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Application of Generalized Maximum Entropy Estimator to the Two-way Nested Error Component Model with Ill-Posed Data

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Abstract

Recently Song and Cheon (2006) and Cheon and Lim (2009) developed the generalized maximum entropy(GME) estimator to solve ill-posed problems for the regression coefficients in the simple panel model. The models discussed consider the individual and a spatial autoregressive disturbance effects. However, in many application in economics the data may contain nested groupings. This paper considers a two-way error component model with nested groupings for the ill-posed data and proposes the GME estimator of the unknown parameters. The performance of this estimator is compared with the existing methods on the simulated dataset. The results indicate that the GME method performs the best in estimating the unknown parameters in terms of its quality when the data are ill-posed.

Keywords: Two way nested error component, Ill-posed, GME estimation.

1. Introduction

In recent years a huge of data have been generated in econometrics, biometrics and medical science, and thus there has been a great deal of interest in handling those large data. Particulary, in many economic applications with large data the data may contain nested groupings. For example, data on firms may be grouped by industry, data on states by region and data on individuals by profession. In this case, one can control for unobserved industry and firm effects using a nested error component model.

In general the above data are complete or balanced. However, the empirical applications face missing observations or incomplete panels. The partial or incomplete data may cause that the number of unknown parameters exceeds that of data points, the data are mutually inconsistent, and the columns of the design matrix are linearly dependent. Under classical methods, these type of problems may not be solved. This is called the "ill-posed" problem.

For the ill-posed problem, Judge and Golan (1992), Golan (1994) and Golan and Judge(1996) investigated the estimation problem in the regression model, and Song and Cheon (2006) recently proposed a robust generalized maximum entropy(GME) estimator less sensitive to the assumption and limited situation in the one-way error component regression model. More recently Cheon and Lim (2009) developed the GME estimator of regression coefficients in a linear regression model with a spatial autoregressive disturbance with ill-posed data. The GME estimator was proposed to recover the unknown parameters and the unobserved or unobservable variables for a range of ill-posed problems. This estimator is based on the classic maximum entropy(ME) approach of Jaynes (1957a, b), and

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they proposed maximizing the entropy, subject to the available sample-moment information and the requirement of proper probabilities to recover the unknown probabilities that characterize a given data set. The GME approach generalizes the maximum entropy problem by considering individual noisy observations. Song and Cheon (2006) dealt with the only individual effect, and Cheon and Lim (2009) did a research with the spatial autoregressive effect. Since in general the panel data can be grouped by other factors, the nested effect needs to be taken into consideration.

This paper considers the two-way nested error component regression model in ill-posed data and proposes the GME estimator. This estimator is compared with ordinary least squares(OLS), generalized least squares(GLS), a few feasible GLS(FGLS) estimators (Wallace and Hussain, 1969; Amemyia, 1971; Swamy and Arora, 1972).

The remaining part of this paper is organized as follows. In Section 2, we describe the two-way nested error component regression model. In Section 3 we propose the GME method and describe existing estimation methods for unknown regression coefficients. In Section 4, we apply GME to the simulated dataset. In Section 5 we conclude the paper with a brief discussion.

2. The Model

We consider the following panel regression model,

$$y_{ijt} = x'_{ijt}\beta + u_{ijt}, \quad i = 1,2,\ldots,M; \; j = 1,2,\ldots,N; \; t = 1,\ldots,T, \tag{2.1}$$

where y_{ijt} could denote the output of the j^{th} firm in the i^{th} industry for the t^{th} time period. x_{ijt} denotes a vector of k nonstochastic inputs. The disturbances of (2.1) are assumed that

$$u_{ijt} = \mu_i + \lambda_{ij} + e_{ijt}, \tag{2.2}$$

where μ_i denotes the i^{th} unobservable industry specific effect which is assumed to be *i.i.d.* $N(0, \sigma_\mu^2)$, λ_{ij} denotes the nested effect of the j^{th} firm within the i^{th} industry which is assumed to be *i.i.d.* $N(0, \sigma_\lambda^2)$, and e_{ijt} denotes the remainder disturbance which is also assumed to be $N(0, \sigma_e^2)$. The μ_i's , λ_{ij}'s and e_{ijt}'s are independent of each other and among themselves. (2.1) and (2.2) can be written in a matrix form as

$$y = X\beta + u, \tag{2.3}$$

$$u = \Delta_1\mu + \Delta_2\lambda + e, \tag{2.4}$$

where $y \sim (MNT\times 1)$, $X \sim (MNT\times k)$, $\beta \sim (k\times 1)$, $\Delta_1 = (I_M \otimes i_{NT})$, $\Delta_2 = (I_{MN} \otimes i_T)$, $\mu' = (\mu_1, \ldots, \mu_N)$, $\lambda' = (\lambda_{11}, \ldots, \lambda_{MN})$, $e' = (e_{111}, \ldots, e_{MNT})$ and i_{NT} and i_T are vectors of ones of dimension NT and T, respectively, and $\otimes$ denotes Kronecker product. The disturbance covariance matrix $E(uu')$ can be written as,

$$\Omega = \sigma_\mu^2(I_M \otimes J_N \otimes J_T) + \sigma_\lambda^2(I_{NN} \otimes J_T) + \sigma_e^2(I_{MNT}), \tag{2.5}$$

where J_N and J_T are matrices of ones of dimension N and T, respectively. We replace J_N by $N\bar{J}_N$, I_N by $E_N + \bar{J}_N$, J_T by $T\bar{J}_T$ and I_T by $E_T + \bar{J}_T$ to get Ω^{-1}, and collect terms with the same matrices where $\bar{J}_N = J_N/N$ and $E_N = I_N - \bar{J}_N$. This gives

$$\Omega = \sigma_e^2 Q_1 + \sigma_2^2 Q_2 + \sigma_3^2 Q_3, \tag{2.6}$$

where $\sigma_2^2 = T\sigma_\lambda^2 + \sigma_e^2$, $\sigma_3^2 = NT\sigma_\mu^2 + T\sigma_\lambda^2$. Correspondingly, $Q_1 = (I_M \otimes I_N \otimes E_T)$, $Q_2 = (I_M \otimes E_N \otimes \bar{J}_T)$ and $Q_3 = (I_M \otimes \bar{J}_N \otimes \bar{J}_T)$, respectively.

Therefore, we can easily obtain Ω^{-1} as

$$\Omega^{-1} = \left(\sigma_e^2\right)^{-1} Q_1 + \left(\sigma_2^2\right)^{-1} Q_2 + \left(\sigma_3^2\right)^{-1} Q_3. \tag{2.7}$$

3. Comparison of Estimators

3.1. OLS and GLS estimators

The ordinary least squares(OLS) estimator in (2.1) is given by

$$\widehat{\beta}_{OLS} = (X'X)^{-1}X'y. \tag{3.1}$$

The OLS estimator is an unbiased and consistent estimator, but may not be efficient because it ignores the variance components. The OLS residuals are denoted by $\widehat{u}_{OLS} = y - X\widehat{\beta}_{OLS}$. If variance components in (2.5) are known, the generalized least squares(GLS) estimator can be obtained as follows.

$$\widehat{\beta}_{GLS} = \left(X'\Omega^{-1}X\right)^{-1} X'\Omega^{-1}y. \tag{3.2}$$

However, since variance components are unknown in reality, it may be a theoretical estimator.

3.2. Feasible GLS estimators

In this section we discuss a few feasible GLS(FGLS) estimators when the variance component Ω is unknown. The FGLS estimator can be obtained by that first the variance components are estimated and they are substituted for Ω in (3.2). Then the estimated regression coefficients are as follows.

$$\widehat{\beta}_{FGLS} = \left(X'\hat{\Omega}^{-1}X\right)^{-1} X'\hat{\Omega}^{-1}y, \tag{3.3}$$

where $\hat{\Omega} == \hat{\sigma}_e^2 Q_1 + \hat{\sigma}_2^2 Q_2 + \hat{\sigma}_3^2 Q_3$. The FGLS estimator has similar properties with the GLS estimator approximately, but the property of FGLS is unknown so far for a small sample.

3.2.1. A modified Wallace and Hussain(WH) estimator

Wallace and Hussain (1969) suggested to use $\hat{u}_{OLS}$ instead of the error term, u, to estimate the variance components in the two-way error component model. The modified variance components are estimated as follows.

$$\widehat{\sigma_e^2} = \frac{\hat{u}'_{OLS} Q_1 \hat{u}_{OLS}}{MN(T-1)} = \frac{1}{MN(T-1)} \sum_{i=1}^{M}\sum_{j=1}^{N}\sum_{t=1}^{T} (\hat{u}_{ijt} - \bar{\hat{u}}_{ij.})^2, \tag{3.4}$$

$$\widehat{\sigma_2^2} = \frac{\hat{u}'_{OLS} Q_2 \hat{u}_{OLS}}{M(N-1)} = \frac{T}{M(N-1)} \sum_{i=1}^{M}\sum_{j=1}^{N} (\bar{\hat{u}}_{ij.} - \bar{\hat{u}}_{i..})^2, \tag{3.5}$$

$$\widehat{\sigma_3^2} = \frac{\hat{u}'_{OLS} Q_3 \hat{u}_{OLS}}{M} = \frac{NT}{M} \sum_{i=1}^{M} \bar{\hat{u}}_{i..}^{\;2}. \tag{3.6}$$

The WH estimator using the above estimated modified variance components can be obtained by substituting (3.4)–(3.6) for (3.3).

3.2.2. A modified Amemiya-Type(AM) estimator

Amemiya (1971) suggested to use the WTN residual for estimating the variance components. The estimated variance components are given by,

$$\widehat{\sigma_e^2} = \frac{\tilde{u}'_{WTN} Q_1 \tilde{u}_{WTN}}{MN(T-1)}, \tag{3.7}$$

$$\widehat{\sigma_2^2} = \frac{\tilde{u}'_{WTN} Q_2 \tilde{u}_{WTN}}{M(N-1)}, \tag{3.8}$$

$$\widehat{\sigma_3^2} = \frac{\tilde{u}'_{WTN} Q_3 \tilde{u}_{WTN}}{M}, \tag{3.9}$$

where $\tilde{u}_{WTN} = y - X_s\tilde{\beta}_s - (\bar{y}_{...} - \bar{X}_{...s}\tilde{\beta}_s)$, $\bar{y}_{...} = \sum\sum\sum y_{ijt}/MNT$, $\bar{X}_{...s} = \sum\sum\sum X_{ijts}\ /MNT$ and $\tilde{\beta}_s = (X_s'Q_1X_s)^{-1}X_s'Q_1y$. Here X_s denotes the exogenous regressor excluding the intercept. The AM estimator is obtained by substituting (3.7)–(3.9) for (3.3).

3.2.3. A modified Swamy and Arora(SA) estimator

Swamy and Arora (1972) fitted regression models using within transformation, between individual regression and between time regression, and then estimated the variance components by mean square error of each model. Applying the results of Swamy and Arora (1972) to (2.1) provides the estimated variance components as follows.

$$\widehat{\sigma_e^2} = \frac{y'Q_1y - y'Q_1X_s(X_s'Q_1X_s)^{-1}X_s'Q_1y}{MN(T-1)-k}, \tag{3.10}$$

$$\widehat{\sigma_2^2} = \frac{y'Q_2y - y'Q_2X_s(X_s'Q_2X_s)^{-1}X_s'Q_2y}{M(N-1)-k}, \tag{3.11}$$

$$\widehat{\sigma_3^2} = \frac{y'Q_3y - y'Q_3X(X'Q_3X)^{-1}X'Q_3y}{M-k-1}. \tag{3.12}$$

The SA estimator is obtained by substituting (3.10)–(3.12) for (3.3).

3.3. The GME estimator

Based on the GME formulation proposed by Song and Cheon (2006), we perform the reparameterization by giving probabilities on each support corresponding to each $\beta_k, \mu_m, \lambda_{mn}, e_{mnt}$. Refer to Song and Cheon (2006) for more details in the GME formulation.

We perform the reparameterization for β, μ, λ and e as follows.

$$\beta = Zp = \begin{bmatrix} z_1' & 0 & \cdots & 0 \\ 0 & z_2' & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & z_K' \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_K \end{bmatrix}, \tag{3.13}$$

$$\mu = Fg = \begin{bmatrix} f_1' & 0 & \cdots & 0 \\ 0 & f_2' & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & f_M' \end{bmatrix} \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_M \end{bmatrix}, \tag{3.14}$$

$$\lambda = Ab = \begin{bmatrix} a'_{11} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & a'_{mn} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a'_{MN} \end{bmatrix} \begin{bmatrix} b_{11} \\ \vdots \\ b_{mn} \\ \vdots \\ b_{MN} \end{bmatrix}, \tag{3.15}$$

$$e = Vw = \begin{bmatrix} v'_{111} & 0 & \vdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \vdots & v'_{mnt} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \vdots & \vdots & \cdots & v'_{MNT} \end{bmatrix} \begin{bmatrix} w_{111} \\ \vdots \\ w_{mnt} \\ \vdots \\ w_{MNT} \end{bmatrix}, \tag{3.16}$$

where $\beta_k = \sum_{a=1}^{A} z_{ka} p_{ka}$, $k = 1, 2, \ldots, K$, $z_k = (z_{k1}, z_{k2}, \ldots, z_{kA})$, $p_k = (p_{k1}, p_{k2}, \ldots, p_{kA})$, $\mu_m = \sum_{i=1}^{I} f_{mi} g_{mi}$, $m = 1, 2, \ldots, M$, $f_m = (f_{m1}, f_{m2}, \ldots, f_{mI})$, $g_m = (g_{m1}, g_{m2}, \ldots, g_{mI})$, $\lambda_{mn} = \sum_{d=1}^{D} a_{mnd} b_{mnd}$, $m = 1, 2, \ldots, M$, $n = 1, 2, \ldots, N$, $a_{mn} = (a_{mn1}, a_{mn2}, \ldots, a_{mnD})$, $b_{mn} = (b_{mn1}, b_{mn2}, \ldots, b_{mnD})$, $e_{mnt} = \sum_{j=1}^{J} v_{mntj} w_{mntj}$, $m = 1, 2, \ldots, M$, $n = 1, 2, \ldots, N$, $t = 1, 2, \ldots, T$, $v_{mnt} = (v_{mnt1}, v_{mnt2}, \ldots, v_{mntJ})$ and $w_{mnt} = (w_{mnt1}, w_{mnt2}, \ldots, w_{mntJ})$.

With the above reparameterization, we propose the GME estimator in the two-way nested error component regression model. The GME formulation is as follows.

$$\max_{p,g,b,w} H(p, g, b, w) = -p' \ln(p) - g' \ln(g) - b' \ln(b) - w' \ln(w), \tag{3.17}$$

subject to the data-consistency relations

$$y = XZp + (I_M \otimes i_{NT})Fg + (I_{MN} \otimes i_T)Ab + Vw, \tag{3.18}$$

where $y_{mnt} = \sum_{k=1}^{K} x_{kmnt} (\sum_{a=1}^{A} z_{ka} p_{ka}) + \sum_{i=1}^{I} f_{mi} g_{mi} + \sum_{d=1}^{D} a_{mnd} b_{mnd} + \sum_{j=1}^{J} v_{mntj}\, w_{mntj}$, $m = 1, 2, \ldots, M$; $n = 1, 2, \ldots, N$; $t = 1, 2, \ldots, T$ and the additivity normalization constraint

$$i_K = (I_K \otimes i'_A)\, p, \qquad i_M = (I_M \otimes i'_I)\, g \tag{3.19}$$

$$i_{MN} = (I_{MN} \otimes i'_D)\, b, \qquad i_{MNT} = (I_{MNT} \otimes i'_J)\, w. \tag{3.20}$$

Now we derive the GME estimator. First we define the Lagrangian equation as below.

$$\begin{aligned} L = & -p' \ln p - g' \ln g - b' \ln b - w' \ln w + \xi' [y - XZp - (I_M \otimes i_{NT}) Fg \\ & - (I_{MN} \otimes i_T) Ab - Vw] + \theta' [i_K - (I_K \otimes i'_A)\, p] + \tau' [i_M - (I_M \otimes i'_I)\, g] \\ & + \gamma' [i_{MN} - (I_{MN} \otimes i'_D)\, b] + \eta' [i_{MNT} - (I_{MNT} \otimes i'_J)\, w] \end{aligned}$$

to find the interior solution. Here $\xi \in R^{MNT}$, $\theta \in R^K$, $\tau \in R^M$, $\gamma \in R^{MN}$ and $\eta \in R^{MNT}$ are the associated vectors of Lagrange multipliers. Taking the first-order differentiation in L, the $\tilde{p}$ estimator is obtained by

$$\tilde{p} = \exp(-Z'X'\xi) \odot \{(I_K \otimes i_A i'_A) \exp(-Z'X'\xi)\}^{-1}, \tag{3.21}$$

where $\odot$ is the Hadamard(elementwise) product. In (3.21), ξ may be obtained by the iterative way within the possible range of ξ. In this paper, we set $\xi \in [-1, 1]$ because the function of ξ decreases and then increases at near to zero as ξ increases in finding the minimum.

Finally we estimate the regression coefficient β with the estimated p in (3.21) as follows.

$$\tilde{\beta} = Z\tilde{p} = Z\exp(-Z'X'\xi) \odot \{(I_K \otimes \iota_A \iota_A')\exp(-Z'X'\xi)\}^{-1}. \tag{3.22}$$

In (3.22), the k^{th} GME estimator is given by

$$\begin{aligned}\tilde{\beta}_{GME(k)} &= \sum_{a=1}^{A} z_{ka}\tilde{p}_{ka} \\ &= \sum_{a=1}^{A} z_{ka}\left[\frac{\exp\left(-z_{ka}\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{t=1}^{T} x_{kmnt}\xi\right)}{\Omega_k(\tilde{\xi})}\right],\end{aligned}$$

where $\Omega_k(\tilde{\xi}) = \sum_{a=1}^{A}\exp\left[-z_{ka}(\sum_{m=1}^{M}\sum_{n=1}^{N}\sum_{t=1}^{T} x_{kmnt}\xi)\right]$.

4. Numerical Results

We consider the following regression equation,

$$\begin{aligned} y_{ijt} &= x_{ijt}\beta + u_{ijt}, \quad i = 1,\ldots,M;\ j = 1,\ldots,N;\ t = 1,\ldots T, \\ u_{ijt} &= \mu_i + \lambda_{ij} + e_{ijt}, \end{aligned}$$

where y is a (300×1) dependent variable vector, $M = 25$, $N = 4$ and $T = 3$, X is a (300×4) independent variable vector and β is a (4×1) regression coefficient vector. μ is (300×1), λ is (300×1) and e is (300×1) error vectors. u_{ijt} was generated from $\mu_i \sim IIN(0, \sigma_\mu^2)$, $\lambda_{ij} \sim IIN(0, \sigma_\lambda^2)$ and $e_{ijt} \sim IIN(0, \sigma_e^2)$. This paper considers the variance components are known, thus we fixed $\sigma^2 = \sigma_\mu^2 + \sigma_\lambda^2 + \sigma_e^2 = 20$; here $\{\sigma_\mu^2, \sigma_\lambda^2, \sigma_e^2\}$ were varied over the set $\{(2, 8, 10), (4, 6, 10), (6, 8, 6), (8, 4, 8), (10, 6, 4), (12, 6, 2), (14, 2, 4), (16, 2, 2)\}$. To form a design matrix with a desired condition number, $c(X'X) = m$, the singular value decomposition(SVD) of X was recovered (Belsley, 1991); *i.e.*, X is changed to $X_a = QL_aR$ according to m. Then, the eigenvalues in L were replaced with

$$a = \left[\sqrt{\frac{2}{1+m}}, 1, 1, \sqrt{\frac{2m}{1+m}}\right].$$

The GME estimator was first applied to estimate β. Supports of the GME estimator are as follows. The parameter support is $z_k = [-5, -3, 0, 3, 5]$ for each k, the individual effect support is $f_i = [-3, 0.3]$ for each i, the time effect support is $a_t = [-3, 0.3]$ for each t and the remainder stochastic disturbance term support is $v_{it} = [-3, 0.3]$ for each i and t. GME was run 1000 times independently. For comparison, the OLS, GLS, WH, AM and SA estimators were also applied and run 1000 times independently.

Table 1 provides that when the distortion of data is very serious, GME performs the best among all estimators; *i.e.*, when $m = 100$, $\sigma_\mu^2 = 10$, $\sigma_\lambda^2 = 6$, $\sigma_e^2 = 4$, MSEs of OLS, GLS, WH, AM and SA are 251.5851, 75.4855, 75.4414, 75.2932 and 75.3617, respectively, but GME has only 2.5083. However, when the condition number is 1 which means the complete data, the efficiency of GME is very poor. For example, when $m = 1$, $\sigma_\mu^2 = 10$, $\sigma_\lambda^2 = 6$, $\sigma_e^2 = 4$, MSEs of OLS, GLS, WH, AM, SA and GME are 0.0672, 0.0192, 0.0192, 0.0197, 0.0205 and 1.9160, respectively. The reason why MSE of GME is large is because the GME estimator has been developed to recover the ill-posed problems. Thus GME may not perform well in the complete data. Note that even if the data is ill-posed, GLS and FGLS have similar MSEs. Furthermore, when the condition number increases, the GME estimators are quite stable, near to 2.5, but MSEs of all other existing estimators increase sharply.

Table 1: The comparison of MSE of estimators

σ_μ^2	σ_λ^2	σ_e^2	m	OLS	WH	GLS	AM	SA	GME
2	8	10	1	0.0674	0.0446	0.0445	0.0451	0.0458	1.8786
			10	42.6648	28.4412	28.3044	28.3190	28.4585	2.5028
			50	150.8582	98.5139	97.9431	97.7482	98.5370	2.5034
			100	267.9455	182.8609	181.5762	181.3080	182.8110	2.4950
4	6	10	1	0.0662	0.0434	0.0433	0.0436	0.0444	1.8808
			10	40.5042	27.7557	27.5721	27.6890	27.7773	2.4959
			50	145.8705	95.3064	94.6524	94.4831	95.2816	2.5014
			100	270.6062	178.1333	175.8912	176.3170	178.3699	2.4981
6	8	6	1	0.0678	0.0293	0.0291	0.0295	0.0302	1.9076
			10	41.9893	17.0074	16.9823	16.9790	16.9968	2.4966
			50	138.2713	60.5151	60.1889	60.2611	60.5057	2.5022
			100	260.5676	109.0608	108.7257	108.5749	109.2259	2.5125
8	4	8	1	0.0688	0.0363	0.0362	0.0367	0.0376	1.9653
			10	40.9193	22.2646	22.1132	22.1630	22.2617	2.5010
			50	142.9272	69.0460	68.9861	68.4853	69.1285	2.5014
			100	252.6091	129.9312	129.6206	128.6221	129.7769	2.5166
10	6	4	1	0.0672	0.0192	0.0192	0.0197	0.0205	1.9160
			10	40.3534	11.5876	11.5539	11.5520	11.5877	2.5119
			50	139.0700	38.1419	37.7319	37.9364	38.0910	2.4948
			100	251.5851	75.4414	75.4855	75.2932	75.3617	2.5083
12	6	2	1	0.0658	0.0100	0.0100	0.0108	0.0115	1.9192
			10	42.1791	5.9634	5.9392	5.9479	5.9588	2.5068
			50	139.0043	20.4561	20.3352	20.3226	20.3786	2.5166
			100	280.1318	34.1239	34.0449	33.8842	34.0070	2.5083
14	2	4	1	0.0711	0.0181	0.0180	0.0185	0.0199	1.9326
			10	37.4508	10.3213	10.2296	10.2425	10.3040	2.5059
			50	130.3356	35.7995	35.7499	35.5827	35.7877	2.5051
			100	282.9393	67.0270	66.9162	66.4742	66.9194	2.5087
16	2	2	1	0.0693	0.0090	0.0090	0.0096	0.0112	1.9149
			10	38.6434	5.9888	5.9838	5.9877	5.9920	2.5130
			50	146.3061	20.2304	19.9994	19.9951	20.0974	2.5145
			100	277.9596	38.1689	37.7730	37.9002	37.9671	2.5165

For more detailed comparison, we drew scatter plots for y *vs.* $\hat{y}$ and β *vs.* $\hat{\beta}$ produced by OLS, GLS, WH, AM, SA and GME when $m = 1$ and $m = 100$, respectively, in a randomly selected sample (Figure 1, 2). The figures support the result of Table 1; *i.e.*, when the data is distorted seriously(m is large), the GME estimator performs the best because $\hat{y}$ and $\hat{\beta}$ by generated GME are closest to y and β, respectively, among all estimators. Thus, it is reasonable that GME is a robust and quite good efficient estimator for ill-posed problems.

5. Conclusion

This paper proposes the GME estimator for ill-posed problems in the two-way nested error component model, and has shown how ill-posed problems are solved using the GME formulation. GME is compared with existing methods on the simulated dataset and numerical results indicate that GME is the best estimator for ill-posed problems in terms of quality of the estimator.

The GME approach employed in this paper appears to offer an advantage over conventional methods that the GME approach has a capability to recover information from small samples of data with a degree of precision when the data are distorted seriously.

Further research should extend the ill-posed error component panel regression model to allow for heteroskedasticity and serial correlation in the disturbances.

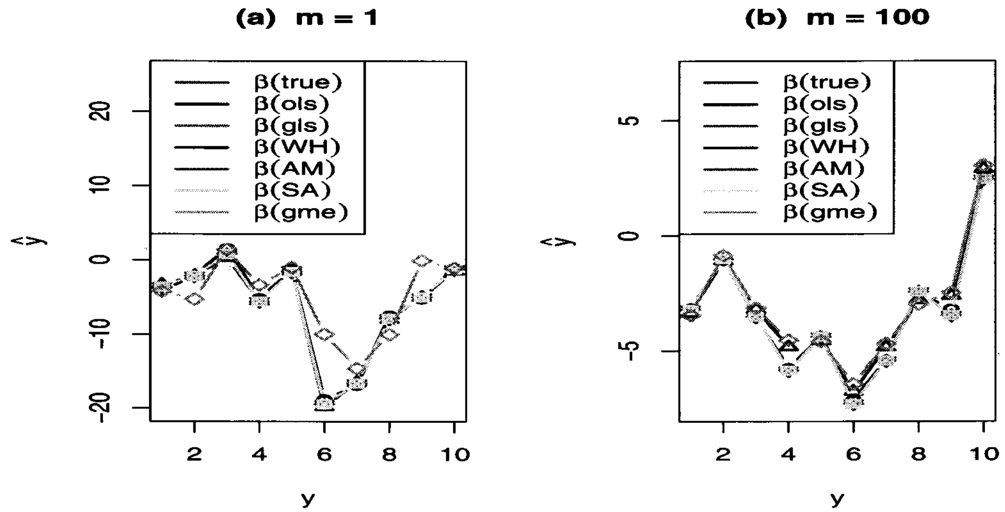

Figure 1: *Comparison of partial scatter plots for y vs. ŷ produced by OLS, GLS, WH, AM, SA and GME when* $\sigma_\mu^2 = 10, \sigma_\lambda^2 = 6, \sigma_e^2 = 4$, *and (a)* $m = 1$ *and (b)* $m = 100$.

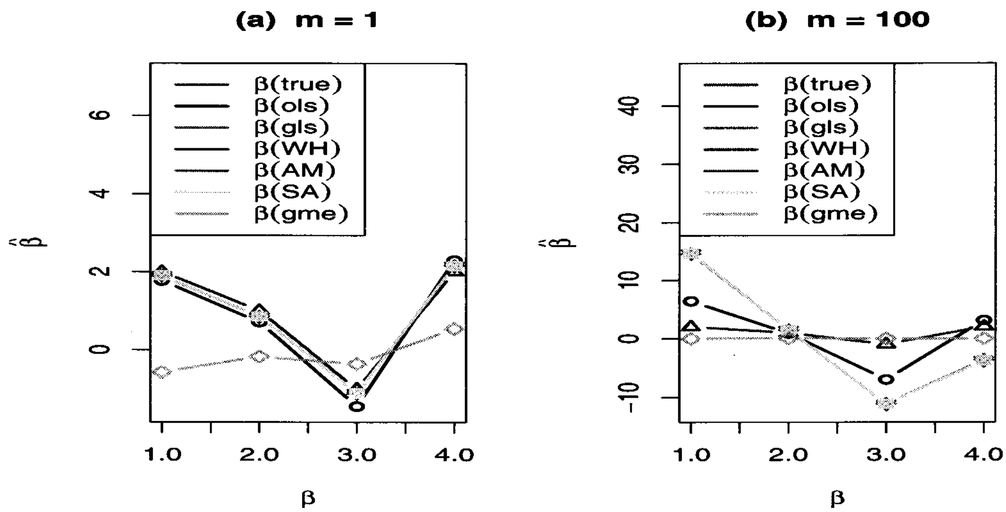

Figure 2: *Comparison of scatter plots for* β *vs.* $\hat{\beta}$ *produced by OLS, GLS, WH, AM, SA and GME when* $\sigma_\mu^2 = 10, \sigma_\lambda^2 = 6, \sigma_e^2 = 4$, *and (a)* $m = 1$ *and (b)* $m = 100$.

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