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The Function of Creativity in the Solutions of Irregular Sequence Problems among Elementary School Mathematics Teachers and Teacher-Trainees in other Disciplines

Gazit, Avikam
Kibbutzim College of Education, Beit-Berl College of Education / Department of Education & Psychology, Open University; Israel;
Email: avikam@openu.ac.il

Patkin, Dorit*
Mathematics Department, Kibbutzim College of Education, Tel Aviv 62507, Israel;
Email: Patkin@netvision.net.il

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The article aims to present findings of a study which has examined the ability of elementary school mathematics teachers and of teacher-trainees in other disciplines to solve irregular challenging problems of sequences in general rather than numerical sequences only. The findings show that mathematics teachers succeed to cope with unusual assignments when the requirements of the problems presented to them are analogous to irregular problems. However, when the problems require a change in the thinking procedure in the direction of creative thinking, there is a considerable decrease in performance. Another finding shows that, although teacher-trainees succeed less in solving the presented problems, they give incorrect solutions which do indicate creative thinking. An inevitable conclusion based on the research findings is that teacher training institutions should enhance and reinforce multi-directional, branching out and creative thinking competences.

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* Corresponding author

THEORETICAL BACKGROUND

Creativity

There is a story, whose origin is unknown but is based on a real case, and two physicists, Nobel Prize laureates, Nils Bohr and Ernest Rutherford are involved in it. Both collaborated in the University of Manchester, where they lectured in physics. One of them asked the other to be the arbiter in a student's appeal. The story was published in the book *The Quark and the Jaguar* under the title: The Barometer Story. The student contested having received a score of 0 in an answer for which he believed he deserved the full score. The lecturer asked his colleague to be the arbiter and the latter read the question under dispute: "How can we know the height of a building by means of a barometer?" The student's answer was: "Go up to the roof of the building taking a barometer with you. Tie the barometer to a long rope and drop it down from the roof until it touches the pavement. Pull up the barometer and measure the rope length! The length of the rope is the height of the building." The arbiter lecturer thought to himself that the answer was interesting and, in fact, correct. However, he hesitated whether giving the student the full score would demonstrate his knowledge or not in the learnt discipline – physics. Since he considered that the student was trying to pass himself off as wise, he suggested that he tried to answer the question once more, after getting his colleague's approval.

The student agreed. Six minutes was allocated for him to demonstrate some mastery in physics content. After five minutes the student wrote nothing and told the arbiter that he actually had several answers to that question and he was trying to decide which was the most suitable. In the remaining minute he wrote his answer: "Go up to the building roof with the barometer and drop it over the parapet while measuring the time of fall by means of a stopwatch. Then, by using the free fall formula, calculate the height of the building." At this stage, the arbiter asked his colleague whether this answer was acceptable and the lecturer agreed to give the student the full score. When the arbiter left the classroom, recalling the student saying he had several answers to the problem, curiosity suddenly hit upon him to know what they were. He returned to the classroom and asked the student what he meant. The student answered: "There are many ways to measure the height of the building by means of a barometer.

For example, one can take the barometer outside on a sunny day and measure the building's height, the height of the shadow it casts and the height of the shadow cast by the building. Then, by means of the rule of three (using Thales inscribed angle theorem) one can calculate the height of the building. There is a simple method; whereby you climb the stairs of the building, marking on the wall the height of the barometer while climbing the stairs. Then you count the lines and multiply them by the length of the barometer. If we want a more complex method, we can tie the barometer to the end of a rope and swing it like a pendulum going down to the pavement. You calculate the g value, which is the free fall constant, and the difference between the g values will constitute the height of the building. In case there is no analogy with a physical answer specifically, we can take the barometer; knock on the concierge's door and offer him the new barometer if he tells us the height of the building." At this point the arbiter stopped the student and asked him if he really did not know the required answer to the question. Thus, the story ends, intending to present the creative element of thinking – as element that, according to the student's claim, the set of tests at school is oriented against creative students who do not necessarily accept what the teacher has taught (Gell-Mann, 1994). The story leads to the question: "What is that creativity manifested in the students' answers? Can it be found and integrated in the solution of mathematical problems?"

Torrance (1962) defines creativity as a process which encompasses a feeling of disturbing, absent elements and the need to consolidate ideas about them. Barron (1969) defines creativity as the ability to actualize something new. Re-design of a given thing. Parsons (1971) relates to the combination of personality, process and outcome. He believes that a person is creative even if he/she has not actualized! Simonton (1988) defines creativity as a form of leadership which includes a personal influence on others. Martindale (1989) argues that a creative idea must be original, pragmatic with regard to the situation in which it has been evoked and be materialized in practice.

The common feature of these various definitions is that creativity means the ability to create something new and original from existing elements. This gives rise to two questions: How is this ability manifested? What is the cognitive process whereby creative thinking transpires?

The creative process

Researchers and theoreticians concur with the stages of the creative process (Butcher, 1968):

1. The preparation stage, during which we gather information and raw materials, identifying the need, the problem.
2. The incubation stage which constitutes an unknown stage, similar to a mental gestation as awaiting the discovery.
3. The enlightenment stage which brings about a sudden discovery; an idea emerges; there is an inspiration – aha!
4. The validation stage, whereby we test the outcome and process it into an actual product.

The enlightenment stage is the most crucial of the four stages. The assumption is that it involves initial cognitive processes, which are not controlled by regular laws of reason and reality. The associative process of creativity is different from regular processes, in which a certain stimulus reminds another stimulus attached to it. Like in mathematics, if you ask students what is the result of a routine mathematical exercise, they will reach the solution by regular associative thinking which they have practiced and to which they were adjusted.

However, if we give students an assignment which is not grounded on an immediate algorithm, then at the solution stage one cannot rely only on a regular associative process but rather on a more complex process. This complex process requires a new associative relation between elements of the problem, a relation which has not been known earlier—a dissociative combination, allowing us to achieve creative solutions (Martindale, 1989).

Some cognitive psychologists maintain that regular algorithmic thinking is identified with a limited number of high wakefulness junctions in the brain. These are the junctions connected directly to the problem elements, whereas most of the other junctions are dormant. In creative thinking, however, there is an intermediate wakefulness of many junctions, not necessarily those connected directly to the specific problem. Such wakefulness allows the problem-solving person a wider range of associations—options for solving the problem (Martindale, 1989).

Mathematics and creativity

As mentioned, traditional teaching of mathematics involves mainly demonstrative processes of solution and practice, whereby students are given problems similar to those presented with a one, pre-known answer to each problem. Schools do not properly prepare their graduates to cope with various mathematical issues with the exception of calculation skills.

Students lack the ability to implement these calculation skills in different important ways. Here, creativity is introduced and, teaching mathematics without elements of the creativity process, does injustice to the entire student population and, particularly, to the gifted and excelling ones (Mann, 2006).

Creativity in mathematics is manifested by an independent formulation of uncomplicated mathematical problems, finding ways and means for solving these problems and identifying original methods for solving irregular problems. One of the ways for creating situations which require creative thinking is to present open-ended problems, to which there is not one single explicit solution. If we ask students how to equally divide 12 apples between 3 bowls, the algorithm is unequivocal and one answer is required in the given conditions. However, if we ask how to equally divide 12 apples among a number of bowls, there is not one single solution and the student has to make assumptions before choosing the solution out of several possible answers (Yee, 2005).

Creative teaching in mathematics offers both the teacher and the students opportunities to present important problems in mathematics in order to explore a variety of ways and methods for solving them (Westcott, 1978). Problem solution constitutes the heart and core of mathematics and it also includes the solution of exercises which do not have a pre-defined acceptable algorithm. The mathematician Littlewood (1953) wrote, that a good mathematical riddle is preferable to a dozen of intermediate exercises (originally, Littlewood used the word as joke and not the riddle). A mathematical riddle sets a challenge for thinking and people look for challenges and enjoy rising up to them. Their pleasure is not caused by wages or other rewards but rather by the very existence of the problem and the experience of solving it. The solution process is not pleasurable and simple and, by its very definition – problem, challenge – it might concern and frustrate us. Nevertheless, with the progress made and the breakthrough towards the solution, the negative feelings vanish and the solving person is satisfied and content.

One can find here the connection between mathematical problem solution and creative process if we relate to the definition made by Torrance (1962), *i.e.*, a process which embodies a feeling of disturbing, absent elements and the need to consolidate ideas about them. A mathematical challenging problem gives rise to a sense of disturbance and lack of data required for solving it. This sense evokes the creative thinking, identified with the thought of distinguished scientists, among them famous mathematicians. Creativity, though, does not belong merely to great scientists because a nucleus of creativity exists in every person, and it is essential to foster it from early age (Grossman, 1988). Elements which can enhance this issue are a suitable atmosphere as well as an environment providing appropriate stimuli, such as a mathematics magazine which organizes contests and sets challenges. A Hungarian magazine, "Mathematical Pages" is an example of that (Ross, 1991). Ross points out that a creative atmosphere, like the one established by the magazine, promoted mathematics at the beginning of the 20th century in Hungary, where several well-known mathematical talents grew, among them: Pólya, Erdös, von Neumann and the physicist Teller.

Mathematical problem solution and creativity

The creative mathematician Euler (1707–1783) described the process of solving complex mathematical problems using a 4-stage model, similar to the four stages of creative thinking and Pólya (1985) formulated them somewhat differently:

1. Understanding the problem.
2. Devising a plan for the solution.
3. Executing the plan.

4. Reviewing in retrospect – assessment and criticism.

The problem solution process requires a focused and categorizing reading of the problem givens, an appropriate choice of a strategy suitable for the combination of data in the problem, the problem solution and, finally, control of the solution. Combining this process and what has been previously written by various researchers, we can focus on the conditions which are necessary for a creative solution of a mathematical challenging problem:

1. Problem solution is a complex cognitive process, in which one should predict, imagine, analyze and attach ideas. This process requires motivation, concentration ability and time.
2. Problem solution demands new, original and, sometimes, exclusive answers. This calls for an extensive imagination, thinking flexibility and ability to scan information in the memory.
3. Problem solution necessitates a wide knowledge basis which includes concepts, principles, facts and mathematical information processing methods. The ability to scan quickly and retrieve ideas and connections suitable for solving the problem enables the problem solving person to make an informed choice of the construct suitable for the problem solution.
4. Creative problem solution requires a thorough reading comprehension as well as a good calculation capability suitable for the needs of the problems. This condition is essential for classifying the problem in the correct category and preventing a random solution which does not meet the requirements.

In order to help learners to exhaust their creative potential in mathematics, teachers should give them challenging problems which develop and lead to creative thinking.

Teachers must master the competences mentioned in the previous item and create an appropriate environment for delivering the competences to the learners. The Hungarian mathematician, Pólya, mentioned earlier as one of the prodigies who grew up in Hungary at the beginning of the 20th century, writes in his book *How to solve it*:

> "If he (the teacher) employs the time available to him in practicing routine actions with his students, then he kills their interest, inhibits the development of their thinking and misses his opportunities. If, however, he stimulates the students' inquisitiveness, presenting to them problems in the range of their perception and assists them to solve their problems by means of guiding questions, he might inculcate in them a taste and affection for independent thought while developing tools for that" (Pólya, 1985, p.1).

Vase (1993) divides the problems asked during mathematics lessons into three categories: questions dealing with facts, questions dealing with thinking and open-ended questions. Most of the questions (about 80%) cope with remembering facts. When students answer such questions, they should only remember information learnt before. They are not required to compare the facts they have acquired with new information or to doubt previous information which they have learnt. Few are open-ended questions or questions dealing with thinking. The latter demand the problem-solving person to structure or reconstruct from the memory information which is logically organized. Vase distinguishes four types of thinking questions: close-ended thinking questions, grounded on information reconstruction; close-ended questions which do not rely on existing information; open-ended thinking questions, enabling the search of suitable information; and observations designed to interpret what can be seen from the data. As for open-ended questions, Vase (1993) adds that one can ask open-ended questions which do not demand thinking as well as those which do.

As already indicated, creativity constitutes a cognitive process, whereby we create something not known before by means of known components. Guilford (Guilford & Hoepfner, 1971), who engaged in the need for developing creative thinking, defined branching out thinking, enabling people to flow in many and varied directions of possibilities. A branching out thinking reinforces the potential to achieve by using it the most appropriate outcome. He suggests a series of tests for checking creativity through branching out thinking, which examines among others thinking flexibility. An example of a branching out production of symbolic relations is a presentation of a numbers sequence, such as: 1, 3, 5, 10, 11, 17, the requirement being to reach the number 8 by using the four rules. Another assignment is a branching out production of form-shaped transformations. A typical example of it are the challenges of matches placed in some format and one has to remove or move a certain number of matches in order to achieve another format (Guilford & Hoepfner, 1971).

In a study conducted by Gazit (2003), three mathematical challenging problems were presented to 10-grade students of two different study levels and to students training to teach mathematics. The problem solution does not require mathematical knowledge beyond what is learnt at elementary school. It does require, however, an informed use of a suitable algorithm, while properly representing the problem. In other words, the problems necessitated an informed use of creative thinking, since the structure of the questions was not similar to the questions they were used to receive in mathematics lessons. The success percentage among the mathematics teacher-trainees was the lowest (25) among the three investigated groups. Moreover, a small, insignificant difference was found in favor of the 10th graders who study at a 3-unit level, some of learning-disabled (47%), compared to the 10th graders who study at a 4- and 5-unit level (43%). A technical-automatic use of an algorithm, unsuitable to the conditions of the problem, was prominent among those whose solution was wrong. Conversely, those who solved the problem correctly, particularly the 10th graders who study at a 3-unit level, demonstrated creativity skills when they represented the problem data in a graphical-iconic way and, thus, could form an algorithm suitable to the new situation they achieved.

It is the teachers' role to develop the students' capability to solve problems, some of which demand creative thinking. It is therefore important to know the abilities and competences of teachers and teacher-trainees in this field also in order to adapt a suitable training program, nurturing creativity in the solution of irregular challenging problems.

THE STUDY

The research objective

The present study aims to examine the capability of mathematics teachers and teacher-trainees in other disciplines to solve irregular challenging problems in the field of sequences, which are not necessarily numerical.

The research questions

1. Do elementary school mathematics teachers and teacher-trainees in other disciplines manage to cope with irregular challenging problems in mathematics and what are the characteristics of the incorrect solutions?
2. Is there a difference in the solution of irregular problems of sequences between numerical and non-numerical sequences, such as letters or geometrical shapes sequences?

Methodology

A comparative quantitative research, in which subjects from two populations responded to a sequence completion questionnaire (Appendix 1). Furthermore, the subjects were asked to express their reflection about the questions, following their coping with the irregular problems.

Research population

15 female mathematics teachers in elementary school with at least 3 years' seniority, and 36 female teacher-trainees in other disciplines.

Research tools

For the purpose of the present study, a questionnaire consisting of two parts was formulated.

Part I. Six Questions

Each question presented a sequence with terms and the subjects had to complete the subsequent term $(a_n + 1)$ in each of the sequences (see Appendix). The first two sequences are numerical and in the first one there are three ways to achieve a correct answer. The first way is based on the fact that the difference between each term and its predecessor is a number equal to the power of 2. The differences obtained are: 2, 4, 8, and consequently, the difference between the next requested term and the previous term is 16. Thus, we can find the next term. In the second solution way we can see in the sequence terms a difference of the power of 2, with the number 1. The third way is a representation of the new term as the sum of the number 1 plus the product of the previous term by 2. The second is a Fibonacci sequence that is each number is the total sum of its two preceding terms. The third is a letter sequence, comprising the first letter of the days of the week in Hebrew. The expected solution was S, because this is the first letter of the 7th day of the week in Hebrew (Shabbat: Saturday) [See *Non-numerical sequences – Linguistic comment* on p. 321]. In the last three sequences, the answer could be given either by words and/or a drawing of the next term. The fourth sequence consists of geometrical shapes, whose sequence relates to the Hebrew names of the shapes presented in an increasing alphabetical order in Hebrew[1]. Hence, there are several possible answers. For example, the rectangular (**m**alben in Hebrew) can be followed by a parallelogram (**m**akbilit in Hebrew), rhombus (**m**eouyan in Hebrew) or square (**r**iboua in Hebrew). The two last sequences consist of polygons. In the fifth sequence, the number of sides in each polygon is decreased each time by two. In the sixth sequence, the number of sides is decreased each time by a difference which is smaller by 1 (18, 13, 9, 6, namely the difference decreases from 5 to 4, to 3, and so on). Thus, the next form must be a quadrangle.

Part II. An Open-ended Question

The subjects were asked to write down anything which came up to mind about sequences (reflection). They had to express their feeling towards the assignment and demonstrate a free attitude towards the situation to which they were asked to respond.

Research procedure and data processing

The questionnaires were administered at the beginning of the academic year to a group of female elementary school mathematics teachers within the framework of the profes-

[1] The geometrical shapes in the sequence are arranged according to the first letter of their name in the Hebrew alphabet: ellipse (ellipse – 1st letter in the Hebrew alphabet), kite-shaped quadrangle (dalton – 4th letter), trapeze (trapeze – 9th letter), and rectangular (malben – 13th letter).

sionalization program of the Ministry of Education. Simultaneously, they were given to two groups of teacher-trainees, not from the field of mathematics, who attend advanced 4th year courses. The data processing was performed both quantitatively and qualitatively, in each population separately and by comparing the two populations.

FINDINGS

Part I. Six Questions

The objective of the present study was to identify creativity in the solution of irregular assignments, which do not have a pre-defined algorithm and/or have several ways of solution. Analysis of the following findings relates to the strategies through the solutions of those who answered correctly and the strategies of whose who gave an incorrect answer.

Table 1. Distribution of answers among the teachers and teacher-trainees*

Question Number	The entire population $N = 51$		Teachers $N = 15$		Teacher-trainees $N = 36$	
	Answered Correctly	Answered incorrectly	Answered Correctly	Answered incorrectly	Answered Correctly	Answered incorrectly
1	48 (94%)	3 (6%)	14 (93%)	1 (7%)	34 (94%)	2 (6%)
2	27 (53%)	18 (35%)	11 (73%)	4 (27%)	16 (44%)	14 (39%)
3	7 (14%)	33 (65%)	3 (20%)	1 (7%)	4 (11%)	32 (89%)
4	27 (53%)	14 (27%)	10 (67%)	3 (20%)	17 (47%)	11 (31%)
5	45 (88%)	4 (8%)	15 (100%)	-	30 (83%)	4 (11%)
6	37 (73%)	11 (22%)	14 (93%)	1 (7%)	23 (64%)	10 (28%)

* Only the first question was answered by all the participants in the study.

A. Numerical sequences

Question 1. Please complete the next term in the sequence: 1, 3, 7, 15, 31, ...

The correct answer to that question is 63.

<u>The various solution strategies of those who gave correct answers</u>

48 out of the 51 subjects gave a correct answer and three solution strategies were identified. Half of the correct answers were obtained from using the algorithm of the form $a_{n+1} = 2^{n+1} - 1$. That is, the sixth term $2^6 - 1$. About half of the other correct answers

used an algorithm of the type $a_{n+1} = a_n + 2^n$. One subject used an algorithm of the type $a_{n+1} = 2a_n + 1$.

The various solution strategies of those who gave incorrect answers

Three subjects (about 4%) gave incorrect answers. One of the incorrect answers – number 53 – stems from a calculation error, as a result of making a calculation without using a calculator. Another similar mistake was the answer 32 (which is the product of 2×16). In this case, the subject noticed the difference between the number 31 and the required number and by mistake, wrote the difference as the value of the required term, not adding the number 31.

The third mistake, indicating 59 as the following term, is unclear. The subject viewed the differences of the sequence as a differences sequence of the power of 2 and, inexplicably, probably added 28 instead of 32 to the number 31. In other words, in spite of the mistake made by these three subjects, they identified the sequence model. Namely, with reference to the procedure, the subjects recognized or traced the correct procedure and pattern (difference of the power of 2). It should be indicated that in some answers the differences sequence was written in as "to the –th power" and in others in the specific way of 2, 4, 8, 16, ...

If we compare the teachers and the teacher-trainees, the findings show that one teacher made a mistake (wrote 32) and the other two mistakes (53 and 59) were given by the teacher-trainees.

Question 2. Please complete the next term in the sequence: 1, 3, 4, 7, 11, ...

The correct answer to that question is 18.

The various solution strategies of those who gave correct answers

The pattern for this answer has the form of: $a_{n+1} + a_n = a_{n+2}$. 27 subjects (53%) out of 51 gave a correct answer.

The various solution strategies of those who gave incorrect answers

Out of the incorrect answers, one erroneous answer 28 was given without specifying the way; two incorrect answers were 9 and 18, *i.e.* hesitation between two options. Another answer was the number 18, obtained by adding the number 3 to the incorrect sum $7 + 3 = 11$. Other answers were also given, such as: 14 and 18, 13 and 14, 13, 16, 15, 17 when the answer 13/14 or 14/18 attests again to hesitation between the differences of the sequence terms. An interesting answer in this context was: "a composite number" without indicating its value. The explanation for the solution is as follows:

1 is a prime number (and that is wrong since 1 is not a prime number),
3 – prime,
4 – composite,
7 – prime, and
11 – prime.

Hence, the next number must be a composite number. The notion is original-creative because the subject found a pattern which could be correct (for example: instead of prime she could have written non-composite, non-composite, prime and so on). She failed to pay attention that the number 5, which is a prime number, does not appear... Her answers sheet shows that she wrote at first the composite number 6 but deleted her answer and left only the word "composite." Another incorrect and interesting answer was grounded on the assumption that the total of differences of 2 different pairs of adjacent terms is 5. For example: $1+4=5=a_3-a_2=a_5-a_4$. Consequently, if the difference between the two subsequent pairs is also 5 (which it actually is), that means this is the sequence pattern and, therefore, the sixth term is 14. Thus, the sequence would look as follows: 1, 3, 4, 7, 1, 13, 14. The main mistake in this assumption relies on the fact that we cannot determine, based on three differences between terms, the general course of a sequence.

A comparison between the teachers and the teacher-trainees illustrated that all the teachers gave an answer, either correct or incorrect. The answer: "A composite number" was given by a teacher. The other teachers, who made a mistake, wrote either the number 16 or 17. On the other hand, the teacher-trainees attempted to find different options. However, in view of the fact that the topic of sequences was relatively unknown to them, those who answered incorrectly looked for connections and contexts or various, wider algorithms, demonstrating some characteristics of creative thinking. Unlike the teachers, six teacher-trainees did not answer at all. Among those who made a mistake, hesitation between two possible answers was predominant, since they failed to decide. It is important to point out that the teachers, during the in-service training courses which they have to attend (like frameworks of professionalism or other courses) were exposed to the issue of "sequences." They therefore searched for a fixed pattern with which they were familiar, like the known sequence of numbers equal to the total of the two preceding numbers – the Fibonacci sequence. The teacher-trainees, who were unfamiliar with sequences and their course, demonstrated a behavior of open search rather than tied to the frame suitable to the characteristics of creative thinking and without any connection to a correct or incorrect solution.

Non-numerical sequences – Linguistic comments

The non-numerical sequences (letters and geometrical shapes) in the present study use letters and not numbers as their terms. Since the word order in the English and Hebrew alphabet is different, the participants' answers, given in Hebrew, do not match the English letters. Hence, a translation of the words from Hebrew into English will not be in line with the relevant answers. In order to resolve this, we wrote the Hebrew words in English letters, indicating their place in the alphabetical order. Thus, the sequence would make sense and the next term would be logical. In the geometrical shapes we used the English word, indicating the Hebrew word in English letters and trying to explain the way the Hebrew alphabet is structured. A specific explanation for each sequence is given with the sequence itself.

B. Letter sequences

Question 3. Please complete the next term in the sequence: r, s, s, r, h, s ...[2]

The anticipated correct answer was s.

<u>The various solution strategies of those who gave correct answers</u>

7 out of all the subjects answered correctly (about 14%). The sequence is comprised of the first letter of the days of the week in Hebrew. Consequently, the next term is s (Shabbat).

<u>The various solution strategies of those who gave incorrect answers</u>

Out of the incorrect answers 15 subjects replied: *s*, *h*. They found the pattern of *r*, *s*, *s*, *r* so they answered with the two letters, namely two terms which completed the sequence to *h*, *s*, *s*, *h*. One of the explanations attached to the answer was that there is a symmetry, reflection, reversal. One student explained that the two pairs of the first terms are a reversal of the letters; thus, the next two terms should also be a reversal. If we ignore the instruction given in the questionnaire in the question relate to the completion of the next term (a single term), then this solution can be considered somewhat creative, and to a certain extent, even correct. What is interesting in this question is that, contrary to the two

[2] The letters represent the first letter of the days of the week in the Hebrew alphabet: Sunday (**R**ishon), Monday (**S**heni), Tuesday (**S**hlishi), Wednesday (**R**evi'i), Thursday (**H**amishi), Friday (**S**hishi). The anticipated missing term is Saturday (Shabbat).

previous numerical questions, many of the subjects took the liberty of giving an answer with more than one letter (2 letters, 3 letters and up to 6 letters). Among the incorrect answers, three had the form of: *s*, *r*. Other three gave an incorrect answer with a 3-letter combination: *r*, *s*, *s* or h, s, s. Six incorrect answers related to a combination of 5 or more letters, for instance *s*, *r*, *t*, *s*, *s*, *r*, *i*, *s* or *r*, *s*, *s*, *r*, *h*, *s*. One incorrect answer was a combination of 4 letters and another incorrect one was the letter *h*.

When one compares the two research group, the findings show that one teacher and 12 teacher-trainees gave the incorrect answer in the form of *s*, *h*. Eleven teachers did not answer that question at all.

C. Geometric shape sequences

Question 4. Please complete the next term in the sequence: ellipse (ellipse), kite-shaped quadrangle (dalton), trapeze (trapeze), rectangular (malben) …[3]

In this case there is more than one correct answer because the intention in this sequence is to arrange the first letter of the shape in ascending order. The correct answer can be any geometrical shape whose first letter comes after the letter "m" (13th letter in the Hebrew alphabet as the 4th term of the sequence is malben in Hebrew), for example – square (riboua – 20th letter in Hebrew). Or/and that is an option taken into consideration particularly if we relate to creativity – the first letter remains "m" but the second would be a letter whose place is after the letter "L", the second letter of the word malben (we do not count the vowels, only the consonants). Thus, a correct answer could be meouyan (rhombus) or maagal (circle), the second letter in both cases being "ayin" – 16th letter of the Hebrew alphabet.

<u>The various solution strategies of those who gave correct answers</u>

27 subjects gave a correct answer – 10 teachers and 17 teacher-trainees. Their answer was given by drawing a shape or by writing the word describing the next term. Out of those giving the correct answer, 16 subjects wrote square (riboua) – 8 teachers and the remainder – teacher-trainees. Only one student explained what underlined her answer. She wrote: "This is a square (riboua) because it is a regular 4-side shape…." The answer was correct but the explanation and argument were not. She found the pattern – rule – of the sequence in the following way: "First, the ellipse has 0 sides; there are no fragments at all. The second term, the kite-shaped quadrangle, is a quadrangle (but she did not see that it has equal sides).

[3] The geometrical shapes are arranged according to the first letter of their name in the Hebrew alphabet: ellipse (ellipse – 1st letter in the Hebrew alphabet), kite-shaped quadrangle (dalton – 4th letter), trapeze (trapeze – 9th letter), and rectangular (malben – 13th letter).

Then, the trapeze has one pair of equal sides (perhaps she referred to an individual case of an equilateral trapeze). The rectangular has 2 equilateral pairs and, therefore, in the square, all the sides are equilateral, making it a regular shape." The concept is interesting and creative but that student obtained a correct answer in an incorrect way. Out of the remaining correct answers, 6 subjects, all teacher-trainees, wrote a triangle (meshulash in Hebrew), three (two teachers and one student), wrote circle (maagal), one student wrote rhombus (meouyan) and one student wrote octagon (metouman). Another subject indicated as a correct answer the regular hexagon (meshusheh). In her arguments she related to the number of pairs of parallel sides. She wrote as follows: "The kite-shaped quadrangle has no pair of parallel sides. The trapeze has one pair, the rectangle has 2 pairs and the regular hexagon has 3 pairs." The answer is original and creative but does not match the conditions due to the fact that an ellipse is not a quadrangle.

The various solution strategies of those who gave incorrect answers

Classification of the incorrect answers into teacher-trainees or teachers showed one answer given by a teacher, who made a mistake and looked for a pattern connected to equilateral sides (somewhat similar to the argument of the student who wrote square). This teacher pointed out that an ellipse is not a quadrangle and, therefore, has 0 sides. She continued and indicated that a kite-shaped quadrangle is also a shape with 0 equilateral sides (this is a mistake since a kite-shaped quadrangle has two pairs of adjacent equilateral sides and perhaps she meant opposite sides). In the case of the trapeze she wrote that there is one pair of equilateral sides (this, too, is inaccurate because it applies only to an equilateral trapeze). The rectangular has 2 pairs. Then she drew two quadrangles, a convex hexagon with 3 pairs of equilateral sides and also indicated the number 3. Furthermore, she drew another quadrangle, indicating 0 equilateral sides. She tried to ground herself on a pattern of 0, 1, 2, 0 or a pattern of 0, 1, 2, 3 (totally ignoring the ellipse which is not a quadrangle). Two students wrote the word ellipse as their answer. Perhaps they were looking for a sequence whereby the first and last term are the same.

Question 5. Please complete the next term in the sequence: a polygon with 13 sides, 11 sides, 9 sides, 7 sides, ...

The correct answer is a pentagon or, in other words, a 5-side polygon.

The various solution strategies of those who gave correct answers

45 correct answers were given, all of them by drawing and in some of the questions the word pentagon or a 5-side polygon was added. All the 15 teachers gave a correct answer. The remainders (30) were teacher-trainees. Out of the 45 correct answers, in 40

of them the drawing related to a convex polygon. In the other five answers a convex pentagon was drawn (3 students and 2 teachers).

Question 6. Please complete the next term in the sequence: polygon with 18 sides, 13 sides, 9 sides, 6 sides, ...

The correct answer is a quadrangle.

The various solution strategies of those who gave correct answers

Out of the 37 correct answers, 14 were given by teachers and the rest by teacher-trainees. The answers were drawn or explicitly indicated the word "quadrangle."

The various solution strategies of those who gave incorrect answers

Out of the 11 incorrect answers (1 teacher and 10 teacher-trainees), a triangle was drawn in 6 answers, given by teacher-trainees. Two students drew a pentagon while 1 student identified a difference of 2 in a decreasing sequence but, by mistake, drew an open broken line, consisting of 2 segments. That is, she identified the difference 2 and drew it instead of drawing the next term. Another student who made a mistake wrote 24. She read the sequence from left to right and instead of reading correctly and seeing a decreasing sequence; she found a subsequent number for an 18-side polygon. The next term, then, would be a polygon whose number of sides was higher by 6 and she drew next to the number 24 a convex polygon with 24 sides. This answer can be considered as a correct solution since the subject identified the increasing difference but it did not comply with the rules of reading comprehension in mathematics (the location of the term to be completed was on the left and, thus, the polygon had to be with a number of sides smaller than 6).

Only one teacher made a mistake and her answer was that there is no such shape – without any explanation. This is an interesting answer since it is unclear why such a shape does not exist.

The various solution strategies of those who gave incorrect answers

Out of the 4 teacher-trainees who gave incorrect answers, two drew a hexagon, one student drew a heptagon and one drew a convex polygon in the form of 12-side star. No explanations were given to the answer and, therefore, the way of thinking about these answers is unclear.

Part II. An Open-ended Question

The open-ended question – reflection on the assignment

Although all the subjects were asked to write some words about their feelings and reflection regarding the questions, only 10 of them responded, all of them teacher-trainees. Moreover, it is important to mention that all the teacher-trainees wrote, out of their own free will, their name on the questionnaire although they had not been asked to do so. Conversely, none of the teachers wrote her name. Analysis of the reflection illustrated two main indices: the first is frustration, difficulty and pressure and the second manifests pleasure, challenge and interest.

Below are some examples demonstrating frustration, difficulty, failure or pressure:

- "The greatest frustration is that I have not yet managed to solve everything. But I am not going to give up" (this student answered correctly 4 questions, incorrectly 1 question and did not answer at all 1 question).
- "I did not understand the third assignment. This is a frustrating sequence (the days of the week letter sequence)" (This is the reflection of a student who gave four correct answers).
- "The 'cracking' stage is worth every moment of hard work. On the other hand, unsolved sequences are frustrating and at some point make me give up and feel despair" (this student too answered 4 questions correctly).
- "Apparently, the exercises seemed simple and easy to solve but they were much more difficult than they had seemed and I had to think in order to solve them" (this student gave 5 correct answers).
- "I found it very difficult and failed in most of the exercises." (This student gave only one correct answer.)
- "At first I managed to solve only the first question but, some time later, I was more successful."
- "This is to some extent a game; to some extent stressful... you need to keep up with the pace."
- "Such a sheet is sometimes frustrating. You don't always succeed."

Examples attesting to pleasure, challenge and interest:

- "The sheet deals with sequences, but not arithmetic or geometric. Interesting"
- "I like such challenges, riddles. A unique combination...."
- "There is a challenge, let's see 'what do I know'..."

DISCUSSION AND CONCLUSIONS

The article aimed to examine the ability of elementary school mathematics teachers and of teacher-trainees in other disciplines to cope with irregular challenging problems of sequences, some of which not numerical. This concerns questions which do not have a pre-defined algorithm and require a certain extent of breaking regular thinking frameworks and shifting to a way of thinking which is encompasses creative thinking characteristics: searching for a new and unknown pattern of solution out of a series of givens, presenting known items but not necessarily known beforehand. Subjects should make independent assumptions without any prejudices before choosing the solution suitable for the open-ended question (Yee, 2005).

The first research question investigated whether mathematics teachers and teacher-trainees would manage to cope with irregular challenging problems and what would be the typical mistakes. The findings show that the extent of success to answer the questions is associated with the extent to which the question deviates from familiar patters of questions in the investigated area—sequence completion. This conclusion is connected to the second research question which examined whether there would be differences in the success percentage between problems presenting numerical and non-numerical sequences, such as letter or geometrical shape sequences.

Mathematical thinking is usually identified with numbers and shapes whereas language, which serves as a mediator for instructions and questions, is perceived as an inseparable part of the question givens. In the field of sequences, however, we generally engage only in numerical sequences while sequences of shapes do not form part of the mathematics learning routine.

The question with the lowest percentage of success was the one with a sequence of letters representing the first letter of the days of the week. A low percentage of subjects answered correctly and the percentage of non-respondents was the highest among the six questions. The content of this question was different from that of the other questions because it dealt with letters rather than numbers and shapes. This leads to the conclusion that questions with givens with which students have not been acquainted through their studies at school or their training – letters and not numbers – creates a barrier for thinking and prevents them from coping properly. This is supported by part of the incorrect answers, which included a combination of 2, 3, 4, 5, 6 and even more letters, although the requirement was to complete the subsequent term (singular). The subjects are used to letters in various combinations and not in a single form. Most of the creative answers were given by the teacher-trainees and not the actual teachers. Perhaps a sequence which is not represented by numbers of another familiar pattern from the immediate mathematical environment inhibits the thinking or pressures the teacher. This puts in action the mechanism directing us not to answer as a preference to giving an incorrect answer.

A second question, to which the percentage of respondents giving a correct answer was low, was the question combining geometrical shapes with the shape names (Question 4). The subjects had to identify a pattern of letters order increasing alphabetically and only about half of the subjects gave a correct answer. This finding supports the conclusion in the previous question, namely a combination of elements to which we are not accustomed in the standard questions asked and also in connection with sequences, disrupts our normal thinking in certain norms of givens and the relations between them.

One can say with a great degree of certainty that, indeed, there was a difference between non-numerical sequences, *e. g.* letters and geometrical shapes combining letters. This difference illustrates some kind of fixation when instructions for completing numerical sequences are concerned—there is only one possible number (with the exclusion of several subjects who hesitated between two solutions) as compared to a letter sequence, with which they are not acquainted and, then, the options are open.

Another question which received a similar percentage of correct answers (only about half) is Question 2, which dealt in fact with a numerical sequence but an irregular one (in spite of its popularity). The question presented a sequence of numbers, each equal to the total of the two preceding numbers. This is a sequence in the format of the famous Fibonacci sequence and such a relation between numbers in a sequence is irregular. Regular sequences are characterized by a fixed pattern of difference, proportion or difference of differences and the pattern is obvious at first sight. This pattern of a total of two numbers can create many sequences and the rule is evident. However since it is irregular, "blindness" – fixation – is formed. Perhaps if in the question, instead of presenting five terms and looking for the sixth one, six terms had been presented and we had been asked to identify the seventh one, it would have been possible to see the absence of a fixed difference and to think about another sequence. Moreover, we could see in the variety of the incorrect answers, with a possible reason, a feature suitable to creative thinking, requiring a flow of solution options, if one does not refer to a correct or incorrect solution.

The proof is that a problem dealing with numerical sequences, albeit irregular but without a particular rule (Question 1) received the highest percentage of success.

The question which received the second highest percentage of success was the question dealing with a sequence of polygon whose number of sides was smaller by 2 from one shape to the other. The pattern is very clear and, in fact, we have here a projection from a numerical arithmetic sequence, presented by means of shapes. The last question (Question 6), which also engaged in a sequence of polygons whose number of sides was smaller by a non-fixed number from one shape to another, was answered correctly by 3/4 of the subjects. Perhaps it was due to the combination of a geometrical shape sequence with differences in the number of sides.

One can say that when the question deals more with a less regular sequence and requires a more open thinking identified with creative thinking, the level of coping declines. Moreover, percentage of success, which was between 94% and 73% for more familiar questions, decreases to 53% in questions presenting less regular situations and reaches almost the point of failure. That is, only 14% success in a question deviating from the field of classical mathematics (number and shapes) and presenting a letter sequence relating to the days of the week—allegedly a familiar situation but in an irregular context.

As to creative thinking, which should be manifested in solving irregular problems: although the teacher-trainees presented a smaller percentage of correct answers in comparison with the mathematics teachers, the incorrect answers included creative irregular notions? Parts of the answers presented in the findings indicate breaking of the thinking framework and abandoning conformity. The variety in the correct or incorrect answers, in a question with a number of possible answers, was manifested by the teacher-trainees. Perhaps the teaching-learning process, together with attendance in in-service training courses enhances the factual knowledge of the teachers. Nevertheless, they inhibit and block creative thinking, which is not required in the curriculum.

To sum up, these findings lead to the conclusion that teachers' training colleges should teach their graduates to cope with irregular challenging problems as well as develop their creative thinking.

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APPENDIX

SIX QUESTIONS

Dear interviewee

Below are six questions.
Please complete the next term in each of the sequences.
You can use words or a drawing.

First sequence: 1, 3, 7, 15, 31, _____

Second sequence: 1, 3, 4, 7, 11, _____

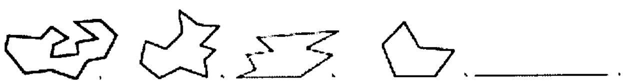
Third sequence[4] : r, s, s, r, h, s _______________

Fourth sequence[5]:

Ellipse	Kite-shaped quadrangle	Trapeze	Rectangle
(Ellipse)	(Dalton)	(Trapeze)	(Malben)

Fifth sequence:

Sixth sequence:



Please express your reflection about the questions. Thank you

[4] The letters represent the first letter of the days of the week in the Hebrew alphabet: Sunday (Rishon), Monday (Sheni), Tuesday (Shlishi), Wednesday (Revi'i), Thursday (Hamishi), Friday (Shishi).

[5] The geometrical shapes are arranged according to the first letter of their name in the Hebrew alphabet: ellipse (ellipse – 1st letter in the Hebrew alphabet), kite-shaped quadrangle (dalton – 4th letter), trapeze (trapeze – 9th letter), rectangular (malben – 13th letter).