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PERMANENCE FOR THREE SPECIES PREDATOR-PREY SYSTEM WITH DELAYED STAGE-STRUCTURE AND IMPULSIVE PERTURBATIONS ON PREDATORS

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ABSTRACT. In this paper, three species stage-structured predator-prey model with time delayed and periodic constant impulsive perturbations of predator at fixed times is proposed and investigated. We show that the conditions for the global attractivity of prey(pest)-extinction periodic solution and permanence of the system. Our model exhibits a new modelling method which is applied to investigate impulsive delay differential equations. Our results give some reasonable suggestions for pest management.

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1. Introduction

Biological control is, generally, man's use of a specially chosen living organism to control a particular pest, which is a component of an integrated pest management strategy. This chosen organism might be a predator, parasite, or disease which will attack the harmful insect. It is a form of manipulating nature to increase a desired effect. It may also be a more economical alternative to some insecticides. Some biological control measures can actually prevent economic damage to agricultural crops. Virtually all insect and mite pests have some natural enemies. One approach to biological control is augmentation, which is manipulation of existing natural enemies to increase their effectiveness. This can be achieved by mass production and periodic release of natural enemies of the pest, and by genetic enhancement of the enemies to increase their effectiveness at control. One of the first successful cases of biological control in greenhouses was that of the parasitoid Encarsia formosa against the greenhouse whitefly Trialeurodes vaporariorum on tomatoes and cucumbers [1,2].

Recently, it is of great interests to investigate the models with impulsive

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perturbations in biological populations. Based on the classical Lotka-Volterra predator-prey system, many paper[3-6] suggested impulsive differential equations to model the process of periodically releasing natural enemies at fixed time for pest control. Impulsive differential equations are found in almost every domain of applied sciences and have been studied in many investigation [7-11].

In the nature world, there are many species (especially insect) whose individual members have a life history that takes them through two stages, immature and mature. There have been some works on modelling and considering the stage-structured models with various life stages[12-15]. Aiello and Freedman [16] constructed and studied a time delay model of single species growth with stage structure as the following:

$$\begin{cases} x_1'(t) = \alpha x_2 - \beta x_1 - \alpha e^{-\beta\tau} x_2(t-\tau) \\ x_2'(t) = \alpha e^{-\beta\tau} x_2(t-\tau) - l x_2^2 \end{cases} \tag{1.1}$$

where $x_1(t), x_2(t)$ represent the immature and mature populations densities respectively, τ represents a constant time to maturity, and α, β, l are positive constants.

In 1970, Parrish and Saila [17] constructed and studied the following predator-prey system with two prey and one predator as the following:

$$\begin{cases} x'(t) = x(t)(l - kx(t) - m_1 y(t) - m_2 z(t)) \\ y'(t) = y(t)(r - d_2 y(t) - a_2 x(t) - b_2 z(t)) \\ z'(t) = z(t)(-d + \lambda m_2 x(t) + \lambda b_2 y(t)) \end{cases} \tag{1.2}$$

where $x(t), y(t), z(t)$ are the densities of the two prey, one predator at time t respectively, $l, k, m_1, m_2, r, d_2, a_2, b_2, d, \lambda$ are positive constants. .

In this paper, we consider the following a delayed prey-dependent consumption two-prey one-predator models with stage-structure for prey and periodic impulsive perturbations of predator:

$$\begin{cases} \left.\begin{array}{l} x_1'(t) = \alpha_1 x_2(t) - \beta_1 x_1(t) - \alpha_1 e^{-\beta_1\tau_1} x_2(t-\tau_1) \\ x_2'(t) = \alpha_1 e^{-\beta_1\tau_1} x_2(t-\tau_1) - d_1 x_2^2 - a_1 x_2(t) y_2(t) - b_1 x_2(t) z(t) \\ y_1'(t) = \alpha_2 y_2(t) - \beta_2 y_1(t) - \alpha_2 e^{-\beta_2\tau_2} y_2(t-\tau_2) \\ y_2'(t) = \alpha_2 e^{-\beta_2\tau_2} y_2(t-\tau_2) - d_2 y_2^2 - a_2 x_2(t) y_2(t) - b_2 y_2(t) z(t) \\ z'(t) = z(t)(-d + \lambda b_1 x_2(t) + \lambda b_2 y_2(t)) \end{array}\right\} \quad t \neq nT, \\ \Delta z(t) = p \qquad\qquad t = nT \end{cases} \tag{1.3}$$

where $x_1(t), x_2(t)$ represent the immature and mature population densities for prey $x(t)$ respectively, $y_1(t), y_2(t)$ represent the immature and mature population densities for prey $y(t)$ respectively, $z(t)$ denotes the density of predator. τ_1, τ_2 represents a constant time to maturity of prey $x_1(t), y_1(t)$, and $\alpha_1, \beta_1, \alpha_2, \beta_2, a_1, a_2, b_1, b_2, d_1, d_2, d$ and λ are positive constants.This model is derived as follows. We assume that at any time $t > 0$, birth into the immature population is proportional to existing mature population with proportionality constant α_1 and α_2 of $x_2(t), y_2(t)$, respectively. We then assume that the death rate of immature population is proportional to the existing immature population with proportionality constant β_1 and β_2 for $x_1(t), y_1(t)$, respectively. $\beta_1, \beta_2, d_1, d_2$ and d are called the death rate of $x_1(t), x_2(t), y_1(t), y_2(t), z(t)$, respectively. λ is the rate of conversing prey into predator. a_1, a_2 are competitive coefficients of mature population $x_2(t)$ and prey $y_2(t)$. $\Delta z(t) = p, p > 0$ is the releasing amount of natural enemies at $t = nT, n \in N$ and $N = \{1, 2, \cdots\}$, T is the period of impulsive of the predator.

The initial data $(x_1(t), x_2(t), y_1(t), y_2(t), z(t))$ for system (1.3) are

$$(x_1, x_2, y_1, y_2, z \in C([-\tau, 0], R^5_+), x_i(0) > 0, , y_i(0) > 0, z(0) \geq 0, i = 1, 2.)$$

where $\tau = \max\{\tau_1, \tau_2\}$,$R^5_+ = \{(x_1, x_2, y_1, y_2, z) : x_1 \geq 0, x_2 \geq 0, y_1 \geq 0, y_2 \geq 0, z \geq 0\}$, and we impose the condition

$$x_1(0) = \int_{-\tau_1}^{0} \alpha_1 x_2(\theta) e^{\beta_1 \theta} d\theta, \;\; y_1(0) = \int_{-\tau_2}^{0} \alpha_2 y_2(\theta) e^{\beta_2 \theta} d\theta,$$

which present the total surviving immature population for mature $x_2(t), y_2(t)$ from the observed births on $-\tau_1 \leq t \leq 0, -\tau_2 \leq t \leq 0$, respectively.

The organization of the paper is as follows. In Section 1, we introduce a time delay and two-prey one-predator models with age-structure for a prey and periodic constant impulsive perturbation of predator. In Section 2, we will give some notations and lemmas. In Section 3 , we analyze the dynamic behavior of such a system. We show that there exists a prey-eradication periodic solution of globally attractive when impulsive effect satisfies the conditions. Moreover, we prove the system is permanent by analytic method. Lastly, we give a brief discussion.

2. Preliminaries

Underside, we will give some definitions, notations and some lemmas which will be useful for our main results.

Let $R_+ = [0, \infty)$. Denote $f = (f_1, f_2, f_3, f_4, f_5)$ the map defined by the right hand of the first second third equations of system (1.3). Let $V : R_+ \times R^5_+ \to R_+$, then V is said to belong to class V_0 if

(1) V is continuous in $(nT, (n+1)T] \times R^5_+$ for each $X \in R^5_+, n \in N$, $\lim_{(t,y) \to (nT^+, X)} V(t, y) = V(nT^+, X)$ exists.

(2) V is locally Lipschitzian in X.

Definition 2.1 Let $V \in V_0$, then for $(t, X) \in (nT, (n+1)T] \times R^5_+$, the upper right derivative of $V(t, x)$ with respect to the impulsive differential system (1.3) is defined as

$$D^+V(t, X) = \lim_{h \to 0^+} \sup \frac{1}{h}[V(t+h, X + hf(t, x)) - V(t, X)]$$

The solution of system (1.3) is a piecewise continuous function $X : R_+ \to R^5_+$, $X(t)$ is continuous on $(nT, (n+1)T]$, $n \in N$ and $X(t^+) = \lim_{t \to t^+} X(t)$ exists. The smoothness properties of f guarantee the global existence and uniqueness of solution of system (1.3), for the details see book [7]. The following lemma is obvious. We will use an important comparison theorem on impulsive differential equation [7].

Lemma 2.1. *Suppose* $V \in V_0$. *Assume that*

$$\begin{cases} D^+V(t,X) \leq g(t,V(t,X)) & t \neq nT \\ V(t,X(t^+)) \leq \psi_n(V(t,X)) & t = nT \end{cases} \tag{2.1}$$

where $g : R_+ \times R_+ \to R$ *is continuous in* $(nT,(n+1)T] \times R_+$ *and for* $u \in R_+, n \in N$, $\lim_{(t,v)\to(nT^+,u)} g(t,v) = g(nT^+,u)$ *exists,* $\psi_n : R_+ \to R_+$ *is non-decreasing. Let* $r(t)$ *be maximal solution of the scalar impulsive differential equation*

$$\begin{cases} u'(t) = g(t,u(t)) & t \neq nT \\ u(t^+) = \psi_n(u(t)) & t = nT \\ u(0^+) = u_0 \end{cases} \tag{2.2}$$

existing on $[0,\infty)$. *Then* $V(0^+,X_0) \leq u_0$, *implies that* $V(t,X(t)) \leq r(t)$, $t \geq 0$, *where* $X(t)$ *is any solution of (1.3).*

We give basic properties about the following subsystem of system (1.3).

$$\begin{cases} z'(t) = -dz(t) & t \neq nT, \\ \Delta z(t) = p & t = nT \end{cases} \tag{2.3}$$

System (2.3) is a periodically impulsive forced linear system, it is easy to obtain positive periodic solution of the system (2.3). That is,

$$z^*(t) = \frac{p\exp(-d(t-nT))}{1-\exp(-dT)}, t \in (nT,(n+1)T], n \in N \tag{2.4}$$

where initial value $z^*(0^+) = \frac{p}{1-\exp(-dT)}$. Since the solution of the system (2.3) is

$$z(t) = (z(0^+) - \frac{p}{1-\exp(-dT)})\exp(-dt) + z^*(t)$$

we get

Lemma 2.2. *For a positive periodic solution* $z^*(t)$ *of system (2.3) and every solution* $z(t)$ *of system (2.3), we have* $|z(t) - z^*(t)| \to 0$, *as* $t \to \infty$.

Lemma 2.3 *Suppose* $X(t) = (x_1(t), x_2(t), y_1(t), y_2(t), z(t))$ *is a any solution of system (1.3) with* $X(0^+) \geq 0$ *for all* $t \geq 0$. *Then* $X(t) > 0$, *for all* $t > 0$ *if* $X(0^+) > 0$.

Proof. It is obvious $z(t) \geq z(0)exp(-dt) > 0$ and

$$x_2(t) \geq x_2(0)exp(-\int_0^t (d_1x_2(\theta) + a_1y_2(\theta) + b_1z(\theta))d\theta) > 0,$$

$$y_2(t) \geq y_2(0)exp(-\int_0^t (d_2y_2(\theta) + a_2x_2(\theta) + b_2z(\theta))d\theta) > 0,$$

Finally we consider the following equation

$$s'(t) = -\beta_1 s(t) - \alpha_1 e^{-\beta_1\tau_1} x_2(t-\tau_1), s(0) = x_1(0) \tag{2.5}$$

and comparing with (1.3), we note that if $s(t)$ is the solution of (2.5), then $x_1(t) > s(t)$ for $0 < t < \tau$. Solving system (2.5) gives

$$s(t) = e^{-\beta_1 t}[x_1(0) - \int_0^t \alpha_1 e^{\beta(u-\tau_1)} x_2(u-\tau_1)du].$$

Hence

$$s(\tau_1) = e^{-\beta_1 t}[\int_{-\tau_1}^0 \alpha_1 x_2(\theta)e^{\beta_1\theta}d\theta - \int_0^{\tau_1} \alpha_1 e^{\beta_1(u-\tau_1)} x_2(u-\tau_1)du].$$

We thus have $s(\tau_1) = 0$ and therefore $s(t) > 0$ for $t \in [0,\tau]$. By induction, we can show that $x_1(t) > 0$ for all $t \geq 0$. By the same method, we can prove $y_1(t) > 0$ for all $t > 0$. □

Lemma 2.4. *Consider the following equation:*

$$s'(t) = as(t-\tau) - b(t)s(t),$$

where $a, \tau > 0$; $s(t) > 0$, *for* $-\tau \leq t \leq 0$,*and* $b(t)$ *is T-periodic continuous for* $t \in [0,T]$, *and* $b(T^+) = b(0^+)$, *we have:*

1) *If* $a > b$, *then* $\lim_{t\to+\infty} s(t) = +\infty$, 2) *If* $a \leq b$, *then* $\lim_{t\to+\infty} s(t) = 0$.

Lemma 2.5. *Consider the following equation:*

$$s'(t) = as(t-\tau) - bs(t) - cs^2(t),$$

where $a, b, c, \tau > 0$; $s(t) > 0$, *for* $-\tau \leq t \leq 0$; *we have*

1) *If* $a > b$, *then* $\lim_{t\to+\infty} s(t) = \frac{a-b}{c}$;

2) *If* $a \leq b$, *then* $\lim_{t\to+\infty} s(t) = 0$.

Definition 2.2 System (1.3) is said to be permanent if there exist constants $M \geq m > 0$ such that $m \leq x_1(t) \leq M, m \leq x_2(t) \leq M, m \leq y_1(t) \leq M, m \leq y_2(t) \leq M, m \leq z(t) \leq M$ for all t sufficiently large, where $X(t)$ is any solution of system (1.3) with $X(0^+) > 0$.

Lemma 2.6 *There exists a constant* $M > 0$, *such that* $x_1(t) \leq M, x_2(t) \leq M, y_1(t) \leq M, y_2(t) \leq M, z(t) \leq M$ *for each solution*

$$X(t) = (x_i(t), x_2(t), y_1(t), y_2(t), z(t))$$

of system (1.3) with all t large enough.

Proof. Define function $V(t, X(t))$ such that

$$V(t, X(t)) = \lambda x_1(t) + \lambda x_2(t) + \lambda y_1(t) + \lambda y_2(t) + z(t)$$

then $V \in V_0$. We calculate the upper right derivative of $V(t,X)$ along a solution of system (1.3) and get the following impulsive differential equation

$$\begin{cases} D^+V(t)+LV(t) = k_1x_2(t) - \lambda(\beta_1 - L)x_1(t) - \lambda d_1 x_2^2(t) - \lambda a_1 x_2(t)y_2(t) + \\ k_2y_2(t) - \lambda(\beta_2 - L)y_1(t) - \lambda d_2 y_2^2(t) - a_2\lambda x_2(t)y_2(t) - (d-L)z(t), t \neq nT, \\ V(t^+) = V(t) + p \qquad t = nT \end{cases}$$

where $k_i = \lambda(L+\alpha_i), i = 1,2$. Let $0 < L < \min\{\beta_1, \beta_2, d\}$, then $D^+V(t)+LV(t)$ is bounded. Select L_0 and L_1 such that

$$\begin{cases} D^+V(t) \leq -L_0V(t) + L_1 \qquad t \neq nT \\ V(t^+) = V(t) + p \qquad t = nT \end{cases}$$

where L_0, L_1 are two positive constants.

According to Lemma 2.3, for $t > 0$, we have

$$V(t) \leq (V(0^+) - \frac{L_1}{L_0})e^{-L_0t} + \frac{P(1-e^{-nL_0T}}{e^{L_0T-1}}e^{L_0T}e^{-L_0(t-nT)} + \frac{L_1}{L_0}$$

Hence

$$\lim_{t\to\infty} V(t) \leq \frac{L_1}{L_0} + \frac{pe^{L_0T}}{e^{L_0T}-1}$$

Therefore, $V(t,x)$ is ultimately bounded, and we obtain that each positive solution of system (1.3) is uniformly ultimately bounded. This completes the proof. □

3. Extinction and permanence

In this section, we study the global attractivity of prey eradication periodic solution. Finally, we prove the permanence of the system (1.3).

Theorem 3.1. *Periodic solution $X^*(t) = (0,0,0,0,z^*(t))$ of the system (1.3) is globally attractive provided $\alpha_1 e^{-\beta_1\tau_1} < \frac{b_1 p\exp(-dT)}{1-\exp(-dT)}$ and $\alpha_2 e^{-\beta_2\tau_2} < \frac{b_2 p\exp(-dT)}{1-\exp(-dT)}$.*

Proof. Let $(x_1(t), x_2(t), y_1(t), y_2(t), z(t))$ be any solution of system (1.3). Since $\alpha_i e^{-\beta_i\tau_i} < \frac{b_i p\exp(-dT)}{1-\exp(-dT)}, i = 1,2$, we choose $\varepsilon_1 > 0$ small enough, such that $\alpha_i e^{-\beta_i\tau_i} - \frac{b_i p\exp(-dT)}{1-\exp(-dT)} + b_i\varepsilon_1 < 0, i = 1,2$.

From the fourth equation of system (1.3), we get $z(t) > \frac{p\exp(-dT)}{1-\exp(-dT)} - \varepsilon_1$.

By system (1.3), we have

$$x_2'(t) \leq \alpha_1 e^{-\beta_1\tau_1}x_2(t-\tau_1) - b_1(\frac{p\exp(-dT)}{1-\exp(-dT)} - \varepsilon_1)x_2(t) - d_1x_2^2(t)$$

$$y_2'(t) \leq \alpha_2 e^{-\beta_2\tau_2}y_2(t-\tau_2) - b_2(\frac{p\exp(-dT)}{1-\exp(-dT)} - \varepsilon_1)y_2(t) - d_2y_2^2(t)$$

Consider the following system, let $v(t)$ be the solution of

$$v_i'(t) = \alpha_i e^{-\beta_i\tau_i}v(t-\tau_i) - b_i(\frac{p\exp(-dT)}{1-\exp(-dT)} - \varepsilon_1)v_i(t) - d_iv_i^2(t), i = 1,2.$$

By lemma 2.4, we have $v_i(t) \to 0, i = 1,2$ as $t \to \infty$. From Lemma 2.1, then $x_2(t) \le v_1(t)$, $y_2(t) \le v_2(t)$ and $x_2(t) \to 0$ $y_2(t) \to 0$ as $t \to \infty$.

From

$$x_1(t) = x_1(0)e^{-\beta_1 t} + \int_{t-\tau_1}^{t} \alpha_1 x_2(\theta) e^{-\beta_1(t-\theta)} d\theta,$$

$$y_1(t) = y_1(0)e^{-\beta_2 t} + \int_{t-\tau_2}^{t} \alpha_2 x_2(\theta) e^{-\beta_2(t-\theta)} d\theta,$$

we obtain that $x_1(t) \to 0, y_1(t) \to 0$ as $t \to \infty$.

Since $\lim_{t\to\infty} x_2(t) = 0$, $\lim_{t\to\infty} y_2(t) = 0$. For any $\varepsilon_3 > 0$, such that $-d + \lambda b_1 \varepsilon_3 + \lambda b_2 \varepsilon_3 < 0$.

We have

$$-dz(t) \le z'(t) \le z(t)(-d + \lambda b_1 \varepsilon_3 + \lambda b_2 \varepsilon_3).$$

By Lemma 2.1 and Lemma 2.2, we obtain $z_1(t) \le z(t) \le z_2(t)$,$z_1(t) \to z^*(t)$, and $z_2(t) \to z_2^*(t)$ as $t \to \infty$, where $z_1(t)$ is solution of system (2.3), $z_2(t)$ is solution of

$$\begin{cases} z'(t) = z(t)(-d + \lambda b_1 \varepsilon_3 + \lambda b_2 \varepsilon_3) & t \neq nT, \\ \Delta z(t) = p \quad t = nT \\ u(0^+) = z(0^+) \ge 0 \end{cases} \tag{3.3}$$

where $z_2^*(t) = \frac{p\exp((-d+\lambda b_1\varepsilon_3+\lambda b_2\varepsilon_3)(t-nT))}{1-\exp((-d+\lambda b_1\varepsilon_3+\lambda b_2\varepsilon_3)T)}$, $nT < t \le (n+1)T$. Let $\varepsilon_3 \to 0$, we get $z(t) \to z^*(t)$ as $t \to \infty$. This completes the proof. □

Theorem 3.2. *The system (1.3) is permanent if* $d > \max\{\lambda b_1 L_1, \lambda b_2 L_2\}$,

$$\alpha_1 e^{-\beta_1\tau_1} - a_1 L_2 - b_1 \frac{p\exp((-d+\lambda b_2 L_2)T)}{1-\exp((-d+\lambda b_2 L_2)T)} > 0$$

and

$$\alpha_2 e^{-\beta_2\tau_2} - a_2 L_1 - b_2 \frac{p\exp((-d+\lambda b_1 L_1)T)}{1-\exp((-d+\lambda b_1 L_1)T)} > 0$$

where $\frac{\alpha_i e^{-\beta_i\tau_i}}{d_i} - \frac{b_i p\exp(-dT)}{d_i(1-exp(-dT))} = L_i, i = 1,2.$

Proof Suppose $X(t) = (x_1(t), x_2(t), y_1(t), y_2(t), z(t))$ is any solution of system (1.3) with $X(0) > 0$. Form Lemma 2.6, we know that the solution of system (1.3) is bounded. Note that

$$x_2'(t) \le \alpha_1 e^{-\beta_1\tau_1} x_2(t-\tau_1) - d_1 x_2^2 - b_1 \frac{p\exp(-dT)}{1-\exp(-dT)} x_2(t)$$

$$y_2'(t) \le \alpha_2 e^{-\beta_2\tau_2} x_2(t-\tau_2) - d_2 y_2^2 - b_2 \frac{p\exp(-dT)}{1-\exp(-dT)} y_2(t)$$

Considering the following comparison equations

$$\begin{cases} v_1'(t) &= \alpha_1 e^{-\beta_1\tau_1} v_1(t-\tau_1) - d_1 v_1^2 - b_1 \frac{p\exp(-dT)}{1-\exp(-dT)} v_1(t) \\ v_1(0) &= x_2(0) \end{cases}$$

$$\begin{cases} v_2'(t) &= \alpha_2 e^{-\beta_2\tau_2} v_2(t-\tau_2) - d_2 v_2^2 - b_2 \frac{p\exp(-dT)}{1-\exp(-dT)} v_2(t) \\ v(0) &= y_2(0) \end{cases}$$

we have $x_2(t) \le v_1(t)$ and $v_1(t) \to \frac{\alpha_1 e^{-\beta_1\tau_1}}{d_1} - \frac{b_1 p\exp(-dT)}{d_1(1-exp(-dT))} = L_1$, then $x_2(t) \le L_1 + \varepsilon_1, \varepsilon_1 > 0$ for t large enough. And, we have $y_2(t) \le v_2(t)$ and $v_2(t) \to \frac{\alpha_2 e^{-\beta_2\tau_2}}{d_2} - \frac{b_2 p\exp(-dT)}{d_2(1-exp(-dT))} = L_2$, then $y_2(t) \le L_2 + \varepsilon_2, \varepsilon_2 > 0$ for t large enough. Let $m_5 = \frac{p\exp(-dT)}{(1-exp(-dT))} - \varepsilon, \varepsilon > 0$. By Lemma 2.1, clearly we have $z(t) > m_5$ for all t large enough. We shall next find $\overline{m_2} > 0$ and $\overline{m_4} > 0$ such that $x_2(t) > \overline{m_2}$ and $y(t) > \overline{m_4}$ for t large enough. We will do it in the following two step:

Step1: We can select m_2, m_4 small enough such that $0 < m_2 < \frac{d-\lambda b_2(L_2+\varepsilon_2)}{\lambda b_1}$, $0 < m_4 < \frac{d-\lambda b_1(L_1+\varepsilon_1)}{\lambda b_2}$ and $\lambda b_1 m_2 + \lambda b_2 m_4 < d$. We will prove there exist $t_1, t_1' \in (0,\infty)$ such that $x_2(t_1) \ge m_2, y_2(t_1') \ge m_4$. Otherwise there will be three cases.

1) There exists a $t_2 > 0$ such that $y_2(t_1') \ge m_4$, but $x_2(t) < m_2$ for all $t > 0$.

2) There exists a $t_1 > 0$ such that $x_2(t_1) \ge m_2$, but $y_2(t) < m_4$ for all $t > 0$.

3) $x_2(t) < m_2, y_2(t) < m_4$ for all $t > 0$.

We first consider case 1). Let $\varepsilon' > 0$ small enough such that $\sigma = \alpha_1 e^{-\beta_1\tau_1} - d_1 m_2 - a_1(L_2+\varepsilon_2) - b_1(\frac{p\exp((-d+\lambda b_1 m_2+\lambda b_2(L_2+\varepsilon_2)T)}{1-\exp((-d+\lambda b_1 m_2+\lambda b_2(L_2+\varepsilon_2)T)} + \varepsilon') > 0$. According to the above assumption, we get

$$z'(t) \le z(t)(-d + \lambda b_1 m_2 + \lambda b_2(L_2+\varepsilon_2)).$$

By Lemma 2.1 and Lemma 2.2, we know $z(t) \le u_1(t)$ and $u_1(t) \to u_1^*(t)$ as $t \to \infty$, where $u_1(t)$ is the solution of

$$\begin{cases} u_1'(t) = u_1(t)(-d + \lambda b_1 m_2 + \lambda b_2(L_2+\varepsilon_2)) \qquad t \ne nT, \\ \Delta u_1(t) = p \qquad t = nT \\ u_1(0^+) = z(0^+) \ge 0 \end{cases} \tag{3.4}$$

and

$$u_1^*(t) = \frac{p\exp((-d+\lambda b_1 m_2 + \lambda b_2(L_2+\varepsilon_2))(t-nT))}{1-\exp((-d+\lambda b_1 m_2+\lambda b_2(L_2+\varepsilon_2))T)}, t \in (nT, (n+1)T].$$

Therefore, there exists a $T_1 > 0$, such that $z(t) \le u_1(t) \le u_1^*(t) + \varepsilon'$ and

$$x_2'(t) \ge \alpha_1 e^{-\beta_1\tau_1} x_2(t-\tau_1) - (d_1 m_2 + a_1(L_2+\varepsilon_2) + b_1(k+\varepsilon'))x_2$$

for $t > T_1$. Where $k = \frac{p\exp((-d+\lambda b_1 m_2+\lambda b_2(L_2+\varepsilon_2))T)}{1-\exp((-d+\lambda b_1 m_2+\lambda b_2(L_2+\varepsilon_2))T)}$. By Lemma 2.4, $x_2(t) \to \infty$ as $t \to \infty$, which is a contradiction to the boundedness of $x_2(t)$.

Case 2) can be analyzed by the same method as in case 1), so we omit it.

Next, we consider case 3). Choose $\varepsilon'' > 0$, such that

$$\alpha_1 e^{-\beta_1\tau_1} - (d_1 m_2 + a_1 m_4 + b_1(\frac{p\exp((-d+\lambda b_1 m_2+\lambda b_2 m_4)T)}{1-\exp((-d+\lambda b_1 m_2+\lambda b_2 m_4)T)} + \varepsilon'') > 0$$

and $\alpha_2 e^{-\beta_2\tau_2} - (d_2 m_4 + a_2 m_2 + b_2(\frac{p\exp((-d+\lambda b_1 m_2+\lambda b_2 m_4)T)}{1-\exp((-d+\lambda b_1 m_2+\lambda b_2 m_4)T)} + \varepsilon'') > 0$. By the assumption case 3), we have

$$z'(t) \le z(t)(-d + \lambda b_1 m_2 + \lambda b_2 m_4).$$

By Lemma 2.1 and Lemma 2.2, we know $z(t) \le u_2(t)$ and $u_2(t) \to u_2^*(t)$ as $t \to \infty$, where $u_2(t)$ is the solution of

$$\begin{cases} u_2'(t) = u_2(t)(-d + \lambda b_1 m_2 + \lambda b_2 m_4) & t \neq nT, \\ \Delta u_2(t) = p \quad t = nT \\ u_2(0^+) = z(0^+) \geq 0 \end{cases} \tag{3.5}$$

and $u_2^*(t) = \frac{p\exp((-d+\lambda b_1 m_2+\lambda b_2 m_4)(t-nT))}{1-\exp((-d+\lambda b_1 m_2+\lambda b_2 m_4))T)}, t \in (nT, (n+1)T]$. Therefore, there exists a $T_3 > 0$, such that

$$z(t) \le u_2(t) \le u_2^*(t) + \varepsilon''$$

and

$$x_2'(t) \ge \alpha_1 e^{-\beta_1 \tau_1} x_2(t-\tau_1) - (d_1 m_2 + a_1 m_4 + b_1(l + \varepsilon''))x_2(t)$$
$$y_2'(t) \ge \alpha_2 e^{-\beta_2 \tau_2} y_2(t-\tau_2) - (d_2 m_4 + a_2 m_2 + b_2(l + \varepsilon''))y_2(t)$$

for $t > T_3$, where

$$l = \frac{p\exp((-d + \lambda b_1 m_2 + \lambda b_2 m_4)T)}{1 - \exp((-d + \lambda b_1 m_2 + \lambda b_2 m_4)T)}.$$

By the same method as in above case, we have $x_2(t) \to \infty$ and $y_2(t) \to \infty$ as $t \to \infty$ which is a contradiction.

From the above three cases, we conclude that ,three exist $t_1 > 0, t_1' > 0$ such that $x_2(t_1) \ge m_2, y_2(t_1') \ge m_4$.

Step 2. If $x_2(t_1) \ge m_2$ for all $t > t_1$, then our aim is obtained. Otherwise, we consider the case that is oscillatory about m_2. Let $t_2 = \inf_{t \ge t_1}\{x_2(t) < m_2\}$. Then $x_2(t) \ge m_2$ for $t \in [t_1, t_2)$, and $x_2(t_2) = m_2$ since $x_2(t)$ is continuous. If $x_2(t) < m_2$ for $t > t_2$, then like Step1,we know that it will lead to a contradiction of boundedness of $x_2(t)$. There exists $t_3 = \inf_{t \ge t_2}\{x_2(t) < m_2\}$ and $t_2 < t_3$. Then $x_2(t) \ge m_2$ for $t \in [t_2, t_3)$, and $x_2(t_3) = m_2$. After finite times of the above process, we stop and we will complete the proof. Otherwise, we can find a time sequence $t_1 < t_2 < \cdots < t_{2k} < t_{2k+1} < \cdots$, which has the following property:

(1) $x_2(t_i) = m_2$ for $i = 2, 3, 4, \cdots$;

(2) $x_2(t) < m_2$ for $t \in (t_{2k}, t_{2k+1}), k = 1, 2, 3, \cdots$;

(3) $x_2(t) > m_2$ for $t \in (t_{2k+1}, t_{2k+2}), k = 1, 2, 3, \cdots$;

In the following, we firstly show that that there exists $T_0 > 0$ such that $\sup\{t_{2k} - t_{2k+1} k \in N\} = T_0 < +\infty$. Otherwise, there exists a subsequence $(t_{2k_i} - t_{2k_i - 1}) \to \infty, k_i \to \infty$. As in the proof of the first step, this will lead to a contradiction of the boundedness of $x_2(t)$.

Notice

$$x_2'(t) \ge (-d_1 x_2(t) - a_1 y_2(t) - b_1 z(t))x_2(t) \ge (-d_1 m_2 - a_1(L_2 + \varepsilon_2) - b_1 M))x_2(t)$$

for $t \in (t_{2k-1}, t_{2k})$. We obtain

$$x_2(t) \ge m_2 \exp(-(d_1 m_2 + a_1(L_2 + \varepsilon_2) + b_1 M)T_0) := \overline{m_2}.$$

Hence $x_2(t) \geq \overline{m_2}$ for $t > t_1$.

By the same method, we can prove that there exists $\overline{m_4}$ such that $y_2(t) > \overline{m_4}$ for $t > t_1'$. Lately, we prove that there is an $m_1 > 0$ such that $x_1(t) > m_1$ for t large enough. We define $m_1 := \alpha_1(1 - e^{-\beta_1\tau_1})m_2$. By first equation of system (1.3), we have

$$x_1'(t) \geq x_1(0)e^{-\beta_1\tau_1} + \alpha_1 \int_{t-\tau_1}^{t} x_2(u)e^{\beta_1(u-t)}du.$$

Hence, for $t_1^* > t_1 + \tau_1$, we have

$$x_1'(t) \geq x_1(0)e^{-\beta_1(t_1^*+\tau_1)} + \alpha_1(1 - e^{-\beta_1\tau_1})m_2.$$

By the same method, we prove that $y_1(t) > m_3$ for t large enough, where $m_3 := \alpha_2(1 - e^{-\beta_2\tau_2})m_4$.

By the above discussion, system (1.3) is permanent. The proof is complete. □

4. Conclusion

In this paper, we introduce a time delay and pulse into the predator-prey models with stage-structure for prey and theoretically analyze the effects of impulsive releasing natural enemy of pest for controlling the pest.

By Theorem 3.1, we know that prey-eradication periodic solution $(0, 0, 0, 0, z^*(t))$ is globally attractive if

$$p > \max\{\frac{\alpha_1 e^{-\beta_1\tau_1}(1 - \exp(-dT))}{b_1 \exp(-dT)}, \frac{\alpha_2 e^{-\beta_2\tau_2}(1 - \exp(-dT))}{b_2 \exp(-dT)}\}.$$

By Theorem 3.2, if $p < \min\{A, B\}$, where $A = \frac{(\alpha_1 e^{-\beta_1\tau_1} - a_1 L_2)(1-\exp(-d+\lambda b_2 L_2))}{b_1 \exp(-d+\lambda b_2 L_2)}$, $B = \frac{(\alpha_2 e^{-\beta_2\tau_2} - a_2 L_1)(1-\exp(-d+\lambda b_1 L_1))}{b_2 \exp(-d+\lambda b_1 L_1)}$, then system (1.3) is permanence. From the prove of Theorem 3.1 and Theorem 3.2, we can derive the following results.

If $\frac{\alpha_1 e^{-\beta_1\tau_1}(1 - \exp(-dT))}{b_1 \exp(-dT)} < p < \frac{(\alpha_2 e^{-\beta_2\tau_2} - a_2 L_1)(1 - \exp(-d + \lambda b_1 L_1))}{b_2 \exp(-d + \lambda b_1 L_1)}$,

then $x_1(t) \to 0, x_2(t) \to 0$ as $t \to \infty$ and $y_1(t), y_2(t)$ are permanent.

If $\frac{\alpha_2 e^{-\beta_2\tau_2}(1 - \exp(-dT))}{b_2 \exp(-dT)} < p < \frac{(\alpha_1 e^{-\beta_1\tau_1} - a_1 L_2)(1 - \exp(-d + \lambda b_2 L_2))}{b_1 \exp(-d + \lambda b_2 L_2)}$,

then $y_1(t) \to \infty$ and $y_2(t) \to \infty$ as $t \to \infty$ and $x_1(t), x_2(t)$ are permanent.

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