

# A Performance Modeling of Wireless Sensor Networks as a Queueing Network with On and Off Servers

Mustafa K. Mehmet Ali and Hao Gu

**Abstract:** In this work, we consider performance modeling of a wireless sensor network with a time division multiple access (TDMA) media access protocol with slot reuse. It is assumed that all the nodes are peers of each other and they have two modes of operation, active and sleep modes. We model the sensor network as a Jackson network with unreliable nodes with on and off states. Active and sleep modes of sensor nodes are modeled with on and off states of unreliable nodes. We determine the joint distribution of the sensor node queue lengths in the network. From this result, we derive the probability distribution of the number of active nodes and blocking probability of node activation. Then, we present the mean packet delay, average sleep period of a node and the network throughput. We present numerical results as well as simulation results to verify the analysis. Finally, we discuss how the derived results may be used in the design of sensor networks.

**Index Terms:** On/off, performance modeling, sensor networks, throughput, queueing networks.

## I. INTRODUCTION

A wireless sensor network (WSN) consists of a large number of tiny sensor nodes which self-organize themselves into a communication network [1]. Sensor nodes are low power small devices that integrate sensing, processing and wireless communications capabilities. WSN is expected to have a wide range of applications in health, environmental and military areas. Sensors may perform a common task such as monitoring a moving object, or environmental parameters such as humidity, temperature, etc.. The sensor network design is influenced by many factors, which include fault tolerance, scalability, operating environment, network topology and energy efficiency. Sensor nodes are usually battery-powered and recharging or replacement of batteries may not be convenient or possible due to operating conditions. Sensor nodes have limited communication ranges and the collected data needs to be forwarded to its destination through multihop communications with intermediate nodes acting as relays. Therefore, sensor nodes are both source of information as well as they take part in delivery of the information to its destination. It is expected that some sensor networks will be deployed over large and inhospitable areas and they may have very large node populations reaching to thousands [1], [2]. Since these networks may not be accessible following deployment, their proper design is very important, so that there is a properly functioning network which can accomplish its task. The perfor-

mance modeling of these networks is significant in advancing their design process.

There are two approaches for information processing in wireless sensor networks, centralized or distributed. In the centralized approach, the information collected by the sensors is forwarded to a central server, sink, for processing [2]. The central server may generate queries requesting information, to which, sensor nodes in the network respond. The main drawback of this approach is that the sensor nodes may be far away from the sink due to their limited transmission range. As a result, the energy cost of communications will be high. In the distributed approach, the nodes are peers of each other resulting in peer-to-peer sensor networks [3]–[7]. In this type of networks, queries may be generated by all the nodes in the network, thus each node may act also as a sink. The responses to the query of a node will come from its closest neighbors which can provide the requested information. It is envisioned that many sensor networks will have Internet interfaces which will provide a bridge between users and the sensor network. The user queries requesting information will be passed to the WSN through these interfaces and in the opposite direction the responses to the queries from the sensor network. In [6], a peer-tree has been developed to respond the queries efficiently in such an environment. In this paper, we assume the distributed approach for information processing, thus each node may be the source, carrier and the destination of the gathered information in the network.

The energy efficiency requires that the sensor devices are not operated continuously, and it is suggested that sensor nodes have at least two modes of operation as active and sleep modes. In active mode a node is fully operational while in the sleep mode the sensor does not take part in the network activities. In sleep mode, a sensor node will not consume energy and it may also be able to recharge its batteries. An important design problem is how to schedule the sleeping and active periods of the network nodes such that the connectivity of the network is preserved. Further, since the transmission medium is wireless, an effective MAC protocol is needed for accessing to the medium [8]. Two classes of MAC protocols have been under consideration for sensor networks, contention and scheduling based protocols. Contention based protocols are flexible in adapting to changes in topology because the bandwidth is assigned on demand and they do not require precise time synchronizations. On the other hand, the channel contention results in nondeterministic, dependent service times [9], [10]. Thus this type of protocols has the major disadvantage of inefficient energy consumption which is at a premium in sensor networks. In scheduling class, a time-division multiple access (TDMA) media access protocol with slot reuse has received attention as a possible MAC protocol [11], [12]. In a TDMA system, the frame will have enough time slots to accommodate all the nodes in the network. How-

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ever, this may not be possible in high node density sensor networks, and therefore slot reuse has been proposed to take advantage of limited transmission capabilities of the nodes as well as their alternation between active and sleep modes. Thus, the nodes within transmission range of each other need to be assigned different slots. The proposed MAC protocol consists of a fully distributed and self-organizing TDMA scheme. Several approaches have been proposed to manage access to the TDMA slots [13]. TDMA scheme has been used in self-organizing medium access control for sensor networks (SMACS) and in low-energy adaptive clustering hierarchy (LEACH) protocols [14], [15]. In LEACH, sensor nodes are organized into clusters and TDMA scheme has been used within each cluster.

The objective of this work is to present a performance analysis of a peer-to-peer wireless sensor networks with the TDMA media access protocol. It is assumed that the packet arrivals to each node are according to a Poisson process and each node serves to the arrivals according to the first-come, first-served (FCFS) service discipline. The path of a packet in the network is determined by a routing matrix. The sleep periods may consist of several exponentially distributed stages and the packet arrivals to the sleeping nodes are disabled during these periods. The sleeping nodes also relinquish their slots in the TDMA frame. We model the sensor networks with open network of queues with unreliable nodes. Unreliable nodes have two states, on and off states, respectively. The active and sleeping periods of sensor nodes are modeled as on and off periods of unreliable nodes, which allows us to make use of the results in [16]. However, the application of those results poses three problems: the sensor networks may have only a subset of the state space of the corresponding Jackson networks, wake up of a node may be blocked due to unavailability of slots in the TDMA frame and in sensor networks a customer does not receive independent service times at different nodes. Following the resolution of these problems, we determine the joint distribution of the queue lengths in a sensor network. Then, the probability distribution of the number of active nodes and blocking probability of node activation are derived. After that, we determine the mean packet delay and average sleeping period of a node. Finally, we present the average throughput of the network.

We note that as an alternative to the assignment of the slots to active sensor nodes until they go to sleep, the slots could have been dynamically assigned to the nodes. A dynamic backpressure algorithm has been proposed in [17] for the assignments of slots which minimizes the unfinished work in all the nodes in the network. This approach requires global knowledge of the queue backlogs for all the nodes in the network in real-time which is a major difficulty in distributed network environment. Further, this approach may lead to poor delay performance.

The remainder of the paper is organized as follows. In Section II, we present the system model and analysis, the section following that gives the derivation of the network performance measures. Section IV presents the numerical results regarding the analysis in the paper and discusses how these results may be used in the design of sensor networks. Finally, Section V presents the conclusions of the paper.

## II. THE SYSTEM MODEL AND ANALYSIS

### A. Modeling of Networks with Unreliable Nodes

In this section, we introduce a model of networks with unreliable nodes. We specifically consider Jackson networks with node breakdowns and repairs studied in [16]. Next, we describe the results from [16] for the special case under consideration. The model assumes a network with  $J$  nodes  $\bar{J} = \{1, 2, \dots, j, \dots, J\}$ , and the  $j$ th station (node) has a single server with FCFS service discipline. The external traffic arrives at the  $j$ th node according to an independent Poisson process with the rate  $\lambda_j$ . The service time at node  $j$  is exponentially distributed with mean  $1/\mu_j$ . All service times are assumed to be chosen independent of each other at a node. Further, the next service time of a customer is independent of its service times at the previous nodes that it has visited.

After receiving service at the node  $i$ , a customer selects node  $j$  as the next node to receive service with probability  $r(i, j)$ ; or selects to depart from the network immediately with probability  $r(i, 0)$  (we use "0" to denote the departures from the network, so  $\sum_{j=0}^J r(i, j) = 1$ ). Let us introduce the following notation,

- $n_j$ : Number of customers at node  $j$ ,  $j \in \bar{J}$ .
- $e_j$ : Total arrival rate of the customers to node  $j$ .
- $\rho_j$ : Utilization of node  $j$ .
- $\vec{n} = [n_1, n_2, \dots, n_J]$ : the joint queue length vector.
- $R = [r_{ij}]$ : Routing matrix with dimensions  $J \times J$ .
- $\bar{J}^* = \{0 \cup \bar{J}\}$ : The augmented node set that also includes node 0 which denotes departures from the network.

The traffic equation for node  $j$  is given by

$$e_j = \lambda_j + \sum_{i=1}^J e_i r(i, j), \quad j = 1, \dots, J. \quad (1)$$

The simultaneous solution of the above system of equations determines the total arrival rate of each node. We let  $\pi(\vec{n})$  denote the joint distribution of node queue lengths which is given by the product form solution below

$$\pi(\vec{n}) = C^{-1} \prod_{j=1}^J \rho_j^{n_j} \quad (2)$$

where  $\rho_j = \frac{e_j}{\mu_j}$  is the utilization of node  $j$  and for the stability of the node queue we require that  $\rho_j < 1$ .  $C$  is determined from the normalization condition,  $\sum_{\vec{n} \in E} \pi(\vec{n}) = 1$  with  $E$  being the state space of the joint queue length vector. The evaluation of this condition results in  $C = \prod_{j=1}^J \frac{1}{1-\rho_j}$ .

Next, we make the assumption that the routing matrix,  $R$ , is reversible. This means that the joint distribution of queue length vector, (2), satisfies the detailed balance equations which mean that the flows between any two nodes are equal [18]

$$e_i r(i, j) = e_j r(j, i), \quad i, j = 1 \dots J. \quad (3)$$

We note that a routing matrix is reversible if and only if its transition probabilities satisfy the Kolmogorov's reversibility criteria [18] given below

$$r(j_1, j_2) r(j_2, j_3) \cdots r(j_{n-1}, j_n) r(j_n, j_1) = r(j_1, j_n) r(j_n, j_{n-1}) \cdots r(j_3, j_2) r(j_2, j_1)$$

for any finite sequence of states  $j_1, j_2, \dots, j_n \in \bar{J}$ . Alternatively, a routing matrix that may be written as the product of a symmetric and a diagonal matrix is also reversible which allows construction of reversible routing matrices [19].

Next, we describe the node breakdown and repair processes which have been assumed to be Markovian. The nodes may be in one of two states, on or off states. A node in the on state is receiving both external and internal traffic (traffic forwarded from the other nodes). If a node is in the off state, all the arrivals to that node are blocked. The external arrivals are set to zero and the internal arrivals destined to the blocked node remain at their present location and receive repeat services. Following the repeat services, they choose new destinations according to the modified routing matrix. Let us denote the modified routing matrix as,  $\tilde{R}$ , with the corresponding routing probabilities given below

$$\tilde{r}(i, j) = \begin{cases} r(i, j), & i, j \in \bar{I}_{\text{on}}^*, i \neq j \\ r(i, i) + \sum_{k \in \bar{I}_{\text{off}}} r(i, k), & i \in \bar{I}_{\text{on}}^*, i = j \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

where  $\bar{I}_{\text{on}}$  and  $\bar{I}_{\text{off}}$  denote the set of nodes in the on and off statuses, respectively and  $\bar{I}_{\text{on}}^* = \{\bar{I}_{\text{on}} \cup 0\}$  also includes the zero node. We consider the special case that breakdown and repair of the nodes are independent of each other and it depends only on the local load. We let  $a_i(n_i)$  and  $b_i(n_i)$  denote the breakdown and repair rate of node  $i$  with local load  $n_i$  when it is in the on and off statuses, respectively. Then, the breakdown and repair intensities of the network at state  $\vec{n}$  are given by

$$A(n_i : i \in \bar{I}_{\text{on}}) = \prod_{i \in \bar{I}_{\text{on}}} a_i(n_i),$$

$$B(n_i : i \in \bar{I}_{\text{off}}) = \prod_{i \in \bar{I}_{\text{off}}} b_i(n_i).$$

Next, we assume that the nodes may breakdown only if they are empty and they have constant, identical breakdown and repair rates given by  $\alpha$  and  $\beta$ , respectively. Thus the amount of time that will elapse before an empty node may breakdown and repair time of an off node are exponentially distributed with parameters  $\alpha$  and  $\beta$ , respectively. Next letting  $|\bar{I}_{\text{on}}|$  and  $|\bar{I}_{\text{off}}|$  denote the number of nodes in sets  $\bar{I}_{\text{on}}$  and  $\bar{I}_{\text{off}}$ , respectively, then the network breakdown and repair intensities are given by

$$A(n_i : i \in \bar{I}_{\text{on}}) = \alpha^{|\bar{I}_{\text{on}}|} \prod_{i \in \bar{I}_{\text{on}}} \delta_{0n_i},$$

$$B(n_i : i \in \bar{I}_{\text{off}}) = \beta^{|\bar{I}_{\text{off}}|} \prod_{i \in \bar{I}_{\text{off}}} \delta_{0n_i} \quad (5)$$

where  $\delta_{0n_i} = 1$  for  $n_i = 0$  and 0 for otherwise.

We let  $\bar{\pi}(\bar{I}_{\text{off}}, \vec{n})$  denote the joint probability distribution of the set of off nodes and queue length vector, then, it is given by

$$\bar{\pi}(\bar{I}_{\text{off}}, \vec{n}) = H^{-1} \frac{A(n_i : i \in \bar{I}_{\text{off}})}{B(n_i : i \in \bar{I}_{\text{off}})} \prod_{j=1}^J \rho_j^{n_j} \quad (6)$$

where  $H$  is the normalization constant. We note that if  $(\bar{I}_{\text{off}} = \phi)$ , then,  $A(n_i : i \in \bar{I}_{\text{on}}) = B(n_i : i \in \bar{I}_{\text{off}}) = 1$ .  $H$  is given by

$$H = \sum_{\{\bar{I}_{\text{off}} \in \bar{J} \cap \vec{n} \in E\}} \frac{A(n_i : i \in \bar{I}_{\text{off}})}{B(n_i : i \in \bar{I}_{\text{off}})} \prod_{j=1}^J \rho_j^{n_j}$$

$$= \sum_{\bar{I}_{\text{off}} \in \bar{J}} \prod_{j=1}^J \omega_j$$

where  $\omega_j = \frac{1}{1-\rho_j}$  if  $j \in \bar{I}_{\text{off}}$  or  $\frac{\alpha_j}{\beta_j}$  if  $j \notin \bar{I}_{\text{off}}$ .

From (4), the transition probability between two on nodes remains same, on the other hand, the transition probability to/from an off node is set to zero. In [16], it is shown that the total arrival rate at an on node is independent of the number off nodes in the system. Thus, in the system with unreliable nodes the condition of reversibility, given in (3), remains valid, therefore, the modified routing matrix,  $\tilde{R}$ , is also reversible. We note that the utilization of the nodes in the on state is independent of the number of nodes in the off state.

### B. Modeling of Sensor Networks

Next, we describe how to model sensor networks with Jackson networks with on and off servers introduced in the above. Each node of the sensor network may be in either active or sleeping mode and the state of the network is determined by the combined states of all the nodes. We may model active and sleep modes with on and off states, respectively. The packet transmission corresponds to service time at a node. The external traffic at a node consists of the data collected by the sensor of that node, internal traffic corresponds to the relay traffic and departures of the packets from the network corresponds to the delivery of the information to a user. While packet lengths do not change during transfer of packets from node to node in the network, service times in a Jackson network are independent identically distributed. As usual, this approximation is justified by the Kleinrock's independence assumption [20]. As the number of nodes feeding into a node increases the approximation will improve because of better mixing of the packets. On the other hand, as the traffic load increases for a given network and traffic routing matrix the dependency among the queues increases because the packets may queue in the same order at subsequent nodes in their routes. As a result, there may be a drop in the accuracy of the independence assumption as the traffic load increases. However, sensor networks may be dense increasing the packets mixing in the queues and further the nodes in a sensor network should not be operated under heavy traffic loads in order to prolong their lifetime, therefore, the Kleinrock's independence assumption may be appropriate for sensor networks. Since in a TDMA system a node does not receive service continuously, we set the mean customer service time in equivalent Jackson network to be  $F'$ th multiple of the mean service time in the sensor network where  $F$  is the number of slots in the TDMA frame. Further, we assume that a node will not be allowed to go into sleep if the modified routing matrix is not irreducible. This condition is imposed to ensure that the connectivity of the network is not lost when there are nodes in the sleep mode. Thus, at no time, there will be any active nodes in the network with none of their routing destinations available to forward their traffic because all of them are in the sleep state.

From the above explanation, we may model a sensor network as a Jackson network with on/off servers. However, a sensor network may only have a truncated state space of a Jackson network. We assume that the transmission ranges of all the nodes are circular with radius  $d$ . The nodes within the transmission range of a node will be referred to as its neighbors. The transmission of a node will interfere with the transmissions of its neighbors. This problem may be solved by assigning nodes

within each others' transmission ranges to different time slots. Therefore, the number of active nodes within the transmission range of a node cannot exceed the number of time slots in a TDMA frame. We let  $\bar{U}_j, |\bar{U}_j|$  denote the set of active nodes within the transmission range of node  $j$  and number of nodes in this set, respectively. We will always require that  $|\bar{U}| \leq F$  meaning that  $|\bar{U}_j| \leq F$  for all nodes  $j \in \bar{J}$ , where  $F$  is the number of slots in a frame. This condition will be referred to as the feasibility criteria. As a result, only those states of Jackson's networks that satisfy the feasibility criteria will be admissible in a sensor network, these states will be referred to as feasible states. We should be able to classify each state as feasible or not and the feasibility has to be path independent. In a sensor network, the order that the nodes become active may determine whether a state is feasible or not. We will assume that if there is any order of node activation that allows a state to be feasible, then this state will be classified as feasible. In practice, this may be achieved through the reassignment of the slots to the nodes in the TDMA frame, however if this is not possible, then our results will provide an upper-bound to the performance of the target sensor network.

Thus, from the above explanation a sensor network will have a truncated state space of the corresponding Jackson network with on/off servers and all the feasible states will be reachable from each other. As discussed before, the routing matrix of such a Jackson network,  $\bar{R}$ , is reversible. Therefore, the joint distribution of the set of nodes in the sleep state and the queue length vector may be obtained from the corresponding distribution of Jackson's networks with on and off servers through normalization [18, pp.26]. We let  $\Omega$  denote the truncated network state space and  $\Lambda$  the truncated state space of the admissible sets of off nodes where  $\Omega \in E$  and  $\Lambda \in \bar{J}$ . Let us denote the joint distribution of the set of off nodes and queue length vector of a sensor network by  $\pi'(\bar{I}_{\text{off}}, \bar{n})$ , then,

$$\pi'(\bar{I}_{\text{off}}, \bar{n}) = \frac{1}{G} \bar{\pi}(\bar{I}_{\text{off}}, \bar{n}), \quad \forall \{\bar{n} \in \Omega \cap \bar{I}_{\text{off}} \in \Lambda\} \quad (7)$$

where  $G = \sum_{\{\bar{n} \in \Omega \cap \bar{I}_{\text{off}} \in \Lambda\}} \bar{\pi}(\bar{I}_{\text{off}}, \bar{n}) = \sum_{\bar{I}_{\text{off}} \in \Lambda} \prod_{j=1}^J \omega_j$  with  $\omega_j = \frac{1}{1-\rho_j}$  if  $j \notin \bar{I}_{\text{off}}$  or  $\frac{\alpha_j}{\beta_j}$  for otherwise.  $\bar{\pi}(\bar{I}_{\text{off}}, \bar{n})$  is given by (6).

We note that during active state, a node alternates between busy periods serving packets and idle periods. At the end of a busy period, a node is scheduled to begin a sleep period after an exponentially distributed idle period with parameter  $\alpha$ . If a new packet arrives during the idle period then the sleep period of the node is cancelled. On the other hand, if the duration of an idle period is less than the packet interarrival time, then, the node makes transition from active to sleep state. Thus an active period only terminates during an idle period. The shorter duration idle periods result in shorter active periods. A node becomes active again following the completion of its sleep period and since a waking up node will be idle, the beginning of a new sleep period is immediately scheduled for this node after an exponentially distributed time interval with parameter  $\alpha$ . Thus if the parameter  $\alpha$  has a large value, then, a node which has just become active may go to sleep without serving any packets. On the other hand, if  $\alpha$  has a too small value, then, the idle periods will be long and a node's sleep periods may be cancelled

frequently because of new packet arrivals. Thus larger values of parameter  $\alpha$  will result in shorter residence time in the active state and converse of this will be true for small values of  $\alpha$ . The extreme situations will not be desirable, therefore, an intermediate value of  $\alpha$  will be more appropriate. The sleep periods of nodes may consists of one or more off periods depending on whether the wake up of a node is blocked. Each off period will be exponentially distributed with parameter  $\beta$ .

### III. DERIVATION OF PERFORMANCE MEASURES

From the joint queue length distribution, we can derive several performance measures.

- *Distribution of the number of active nodes*

Let  $s$  denote the number of active nodes and  $p(s)$  its probability distribution. Then,  $p(s)$  may be determined by summing up the probabilities of the states which has the same number of active nodes,

$$\begin{aligned} p(s) &= \Pr(s = k) \\ &= \frac{1}{G} \sum_{\{\bar{I}_{\text{off}} \in \Lambda \cap |\bar{I}_{\text{off}}| = J-k \cap \bar{n} \in \Omega\}} \pi'(\bar{I}_{\text{off}}, \bar{n}). \end{aligned} \quad (8)$$

Then, the average number of active nodes in the system is given by

$$\bar{s} = \sum_{k=0}^J k * \Pr(s = k). \quad (9)$$

Let us define the following probabilities,  $\varphi_j = \Pr(\text{node } j \text{ is active})$ ,  $\bar{\varphi} = \Pr(\text{a node is active})$

Then, the average number of active nodes, given by (9), may also be expressed in terms of active node probabilities,

$$\bar{s} = \sum_{j=1}^J \varphi_j.$$

Next, we determine the probability that a node is active,

$$\bar{\varphi} = \frac{\bar{s}}{J}. \quad (10)$$

We note that  $1 - \bar{\varphi}$  gives the percentage of time that a node spends in the sleep state.

- *The wake up blocking probability of a node*

Next, we derive the probability that a sleeping node's activation will be blocked. Since the off periods are exponentially distributed, an awakening node will see the system at equilibrium like a random observer. If node  $j$  wakes up in a network state such that the number of active nodes within the transmission range of each node will not be less than or equal to  $F$ ,  $|\bar{U}| \leq F$ , then its activation will be blocked, therefore defining,  $P_j = \Pr[\text{activation of node } j \text{ will be blocked}]$

$$\begin{aligned} P_j &= \frac{1}{\sum_{\{\bar{n} \in \Omega \cap j \in \bar{I}_{\text{off}}\}} \pi'(\bar{I}_{\text{off}}, \bar{n})} \\ &\quad \sum_{\{\bar{n} \in \Omega \cap j \in \bar{I}_{\text{off}} \cap |\bar{U}| \leq F\}} \pi'(\bar{I}_{\text{off}}, \bar{n}). \end{aligned} \quad (11)$$

In the above, the denominator corresponds to the sum of the probabilities of the states that node  $j$  is in the sleep mode and the numerator to the sum of the probabilities of subset of those states that activation of node  $j$  will be blocked.

#### - Mean sleep period of a node

Next, we determine the average time that a node spends in the sleep state. The sleep period of each node may consist of a random number of exponentially distributed off periods with parameter  $\beta$ . As explained above the wake up of a node may be blocked if  $|\bar{U}| \leq F$  is not satisfied in the new state, then the node is forced to remain in the off state for another period and repeat this process at the next wake up time. Assuming that the blocking of the consecutive wake ups are independent of each other, then, the number of trials for wake up of node  $j$  will be geometrically distributed with parameter  $P_j$ . The independence assumption will hold better with longer duration off periods and shorter packet transmission times. In these two cases, the chances that the state of the system will change will increase between the end of two consecutive sleep periods. Then, the average sleep period of node  $j$  is given by the product of the mean of the geometric distribution and the mean off period,

$$\bar{\theta}_j = \frac{1}{\beta(1 - P_j)}. \quad (12)$$

We note that the mean sleep period equals to mean off period if the wake up blocking probability is zero.

#### - Mean packet delay

Next, we will determine the mean delay of a packet in the network. We let  $q(n_j)$  denote the marginal distribution of the queue length at node  $j$ , then, it may be obtained from equation (7) by summing up with respect to the queue lengths of other nodes,

$$q(n_j) = \Pr(n_j = k) = \begin{cases} \bar{\phi}_j \frac{\alpha_j}{\beta_j}, & \text{node } j \text{ is in sleep, } k = 0 \\ \bar{\phi}'_j \rho_j^{n_j}, & \text{node } j \text{ is active, } k \geq 0 \end{cases} \quad (13)$$

where

$$\bar{\phi}_j = \frac{1}{G} \sum_{\{\bar{n} \in \Omega \cap j \in \bar{I}_{\text{off}}\}} \prod_{i \neq j} \omega_i,$$

$$\bar{\phi}'_j = \frac{1}{G} \sum_{\{\bar{n} \in \Omega \cap j \notin \bar{I}_{\text{off}}\}} \prod_{i \neq j} \omega_i$$

with  $\omega_i$  as defined in (7). We note that when the queue length of a node is zero, the node may be in either sleep or active state, in the latter case the beginning of a sleep period for this node has already been scheduled. The average queue length at node  $j$  may be determined from (13),

$$\bar{n}_j = \sum_{k=0}^{\infty} k \Pr(n_j = k) = \frac{\bar{\phi}'_j \rho_j}{(1 - \rho_j)^2}. \quad (14)$$

Then, from the application of Little's result, the mean packet delay is given by

$$\bar{d} = \frac{1}{\lambda \bar{\varphi}} \sum_{j=1}^J \bar{n}_j \quad (15)$$

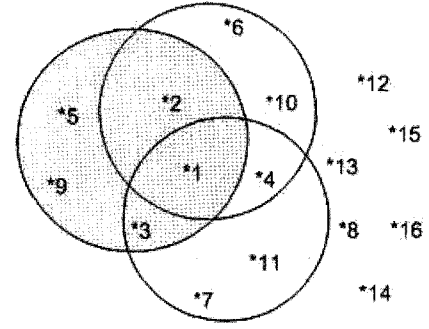


Fig. 1. The topology of a 16-node sensor network.

where  $\lambda$  is the total external arrival rate of the packets,  $\lambda = \sum_{j=1}^J \lambda_j$  and  $\lambda \bar{\varphi}$  gives the total rate of packet admission to the network.

#### - Throughput of the network

Finally, we determine the average throughput of the network. The throughput equals to the input rate of the traffic which has been accepted to the network,

$$S = \lambda \bar{\varphi} = \frac{\lambda \bar{s}}{J} \quad (16)$$

where in the last equation we have substituted for  $\bar{\varphi}$  from (10).

## IV. NUMERICAL RESULTS

In this section, we present some numerical results regarding the analysis as well as simulation results to verify the approximations in the analysis. We have studied a 16 node sensor network with the nodes numbered as  $j = 1, \dots, 16$ . We have considered two network topologies, the first topology is symmetric with uniform traffic loading, while the second topology is non-symmetric with nonuniform traffic loading. In the following, we assume that the mean service rate is constant with the value of  $\mu = 5$  packets per unit time, for the equivalent Jackson network, the service rate at the corresponding sensor network will be given by  $\mu F$ . The mean off period,  $1/\beta$ , has been set to one unit of time.

A discrete event simulation program has been written to obtain the simulation results. The simulation makes the same assumptions as the analysis. Thus arrival of the packets is according to the Poisson process and packet service times are chosen according to the exponential distribution. A packet, following service completion at a node, is forwarded according to the routing matrix and it receives the same service time in all the nodes that it visits until it departs from the network. A node goes into sleep for an exponentially distributed time if during the idle period following a busy period it does not receive any new traffic. During the sleep period, the external arrivals to that node are disabled and the internal arrivals receive repeat services and after that choose new destinations according to modified routing probabilities. The wake up of a node is blocked if all the slots in the TDMA frame are occupied and it begins a new sleep period.

**Table 1.** Connectivity matrix for the symmetric network topology, ( $g(i, j) = 1$  or  $0$  mean that node  $i$  and  $j$  have and do not have connectivity with each other, respectively).

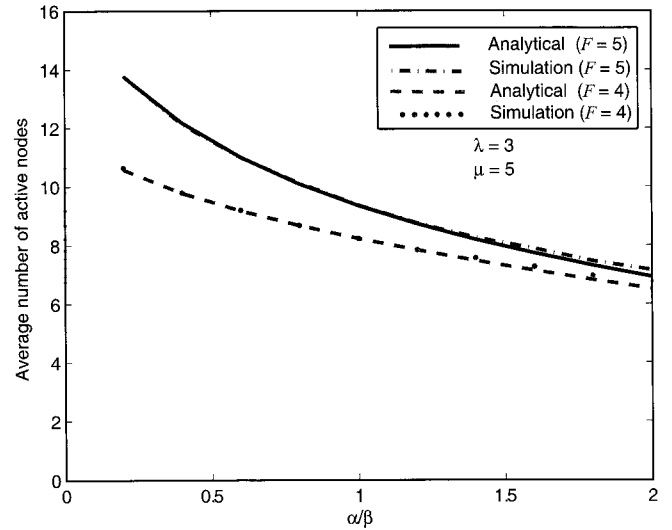
1	1	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
1	1	0	1	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
1	0	1	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0
1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0
0	1	0	0	1	1	0	1	0	0	0	0	0	0	0	0	1	0	0	0
0	0	1	0	1	0	1	1	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	1	0
1	0	0	0	0	0	0	0	0	1	1	1	0	1	0	1	0	0	0	0
0	1	0	0	0	0	0	0	0	1	1	0	1	0	1	0	1	0	0	0
0	0	1	0	0	0	0	0	0	1	1	1	0	1	0	0	1	0	0	1
0	0	0	1	0	0	0	0	0	1	1	1	0	1	0	0	0	1	0	1
0	0	0	0	1	0	0	0	1	0	0	0	1	1	0	0	1	1	1	0
0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0	1	1	0	1
0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	1	0	1	1
0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	1	0	1	1

**Table 2.** Routing matrix for the symmetric network topology, ( $h(i, j) = 1$  or  $0$  mean that node  $j$  is and is not one of the routing destinations of node  $i$ , respectively).

0	1	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0
1	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	0	0	0	0
0	1	0	0	1	0	0	1	0	0	0	0	0	0	0	0	1	0	0	0
0	0	1	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0
0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	0	0	1	1	0	1	0	1	0	0	0	0	0
0	1	0	0	0	0	0	0	1	0	0	1	0	1	0	1	0	0	0	0
0	0	1	0	0	0	0	0	1	0	0	1	0	0	1	0	0	1	0	0
0	0	0	1	0	0	0	0	1	1	0	0	0	0	0	0	1	1	0	0
0	0	0	0	1	0	0	0	1	0	0	0	0	0	1	1	0	0	0	1
0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0	1	0	0	1
0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	1	0	0	1
0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	1	1	0

**A. A Symmetric Network Topology with Equal Traffic Loading of the Nodes**

First, we present the results for the symmetric network topology with equal traffic loading of the nodes. This topology assumes that each node has equal number of neighbors and equal traffic rates. The network topology is shown in Fig. 1 and connectivity, routing matrices are given in Tables 1 and 2. In Fig. 1, we have shown only the coverage areas of the nodes 1, 2, and 3 in order to prevent crowding of the figure. Table 1 presents the connectivity matrix of the network with elements  $g(i, j), i, j = 1, \dots, J$ .  $g(i, j) = 1$  or  $0$  mean that the nodes  $i$  and  $j$  are within and outside the transmission range of each other, respectively. It has been assumed that each node has four nodes within its transmission range. Table 2 presents the routing matrix of the network with each row giving the routing destina-



**Fig. 2.** Average number of active nodes as a function of the ratio of node idle/off rates with frame size as a parameter.

tions of a node.  $h(i, j) = 1$  or  $0$  mean that the node  $j$  is and is not one of the routing destinations of node  $i$ , respectively. It has been assumed that following the service at a node a packet may be forwarded to each of its four neighbors or depart from the network with equal probabilities of 0.2. We also assume that the traffic arrival rates to all of the nodes are equal. In the following, the results are obtained for two values of frame size,  $F = 4, 5$  slots. Since the number of nodes within the transmission range of each node equal to five itself included, then, there will not be wake up blockings when the frame size equals to five.

Fig. 2 presents the average number of nodes in the active state as a function of the ratio of node idle/off rates,  $\alpha/\beta$ , for a given value of total packet arrival rate and frame size as a parameter. For a given value of  $\beta$ ,  $\alpha$  has to increase as the ratio  $\alpha/\beta$  increases as a result each node spends less time in the active state. Therefore, as the ratio  $\alpha/\beta$  increases the amount of time that nodes spend in the active state decreases. This explains why in the figure, the average number of active nodes decreases as the parameter  $\alpha/\beta$  increases for any given frame size. On the other hand, average number of active nodes is lower for the lower frame size compare to the higher one for any given value of  $\alpha/\beta$  because of wake up blockings. When normalized with the total number of nodes in the network the results of this figure gives percentage of time that a node is active which drops below 50 as the ratio  $\alpha/\beta$  increases. As may be seen, there is a close agreement between analytical and simulation results.

Fig. 3 also presents the average number of nodes in the active state as a function of the total packet arrival rate for a given value of the ratio of node idle/off rates,  $\alpha/\beta$ , with frame size as a parameter. The average number of active nodes slowly increases with the total packet arrival rate for any frame size. This is due to the nodes spending more time in the active state to serve the increasing number of packet arrivals. Again, for a given value of the total packet arrival rate, average number of active nodes is lower for lower frame size compared to the higher one because of wake up blockings. The agreement between analytical and simulation results are very good.

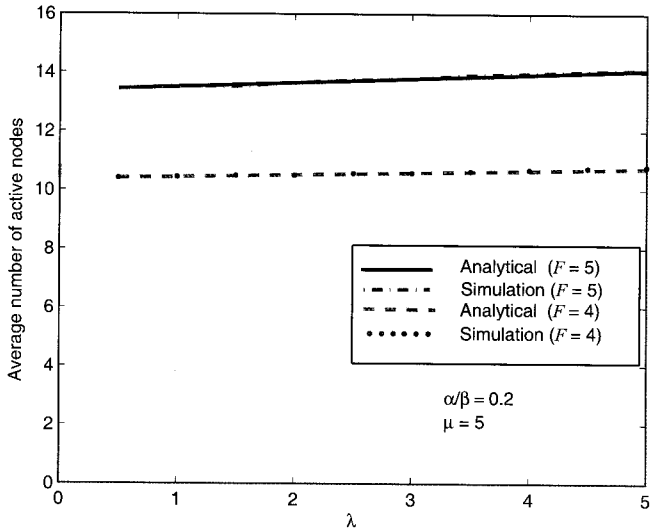


Fig. 3. Average number of active nodes as a function of total packet arrival rate with frame size as a parameter.

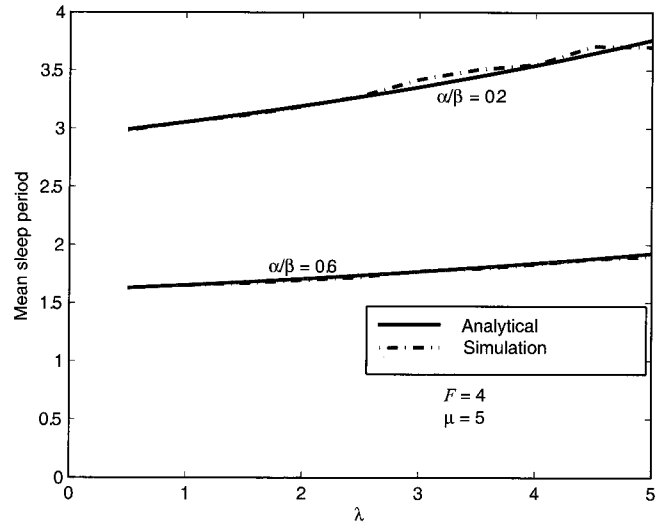


Fig. 5. Mean sleep period as a function of total packet arrival rate with the ratio of idle/off rates as a parameter.

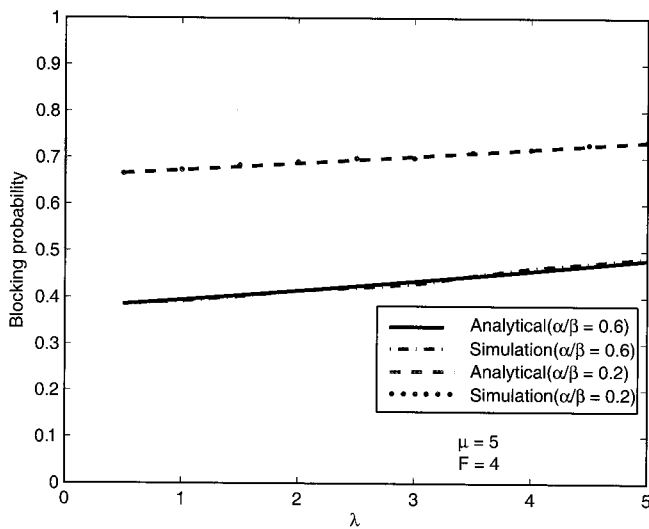


Fig. 4. The wake up blocking probability of a node as a function of total packet arrival rate with the ratio of node idle/off rates as a parameter.

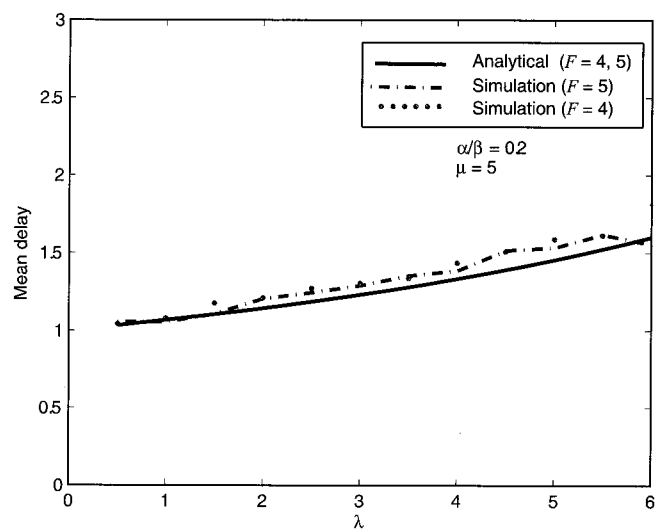


Fig. 6. Mean delay as a function of total packet arrival rate for equal loading of the nodes with frame size as a parameter.

Fig. 4 presents the wake up blocking probability as a function of the total packet arrival rate for frame size of four slots with the ratio of idle/off rates,  $\alpha/\beta$ , as a parameter. From the figure, blocking probability is lower for higher than lower value of  $\alpha/\beta$  for any given value of the total packet arrival rate. This is consistent with the results in Fig. 2, with the average number of active nodes decreasing as the value of the ratio  $\alpha/\beta$  increases. As may be seen, the wake up blocking probability slowly increases with the total packet arrival rate for a given value of  $\alpha/\beta$  because the nodes spend more time in the active state to serve the increasing packet arrivals.

Fig. 5 presents the mean sleep period as a function of the total packet arrival rate for a given value of the ratio of node idle/off rates,  $\alpha/\beta$ , for frame size of four slots. We note that a sleep period consists of a random number of consecutive off periods because of wake up blockings. As expected, the mean sleep period has a higher value than the mean off period which has been assumed to have one time unit value. The curves in Fig. 5 are

consistent with those in Fig. 4, higher blocking probability resulting in higher mean sleep period.

Fig. 6 presents the mean packet delay as a function of total packet arrival rate for  $\alpha/\beta = 0.2$  with frame size as a parameter. The mean delay increases with the total packet arrival rate. From the analytical results, it is seen that the mean delay remains same when the frame size changes because the modified routing algorithm keeps the traffic load of the active nodes constant. The agreement between the analytical and simulation results is close.

Fig. 7 presents the average throughput as a function of  $\alpha/\beta$  for a given value of total packet arrival rate with frame size as a parameter. The curves in this figure follow those in Fig. 2, because from (16) the throughput is proportional to the average number of active nodes for a given value of the total packet arrival rate. Finally, Fig. 8 presents the average throughput as a function of the total arrival rate for a given value of the ratio  $\alpha/\beta$  with frame size as a parameter. The throughput linearly in-





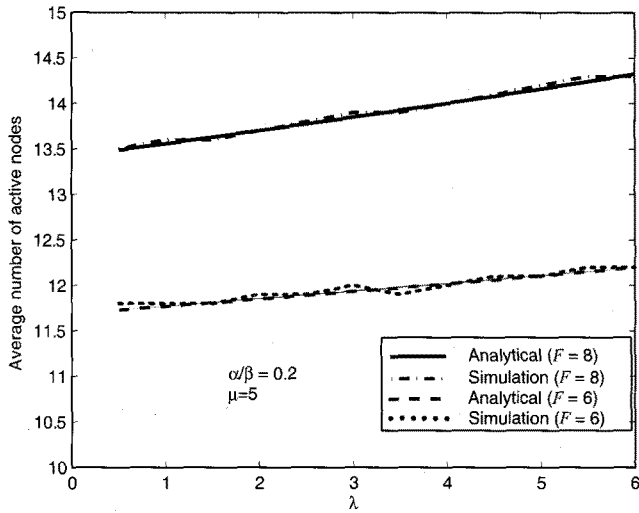


Fig. 9. Average number of active nodes as a function of total packet arrival rate for unequal loading of the nodes with frame size as a parameter.

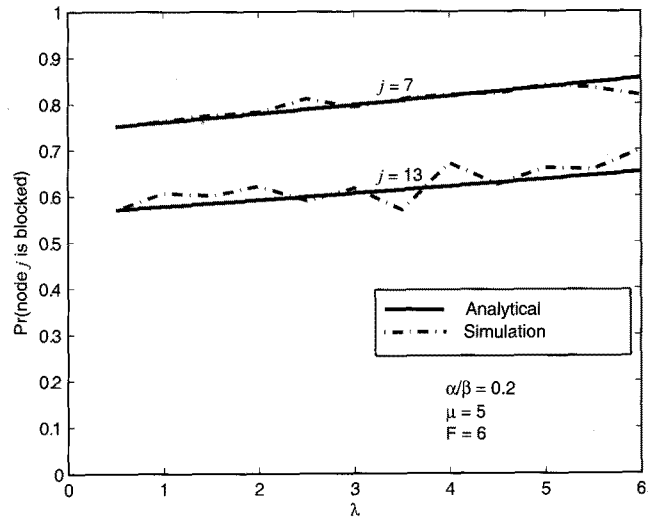


Fig. 11. The wake up blocking probability of node  $j$  as a function of total packet arrival rate with unequal loading of the nodes.

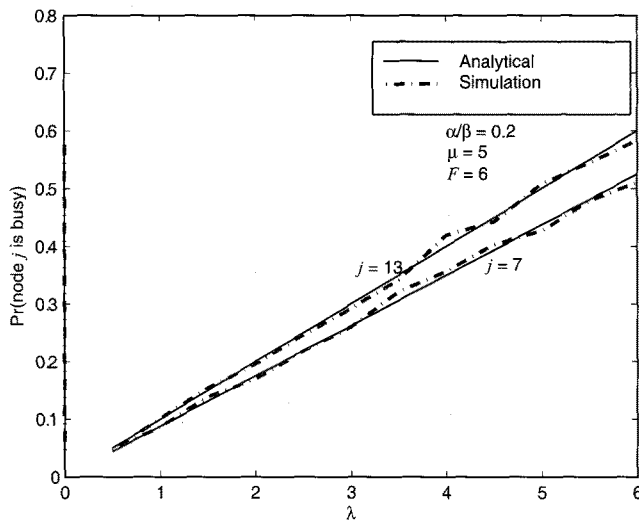


Fig. 10. Probability that node  $j$  is busy as a function of total packet arrival rate with unequal loading of the nodes.

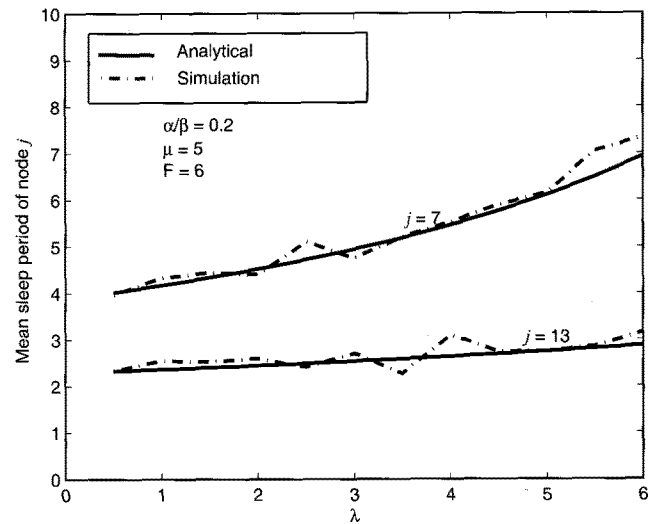


Fig. 12. Mean sleep period of node  $j$  as a function of total packet arrival rate with unequal loading of the nodes.

may be seen to increase with the total traffic arrival rate. As in the previous case, the average number of active nodes is also higher for larger frame size.

Fig. 10 presents the probability that a node is busy for the nodes  $j = 7$ , and 13 for the frame size,  $F = 6$  slots. It may be seen that the traffic loading of the two nodes are different with node 13 having the higher load. From Table 3, the nodes 7 and 13 have 7 and 8 neighbors, number of nodes within their transmission ranges, respectively. Figs. 11 and 12 show the wake up blocking probability of a node and mean sleep period for the same nodes, respectively. It may be seen that the blocking probability and mean sleep period of the node 7 is higher than node 13. The blocking probability of a node depends on the number of its neighbors as well as the traffic load of each of the neighbors. A node with higher traffic load will remain in the active state longer duration of time. Therefore, the higher number of neighbours and their traffic loads will result a node in experi-

encing higher blocking probability. Finally, Fig. 13 presents the mean packet delay for the two frame sizes. We note that the analytical results for the two frame sizes is very close, therefore, we have plotted only one of them. As may be seen the packet delay increases with the total arrival rate. There is some divergence between the analytical and simulation results at higher traffic loads, this may be due to the approximation in the independence assumption at the higher loads as discussed earlier on.

In the above, in general, the numerical and simulation results have close agreement.

### C. Application of the Results in the Design of Sensor Networks

Next, we briefly discuss how the results of the paper may be used in the design of sensor networks. The main design parameters of the network are average sleep period, throughput, delay and the TDMA frame size in number of slots. Clearly, we would like to keep the nodes in sleep mode as much as possible in order to conserve energy. From Figs. 2 and 7, both the average

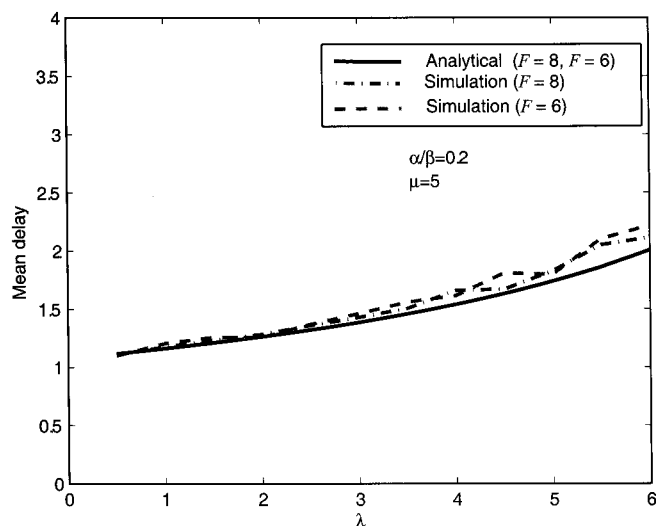


Fig. 13. Mean delay as a function of total packet arrival rate for unequal loading of the nodes with frame size as a parameter.

number of active nodes and the throughput decreases as nodes idle/off ratio increases. As the average number of active nodes decreases a node spends more time in sleep state. The presented results will enable the choice of appropriate value of the ratio which balances the throughput and the sleep period. From Figs. 3 and 8, for a given choice of the value of the ratio of idle/off rates, both the average number of active nodes and the throughput increase with the increase of traffic load. However, the increasing value of the average number of nodes means that each node spends less time in sleeping state. Further, mean packet delay increases with the increasing traffic load. Thus the twin goals of keeping energy consumption and packet delay down determine the appropriate traffic load. Finally, as the frame size is reduced the nodes spend more time in the sleep state and average throughput is reduced, therefore, in practice, it will be desirable to use the largest frame size possible to prevent wake up blockings.

## V. CONCLUSIONS

In this paper, we have provided a performance modeling of a TDMA scheme with slot reuse for peer-to-peer sensor networks. It has been shown that this type of networks may be modeled as a queueing network with unreliable nodes. The sleep periods of sensors have been modeled as node breakdowns. Then, we have derived main network performance measures such as average number of active nodes, probability of wake up blocking, mean delay and throughput. We presented numerical results which have close agreement with simulation results. Finally, we have demonstrated how the results may be used in the design of sensor networks.

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