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In-Service Mathematics Teacher Training from the Van Hiele Theory Perspective[1]

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In this work we present a study undertaken with in-service mathematics teachers of primary and secondary school where we describe and analyze the didactical competences needed to implement an innovative design in geometry applying Van Hiele's models. The relationship between such competences and an ideal teacher profile is also studied.
Teachers' epistemology is established in terms of didactical competences and we can see that this epistemology is an element that helps us understand the difficulties that teachers face in practice when implementing an innovative curriculum, in this case, geometry based on the Van Hiele theory.

Keywords: mathematics teacher training, training programme, didactic competences, Van Hiele theory, Van Hiele's model
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GENERAL CONSIDERATIONS

Geometry is a basic educational material worked on by students from the first years of mathematics learning in Primary and Secondary schools. Research undertaken in the mathematical education sector on teaching and learning processes is growing and requires studies that contribute results to understand aspects of teachers' epistemology and the role that this plays when taking decisions to enrich the curricular development of geometry when an innovative curriculum is implemented. The idea of researching questions of teaching and learning based on the Van Hiele's model is relevant in this sense (Afonso, 2004).

We study the use that the participating teachers make of several learning units from the teaching programme (angles, measurement of angles, and rotations) when the teachers analyze these units as curricular material for their students and when they use them with their students in the classroom.

In the general research understood as the design, development and evaluation of a "Mathematics Teacher Training" programme the following objectives were set out:

(1) To design, implement and evaluate an in-service mathematics teacher training programme that uses the didactical competences deriving from Van Hiele's model.
(2) To analyze the teachers' didactical competences before and after undergoing the training programme.
(3) To analyze the teachers' predisposition to use the Van Hiele's model after undertaking the training programme.

With regard to these general objectives we made the following conjectures:

- Combination of the Usiskin Test (UT) and the Jaime Test (JT) is suitable for demonstrating both the level of geometric thinking of the teachers taking part in the training Programme and of the students who work on the learning units designed.
- The Van Hiele's model will be taken on by the teachers through quality training programmes which include practice (immersion) as an active ingredient.
- The categories of analysis taken from the Semiotic Logic Approach (Socas, 2007) are suitable for analyzing the teachers' didactical competences.

Five significant research stages were established

1. Preparatory: A preliminary study made before final research design to study the three general aims.
2. Initial: A study to determine the initial conditions of the participating teachers regarding the aims set forth in the training programme.

3. Intermediate: A study to determine the conditions of the participants at the end of the immersion orientation course.
4. Final: A study made when finishing the implementation with their students of the learning units chosen by the teachers.
5. Subsequent: A study undertaken six months after the conclusion of the training programme in order to analyze the didactical consequences of the training programme for the participating teachers.

In this work we focus on Aims 2 and 3 at the four stages: initial, intermediate, final and subsequent. The researchers' objective here is to establish a technique that allows us to find a way to describe the specific competences of a number of teachers in relation to mathematics teachers' general competences. Thus, the teachers' didactical competences are analyzed and then related to the ideal profile of a teacher who can put into practice a geometry teaching proposal from the perspective of the Van Hiele's model. To gather information about the process followed by the eleven teachers, analysis is made in five study fields: context, geometric cognition, curricular adaptation, didactic cognition and interactions. We would determine not only the institutional situation the teachers work in but also their didactical competences with regard to the Van Hiele's model of learning and teaching.

In a previous work (Afonso, Camacho & Socas, 1999) an ideal teacher profile was established within the framework of the 1989 education Reform for putting into practice a geometry proposal based on the Van Hiele's model of reasoning. In this study, we relate the ideal teacher profile to the didactical competences that the teachers taking part in the study demonstrate, using a technique that allows us to establish a procedure that describes the teachers' competences and their relation to the ideal profile for a given didactical action.

Finally, we include some conclusions which supply us with a series of elements that help us to characterize the teachers' epistemology and the role that this plays in the taking of decisions to enrich the curricular development of geometry and implement an innovative curriculum.

This work is situated in the research paradigm that focuses on teachers' thinking and attempts to describe the cognitive representations that teachers make of their tasks and the way in which these representations have a repercussion on students' behaviour. It also seeks to define the relations that exist between these representations and the teachers' and students' actions. Teachers' thinking and their decision making, then, are key elements in this work.

THEORETICAL FRAMEWORK

The theoretical framework within which our research is carried out is based on five components:

The Van Hiele's model;
In-service mathematics teacher training;
The mathematics teacher's geometry knowledge and didactical competences; and
The teacher profile and evaluation of educational programmes.

We now briefly describe each of these components:

The Van Hiele's model

According to the Van Hiele theory, mathematical thinking follows a concrete model that is formed by two parts:

- A descriptive part by which a sequence of types of reasoning called "levels of reasoning" is identified:

  Level-1 (Visualization),
  Level-2 (Analysis),
  Level-3 (Informal deduction),
  Level-4 (Formal Deduction), and
  Level-5 (Rigor),

- Another instructional part which gives the teachers guidelines so that they can help students attain with greater ease a higher level of reasoning, and which are called "learning phases".

The Van Hiele recommended that geometry teachers organize their teaching following these phases so that students receive some given guidelines and also go through each and every one of these phases to attain a higher level of reasoning. Fuys, Geddes & Tischler (1988) describe in detail the main features of the Van Hiele levels of geometric reasoning.

There are several studies on different aspects of this teaching and learning model. We are interested in research with relation to secondary and primary school teachers. The majority of research made with teachers, has been devoted mainly to determine the level of reasoning geometric in which are found. In this way, the investigations of Gutierrez, Jaime & Fortuny (1991), have done with pre-service primary school teachers and Knight (2006), also with pre-service Secondary school teachers.

Recently Halat and Sahin (2008) compared the levels of reasoning that are present in pre-service and in-service elementary school teachers.

In-service teacher training

Studies (relating teachers' thinking to their decision making) allow us to lay a solid foundation for teacher training and for carrying out educational innovations. Accordingly, such research has been taken into greater consideration over the past years.

In order to design our in-service teacher training programme we have followed the research and development model for curricular adaptation used by Socas, Afonso, Hernández & Palarea (1994), regarding the variables that influence teachers' decision making.

Mathematics teachers' geometry knowledge and didactical competences

Teachers' knowledge depends on knowledge of various disciplines (mathematics, mathematics didactics, psycho-pedagogy) as well as the teachers' conceptions and beliefs regarding mathematics and teaching of mathematics. These conceptions, beliefs and knowledge evolve favourably when they are involved in a process of reflection and action regarding teaching practices.

In research on mathematics teachers' professional knowledge, Llinares (1998) identifies two research agendas called the mathematics teachers' apprenticeship and the teachers' professional practice. Llinares (1998) also underlines several research problems concerning, among other things, cognitive elements in teachers' mathematical knowledge, specific mathematics pedagogic knowledge and problem solving.

Boero, Dapueto & Parenti (1996) refer to three more general and extended positions that are held regarding mathematics teachers' professional knowledge.

The first position is that held by those who state that "the teacher should be as competent as possible in mathematical knowledge." This vision sustains the idea that "whoever knows mathematics can teach it."

The second position is held by those who believe that "the teacher should use his or her professional competence like a craftsman" (if possible, like an artist), a metaphor that is used to evoke the idea that a teacher is able to confront professional problems flexibly (craftsman) or create substantial innovations (artist). Thus, a good mathematics teacher should be able to handle mathematics and should be informed about the art of teaching it.

The third position sustains that "the teacher's professional competence should be acquired though various scientific domains (mathematics, Education Sciences and mathematics Didactics)." The teacher's education should broaden his or her knowledge of mathematics, together with other subjects arising from Education Sciences (Educational Psychology, Educational Sociology) and mathematics Education (as a specific field of professional competence and as a field of research), This final position emerges as the most coherent and well-founded of the three.

The curricular changes that have occurred in mathematics teaching at the level of compulsory education play a significant role in epistemological change regarding contents and are especially relevant in the way of understanding the process of mathematics teaching/learning, that is, in the proposal that they make with regard to the process of construction of this. These changes are not alien to the mathematics teacher; rather, the teacher should assume them and incorporate them as part of his or her professional knowledge.

We wonder whether it is possible to make a general categorization of mathematics teachers' knowledge within the framework of current reforms and specify this categorization by describing the knowledge that should be possessed by those mathematics teachers who are going to implement geometry learning units from the Van Hiele perspective.

This knowledge derives from the model of analysis we have chosen, the *Semiotic Logic Approach* (Socas 2007). This approach is peculiarly relevant in the socio-cultural and institutional context, mathematics curricular knowledge, the student as learner, and the teacher as educator. This approach is also relevant for the three basic relations that we term:

1) "School mathematics learning as conceptual change";
2) "Adaptation of mathematical curricular content into material for teaching," and
3) "Interactions".

These relations are grouped into three types of interrelated knowledge: mathematical knowledge, mathematics didactical knowledge and knowledge of educational practice, which we take as a general reference in order to configure the mathematics teacher's professional knowledge.

One part of teachers' professional knowledge is made up of their mathematics didactical knowledge. In this work we refer to mathematics didactical knowledge as that knowledge needed for planning (design), putting into practice (development) and valuation (assessment) of the teaching activities, or more generally the learning units.

Marcelo (1993) points out that there are some limitations in teacher training. So, for example, Marcelo believes that mathematics teachers' didactical deficiencies lead them to have recourse in their professional field to trial and error as the main instrument in learning to teach.

Escudero & Sánchez (2007) also emphasize the complexity of integrating knowledge of the subject (mathematics) and pedagogical knowledge, and they show how two teachers with similar experience and knowledge differ significantly when it comes to the characteristics of the domains of knowledge they integrate into their lesson plans and into the implementation of those plans.

One possible way of helping teachers to overcome these limitations would be to set up in-service training programmes using models of teaching/learning situations in immersion environments which could bring about changes in the way teachers go about their real professional activities by incorporating new geometry teaching/learning domains which might make the learning process more fruitful.

We now describe the notion of didactical competence regarding the knowledge we have characterized for mathematics teachers.

Didactical competence

Generally speaking, we take the term competence to be a person's disposition towards that knowledge or those abilities that allow the person to carry out an activity in a suitable form (Short, 1985).

When considering mathematics teachers' professional competences we should take into account the knowledge that affords teachers options to use and assess a greater number of conceptual tools that allow them to decide upon and establish teaching/learning sequences in order to present mathematical concepts and procedures and that suggest new ways of assessing and interacting with students.

This knowledge is made up, as indicated in NCTM (1991), Simon (1994) & Llinares (1998), of four perspectives that takes into account mathematics teachers' general competences, skills and abilities:

- Curricular (knowledge of and competence in planning and organization of the geometry curriculum from the Van Hiele perspective);
- Mathematical (knowledge of elementary geometry and control of geometric competences – levels of thinking – that students must work on);
- Learning (knowledge of and competence in students' geometric conceptions and processes regarding specific geometric content in order to diagnose and give meaning to students' productions - the learning model), and finally,
- Teaching (knowledge of and competence in planning and organization of the geometric content for teaching – learning model – in order to manage communication and interactions in the classroom).

In this case, teachers' knowledge should help them to assume the Van Hiele idea of levels of geometric thinking and learning phases as another viable geometry teaching model.

In our study, didactical competences refer especially to knowledge, abilities and skills related to Van Hiele's levels of geometric thinking and learning model.

We understand as didactic competence for putting into practice a geometry programme from the Van Hiele perspective the teacher's capacity to select with well-founded criteria a piece of specific knowledge or ability pertaining to geometry in order to apply this in the teaching/learning situation in accordance with the Van Hiele's model.

If we take into account the model of analysis chosen (Semiotic Logic), there are five categories or descriptors that lead to classification of the teacher's Didactical Competences: Context, Geometric Cognition, Curricular Adaptation, Didactic Cognition and Interactions.

The *Context* is determined by the set of initial data that situates the working teacher in relation to the School Institution, as well as the value judgments that the teacher attributes to other elements in the educational micro system: students, mathematics and geometry.

Geometric Cognition in this work refers both to teachers' levels of reasoning in terms of the Van Hiele levels, as well as to analysis of the teacher's state of opinion as to the importance and conceptions of geometry .

We used tests devised by Jaime (1993) and by Usiskin (1982) to determine the levels of teachers' geometric thinking. There are two reasons for using both of these tests: first, to determine the degree to which teachers attain their level, and secondly, to contrast the information supplied by the Jaime Test with the results obtained from the Usiskin Test.

Moreover, for all the teachers we will study: the role that is played for them by deduction, informal work, the setting and solving of open questions, the development of spatial intuition in geometry, as well as the importance of these in relationship to geometry itself and other subjects.

Curricular Adaptation refers to the design of teaching/learning activities made by teachers for their students. Analysis of class design is carried out by taking into account the different curricular components: aims, contents, methodology and evaluation. The characteristics of the aims set by teachers have given us a measurement of the direction that teachers take in their teaching programme. The contents have shown us the conceptual and procedural aspects underlined in their proposals. Methodology has allowed us to observe the teaching and learning activities designed by the teacher, whether they are individual or group activities. Evaluation, as a curricular component, has shown us – when teachers have formally explained it - the diagnostic, formative or summative sense that evaluation has in the programme set up. All of these have allowed us to observe the decisions made by the teachers in their class design.

In this sense, the structuring of the lesson plans supplied by teachers allows us to examine the type of conceptual or curricular organization they propose. Conceptual organization is achieved when the teacher considers contents basically as an instructional element and organizes them from the viewpoint of their internal logic. Curricular organization is achieved when the contents are considered basically as an educational element and are organized from a curricular point of view. The contents are considered from epistemological and phenomenological perspectives as an educational tool used to acquire given capacities, and which also require pedagogic and didactical organization (methodology) and organization of the evaluation process in order to measure the capacities acquired.

Didactical Cognition attempts to delimit the didactical decisions taken by the teachers when carrying out their work as educators, allowing us to define the teachers' "didactical tendencies": traditional, technological and research (Contreras, 1999). Also, didactical cognition takes into account the supposed degree of acceptance by students of geometry, examining the reasons they give for this, as well as the lesson plans used and the way classes are carried out.

More specifically, the traditional tendency can be described as an explanation of the contents fundamentally based on the use of textbooks, using concrete materials, diagrams and graphs. Also, the examples used are the examples found in the textbooks themselves and which explain the concrete solutions included. The technological tendency is determined by the use of a closed programme, as well as materials and examples based mainly on textbooks. The research tendency (including here the spontaneous tendency as there are not enough elements to differentiate these two tendencies) is characterized by the importance given by teachers to the role they believe students play in learning: the teachers encourage their students to look for their own solutions, carry out research with the objects surrounding them, allow and use the solutions put forward by the students to achieve the correct solution.

Interactions: In this article we do not aim to analyze the contribution made by each participant in the process of generation of mathematical knowledge. We are exclusively interested in whether there is or not interaction and in the contents of the interventions made by teachers and students in the exchanges of geometric information from the perspective of the Van Hiele's model.

It is obvious that the teacher's cognitive perspective (geometric and didactical) plays a fundamental role in the generation of interactions generated in the geometry class.

The five analytical categories described allow us to differentiate two subjects: the teacher and the student, who have different aims with regard, for example, to geometric and didactical cognition, curricular adaptation, interactions and context. We wish to focus especially on the teacher but we will also analyze the student especially with regard to interactions.

Teacher profile: Mathematics education reforms and the different actions undertaken in teacher training underline the need to analyze teachers' epistemology.

For example, the non-university Education Reform undertaken as from the year 1989–1990 (MEC, 1989) requires a teaching body that can assimilate these curricular changes and face up to new tasks. Among these tasks are the requirement to work with an open curriculum that necessitates the evaluation and selection of the various pedagogic alternatives most suited to the immediate educational needs and the need to work on more complex tasks than required in a closed curriculum. This curriculum is based on the theoretical decisions made by the curriculum designers taking into account what students have to learn, in what order and to what end.

The mathematics curricular reforms tended towards a basic curriculum which takes mathematics as a discipline in continual evolution and where mathematical activity plays an essential role in the construction of mathematical knowledge. Also, emphasis is put on problem solving as the main focus for developing mathematical concepts, the development of a positive attitude towards mathematics, the consideration of mathematics as expression and creativity, as well as to facilitate mathematics for all by reducing the more abstract aspects as much as possible.

Afonso, Camacho & Socas (1999) analyzed, within the framework of the mathematics teacher profile outlined in the 1989 Educational Reforms, a profile for the mathematics teacher who is "prepared" to put into practice successfully an innovative curricular plan for mathematics, centred around an interpretation of the geometry curriculum based on Van Hiele's geometric and didactical models. The authors conclude that Van Hiele's models require significant changes in teacher training with direct implications regarding their classroom work. In other words, curricular planning based on Van Hiele required teachers who possess a determined set of skills and attitudes (teacher Profile), implying significant changes in their epistemology, which can be summed up as:

1. Scientific training in geometry, at least to a level in geometry higher than the level to be worked at with the students.
2. Concept of learning as directed research.
3. Ability to work with a highly heterogeneous group of students with respect to basic skills, interests and needs in geometry.
4. Conception of the geometry curriculum as an educational tool that allows different levels of geometric reasoning.
5. Value put on and performance in teamwork.
6. Ability to facilitate mathematics for all, reducing the more abstract aspects as much as possible.

Once the profile required by a geometry curriculum in Van Hiele's terms was established, and once the teacher's didactical competences based on a Semiotic Logic Approach were formulated, it was necessary in our model of analysis to establish a relationship between the didactical competences and the ideal teacher profile referred to above, in order to observe how a teaching proposal could be put into practice. The relationship between the didactical competences and the teacher profile was set out as follows:

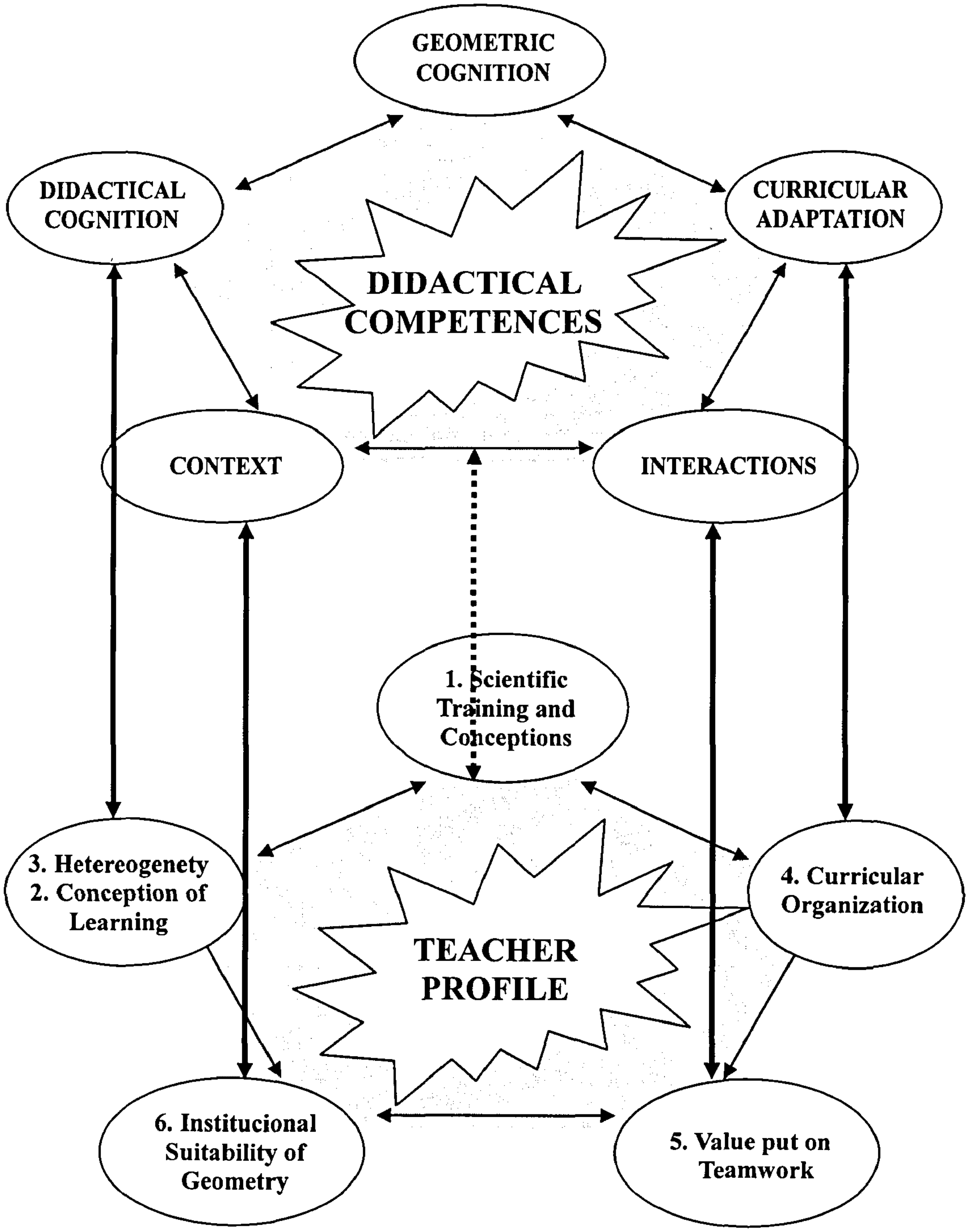

Figure 1. Didactical competences and teacher profile

Geometric cognition is related to scientific training in geometry and to conceptions of geometry: deductive, manipulative-based and informal work; didactical cognition that respects heterogeneity, and the idea of learning in terms of directed research. For its part, curricular adaptation is related to organization of geometry from a curricular perspective; interactions are related to the value put on and performance of teamwork, and the context with the importance of mathematics and its institutional suitability. Figure 1 visualizes these connections.

Finally, the last component in our conceptual framework relates to *Evaluation of Educational Programmes*. Evaluation of mathematics educational programmes constitutes an immediate action which is meant to steer changes towards higher quality in teaching and learning. Authors such as Pérez Juste (2000) have carried out research in these terms. We set out to attain one of our goals which were to observe the improvement in quality in the planning of geometry teaching, as well as determine the conditions in which this improvement is realized.

GENERAL RESEARCH DESIGN

As we have described above, there are five significant stages that we have organized into two major phases:

Phase 1: Analysis and training (stages: preparatory, initial and intermediate), and
Phase 2: Classroom implementation (stages: final and subsequent).

In the first phase we diagnose and analyze the participating teachers and a course is designed and held (which we call the Orientation Course) that constitutes a period of training and debate. The second phase was devoted to classroom implementation by the teachers of the material used in the Orientation Course, analyzing in this phase the teachers' actions and the difficulties and potentials in the use of the materials that had previously been designed.

In general terms, we aim for teachers to use with their students curricular material designed by the research team prior to classroom implementation and review with the teachers themselves, which constitutes what we call the Orientation Course. The course was divided into three topics that we decided to cover and which we had worked on in the first phase: angles (OCA), measurement of angles (OCM) and rotations (OCR).

Hereafter we describe the different stages our work was divided into, framing them into each of the phases and briefly submitting the instruments of analysis used.

Phase 1: Analysis and training

The Preparatory study was carried out before the final design and was undertaken by the research team (three researchers) and four in-service teachers (two from Primary Education and two from Compulsory Secondary Education). This phase was structured though meetings within the framework of a Permanent Geometrics Didactical Seminar in order to analyze the reality of geometry teaching in the schools and with the goal of carrying our innovations in the geometry curriculum for students from 10 to 14 years old. The question guiding this stage was:

Which theoretical model might respond to the curricular need of geometry?

These results led to restructuring of the initial design. We found that:

- The Van Hiele theory responds to the requirement to make the Primary and Secondary education curriculum more concrete, so that it became feasible to design learning units based on this idea.
- The learning units used should be designed by the research team and later modified if necessary once set for the teachers during an Orientation Course, as opposed to our initial idea which was to allow the teachers themselves to design the instruction.
- When generalizing further the results of the training programme it would be best to carry out our programme with a larger group of teachers in order to broaden the range of possible didactical competences of the group.
- It was necessary to find out the state of opinion of the participating teachers before, during and after the experiment, for which suitable instruments would have to be designed.

In the first phase various instruments were designed and implemented in parallel with the preparation of the *Orientation Course* (Afonso, 2004):

- A *Closed Protocol* (CP), the aim of which was, by means of semi-structured interviews with the participating teachers, to get to know the state of opinion of these teachers regarding geometry and the teaching/learning of this subject.
- The adaptation of the Usiskin Test (UT) and Jaime Test (JT) in order to analyze the level of geometric reasoning of the teachers taking part in the experiment, as we would follow the Van Hiele's model as a conceptual framework which would guide the micro curriculum that both the teachers and students were going to follow.

Apart from the above, we undertook two other actions which should be mentioned. The first was to analyze the mathematics teachers' normal classes. The teachers were thus asked to prepare lesson plans, preferably for their geometry classes, and two one-hour class sessions following these plans were videotaped. The second action, because our aim was that the teachers use curricular material designed by the research team, prior to implementation and revision by the teachers, was to implement the micro-curriculum prepared for *Angles, Measurement of Angles, and Rotations*. That is, the *Orientation Course* was carried out with the participating teachers using the "immersion" technique: the participating teachers followed the sequences of learning which they themselves set for their students in order to know the design in greater depth and experience any possible difficulties that their own students would face later.

Apart from the instruments detailed above, use was also made of: the teachers' productions which were recorded in the curricular material used in the Orientation Course, the researcher's diary, the audio recordings of the sessions undertaken and the group discussions held at different stages during the Orientation Course.

Regarding the Orientation Course, we should briefly point out that this course implied making the in-service teachers face up to a teaching and learning proposal for geometry that generates reflection about reasoning and strengthens future students' geometric abilities which are normally covered by formal teaching, but using different strategies.

As Marcelo (1993) states, the didactical deficiencies of mathematics teachers lead them to have recourse to trial and error as their main tool in learning how to teach. Therefore, the change we have proposed though the research undertaken implies both curricular and methodological changes for the teachers. So, we start off with training that is not wholly theoretical (Van Hiele's *Theory of Geometric Thinking*) as there needs to be a firm belief that what is undertaken in the classroom favours students' learning. The Orientation Course, then, is not merely a recipe for how to put into action a training plan for Primary and Secondary students in the areas of Angles, Measurement of Angles and Rotations, nor does it aim to reproduce a micro-curriculum designed by a research team. Rather, the aim was for it to be a vehicle for interpretation, justification, clarification and orientation founded on practice itself ("immersion") of the transformations that arise during a process of teaching and learning of geometry. This aim is achieved not only through teaching practice but also the fact that the Orientation Course incorporates the results selected from research carried out until the present based on the Van Hiele theory, the opinions and experience of the teachers taking part in the first phase of the research project and the opinions and experiences of the 11 in-service teachers taking part.

We note again that the Orientation Course was held using the same curricular material (instruction designs) that the teachers then set for their students in the classroom, except for the minor modifications that were made based on the suggestions arising during the various discussions that took place.

The curricular material used by the teachers was: The *three instruction designs.*

The three instruction designs we have worked on in the *Orientation Course, Angles, Measurement of Angles, and Reflections* pitched the learning unit at Van Hiele Levels 2 and 3 with the concrete aim of going from one level of geometric thinking to the next. In this case it was designed to pass students from Level 1 to Level 2, and then from Level 2 to Level 3.

Phase 2: Classroom implementation

Regarding the second phase, which we call the classroom implementation phase, we differentiated two different stages: the final and subsequent. The final stage focussed on the classroom implementation of the micro-curriculum designed and worked on by the teachers. The idea was to analyze the actions undertaken by the teachers in the classroom when they put into practice the teaching of *Angles, Measurement of Angles and Reflections*. Not all the teachers used all the learning units, but each teacher worked on a concrete topic with two learning units in order that two teachers per topic could be later selected for the complete study. The overall idea was to analyze the way in which the teachers implemented the design made and study the decision making process, the interactions of students with geometry and between teachers and students while they worked on the material prepared for this end. The instruments used for collecting information at this stage of the research were: the lesson plans written by the teachers to be used in class; video recordings of two sessions per teacher working on the learning unit chosen; the productions of the students when they solved the activities that had been designed by the research team and then selected by the teachers; the researcher's diary; the class diary filled in by the teachers; the taped recordings of the final interviews, and the questionnaire to evaluate the experiment which was given to them in the final meeting.

The subsequent stage took place six months after the conclusion of the training programme and took the form of a third individual interview (Final Interview II) which finally concluded the collection of information for this study.

There are two points of interest in the final interview (Final Interview II). First, we wished to confirm the information about the application of the design in the previous school year as well as the choice of students for the experiment; secondly, we aimed to analyze the didactical consequences of the training programme for the participating teachers. We classified these consequences into terms of didactical decisions projected, didactical implications and personal evaluation of the implementation of the design in the classroom. Figure 2 sums up the design worked on.

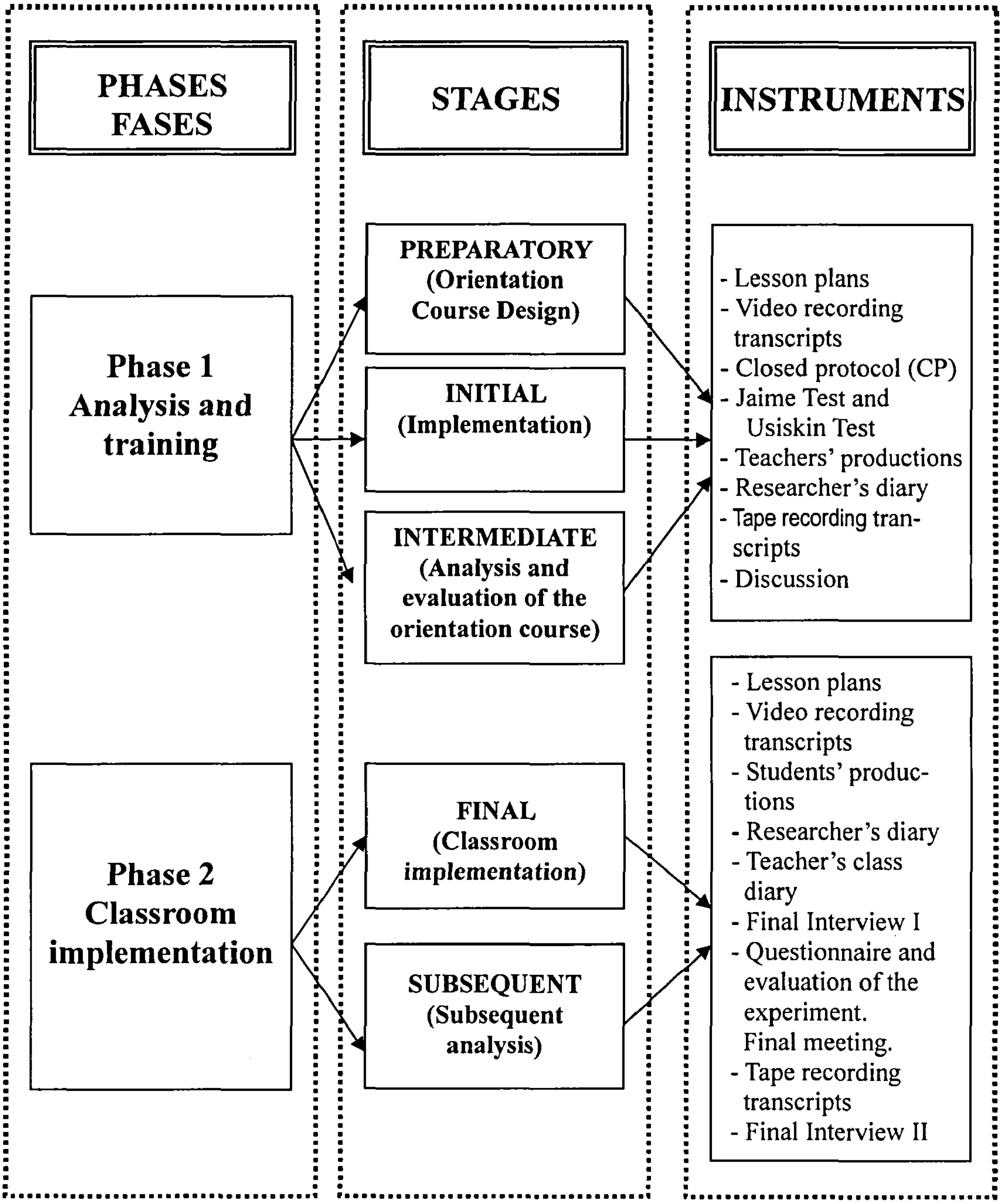

Figure 2. Relationship between phases, stages and design instruments

RESULTS AND DISCUSSION

We will give the overall results of our research and discuss the data obtained based on the different categories of analysis we have used, attempting thus to characterize in terms of didactic competences the teachers T1, T2, ..., T11, in order to be able, then, to establish the suitability of each of the teachers with respect to the profile of the ideal teacher who is able to carry out successfully a teaching-learning proposal based on the Van Hiele theory. In relation to the *context*, we could see that practically all the participating teachers were working in a suitable institutional context.

In relation to *geometric cognition*, as could be predicted, there were real difficulties when it came to the teachers completing a test like that proposed by Jaime (1993), which turned out to be long, quite complex and difficult to solve by the teachers. In our case, most teachers did not attempt to solve the Level 4 tasks on geometric thinking in the Jaime Test, and four of the eleven teachers (T3, T4, T9 and T10) did not even attempt Level 2 or 3 tasks. This was not the case in the Usiskin Test (1982) in spite of the broad range of limitations evident in this type of instrument regarding cognitive interpretation. Nevertheless, we were able to analyze geometric cognition using both tests, as if only the Usiskin Test is taken as the sole reference, we could find, for example that T3 as at Level 1 of geometric thinking, which is undoubtedly unreliable when compared to the results obtained. As for the rest of the teachers, only two of them (T1 and T5) were found to be at Level 4 in geometric thinking and both tests confirm this. Teachers T2, T6 and T11 were found to be between Levels 2 and 3 and the rest (T4, T7, T8, T9 and T10) were placed at Level 2 of geometric thinking.

In terms of the first descriptor regarding the level of geometric reasoning of the profile of the teacher ideally suited to carry out in the classroom an educational proposal based on the Van Hiele theory, we can see that only Teachers T1 and T5 might be considered suitable for putting into practice a curriculum from such a perspective.

However, regarding the second descriptor on geometric cognition, most teachers (T1, T2, T3, T6, T8, T10 and T11) conceive of geometry as a subject where deduction plays a fundamental role and that deduction should be based on much informal, manipulative-based work.

With regard to *didactical cognition*, we can see that teachers in general display some degree of rigidity and keep too much control over classroom dynamics. However, they do adopt a very flexible position when it comes to communicating contents to students and use suitable vocabulary and give answers that are right and correct to students' doubts and questions. We could see that most of the participating teachers undertake their classroom work with a clear tendency towards homogenization of their responses rather than giving individualized treatment (T9, T10 and T11), with a notable tendency towards responding to the group (T2, T3 and T4) and on some occasions towards striking a balance between individualized and group responses (T5, T6, T7 and T8).

Also, taking into account the answers supplied by the teachers in the initial interview, we could see that teachers reveal that in their teaching they vary from one teaching style to another, and that Teachers T1, T3, T4, T8, T9 and T11 adopt more a research style rather than a traditional or technological one.

Teachers T2 and T10 do not reveal any clear didactical tendencies, but rather vary between the three tendencies, although both of them manifest a highly research style. Teacher T5 reveals a highly traditional didactical tendency.

Finally, Teacher T6 varies between the traditional and research tendencies while T7 varies between the traditional and technological tendencies.

Analysis of the transcriptions of the class session video recordings allowed us to see that of the eleven teachers only seven confirm the predominant didactical tendencies they laid claim to in the interview. Teachers, T1, T2, T3, T4, T8 and T9 undertake their classroom teaching with the clear research tendency they manifested in their initial interview, while T5 reveals in the classroom the traditional style that was vouched for in the interview. However, Teachers T6, T7, T10 and T11, who claimed research styles in the initial interview, can be classified as teachers with either technological or traditional styles based on the classroom sessions undertaken.

In sum, Teachers T1, T2, T3, T4, T8 and T9 can be seen to organize tasks from a research point of view, and play the role of orienteer in those tasks. The remaining teachers go about tasks in a routine manner, playing the role of transmitters of knowledge.

With regard to the analysis category *curricular adaptation*, there are few teachers who organize the tasks from a curricular perspective (T1, T2, T4 and T8). The other teachers organize tasks conceptually. Curiously, Teacher T9 manifests some contradictions in that she organizes her classes from a research perspective and plays the role of orienteer for her students, in spite of manifesting a conceptual organization in her lesson plan. This might be the result of the prescriptive nature of her school's organization. Teacher T2 starts with a curricular organization of tasks but in fact puts into practice a routine form of teaching, mainly owing, we believe, to a poor definition of a clear didactic tendency (this teacher varies between traditional, technological and research tendencies).

Based on analysis of the video recorded sessions and with regard to the category of *interactions*, we can see how the groupings for carrying out tasks confirm a predisposition towards and valuing of group work for Teachers T3, T4, T8 and T9. The other teachers did not show that their work was effectively group work (T1, T2, T5, T6, T7, T10, T11), which fails to ratify the state of opinion expressed by some of the teachers in the initial interview (T5, T6 and T7 stated that they almost always do group work).

In the Table 1 we collect together some of the more relevant results that we have just commented on for the *Initial stage*. In one column reference is made to the eleven teachers in terms of suitability in accordance with the descriptor (YES) or lack of suitability (NO). The teachers with an asterisk (*) next to them are those teachers for whom we have only used the Usiskin Test to evaluate their level of geometric reasoning. In the row covering descriptor 2 of the *Profile*, we have written: Tendency manifested in the initial interview—Tendency observed in the video recordings.

For Teachers T5, T6, T7 and T8, the descriptor 3 means that they sometimes attend to heterogeneity and other times they do not.

Based on the initial analysis carried out and on the categories established, we can point out that of the group of participating teachers T1 and T8 might be considered participants whose didactic competences are suitable for putting into practice a curriculum from the Van Hiele point of view. For Teacher T1, a powerful geometric cognition and cognition of learning in terms of directed research as well his role as learning orienteer are basic elements in his suitability in accordance with the profile. His group work and respect for heterogeneity would have to be fomented.

Teacher T8 demonstrates didactical competences mainly in keeping with the desired profile, but fundamentally based on this teacher's powerful didactical cognition and on the interactions achieved with students. In this case, we believe that institutional factors determined by the level at which this teacher works (4th Primary) are decisive in being able to characterize some of this teacher's competences. The lower academic level of the students might influence interactions and in some cases didactical cognition. Moreover, this teacher's geometric cognition is two levels lower than that prescribed in our profile.

Teachers T5 and T7 are furthest from attaining the profile prescribed. However, there is a fundamental difference between the two. While T5 can be seen to possess a perfectly suitable geometric cognition (level of geometric cognition), T7 has low geometric cognition as well as a didactical cognition far from the profile prescribed, as is also the case in the other categories of analysis.

Teachers T6, T10 and T11 also fail to meet the desired profile. Their geometric cognition shows them to have scientific training below the expected level when working professionally in the first year of Secondary Education. There are no significant differences between the teachers. T6, T10 or T11 are not willing to any great degree to respond to the large group of students but rather try to establish a certain balance between group and individual responses. In order for these teachers to approximate to the profile described, they would need to value and exercise group work as well as go beyond being routine teachers and mere transmitters of knowledge. The teachers T2, T3, T4 and T9 also demonstrate didactical competences that fail to fit the ideal profile, but are perhaps nearer this profile than teachers T5, T6, T7, T10 and T11.

Table 1. Didactical Competences and Teacher Profiles in the Initial Stage.

DIDACTICAL COMPETENCES	TEACHER PROFILE	T1	T2	T3	T4	T5	T6	T7	T8	T9	T10	T11
Geometric cognition	1. Scientific training (Level of geometric reasoning)	YES 4	NO (2-3)	NO 1(*)	NO 2(*)	YES 4	NO (2-3)	NO 2	NO 2	NO 2(*)	NO 2(*)	NO 2-3
	Conceptions of geometry (deductive, manipulative-based, informal	YES	YES	YES	NO	NO	YES	NO	YES	NO	YES	YES
Didactical cognition	3.Respect for heterogeneity	NO	NO	NO	NO	YES-NO	YES-NO	YES-NO	YES-YES	YES	YES	YES
	2 .Conception of learning in terms of directed research	YES-YES	YES-YES	YES-YES	YES-YES	NO-NO	YES–NO	YES-NO	YES-YES	YES-YES	YES-NO	YES-NO
Curricular adaptation	4. Organization of geometry from a curricular perspective	YES	YES	NO	YES	NO	NO	NO	YES	NO	NO	NO
Interactions	5. Value put on and practice in group work	NO	NO	YES	YES	NO	NO	NO	YES	YES	NO	NO
Context	Importance of mathematics, institutional	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES	YES
	Suitability											

The formers' didactical cognition and possession of a research concept of teaching and learning (these four teachers being described as having a high orienteering tendency for geometric knowledge) might lead us to think that these would help compensate their deficiencies with regard to geometric cognition. This observation could be essential in the subsequent research.

The results from the *Intermediate stage* once the Orientation Course is over clearly demonstrate that the teachers' geometric and didactical knowledge help increase their didactical competences.

From analysis of the participating teachers we can at present underline their observation and acknowledgement of the resources and the high value put on the teaching/learning proposal contained in the model. They acknowledge and value the resources used and identify the five phases of the process.

Evidence of the progress made by the participating teachers can be clearly seen in the teachers' productions and in the discussions held throughout the Orientation Course. Some level of integration of the Van Hiele's model can be seen in the teachers taking part in the experiment. Improvements in participants' arguments and judgements expressed in different sessions of the Orientation Course reveal a number of changes that we deem significant and that relate the teachers to the Van Hiele's model.

On balance we can claim that the programme as put into practice in relationship to the Orientation Course is favourable. However, we should point out that the teachers noted the shortage of time required to undertake the activities as well as the reiterated use of some of them. These elements should be improved by readjusting the activities and taking note of the time required to carry them out, which of course has an effect on the number of hours needed.

In the *final stage* we take into account the practical and operative aspects of the training programme in relation to the fulfilment of the programming of the learning units selected and of the geometric and didactical model followed.

At this stage, the teachers organize themselves in order to select the learning units that they are going to implement and establish the levels at which they are going to undertake the activities. The result of this choice is six teachers choosing angles (T4, T5, T6, T7, T8 and T10), three measurement of angles (T3, T9 and T11), and two rotations (T1 and T2).

The eleven teachers chosen demonstrate various competences, as we have seen earlier. For example, in relation to Van Hiele's levels of thinking, Teacher T1 achieves Level 4 and Teachers T1 and T3 only achieve Levels 2 and 1, respectively. However, the three teachers have conceptions of geometry and didactical concepts approaching the profile of the ideal teacher. Teacher T5 demonstrates a high level of geometric thinking (4); however, in practice, this teacher adopts a traditional style of teaching based on a conceptually organized curriculum. Teacher T6 achieves a lower level of geometric thinking (between

2 and 3) together with a tendency towards a traditional style of teaching that is far from the ideal profile. Finally, Teacher T11 manifests a level of geometric thinking between 2 and 3 and a didactic organization that betrays an evident contradiction between what this teacher says and does.

We also find that not all the teachers in our study incorporate the didactical models proposed by Van Hiele into their didactical competences. For example, Student T5 has institutional problems, while T3 came across problems when carrying out the learning sequences as this teacher had to take 5th Year Primary.

In general, and to sum up, we can point out that when we take into account the data produced and the analysis made, the following can be noted once the training Course is completed:

- The participating teachers generally accept the learning concepts deriving from the Van Hiele didactical models.
- The participating teachers find no difficulties in coming up with new activities when these become necessary to clarify and complete the learning unit they are working on, without detracting from the Van Hiele didactical models.
- Control over the learning phases constitutes one part of the model assumed by most teachers, although we find teachers who admit that they have clear difficulties in differentiating them.
- The grouping of students made by the participating teachers when working with the learning unit demonstrates the value they have given to this factor in order to implement the unit.
- The participating teachers demonstrate geometric competences to implement without any difficulty the learning units selected, both at Level 2 and 3, in spite of their initial low level of geometric reasoning.
- Except for some isolated cases, adaptation of the curriculum based on the Van Hiele's model is accepted by most of the participants.

As we have stated, the *subsequent stage* took place six months after completion of the training programme and was organized by means of an individual interview (Final Interview II). There are two points of interest regarding this interview. First, we wished to confirm information on the application of the design in the previous school year as well as the selection of students. Secondly, we aimed to analyze the didactical consequences of the training programme for the participating teachers, which we organized into the terms of projected didactical decisions, didactical implications and personal valuation of the implementation of the design in the classroom. Here we resume and comment on the relevant aspects.

For the didactical decisions projected we considered:

- Working in the following school year with the design prepared (D1)
- Working in the following school year with some activities of the design prepared (D2)
- Intending to work with the design (D3)
- Intending to work with some activities from the design (D4)
- Neither working with the design nor the activities from the design (D5)

For the didactical implications analysis is made of aspects referring to:

Relations between the design and the work carried out in the activities (R)

Finally, we consider:

Valuation of the implementation of the design in the classroom (V)

In Table 2 we gather together the results obtained.

Table 2. Subsequent Stage: Didactical consequences of the training programme

Participating Teachers	**Didactic Decisions Projected (D)**	**Valuation of Implementation of the Design (D)**	**Didactical Implications (R)**	**Observations**
T1 Secondary-2nd (Rotations)	D3 at Level 2	Positive valuation Improvements in text and graphics Selection of activities	Adapts the curricular material to teaching strategies	
T2 Secondary-2nd (Rotations)	D1 and D3 at Level 2	Positive valuation No need for improvement	Accepts the design as curricular material suitable for own needs	Main motivation is for students to reason and think with this style of work
T3 Primary-5th (Measurement of Angles)	D2 and D4 at Level 2	Positive valuation There are some activities with high level of difficulty	Identifies the Van Hiele teaching/learning model with own teaching strategy	
T4 Primary-6th (Angles)	D5	Positive valuation Excessively long work. Difficulties in reading activities	Does not incorporate any aspect of the model, although identifies it with own methodology	Has curricular material organized for all Primary School mathematics

(Continued)

Participating Teachers	Didactic Decisions Projected (D)	Valuation of Implementation of the Design (D)	Didactical Implications (R)	Observations
T5 Secondary-1st (Angles)	D5	Positive valuation (by students) Amount of time proposed seems excessive	None	No loner works in Primary Education, now working in Social Guarantees Education
T6 Secondary-1st (Angles)	D3 at Level 2	Positive valuation Intuitive method in which student discovers by deduction	Accepts the design as curricular material suitable for own needs	Angles throughout whole Level 2
T7 Primary-6th (Angles)	D5	Positive valuation Emphasizes the type of research work involving the student	None	Transfers to 1st Primary Education
T8 Primary-4th (Angles)	D5	Positive valuation Underlines the manipulative-based, dynamic and constructive work done by student	Establishes relationship between own methodology and the Van Hiele's model proposal	Transfers to 1st Primary Education
T9 Primary-5th (Measurement of Angles)	D3 and D4	Positive valuation Thinks, however, that it is too long and that maintains the same work dynamics	Relates own methodology with the Van Hiele's model proposal	
T10 Primary-6th (Angles)	D4?	Positive valuation Notes the excessive amount of time required as a difficulty	None, given excessive number of students	Does not rule out working with certain activities from the design that can be fitted into the programme currently working on
T11 Secondary-2nd (Measurement of Angles)	D3?	Positive valuation Emphasizes the suitability of the design for both the student and the teacher	None	Does not rule out using design at Level 2 if the course conditions so permit. Has had to work in 3rd Cycle of Primary Education

To sum up, we find that four of the teachers who took part in the *Orientation Course* after having completed the training programme did not work with the curricular material used in the design. The reasons in three of the cases were of an institutional nature (they changed to the first cycle of primary education). The fourth teacher declared that this curricular material poses some difficulties and finds it advisable not to incorporate it as the school where this teacher works had already organized a complete programme for mathematics for all primary education.

Seven of the eleven teachers taking part in the research were working or intended to work with learning units or with activities selected from these. One of the teachers at the time of the final interview (Final Interview I) was then working on the learning unit *Angle Level 2* and aimed to continue working with the learning unit *Angle Level 3*. Another teacher was working with a number of activities taken from the learning unit *Measurement of Angles Level 2* and aimed to continue with *Level 3*, although this teacher did not intend to use the learning units completely. Four of the eleven teachers foresaw in their annual general programme working with learning units *Rotations Level 2*, *Angles Level 2* and *Measurement of Angles Level 2* in the second term of this year, while another teacher also foresaw using *Measurement of Angles Level 3* in the third term. Finally, one teacher incorporated into the programme a number of activities for *Measurement of Angles*, although not the whole unit.

FINAL CONSIDERATIONS

In our study we have started with an examination of teachers' geometric knowledge and then implemented an immersion programme that incorporates aspects of the knowledge that allow use of the Van Hiele's model, analysing the influence of this programme on the participants' teaching and the relationship with their knowledge of geometry.

The first conjecture established that the combination of the Usiskin Test and the Jaime Test was suitable to show us the level of geometric thinking of the teachers taking part in the training programme and of the students who worked with the learning units we designed. The Usiskin Test and the Jaime Test became, then, complementary instruments for analyzing teachers' geometric thinking, but gave rise to a number of practical difficulties, especially the Jaime Test. At first we believed that this test was going to serve as a significant indicator of the reliability of the Usiskin Test, in spite of the fact that the two tests were based on different premises. However, when administering the Jaime Test, the teachers pointed out the complexity of some of the questions, especially those where the descriptors were at the fourth level (Items 12, 13, 16 and 17). Some teachers decided to answer the questions or only answer the questions in part. Teachers T3, T4, T9 and T10 did this declaring that in general these questions were unclear. In spite of these problems, the other teachers completed the test and, generally speaking, the test helped us to evaluate more precisely the teachers' levels of geometric thinking as the results from both tests coincided in practically all cases.

The second conjecture was that the Van Hiele's model would be accepted by the in-service teachers through high-quality training programmes where practice (immersion) formed an essential part of the programme. We found that the participating teachers accepted the concept of learning deriving from Van Hiele's model and did not encounter any difficulties in coming up with new activities when necessary without detracting from the Van Hiele didactical model. It also became clear that teachers exercised control of the learning phases as part of the models taken on by most of the teachers, although some teachers found it difficult to differentiate the phases clearly.

Finally, we should point out that six months later seven of the eleven teachers were still making autonomous use of the Van Hiele's model. Three were unable to use them for institutional reasons and only one deemed it impossible to integrate them in the normal design of mathematics classes. We believe this to be an acceptable verification of the conjecture formulated.

The third conjecture was that the categories of analysis based on a Semiotic Logic approach are suitable for analyzing the didactical competences of in-service teachers taking part in the didactical experiments.

In relation to this conjecture, we saw how context, geometric cognition, didactical cognition, curricular adaptation and interactions became basic elements that allowed us to articulate coherently the qualitative study carried out. The descriptors established in terms of the ideal profile of a teacher able to carry out a reform of the geometry curriculum based on the Van Hiele theory appear as elements that respond to these categories of analysis.

The various instruments designed to determine the didactical competences of each of the teachers turned out to be suitable and operational to allow a general description of both the teachers' states of opinion as well as their actions in the different phases of the training programme so that we could then analyze and categorize the epistemology of each teacher and relate this to the profile for mathematics teachers deriving from the LOGSE educational reforms which attempt to implement geometry based on the Van Hiele's model.

We can thus conclude that categories of analysis used, based on a Semiotic Logic approach, and the instruments employed to determine the components that describe these categories, can be considered as useful elements in analyzing the design of and putting into practice a teacher training programme, not only for a geometry curriculum taken from a Van Hiele perspective, but for any other part of the mathematics curriculum with different theories and methodologies.

As a general summary we can point out that analysis of the results shows us that the epistemology, which has been established in terms of didactical competences, is an element that we must take into account as it supplies us with relevant information about the difficulties faced when implementing an innovative curriculum, in this case a geometry curriculum designed with elements which describe the Van Hiele theory, mainly owing to the various forms of lack of balance experienced by the teachers, as we have just described, and which we can establish, according to the analytical model, in connection with the five categories that describe the ideal profile of the teacher able to put into practice this type of teaching.

The training programme we designed contributed to the development of the in-service teachers' competences and allowed them to take on the Van Hiele's model and implement with some degree of success a geometry micro-curriculum: *Angles, Measurement of Angles and Rotations*. The study has also supplied us with information that helps us direct interventions in the permanent training of in-service teachers when we wish them to implement an innovative curriculum in the classroom which entails changes in some elements of their didactical competences. Our research leads us to believe that in order to confront with some hope of success future proposals for curricular innovations that imply didactical models other than those normally used by in-service teachers, it is necessary to implement beforehand comprehensive teacher training programmes through immersion which are not just a local part of the curriculum to be implemented, nor a recipe to put into practice the Van Hiele's model, but which rather facilitate interpretation, justification and orientation based on practice itself.

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