JPE 9-5-13

Robust Disturbance Compensation for Servo Drives Fed by a Matrix Converter

Kiwoo Park^{*}, Dongkyoung Chwa^{*}, and Kyo-Beum Lee[†]

^{†*}Division of Electrical and Computer Eng., Ajou University, Suwon, Korea

ABSTRACT

This paper presents a time-varying sinusoidal disturbance compensation method (based on an adaptive estimation scheme) for induction motor drives fed by a matrix converter. In previous disturbance accommodation methods, sinusoidal disturbances with unknown time-invariant frequencies have been considered. However, in the new method proposed here, disturbances with unknown time-varying frequencies are considered. The disturbances can be estimated by using a disturbance accommodating observer, and an additional control input is added to the induction machine drive. The stability analysis is carried out considering the disturbance estimation error and simulation results are shown to illustrate the performance of the proposed solution.

Keywords: Time-varying sinusoidal disturbance, Compensation, Matrix converter

1. Introduction

The induction motor drive fed by a matrix converter is superior to the one fed by a conventional inverter, because it does not require bulky electrolytic capacitors in the dc-link, which have limited lifetimes. It has other advantages, such as bidirectional power flow capabilities, sinusoidal input/output currents and adjustable input power factors. Furthermore, due to the high integration capability, the matrix converter topology is recommended for extreme temperatures and weight/volume critical applications ^[1-3].

Typical systems where sinusoidal disturbances may occur or dominate are those with the rotational machinery. In fact, for many practical systems, sinusoidal disturbances are induced by rotational motions and so, the disturbances are induced by rotational motions. Therefore, the disturbance frequencies are close to the rotational speeds, which can be directly measured on-line. In this situation, the frequency does not have to be estimated by adaptation mechanisms, thus, a relatively simple controller can be used. However, as referenced in [4], the frequency could have time-varying sinusoidal components in a practical system. Disturbance compensation techniques, as proposed in [5-7], are effective in handling sinusoidal disturbances, but these are not applicable to the disturbances with unknown amplitude, phase and time-varying frequencies.

In order to realize a high-performance control of induction motor drives fed by a matrix converter, a matrix converter model for a time-varying sinusoidal disturbance compensation method using a disturbance accommodation technique is proposed in this paper. The structure of the

Manuscript received Mar. 16, 2009; revised July 30, 2009 [†]Corresponding Author: kyl@ajou.ac.kr

Tel: +82-31-219-2376, Fax: +82-31-212-9531, Ajou Univ. *Division of Electrical and Comp. Eng., Ajou Univ.



Fig. 1. Scheme of disturbance compensation.

disturbance accommodation technique is shown in Fig. 1 where, after estimating the time-varying disturbances using a disturbance observer, they are, then, used for controller design and input realization. By using this scheme, disturbance compensation of the control system can be covered in non-minimum phase systems.

Simulation results are presented to verify the effectiveness and feasibility of the proposed new control system.

2. Vector Control of Induction Motor Drives Fed by a matrix converter

The main circuit of a matrix converter is shown in Fig. 2 where it can be seen that the converter comprises an input filter, bidirectional power switches connected to a three-phase matrix and a clamp circuit. The input inductance-capacitance (LC) filter has a second-order low-pass topology, which is used to reduce the high-frequency ripple from the input current. The clamp circuit consists of two diode bridges connecting the input and the output to a clamp capacitor, in order to protect the switches against possible over-voltages that may appear during transients.

The most general modulation method for a matrix converter is the indirect space vector modulation (ISVM), which considers the matrix converter as a two-stage transformation converter. This consists of a rectification stage to provide a constant imaginary dc-link voltage per switching period, and an inverter stage to produce the three output voltages. The input current vector I_{in} , which corresponds to the rectification stage, and the output voltage vector U_{out} , which corresponds to the inversion stage, are the reference vectors.

The indirect space vector modulation uses a combination of two adjacent vectors and a zero vector to produce the reference vector. The ratio between the duty



Fig. 2. Topology of a matrix converter drive.

cycles of the two adjacent vectors determines the magnitude of the reference vector. The duty cycles of the active switching vectors in the rectification stage are calculated with (1), and the duty cycles of the active switching vectors in the inversion stage are calculated with (2).

$$d_{\gamma} = m_I \sin\left(\pi/3 - \theta_{in}^*\right)$$
 and $d_{\delta} = m_I \sin\theta_{in}^*$ (1)

$$d_{\alpha} = m_U \sin\left(\pi/3 - \theta_{out}^*\right)$$
 and $d_{\beta} = m_U \sin\theta_{out}^*$ (2)

where m_I and m_U are the rectification and inversion stage modulation indices, respectively, and θ_{in}^* and θ_{out}^* are the angles within their respective sectors of the input current and output voltage reference vectors (see Fig. 3). The input current reference vector is synthesized by impressing adjacent switching vectors, I_{γ} and I_{δ} , with duty-cycle d_{γ} and d_{δ} in the rectifying stage. The reference voltage vector is modulated by the adjacent voltage vectors, U_{α} and U_{β} , with duty-cycle d_{α} and d_{β} . To obtain a correct balance of the input currents and the output voltages in the same switching period, the modulation pattern should produce all combinations of the rectification (γ - δ - θ) and the inversion (α - β - θ) switching stages, resulting in the following switching pattern: $\alpha\gamma$ - $\alpha\delta$ - $\beta\delta$ - $\beta\gamma$.



(a) In the rectification stage. (b) In the inversion stage.

Fig. 3. Generation of the reference voltage vectors using ISVM.

Therefore, each duty cycle sequence is a result of the cross products of the rectification and the inversion stage duty cycles, while the duration of the zero vectors complete the switching sequence as:

$$d_{\alpha\gamma} = d_{\alpha}d_{\gamma}$$

$$d_{\alpha\delta} = d_{\alpha}d_{\delta}$$

$$d_{\beta\delta} = d_{\beta}d_{\delta}$$

$$d_{\beta\gamma} = d_{\beta}d_{\gamma}$$

$$d_{0} = 1 - \left(d_{\alpha\gamma} + d_{\alpha\delta} + d_{\beta\delta} + d_{\beta\gamma}\right)$$
(3)

The duration of each sequence is found by multiplying the corresponding duty cycle to the switching period.

3. Time-Varying Sinusoidal Disturbance Compensation Method Using a Disturbance Accommodation Scheme

In this section, the control of the disturbance compensation is briefly described ^[5-7]. The compensation method, when the frequency of the sinusoidal disturbance is already known, is described first, and a new method for the unknown frequency is presented afterwards.

3.1 Disturbance Compensation Model

Here, the following non-linear system is considered:

$$\dot{x} = Ax + Bu + F\omega_d \tag{4}$$
$$y = Cx$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $y \in \mathbb{R}^m$ are vectors of state variables, control inputs and outputs, respectively. *A*, *B*, *C*, *F* are matrices of an appropriate dimension, ω_d is the disturbance composed of $[\omega_{d1} \dots \omega_{dp}]^T$. Each ω_{di} is a linear combination of basis functions f_{ij} which has a following form ^[5].

$$\omega_{di}(t) = c_{i1}f_{i1}(t) + c_{i2}f_{i2}(t) + \dots + c_{iq}f_{iq}(t)$$
(5)

where f_{ij} are known functions, and c_{ij} are constants

estimated by a disturbance observer.

It is assumed that each f_{ij} satisfies the linear differential equation, and ω_d can be defined by the following z-dynamics form.

$$\dot{z} = Dz + \sigma(t)$$

 $\omega_d = Hz$
(6)

where z is $[z_1 \dots z_p]^T$, H and D are known values and $\sigma(t)$ is the impulse vector. It is noted that many practical disturbances with constant, ramp, and sinusoidal characteristics can be represented as above.

3.2 Disturbance Compensation Control

The objective of this control is to make the actual system follow the nominal model dynamics, even under severe nonlinearities. A reference model y_m can be obtained from

$$\dot{x}_m = A_m x_m, \quad y_m = C_m x_m \tag{7}$$

where x_m is the state variable of the reference model. Here, the desired performance can be achieved by choosing proper A_m and C_m . The control input is defined as

$$u = u_d + u_p \tag{8}$$

where u_d is a control input term for the disturbance rejection, u_p is a control input for the non-linear system that is used to control the system as if it had the desired dynamics; therefore, the system can be modeled without considering ω_d , so as to follow the nominal model dynamics. Many linear or nonlinear control methods can be applied to the design of the control input u_p .

$$Bu_d = -F\omega_d = -FHz \tag{9}$$

$$u_d = -(FH/B)\hat{z} \tag{10}$$

where \hat{z} is an estimated value of z from the disturbance observer, which will be described in the next section.

3.3 Disturbance Observer

If the following equation is true, then it is obvious that the approximated disturbance error $(\tilde{z} := z - \hat{z})$ converges to zero.

$$\dot{\hat{z}} = D\hat{z} - k_0 \left(z - \hat{z}\right) \tag{11}$$

where *D* is the Hurwitz matrix. As *z* is a variable to be approximated, a new state variable *Q* can be defined and introduced as follows ^[7].

$$Q = \hat{z} - k_1 y \tag{12}$$

From (11), the following equation can be derived.

$$\dot{\hat{z}} - k_1 \dot{y} = (D + k_0) \hat{z} - (k_0 + k_1 CFH) z - k_1 (CAx + CBu)$$
(13)

Since z is an unknown value, the z-term can be removed by choosing appropriate design constants k_0 and k_1 as (14), which leads to a simpler form of (13).

$$k_0 + k_1 CFH = 0 \tag{14}$$

$$\dot{Q} = (D+k_0)Q + (D+k_0)k_1y$$

$$-k_1(CAx + CBu)$$
(15)

The initial value of Q can be defined as $Q_0 = -k_1 y$. From (6) and (12), the disturbance can be derived as the following equation.

$$\hat{\omega}_d = H\hat{z} = H\left(Q + k_1 y\right) \tag{16}$$

Therefore, the control input for the disturbance has the following form

$$u_d = -\hat{\omega}_d \tag{17}$$

3.4 Compensation of Unknown Time-Varying Sinusoidal Disturbances in Nonlinear Systems

To compensate unknown time-varying sinusoidal disturbances, a general form of disturbance given by

$$\omega_d(t) = A\sin\left(\omega(t) + \phi\right) \tag{18}$$

is assumed, where $\omega(t)$ is an unknown time-varying

frequency which satisfies $|\dot{\omega}(t)|, |\ddot{\omega}(t)| < \infty$ for $\forall t \ge 0$.

Assume that $\omega_d(t)$ is z_1 in (17) and derivatives of z_1 can be expressed as follows:

$$\dot{z}_{1} = A\cos(\omega(t) + \phi) \cdot \dot{\omega}(t)$$

$$\ddot{z}_{1} = (\dot{\omega}(t))^{2} z_{1} + A\cos(\omega(t) + \phi) \cdot \ddot{\omega}(t)$$
(19)

Instead of (6), (18) can be represented as

$$\dot{z} = \left(\overline{D} - \Omega\right)z + \Phi \tag{20}$$

where

$$\begin{split} \overline{D} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ \Omega &= \begin{bmatrix} 0 & 0 \\ \left(\dot{\omega}(t)\right)^2 & 0 \end{bmatrix}, \\ \Phi &= \begin{bmatrix} 0 \\ A\cos(\omega(t) + \phi) \cdot \ddot{\omega}(t) \end{bmatrix} \end{split}$$

Here, if $\omega(t) = \sigma_1 + \sigma_2 t$ is satisfied with any constants σ_1 and σ_2 , $|\dot{\omega}(t)|, |\ddot{\omega}(t)| < \infty$ is also satisfied. Equation (20) can be transformed to another equation as

$$\dot{z} = \left(\overline{D} - \Psi\right)z + \left(\Psi - \Omega\right)z + \Phi \tag{21}$$

where $\psi = \begin{bmatrix} 0 & 0 \\ 0 & \psi \end{bmatrix}$, $\psi > 0$. If $\dot{\hat{z}}$ is modeled as (22) instead of (11), an observer for time-varying sinusoidal disturbances can be derived as

$$\dot{\hat{z}} = \left(\overline{D} - \psi\right)\hat{z} - k_0\left(z - \hat{z}\right) \tag{22}$$

where k_0 is a gain matrix of the observer and can be designed to satisfy $(\overline{D} - \Psi + k_0) + (\overline{D} - \Psi + k_0)^T = -Q$ for a positive limit matrix $Q = diag(q_1, q_2)$. The variables q_1 and q_2 , which are components of Q, can be adjusted by k_0 and ψ .

From (21) and (22), the following equation is derived.

$$\dot{\tilde{z}} = \left(\overline{D} - \psi + k_0\right)\tilde{z} + \left(\psi - \Omega\right)z + \Phi$$
(23)

To ensure the stability of the system, a *Lyapunov function* is defined as the following form.

$$V = \tilde{z}^T \tilde{z} \tag{24}$$



Fig. 4. The proposed disturbance observer for matrix converter drives.

The derivative of (24) is as follows:

$$\dot{V} = -\tilde{z}^{T}Q\tilde{z} + \tilde{z}^{T}(\psi - \Omega)z + \tilde{z}^{T}\Phi$$

$$= -q_{1}\tilde{z}_{1}^{2} - q_{2}\tilde{z}_{2}^{2}$$

$$+ \left[-(\dot{\omega}(t))^{2}z_{1} + \psi z_{2} + A\cos(\omega(t) + \phi) \cdot \ddot{\omega}(t) \right]\tilde{z}_{2}$$

$$= -q_{1}\tilde{z}_{1}^{2}$$

$$- q_{2} \left[\tilde{z}_{2} - (\dot{\omega}(t))^{2}z_{1} + \psi z_{2} + \frac{A\cos(\omega(t) + \phi) \cdot \ddot{\omega}(t)}{2q_{2}} \right]^{2}$$

$$+ \frac{\left[-(\dot{\omega}(t))^{2}z_{1} + \psi z_{2} + A\cos(\omega(t) + \phi) \cdot \ddot{\omega}(t) \right]^{2}}{4q_{2}}$$
(25)

As the last term of the last equation in (25) is bounded, the approximated disturbance error (\tilde{z}_1) is ultimately bounded as well, shown in the following equation.

$$\left|\tilde{z}_{1}\right| \leq \frac{\left|-\left(\dot{\omega}(t)\right)^{2} z_{1}+\psi z_{2}+A\cos\left(\omega(t)+\phi\right)\cdot\ddot{\omega}(t)\right|}{2\sqrt{q_{1}q_{2}}}$$
(26)

The approximated difference \tilde{z}_1 can be minimized by adjusting q_1 and q_2 . Then, $\hat{\omega}_d$ can be derived by H and \tilde{z}_1 in (16).

4. Induction Motor Drives Fed By a Matrix Converter Using a Disturbance Accommodation Scheme

The proposed disturbance observer, with a matrix converter, is presented in Fig. 4. For the purpose of applying a non-linear compensation method to the induction motor drives, a mechanical system of motor drives can be represented, as shown in Fig. 5. From this figure, the following equation is obtained ^[10].

$$\dot{x}_m = Ax_m + Bu_m + CT_d \tag{27}$$

where $x_m = \omega, A = -B_m/J, B = 1/J, u_m = T_e, C = -(1/J)$.

In practical cases, most motors, having rotational motion, could have a time-varying sinusoidal disturbance. The disturbance and the state equation of a mechanical



Fig. 5. Dynamic model of the mechanical system.

system can be re-written in the following form.

$$T_d = C\sin(\omega(t) + \phi) \tag{28}$$

$$\dot{x}_m = Ax_m + B\left(u_p + u_d\right) + C\sin\left(\omega(t) + \phi\right)$$
(29)

As for the control structure, two control inputs are designed separately in such a way that one of them, u_p , is designed for the nonlinear system control without considering the disturbance, and the other one, u_d , is designed using the disturbances estimated from the disturbance accommodating observer.

To identify the time-varying sinusoidal disturbance of the induction motor drive system, the disturbance observer can be modeled as follows:

$$\dot{Q} = \begin{bmatrix} -k_{01} & 1\\ -k_{02} & -\psi \end{bmatrix} Q + \begin{bmatrix} -k_{01} & 1\\ -k_{02} & -\psi \end{bmatrix} \begin{bmatrix} k_{11}\\ k_{12} \end{bmatrix} y - \begin{bmatrix} k_{11}\\ k_{12} \end{bmatrix} (Ax + Bu)$$
(30)

where the modeling constant k_1 , k_2 and ψ are

$$k_{1} = \begin{bmatrix} k_{11} & k_{12} \end{bmatrix}^{T} = \begin{bmatrix} 100 & 100 \end{bmatrix}^{T},$$

$$k_{0} = \begin{bmatrix} -k_{01} & 0 \\ -k_{02} & 0 \end{bmatrix} = \begin{bmatrix} -100 / J & 0 \\ -100 / J & 0 \end{bmatrix},$$

$$\psi = 1.$$
(31)

If the additional compensating input, u_d , is controlled as $u_d = -\hat{T}_d$, then the disturbance can be removed. In addition, the estimated disturbance \hat{T}_d can be approximated by the disturbance observer.

The electrical torque of an induction motor drive is expressed in (32) by the stator current and the rotor flux. In the vector control, the magnitude of the rotor flux is proportional to i_{ds}^e when it is kept constant. If the rotor flux is observed in the rotor flux reference frame, the q-axis of the rotor flux is equal to 0. It may be considered that i_{qs}^e is a torque current component ^[11]:

$$T_e = \frac{3}{2} \frac{P}{2} \frac{L_m}{L_r} (\lambda_{dr} i_{qs} - \lambda_{qr} i_{ds})$$
(32)

$$T_e = \frac{3}{2} \frac{P}{2} \frac{L_m}{L_r} \lambda_{dr}^e i_{qs}^e$$
(33)

From (33), the q-axis current that compensates the disturbance, which is approximated by the disturbance observer, is derived as follows:

$$i_{qs_comp}^{e} = \frac{2}{3} \frac{2}{P} \frac{L_r}{L_m} \frac{\hat{T}_e}{\lambda_{dr}^e}$$
(34)

The control block diagram of the proposed disturbance observer is shown in Fig. 6.

5. Simulation Results

To confirm the validity of the proposed control algorithm, simulations are carried out. The system consists of a 3-phase, 380 V, 60 Hz, 4-pole, 3 kW induction motor. In the simulation, the sampling period is 100µs for the proposed method. Fig. 7 and Fig. 8 show the simulation results when the time-varying sinusoidal disturbance is considered. In the case of the vector control without the compensation method, the disturbance is unable to be eliminated. However, the speed control with the proposed disturbance observer has a robust characteristic against the disturbance. The majority of the time-varying sinusoidal disturbance is well estimated and compensated effectively. In Fig. 9, the sinusoidal disturbance with an unknown amplitude, phase and time-varying frequency load has occurred abruptly. For the speed control, the system with the proposed disturbance observer has shown better performance than that with only a PI-controller. Fig. 10 shows the simulation results when the disturbance load is not a time-varying sinusoidal but a step response. Since all



Fig. 6. Schematic diagram of the disturbance observer.



Fig. 7. Speed response without disturbance observer: (a) speed and (b) disturbance torque.



Fig. 9. Speed response with unknown sinusoidal disturbance: (a) speed, (b) speed error, and (c) disturbance torque.

of the waveforms can be divided into sinusoidal and dc signals by the Fourier transformation, the system with the disturbance observer has better performance than that with only a PI-controller.



Fig. 8. Speed response with disturbance observer: (a) speed and (b) estimated disturbance torque.



Fig. 10. Speed response with a step disturbance: (a) speed, (b) speed error, and (c) control input.

6. Conclusion

In order to realize the high performance control of induction motor drives, a new disturbance observer for

time-varying sinusoidal disturbances is proposed. By using the proposed disturbance observer, the sinusoidal disturbances with unknown frequencies are well estimated, and compensated. The disturbance observer has been successfully applied to induction motor drive systems.

Acknowledgements

This work was supported by the new faculty research fund of Ajou University.

References

- C. Klumpner, P. Nielsen, I. Boldea and F. Blaabjerg, "A New Matrix Converter-Motor (MCM) for Industry Applications," *IEEE Trans. Industrial Electronics*, Vol. 49, No. 2, pp. 325-335, Apr. 2002.
- [2] P. W. Wheeler, J. Rodriguez, J. C. Clare, L. Empringham, and A. Weinstein, "Matrix converter - A technology review," *IEEE Trans. Industrial Electronics*, Vol. 49, No. 2, pp 325-335, Apr. 2002.
- [3] K. B. Lee and F. Blaabjerg, "A Non-Linearity Compensation Method for Matrix Converter Drives," *IEEE Power Electronics Letters*, Vol. 3, No. 1, pp. 19-23, Mar. 2005.
- [4] H. Du, L. Zhang, Z. Lu, and X. Shi, "LPV technique for the rejection of sinusoidal disturbance with time-varying frequency," *IEE Proceedings-Control Theory and Applications*, Vol. 150, No. 2, pp. 132-138, 2003.
- [5] C. D. Johnson, "Improved discrete-time, disturbance -accommodating control performance using discrete /continuous control theory. part I. Complete disturbance -cancellation (rejection)," *in Proceedings of the 35th Southeastern Symposium on System Theory*, pp. 127-133, 2003.
- [6] C. D. Johnson, "Disturbance-Accommodating Control -An overview," in Proceedings of the American Control Conference, Seattle, WA, pp. 526-536, June 1986.
- [7] J. Kim, "Disturbance Accommodating Sliding Mode Control," in Proceedings of the American Control Conference, Chicago, IL, pp. 888-890, June 1992.
- [8] D. Chwa, J. I. Kwon, "Compensation of Sinusoidal Disturbance with Time-Varying Frequency in Uncertain Nonlinear Systems using Disturbance Accommodation Technique," *The 2nd International Conf. on Ubiquitous Information Management and Communication*, CD format, 2008.

- [9] K. B. Lee and F. Blaabjerg, "An Improved DTC-SVM Method for Sensorless Matrix Converter Drives Using an Overmodulation and Simple Non-Linearity Compensation," *IEEE Trans. Industrial Electronics*, Vol. 54, No. 6, pp. 3155-3166, Dec. 2007.
- [10] K. B. Lee, S. H. Huh, J. Y. Yoo, and F. Blaabjerg, "Performance Improvement of DTC for Induction Motor-Fed by Three-Level Inverter With an Uncertainty Observer Using RBFN," *IEEE Trans. Energy Conversion*, Vol. 20, No. 2, pp. 276-283, June 2005.
- [11] B. K. Bose, "Power Electronics and Motor Drives Advances and Trends," *Academic Press*, 2006.



Kiwoo Park was born in Inchon, Korea, in 1981. He received the B.S. degree in Electrical and Computer Engineering from Ajou University, Suwon, Korea, in 2009. He is currently working toward the M.S. degree at Ajou University. His research interests are

power conversion and electric machine drives.



Dongkyoung Chwa received the B. S. and M. S. degrees in Department of Control and Instrumentation Engineering in 1995 and 1997, respectively, and Ph.D. degree in School of Electrical and Computer Engineering in 2001, all from Seoul National

University, Seoul, Korea. From 2001 to 2003, he has been the Post-Doctoral Researcher in the same university. In 2003, he was a Visiting Research Fellow in the UNSW@ADFA (The University of New South Wales at Australian Defence Force Academy) and the Honorary Visiting Academic in the University of Melbourne, in Australia. In 2004, he was a BK21 Assistant Professor in Seoul National University. Since 2005, he has been with the Division of Electrical and Computer Engineering at Ajou University, where he is currently an Associate Professor. His research interests are nonlinear, robust, and adaptive control theories, and their applications to the robotics, underactuated systems including wheeled mobile robots, underactuated ships, cranes, and guidance and control of flight systems.



Kyo-Beum Lee was born in Seoul, Korea, in 1972. He received the B.S. and M.S. degrees in electrical and electronic engineering from the Ajou University, Korea, in 1997 and 1999, respectively. He received the Ph.D. degree in electrical engineering from the

Korea University, Korea in 2003. From 2003 to 2006, he was with the Institute of Energy Technology, Aalborg University, Aalborg, Denmark. From 2006 to 2007, he was with the Division of Electronics and Information Engineering, Chonbuk National University, Jeonju, Korea. In 2007 he joined the Division of Electrical and Computer Engineering, Ajou University, Suwon, Korea. He is an associate editor of the IEEE Transactions on Power Electronics and the Journal of Power Electronics. He has received two IEEE prize paper awards. His research interests include electric machine drives and wind power generations.