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Aerial Triangulation with 3D Linear Features and Arc-Length Parameterization

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Abstract

Point-based methods with experienced human operators are processed well in traditional photogrammetric activities but not the autonomous environment of digital photogrammetry. To develop more robust and accurate techniques, higher level objects of straight linear features accommodating element other than points are adopted instead of points in aerial triangulation. Even though recent advanced algorithms provide accurate and reliable linear feature extraction, extracting linear features is more difficult than extracting a discrete set of points which can consist of any form of curves. Control points which are the initial input data and break points which are end points of piecewise curves are easily obtained with manual digitizing, edge operators or interest operators. Employing high level features increase the feasibility of geometric information and provide the analytical and suitable solution for the advanced computer technology.

Keywords : Aerial triangulation, Linear features, Line-photogrammetry, Arc-length, 3D natural cubic splines

要 旨

기존의 도화사 수작업에 의한 기준점기반 항공삼각측량 기법은 전통적인 사진측량에서 유용하게 이용되어 왔으나, 자동화된 수치사진측량 기법이 확산되면서 기존방법의 문제점이 대두되었다. 따라서 본 논문은 보다 발전된 자동 화 기술 개발을 위하여 상위 레벨인 선형객체들을 이용한 항공삼각측량 가능성을 제시하고 검증하고자 하였다. 최 근에 발달된 선형 객체 추출기법 알고리즘은 보다 정확한 선형 객체 추출기법을 제공하였지만, 포인트 추출기법에 비하여 추출하기 어려운 단점이 존재한다. 따라서 본 논문은 이를 극복하기 위하여 수동 디지타이징이나 에지 연 산자를 통하여 쉽게 획득되는 포인트를 이용하여 상위레벨 선형객체를 생성하고, 이를 통해 지형공간정보 이용 가 능성을 높이고자 하였다. 본 연구 결과를 통해 진화하고 있는 컴퓨터 환경에 적합한 선형객체를 이용한 항공삼각 측량 기법을 발전시킬 것으로 기대된다.

핵심용어 : 항공삼각측량, 선형 객체, 선형 사진측량, Arc-length, 3D natural cubic splines

1. Introduction

A number of researchers have published studies of the automatic feature extraction and its application for various photogrammetric tasks, Ackerman and Tsingas(1994), Ebner and Ohlhof(1994), Haala and Vosselman(1992), Hannah(1989), Schenk(1998), Schenk and Toth(1993), Schenk et al.(1993), Schickler (1992). However, point-based photogrammetry based on the manual measurement and identification of interest points is not established successfully in the autonomous environment of digital photogrammetry but the labor-intensive interpretation since it has the limitation of occlusion, ambiguity and semantic information in view of robust and suitable automation. In addition, many researchers have investigated the feasibility of linear features to autonomous photogrammetry(Habib et al.(2000), Mikhail(1993), Zalmanson and Schenk(2007)), but the modeling of linear features are limited into straight features and conic sections. In this work, the integrated model of the extended collinearity equation utilizing 3D natural cubic spline and arc-length parameterization is derived to recover 3D natural cubic spline parameters and spline location parameters.

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- 2. Feature-based aerial triangulation with 3D natural cubic splines
- 2.1 Extended collinearity equation model for splines.

The correspondence between the 3D curve in the object space coordinate system and its projected 2D curve in the image coordinate system is implemented using a natural cubic spline accommodating curve feature because of its boundary conditions the zero second derivatives at the end points. A natural cubic spline is composed of a sequence of cubic polynomial segments. The collinearity equations are the commonly used condition equations for relative orientation, the space intersection which calculates a point location in the object space using projection ray intersection from two or more images and the space resection which determines the coordinates of a point on an image and EOPs with respect to the object space coordinate system. The space intersection and the space resection are the fundamental operations in photogrammetry for further processes such as triangulation. The basic concept of the collinearity equation is that all points on the image, a perspective center and the corresponding point in the object space are on a straight line. The relationship between the image coordinate system and the object coordinate system is expressed by three position parameters and three orientation parameters.

The collinearity equations play an important role in photogrammetry since each control point in the object space produces two collinearity equations for every photograph in which the point appears. If *m* points appear in *n* images, then $2 \times m \times n$ collinearity equations can be employed in aerial triangulation. The extended collinearity equations relating to a natural cubic spline in object space with ground coordinates $(X_i(t), Y_i(t), Z_i(t))$ into image space with photo coordinates (x_{pi}, y_{pi}) are described in equation 1. A natural cubic spline allows the utilization of the collinearity model for expressing curve parameters.

$$x_{\pi} = -f \frac{(X_i(t) - X_c)r_{11} + (Y_i(t) - Y_c)r_{12} + (Z_i(t) - Z_c)r_{13}}{(X_i(t) - X_c)r_{31} + (Y_i(t) - Y_c)r_{32} + (Z_i(t) - Z_c)r_{33_c}}$$

$$\begin{split} y_{\pi} &= -f \frac{(X_i(t) - X_c)r_{21} + (Y_i(t) - Y_c)r_{22} + (Z_i(t) - Z_c)r_{23}}{(X_i(t) - X_c)r_{31} + (Y_i(t) - Y_c)r_{32} + (Z_i(t) - Z_c)r_{33}} \\ where, f: \text{focal length} \\ x_{pi}, y_{pi} : \text{photo coordinates of } i\text{th segment} \\ r_{ij}: \text{ the element of } 3\text{D orthogonal rotation} \\ matirx \ R^{\ T}(\omega, \phi, \kappa) \\ X_c, \ Y_c, \ Z_c: \text{ camera perspective center} \\ a, b, c: \text{ spline parameters} \\ X_i(t) &= a_{i0} + a_{i1}t + a_{i2}t^2 + a_{i3}t^3, \ t \in [0,1] \\ Y_i(t) &= b_{i0} + b_{i1}t + b_{i2}t^2 + b_{i3}t^3, \ t \in [0,1] \end{split}$$

 $Z_i(t) = c_{i0} + c_{i1}t + c_{i2}t^2 + c_{i3}t^3, t \in [0,1]$

To recover the 3D natural cubic spline parameters in aerial triangulation, a non-linear mathematical model of the extended collinearity equation is differentiated. The models of exterior orientation recovery are classified into linear and non-liner methods. While linear methods decrease the computation load, accuracy and robustness of linear algorithms are not excellent. Otherwise non-linear methods are more accurate and robust. However, non-linear methods require initial estimates and increase the computational complexity. The relationship between a point in the image space and a corresponding point in the object space is established by the extended collinearity equation. Prior knowledge about correspondences between individual points in the 3D object space and their projected features in the 2D image space is not required in extended collinearity equations with 3D natural cubic splines.



Fig. 1. The projection of a point on a spline

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2.2 Arc-length parameterization

The assumption in aerial triangulation by the Gauss-Markov model is that all estimated parameters are uncorrelated. Hence design matrix of aerial triangulation must be of full rank, the non-singular normal matrix. However, since spline parameters are not independent to spline location parameters, additional observations are required to obtain the estimations of parameters. In point-based approaches the point location relationship between image and object space is established to determine pose estimation including positions and orientations of a camera, which is a fundamental application in photogrammetry, the remote sensing and the computer vision. In this research, the arc-length parameterization is applied as an additional condition equation to solve the rank deficient problem in the extended collinearity equations using 3D natural cubic splines. The concept of differentiable parameterization is that the arc-length of a curve can be divided into tiny pieces and then the pieces can be added up so the length of each piece will be approximately linear. The sum of the squares of derivatives is the same with a velocity since a parametric curve can be considered as the point trajectory. A velocity vector describes the path of a curve and moving characteristics. If the particle on a curve moves at a constant rate, the curve is parameterized by the arc-length. While the extended collinearity equation provides the only piece of information, curves contain additional geometric constraints such as the arc-length, the tangent of location, and the curvature, which are supportive to space resection under the assumption of measuring proper additional independent observations both the image space and the object space. The arc-length in the image space is calculated by a geometric integration of the construction from the differentiable parameterization of the photo coordinates from a spline in the object space as following equations 2.

Since the problem of the arc-length parameterization of splines does not have an analytical solution, several numerical approximations of reparameterization techniques for splines or other curve representations have been developed.

$$\begin{split} Arc(t) &= \int \sqrt{(x'_{p}(t))^{2} + (y'_{p}(t))^{2}} \, dt \\ &= \int \sqrt{\left\{ \left(-f \frac{u(t)}{w(t)} \right)^{2} + \left\{ \left(-f \frac{v(t)}{w(t)} \right)^{2} \right\}^{2}} \, dt \\ &= \int \sqrt{\left(-f \frac{u'(t)w(t) - u(t)w'(t)}{w^{2}(t)} \right)^{2}} \, dt \\ &= \frac{\int \sqrt{\left(-f \frac{v'(t)w(t) - v(t)w'(t)}{w^{2}(t)} \right)^{2}} \, dt \end{split}$$

where,

a, b, c: spline parameters

f: focal length

 x_p, y_p : photo coordinates

 $R^{T}(\omega,\phi,\kappa)$: 3D orthogonal rotation

 X_c, Y_c, Z_c : camera perspective center

$$\begin{bmatrix} u(t) \\ v(t) \\ w(t) \end{bmatrix} = R^{T}(\omega,\phi,\kappa) \begin{vmatrix} a_{0} + a_{1}t + a_{2}t^{2} + a_{3}t^{3} - X_{c} \\ b_{0} + b_{1}t + b_{2}t^{2} + b_{3}t^{3} - Y_{c} \\ c_{0} + c_{1}t + c_{2}t^{2} + c_{3}t^{3} - Z_{c} \end{vmatrix} \\ \begin{bmatrix} u'(t) \\ v'(t) \\ w'(t) \end{bmatrix} = R^{T}(\omega,\phi,\kappa) \begin{bmatrix} a_{1} + 2a_{2}t + 3a_{3}t^{2} \\ b_{1} + 2b_{2}t + 3b_{3}t^{2} \\ c_{1} + 2c_{2}t + 3c_{3}t^{2} \end{bmatrix}$$
(2)

Arc-length parameterization of 3D natural cubic splines using Simpson's rule is developed to solve over-parameterization of 3D natural cubic splines. Additional equation to the extended collinearity equation expands bundle block adjustment from limited conditions such as straight lines or conic sections(circles, ellipses, parabolas and hyperbolas) to general cases.

3. Experiments

Actual experiments with real data are implemented to verify the feasibility of the proposed aerial triangulation algorithm using 3D natural cubic splines for the recovery of spline parameters. The line photogrammetric

bundle adjustment in this research aims at the estimation of 3D natural cubic spline parameters using correspondence between splines in the object space and spline observations of multiple images in the image space. Nonlinear functions of orientation parameters, spline parameters and spline location parameters are represented by the extended collinearity equations and the arc-length parameterization equations. Five

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observation equations are produced by each two points, which are four extended collinearity equations and one arc-length parameterization equation. Integrated model provides not only the recovery of image orientation parameters but also surface reconstruction using 3D curves. Of course since the equation system of the integrated model has datum defects of seven, the control information about the coordinate system is required to obtain parameters. This is a step toward higher level vision tasks such as object recognition and surface reconstruction.

Estimates of the unknown parameters are obtained by the least squares solution which minimizes the sum of squared deviations. A non-linear least squares system is required in the conventional non-linear photogrammetric solution to obtain orientation parameters. Many observations in photogrammetry are random variables which are considered as different values in the case of repeated observations such as image coordinates of points on images. Each measured observation represents an estimate of random variables of the image coordinates of points on images. If image coordinates of points are measured using the digital photogrammetric workstation, the values would be measured slightly differently for each measurement.

Medium scale aerial images covering the area of Jakobshavn Isbrae in West Greenland are employed for this study. The aerial photographs were obtained by Kort and Matrikelstyrelsen (KMS: Danish National Survey and Cadastre) in 1985. KMS established aerial triangulation using GPS ground control points with ± 1 pixel RMS (Root Mean Square) error under favorable circumstances and images were oriented to the WGS84 reference frame. Technical information on aerial images are described in table 1.

Table 1. Information about aerial images

Vertical aerial photograph					
Data	9 Jul 1985				
Origin	KMS				
Focal Length	87.75mm				
Photo Scale	1:150,000				
Pixel Size	12µm				
Scanning Resolution	12µm				
Ground Sampling Distance	1.9m				

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(c) Image 766 (d) Target area Fig. 2. Test images

The diapositive films are scanned with the RasterMaster photogrammetric precision scanner which has the maximum image resolution of $12\mu m$ and the scan dimension of $23 \text{ cm} \times 23 \text{ cm}$ to obtain digital images for softcopy workstation as figure 2.

The first experiment is the recovery of spline parameters with known EOPs obtained by manual operation using softcopy workstation. Spline consists of four parts and the second segment parameters are recovered. The total number of equations is 2×3 (the number of images) $\times 3$ (the number of points) + 2(the number of arc-length) $\times 3$ (the number of images) = 24 and the total number of unknowns is 12(the number of spline parameters) +9 (the number of spline location parameters) = 21 so the redundancy is 3. Table 2 expressed the convergence achievement of spline and spline location parameters.

Estimation of spline parameters including location parameters is established by the relationship between splines in the object space and their projection in the image space without the knowledge of the point-to-point correspondence. Since aerial triangulation using splines does not require conjugate points generated by the knowledge of the point-to-point correspondence, the more robust and flexible matching algorithm can be adopted. Table 2 represents the result which the object space information without the knowledge of the point-to-point correspondence is avail-

Spline location parameters								
Image 762								
	t_1		t_4	l	t_7			
ξ^0	0.08		0.3	\$8		0.72		
ĉ	0.0844		0.4258		0.6934			
ξ	± 0.0046		± 0.0	0058		± 0.0072		
Image 764								
	t_2		$t_{ m g}$	j		t_8		
ξ^0	0.22		0.53			0.82		
ĉ	0.2224		0.5170		0.8272			
ξ	± 0.0175		± 0.0104			± 0.0156		
Image 766								
	t_3		t_6		t_9			
ξ^0	0.32		0.59		0.88			
ĉ	0.3075	0.6176		176	0.9158			
ξ	± 0.0097	± 0.0)148 :		± 0.0080		
Spline parameters								
	a_{10}		a_{11}	a_{12}		a_{13}		
ξ^0	535000.00	:	830.00	-150.00		50.00		
2	535394.1732	8	67.6307 -173.1357		357	24.3213		
ξ	± 0.1273	<u>+</u>	= 0.7142 ± 7.654		540	± 21.3379		
	b_{10}		b_{11}	b_{12}		b_{13}		
ξ^0	76710000.00		150.00 140.0		00	-300.00		
ξ	7672048.3173	14	43.1734	130.8147		-290.1270		
	± 0.2237	<u>+</u>	1.6149	6149 ± 10.9058		± 26.7324		
	c_{10}		c_{11}	c_{12}		c_{13}		
ξ^0	0.00		-10.00 -50.0		0	50.00		
ξ	2.1913	-	3.7669	9 -39.8003		27.7922		
	± 0.0547	<u>+</u>	0.1576	± 9.1572		± 19.6787		

Table 2. Spline parameter and spline location parameter recovery

able. ξ^0 is the initial values and $\hat{\xi}$ is the estimates. The estimated spline (a,b,c) and spline location parameters (t) along with their standard deviations are established. However, the correspondence between the image location and object spline segment is not established. All locations are assumed as on the second spline segment and the second spline segment calculated from softcopy workstation is used as control information.

4. Conclusions

In this paper, the traditional least squares of a aer-

ial triangulation process have been augmented to support splines instead of conventional point features. Estimation of spline parameters including location parameters is established by the relationship between splines in the object space and their projection in the image space without the knowledge of the point-to-point correspondence. Since aerial triangulation using splines does not require conjugate points generated by the knowledge of the point-to-point correspondence, the more robust and flexible matching algorithm can be adopted. Automation of aerial triangulation and the pose estimation is obstacled by the correspondence problem, but employing splines is one way to overcome the occlusion and ambiguity problems. The advantage of employing splines is that adopting high level features increases the feasibility of geometric information and provides an analytical and suitable solution to increase the redundancy of aerial triangulation. 3D linear features expressed by 3D natural cubic splines are employed as the mathematical model of linear features in the object space and its counterpart in the projected image space for aerial triangulation. To solve over-parameterization of 3D natural cubic splines, arc-length parameterization using Simpson' rule is developed and in case of straight lines and conic sections, tangents of spline can be additional equations to the overparameterized system. Photogrammetric triangulation by the proposed model including the extended collinearity equation and arc-length parameterization equation is developed to show the feasibility of tie splines and control splines for spline and spline location parameters.

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