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Poisson Effect on Electromechanical Impedance of Unconstrained Piezoelectric Patch

Sung Woo Shin* and Oh-Heon Kwon

Division of Safety Engineering, Pukyong National University, Busan 608-739, Korea

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Abstract : In this study, the Poisson effect on resonant frequency behaviors of the unconstrained piezoelectric patch is investigated. The electromechanical impedance models for the un-bonded patch are derived from the two existing bonded patch models and numerical analysis for a given piezoelectric material is performed. From the analysis, it is found that the Poisson effect is not important as long as the electromechanical impedance model is used to predict the locations of resonant frequencies. However, Poisson effect should be considered when predicting the location of the largest resonant frequency of the patch since the amplitude responses are different with the model used.

Key words: poisson effect, piezoelectric patch, electromechanical impedance

1. Introduction

Recent advances in smart materials such as piezoelectric materials and shape memory alloy have added new dimensions of structural health monitoring technology. Especially electromechanical impedance (EMI) sensing method utilizing piezoelectric materials has emerged as a promising damage detection technique for structural health monitoring of civil infrastructures [1]. The working principle of the EMI method to detect damages in structures is very simple. In EMI method, a piezoceramic (PZT) patch is bonded on to the surface of the monitored structure and excited by an alternating voltage sweep signal, typically in the kilohertz range, by an impedance analyzer. Then, an alternating voltage sweep signal allows to vibrate the piezo-patch due to its electromechanical coupling property [2]. Then, the vibrating patch transfers its vibrations to the host structure and simultaneously, the structure influences the electrical circuit comprising the bonded patch and the ac source. As a result, electrical responses (such as electrical impedance) of the bonded patch inherently reflect the vibration property (such as mechanical impedance) of the host structure [3]. Which means if mechanical property of the host structure is changed due to damages, the electrical impedance of the patch bonded on to the host structure is changed.

*Corresponding author: shinsw@pknu.ac.kr

Finally, damages in the structure can be detected by comparing the damaged and undamaged responses of the bonded patch.

On the other hand, since a sweep of alternating voltage is applied on the bonded patch, the impedance responses of the patch are frequency dependent and which is characterized by both the mechanical and electrical properties of the patch and the host structure. This suggests that the sensitivity of the impedance response to the damage is different with a frequency range to be compared [4] and is very important when the EMI method is applied for damage detection of the structure. For example, a false-alarm may occur when a selected frequency range has low sensitivity and which may lead to fail in the detection of damages though it is damaged. One possible solution to alleviate this problem is to select a frequency range with high sensitivity. With this regard, a range possessing resonant frequency of the patch is seemed to be favorable since high output can be obtained in a resonant frequency. To identify a resonant frequency of the patch, a relevant model is needed for the un-bonded patch. Many models have been proposed by the previous researchers [5]. Among them, two models proposed by Liang et al. [6] and

Zhou et al. [7] have been widely used to predict the electromechanical impedance of the bonded patch. The main difference between two models is whether the Poisson effect is considered or not. In this study, the Poisson effect on resonant frequency behaviors of the unconstrained (ie., unbonded) piezoelectric patch is investigated and the relevance of the two models is discussed. To this end, the electromechanical impedance models for the un-bonded patch are derived from the two existing bonded patch models and numerical analysis for a given piezoelectric material is performed.

2. Electromechanical Impedance Models for Unbonded Piezoelectric Patch

Piezoelectric materials such as lead-ziconate-titanate (PZT) generate electric dipoles when those are subjected to mechanical loading and they also undergo mechanical deformations when subjected to electric fields. The constitutive relations for piezoelectric materials under small field condition are [2]

$$\begin{bmatrix} D \\ S \end{bmatrix} = \begin{bmatrix} \overline{e}^T & d^d \\ d^c & \overline{s}^E \end{bmatrix} \begin{bmatrix} E \\ T \end{bmatrix} \tag{1}$$

where $\mathbf{D}(3\times1)$ = electric displacement vector (C/m^2), $\mathbf{S}$ (6×1) = mechanical strain vector, $\mathbf{E}(3\times1)$ = applied external electric field vector (V/m), and $\mathbf{T}(6\times1)$ = mechanical stress vector (N/m^2). $\overline{e}^T_{ij} = e^T_{ij}(1 - \eta j)(3 \times 3)$ = constant dielectric permittivity matrix at constant stress field; $d^d_{im}(6 \times 3)$ and $d^c_{jk}(6 \times 3)$ = matrices of the piezoelectric strain coefficients; and $\overline{s}^E_{km} = s^E_{km}(1 + \delta j)$ (6×6) = the complex elastic compliance matrix at constant electric field. η and δ are the dielectric and the mechanical loss factors, respectively. The superscripts T and E indicate that the quantity has been measured at constant stress and constant electric field, respectively. The firs subscript denotes the direction of the electric field and the second the direction of the associated mechanical strain.

Liang et al. [6] proposed complex admittance (Y: inverse of impedance) equation of the bonded PZT patch which is assumed to be thin rectangular bar (length l, width w, and thickness $h \ll l, w$) undergoing axial vibrations under a harmonic electric field E_3.

$$Y = G + Bj = \omega j \frac{wl}{h}\left[(\overline{e}^T_{33} - d^2_{31}\overline{Y}^E) + \left(\frac{Z^p}{Z^s + Z^p}\right) d^2_{31}\overline{Y}^E \frac{\tan(\kappa l)}{\kappa l}\right] \tag{2}$$

where G = electrical conductance, B = electrical susceptance, ω = angular frequency, $\overline{Y}^E$ = complex Young's modulus of the patch at constant electric field, Z^p = mechanical impedance of the patch, Z^s = mechanical impedance of the host structure, κ = wave number $(= \omega\sqrt{\rho/\overline{Y}^E})$, ρ = density of the patch. As shown in equation (2), the PZT patch bonded to the structure couples the mechanical impedance to the electrical admittance. It can be inferred from this equation that if the mechanical property of the PZT patch does not change over the monitoring period, then the electrical admittance of the PZT-structure interaction system solely depends on the mechanical impedance of the host structure (Z_s). Therefore, any change of the mechanical property in the host structure causes changes in the mechanical impedance, and also induces changes in the electrical admittance of the patch bonded to the structure. The only limitation of this model is that it ignores the lateral motion of the thin bar. To overcome this limitation, Zhou et al. [7] proposed admittance equation of the bonded PZT patch (dimension of the patch is same with Liang model) considering Poisson effect of the thin bar.

$$Y = G + Bj = \omega j \frac{wl}{h}\left[\left(\overline{e}^T_{33} \frac{2d^2_{31}\overline{Y}^E}{(1-\nu)}\right) + \frac{d^2_{31}\overline{Y}^E}{(1-\nu)}\left\{\frac{\sin\kappa l}{l} \frac{\sin\kappa w}{w}\right\} \mathbf{N}^{-1}\begin{Bmatrix} 1 \\ 1 \end{Bmatrix}\right] \tag{3}$$

where ν = Poisson's ratio of the patch, κ = 2-D wave number $(= \omega\sqrt{\rho(1-\nu^2)/\overline{Y}^E})$, and $\mathbf{N}$=(2×2) matrix, given by

$$\mathbf{N} = \begin{bmatrix} \kappa\cos\kappa l\left(1 - \nu\frac{w}{l}\frac{Z^s_{xy}}{Z^p_{xx}} + \frac{Z^s_{xx}}{Z^p_{xx}}\right) & \kappa\cos\kappa l\left(\frac{l}{w}\frac{Z^s_{yx}}{Z^p_{yy}} - \nu\frac{Z^s_{yy}}{Z^p_{yy}}\right) \\ \kappa\cos\kappa l\left(\frac{w}{l}\frac{Z^s_{xy}}{Z^p_{xx}} - \nu\frac{Z^s_{xx}}{Z^p_{xx}}\right) & \kappa\cos\kappa l\left(1 - \nu\frac{l}{w}\frac{Z^s_{yx}}{Z^p_{yy}} + \frac{Z^s_{yy}}{Z^p_{yy}}\right) \end{bmatrix} \tag{4}$$

where the superscripts s and p denote host structure and piezo-patch, respectively and the subscript x and y are two principal directions coinciding with longitudinal (length) and lateral (width) directions, respectively.

Both equations (2) and (3) are the models for the bonded patch. The main purpose of this study is to investigate the behaviors of the resonant frequency of the unbonded patch considering Poisson effect. Therefore, the unbonded models with and without considering Poisson effect should be derived and which can be obtained when the impedances of the host structure in equations (2) and (3) are made to go zero ($Z^s \to 0$). Then, the equations (2) and (3) are reduced to equations

(5-1) and (5-2), respectively.

$$Y_{free,Liang} = G_{free} + B_{free}j = \omega j \frac{wl}{h}\left[\overline{e^T_{33}} + d^2_{31}\overline{Y^E}\left(\frac{\tan(\kappa l)}{\kappa l} - 1\right)\right] \quad (5\text{-}1)$$

$$Y_{free,Zhou} = G_{free} + B_{free}j = \omega j \frac{wl}{h}$$
$$\left[\overline{e^T_{33}} + \frac{d^2_{31}\overline{Y^E}}{(1-\nu)}\left(\frac{\tan \kappa l}{\kappa l} + \frac{\tan \kappa w}{\kappa w} - 2\right)\right] \quad (5\text{-}2)$$

Equation (5-1) is the EMI model for unbonded patch without considering Poission effect while equation (5-2) is not. It can be inferred from comparing equation (5-1) with (5-2) that when both the Poisson's ratio and the width of the patch are close to zero, then the equation (5-2) is reduced to equation (5-1).

3. Numerical Investigation

In this chapter, Poisson effect on the electromechanical impedance of the unbonded patch is numerically investigated using equations (5-1) and (5-2). To this end, a commercially available PIC 151 patch (a product of PI Ceramic Inc.) is used for analysis [8]. Material properties of PIC 151 are shown in Table 1. Three sizes of the patch in analysis are shown in Table 2.

Calculated electromechanical impedances of the unbonded patch with various aspect ratios are shown in Figures 1 (a) – (e). It is observed that the magnitudes of the resonant frequencies for the patch without considering Poisson effect are lower than the same patch with considering Poisson effect for all cases. Distinct behaviors of the resonant frequencies are observed when the aspect ratios are different. For the cases 3 (aspect ratio - 1:3) and 5 (aspect ratio 1:5), the first resonant frequencies are observed around 12 kHz and 8 kHz when Poisson effect is considered. However, for the cases 2 (aspect ratio - 3:1) and 4 (aspect ratio 5:1), the first resonant frequencies are not appeared when Poisson effect is not considered, while the considered models capture the first resonant frequencies at the same locations with the cases 3 and 5. This distinct behavior is contributed by $\frac{\tan \kappa w}{\kappa w}$ term in equation (5). This result suggests that if the aspect ratio is changed only in length (longitudinal direction), both models can predict the resonant modes relating longitudinal direc-

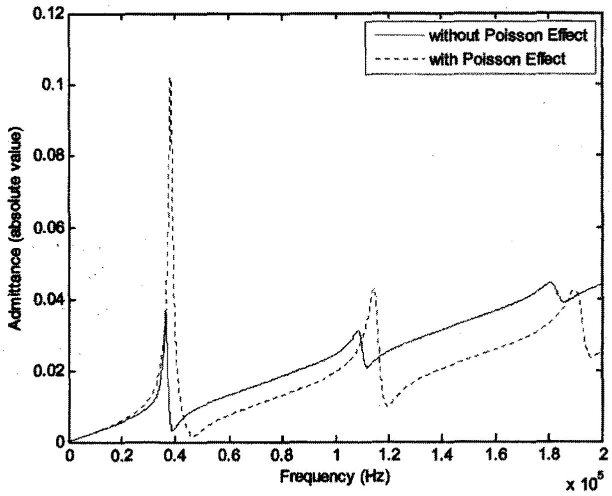
Table 1. Material Properties of PZT Patch

Physical Parameters	Values*
Density, ρ (kg/m^3)	7800
Electric Permittivity, e^T_{33} $(Farad/m)$	2.0796×10^{-8}
Piezoelectric strain coefficient, d_{31} (m/V)	-2.10×10^{-10}
Young's modulus, Y^E (N/m^2)	6.67×10^{10}
Poisson's ratio, ν	0.3
Mechanical loss factor, η	0.0325
Dielectric loss factor, δ	0.016335

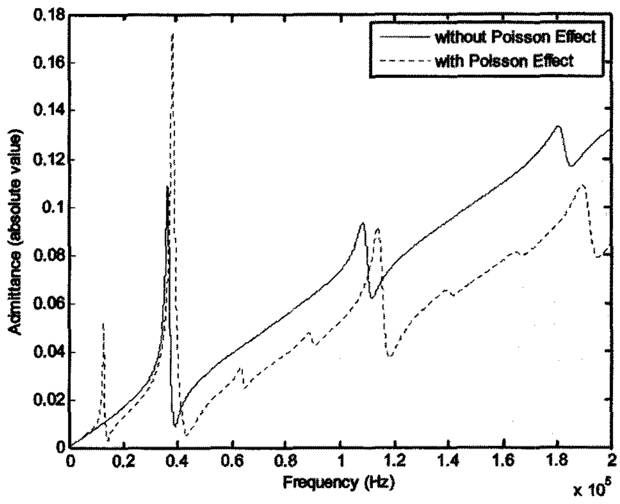
* Provided by the manufacturer[9]

Table 2. Dimensions of PZT Patch with Analysis Cases

Analysis Cases	Dimensions (mm)			Aspect Ratios
	Length	Width	Thickness	
Case 1	20	20	0.02	1:1
Case 2	20	60	0.02	1:3
Case 3	60	20	0.02	3:1
Case 4	20	100	0.02	1:5
Case 5	100	20	0.02	5:1

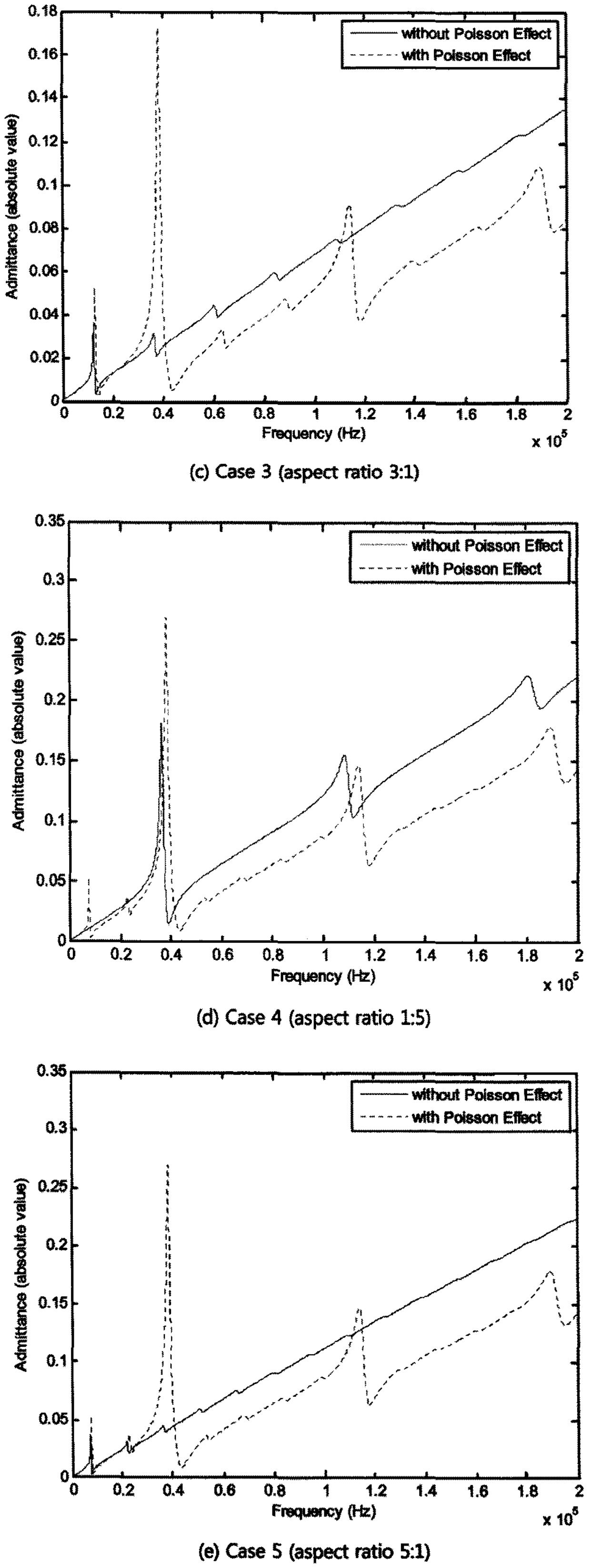


(a) Case 1 (aspect ratio 1:1)



(b) Case 2 (aspect ratio 1:3)

tion because of both models have $\frac{\tan\kappa l}{\kappa l}$ terms. However, the model without considering Poisson effect does not have $\frac{\tan\kappa w}{\kappa w}$ term in equation, the resonant modes relating longitudinal direction cannot be predicted and which may lead wrong predictions of the resonant modes of the patch. On the other hand, very interesting results are observed in the cases 3 and 5. In those cases, although the magnitudes are different, the locations of the resonant frequencies for both models are getting matched well as the aspect ratio increases. A possible explanation for this behavior is as follows. In elastic theory of solids, Poisson effect may be ignored when the lateral deformation is very small in comparison with axial deformation. In other words, when either the aspect-ratio (in this study length to width ratio) is very large, a contribution of lateral motion due to the

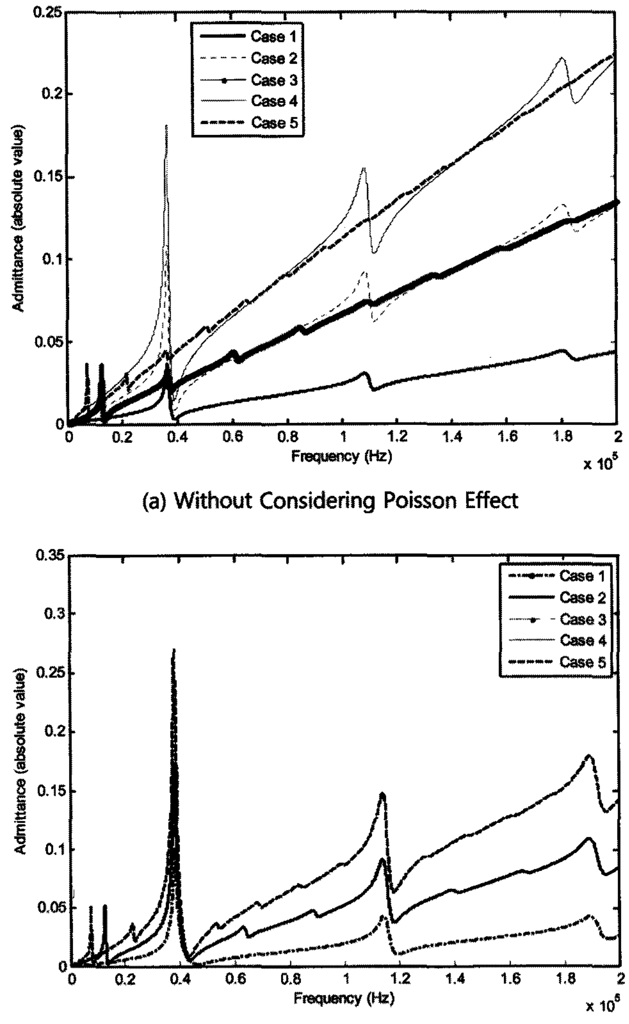


(c) Case 3 (aspect ratio 3:1)

(d) Case 4 (aspect ratio 1:5)

(e) Case 5 (aspect ratio 5:1)

Fig. 1. Admittance (Absolute Value) of Unbonded PZT Patches.



(a) Without Considering Poisson Effect

(b) With Considering Poisson Effect

Fig. 2. All Calculated Admittances (Absolute Value) of Unbonded PZT Patches.

Poisson effect is very small comparing with that of longitudinal motion. Therefore, as long as a model is used to predict the location of resonant frequencies, Poisson effect is not important when the size for the length-wise is larger than that of width-wise. However, if a goal of the prediction is to find a resonant frequency having largest magnitude, then the Poisson effect plays significant role. This suggestion can be confirmed by figures 2 (a) and (b). Figure 2 (a) shows all admittances for the model without considering Poisson effect, while Figure 2 (b) shows those for the considered model. It is observed that the locations of the largest resonant frequencies are around 40 kHz for the considered model. However, the largest resonant frequencies occur in different locations for the model without considering Poisson effect. This observation confirms that Poisson effect should be considered when predicting the largest resonant frequency of the patch.

4. Conclusions

In this study, the Poisson effect on resonant frequency behaviors of the unconstrained (ie., unbounded) piezoelectric patch is investigated and the relevance of the two models is discussed. The electromechanical impedance models for the un-bonded patch are derived from the two existing bonded patch models and numerical analysis for a given piezoelectric material is performed. From the results of the numerical analysis, investigation, Poisson effect is not important as long as the electromechanical impedance model is used to predict the locations of resonant frequencies. However, Poisson effect should be considered when predicting the location of the largest resonant frequency of the patch since the amplitude responses are different with the model used.

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