuitionistic fuzzy minimal compact set, then $f(A)$ is also an almost intuitionistic fuzzy minimal compact set.*

Proof. Let $\{B_i \in I(Y) : i \in J\}$ be an intuitionistic fuzzy minimal open cover of $f(A)$ in Y. Then $\{f^{-1}(B_i) : i \in J\}$ is an intuitionistic fuzzy minimal open cover of A in X. By Definition 4.4, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{j \in J_0} mC(f^{-1}(B_j))$. From Theorem 3.8 (4), it follows

$$\cup_{j \in J_0} mC(f^{-1}(B_j)) \subseteq \cup_{j \in J_0} f^{-1}(mC(B_j)) = f^{-1}(\cup_{j \in J_0} mC(B_j)).$$

Hence $f(A) \subseteq \cup_{j \in J_0} mC(B_j)$. It completes the proof. □

Definition 4.8. Let $(X, \mathcal{M})$ be an IFMS. An intuitionistic fuzzy set A in X is said to be *nearly intuitionistic fuzzy minimal compact* if for every intuitionistic fuzzy minimal open cover $\mathcal{A} = \{A_i : i \in J\}$ of A, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{j \in J_0} mI(mC(A_j))$.

Theorem 4.9. *Let $(X, \mathcal{M})$ be an IFMS. If an intuitionistic fuzzy set A in X is intuitionistic fuzzy minimal compact, then it is also nearly intuitionistic fuzzy minimal compact.*

Proof. For any an intuitionistic fuzzy minimal open set U in X, from Theorem 3.5, it follows $A = mI(A) \subseteq mI(mC(A))$. Thus we get the result. □

Theorem 4.10. *Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be intuitionistic fuzzy M-continuous and intuitionistic fuzzy M-open on two IFMS's. If A is a nearly intuitionistic fuzzy minimal compact set, then $f(A)$ is also a nearly intuitionistic fuzzy minimal compact set.*

Proof. Let $\{B_i \in I(Y) : i \in J\}$ be an intuitionistic fuzzy minimal open cover of $f(A)$ in Y. Then $\{f^{-1}(B_i) : i \in J\}$ is an intuitionistic fuzzy minimal open cover of A in X. By nearly intuitionistic fuzzy minimal compactness, there exists $J_0 = \{1, 2, \cdots n\} \subseteq J$ such that $A \subseteq \cup_{j \in J_0} mI(mC(f^{-1}(B_j)))$. From Theorem 3.8 and Theorem 3.16, it follows

$$
\begin{aligned}
f(A) &\subseteq \cup_{j\in J_0} f(mI(mC(f^{-1}(B_j)))) \\
&\subseteq \cup_{j\in J_0} mI(f(mC(f^{-1}(B_j)))) \\
&\subseteq \cup_{j\in J_0} mI(f(f^{-1}(mC(B_j)))) \\
&\subseteq \cup_{j\in J_0} mI(mC(B_j)).
\end{aligned}
$$

Hence $f(A)$ is a nearly intuitionistic fuzzy minimal compact set. □

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