Sensitivity Correlations of Electrical Vehicle

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전기 차량의 민감도 상관관계

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Abstract

Generally, finite element models used in structural analysis have some uncertainties of the geometric dimensions, applied loads and boundary conditions, as well as in material properties due to the manufacturability of aluminum intensive body. Therefore, it is very important to refine or update a finite element model by correlating it with dynamic and static tests. The structural optimization problems of automotive body are considered for mechanical structures with initial stiffness due to preloading and in operation condition or manufacturing. As the mean compliance and deflection under preloading are chosen as the objective function and constraints, their sensitivities must be derived. The optimization problem is iteratively solved by a sequential convex approximation method in the commercial software. The design variables are corrected by the strain energy scale factor in the element levels. This paper presents an updated method based on the sensitivities of structural responses and the residual error vectors between experimental and simulation models.

Key Words : Aluminum intensive body(알루미늄 집중 차체), Electrical vehicle(전기 차량), Cross-orthogonality(전단 직교성), MAC(modal assurance criterion:모형 확정 기준), DAC(displacement assurance criterion:변위 확정 기준), DDAC (dynamic displacement assurance criterion:동적 변위 확정 기준), DPR(driving point residue:조향 점 간극)

Nomenclature

[K] Conventional stiffness matrix

- $[K_0]$ Initial stiffness matrix due to preloading
- [*u*] Displacement vector
- $\{F\}$ External force vector

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[G]	Differentiation operator of shape function
[S]	Stress matrix due to preloading
U_0	Strain energy due to preloading
U	Strain energy
ρ	Design variable
$\{\Delta P\}$	Design variable variation
$\{\Delta R\}$	Response error vectors
[Q]	Structural sensitivity matrix
x	Static response
У	Dynamic response
Φ	Displacement assurance criterion
ϕ	Normalized static displacement
Ψ	Dynamic displacement assurance criterion
Ψ	Normalized dynamic displacement
$\varTheta_{i,n}$	Driving point residue
ς	Modal damping ratio
r _n	Frequency ratio

1. Introduction

Recently, in the current vehicle maker, the switch from steel to aluminum in an automotive industry is increasing more and more for the weight reduction and the recycling. The efforts on manufacturing processes for aluminum materials are improving quality, durability and even safety. Most of applications of aluminum materials are increasing in the chassis and body component designs. Automakers have doubled the aluminum used in the vehicles since 1991 and are predicting that those applications will be up to 50 percent in next five years. Ford and Daimler Chrysler plan to produce aluminum vehicle prototypes by 2004 and Audi's and Volkswagon's vehicles with all aluminum bodies are on the road. In particular, the needs for environmental vehicles like electric, hybrid and fuel cell encourage automaker to use the aluminum material for recycling and weight reduction. In South Korea, two types of electric vehicles are used for the environmental friendly vehicle development. One is an aluminum space frame and the other is an electric vehicle modified from steel vehicle. Therefore, in the views of material and manufacturabilitybased geometrical dimensions, the model correlation is

very important for the development of vehicle bodies and this correlated finite element model is applied to other development process such as vehicle crash worthiness. Under these manufacturing environments, the speedy and efficient design processes must be presented and the feasible simulation model be made through the test correlations. Most work has been limited to correlations with test data of a single-structure configuration. However, many correlations are required for material distribution, boundary conditions and physical dimensions. Since their changes that improve one configuration may worsen the correlation of another, the multidisciplinary configuration correlations are essential in the structure under the various design environments. However, the numerical predictions of a structure's static and dynamic responses are often inaccurate due to simplifying assumptions, uncertainty and ignorance. The larger the manufacturing tolerances and environmental changes are, the greater the discrepancy between predicted and measured responses. Generally, structural properties have either local design variables or global design variables in each design stage. Using local and global design variables, detailed structural dimensions are used to generate intermediate structural properties for the correlations (1-6). The local and global design variables are interrelated in an integrated optimization procedure for incorporating static and dynamic structural performances. Because of the computational and decisional efficiencies, both types of design variables may have limitations regarding number and region. Therefore, in order to evaluate static and dynamic qualities in an engineering design, criteria must be defined to interpret the computed results. In general, structural percentage differences of less than 5% should be considered acceptable. Currently, many industries require the use of finite element models to predict the behavior of actual structures. It is very important that these models be constructed with high quality. Errors of discretization must be kept to a minimum^(4,5).

In mechanical structures, a design configuration with structural requirements such as stiffness, strength and durability is needed for most structural development stages and various components. The concept design must take into account a minimum weight structure with maximum performance under the given constraints and loading environments. For the design process, we use an optimization method for the distributions of material properties and physical dimensions with mathematical programming. And since the initial stiffness due to preloading has influence on the overall structural stiffness, the preloading conditions must be considered under the operation and manufacturing environments. If we deal with a structure where the deflection and stress are affected by the preloading conditions, we chose to control their flexibilities, that is, the work of external forces and deflections due to the structure.

This paper presents a model update method based on the structural sensitivity matrix and residual error vectors between test and finite element model for the aluminumintensive electrical vehicle body⁽⁷⁻⁹⁾.

2. Sensitivity based correlation

A sensitivity-based correlation algorithm requires the computation of design variable variations for each configuration based on the corresponding analysis data. The correlation equation can be given by

$$\begin{bmatrix} \mathcal{Q}^{T} [\mathcal{Q}] \{\Delta P\} = [\mathcal{Q}]^{T} \{\Delta R\}$$

$$\begin{bmatrix} \mathcal{Q}^{1} \\ \mathcal{Q}^{2} \\ \vdots \\ \mathcal{Q}^{n} \end{bmatrix}^{T} \begin{bmatrix} \mathcal{Q}^{1} \\ \mathcal{Q}^{2} \\ \vdots \\ \mathcal{Q}^{n} \end{bmatrix}^{T} \begin{bmatrix} \mathcal{Q}^{1} \\ \mathcal{Q}^{2} \\ \vdots \\ \mathcal{Q}^{n} \end{bmatrix}^{T} \begin{bmatrix} \mathcal{Q}^{1} \\ \mathcal{Q}^{2} \\ \vdots \\ \mathcal{Q}^{n} \end{bmatrix}^{T} \begin{bmatrix} \{\Delta R^{1} \} \\ \{\Delta R^{2} \} \\ \vdots \\ \{\Delta R^{n} \} \end{bmatrix}$$

$$(1)$$

where [Q] and $\{\Delta R\}$ are the overall structural sensitivity matrix and response error vectors between test and analysis.

 $\{\Delta P\}$ is the vector of design variable variations.

[Q] and $\{\Delta R\}$ are described as follows:

 $\{\Delta R^n\}$ represents the corresponding error vector of the n-th configuration response.

2.1 Stress stiffening consideration

One can take the view that external forces change the stiffness of a structure. If the forces reverse some physically possible deformation mode, the stiffness of the structure increases or decreases. The effects of preloading are accounted for by a matrix that augments the conventional stiffness matrix. The matrix due to these effects is called the stress stiffness matrix and is defined by an element's geometry, displacement field and stress state. The stress stiffness matrix is independent of elastic properties. Including the stress stiffness matrix under preloading, the force-displacement relationship for the structure is given by

$$([K] + [K_0]) \{u\} = \{F\}$$
(3)

where $\{u\}$ is the displacement vector, $\{F\}$ is the external force vector, [K] is the conventional stiffness matrix concerning the elastic properties, and $[K_0]$ is the initial stress stiffness matrix.

$$\begin{bmatrix} K_0 \end{bmatrix} = \sum_{e=1}^{\infty} \begin{bmatrix} k_0 \end{bmatrix}_e$$
(4)

Let the element displacement field be given by $\{u\} = [N]\{d\}$ and $\nabla\{u\} = \nabla[N]\{d\} = [G]\{d\}$. The element's stress stiffness matrix can be given by

$$\begin{bmatrix} k_0 \end{bmatrix}_e = \int_{V_e} \begin{bmatrix} G \end{bmatrix}^T \begin{bmatrix} S \end{bmatrix} \begin{bmatrix} G \end{bmatrix} dV$$
(5)

where [G] is obtained from shape function, [N] by appropriate differentiation and [S] contains the stress level due to the preloading. [N] is the shape function matrix.

$$[S] = \begin{bmatrix} \widetilde{S} & 0 & 0 \\ 0 & \widetilde{S} & 0 \\ 0 & 0 & \widetilde{S} \end{bmatrix}, \quad \begin{bmatrix} \widetilde{S} \end{bmatrix} = \begin{bmatrix} \sigma_{x0} & \tau_{xy0} & \tau_{zx0} \\ \tau_{xy0} & \sigma_{y0} & \tau_{yz0} \\ \tau_{zx0} & \tau_{yz0} & \sigma_{z0} \end{bmatrix}$$
(6)

The stored strain energy due to the preloading is given by

$$U_{0} = \int_{V} \left(\frac{1}{2} \left(u_{,x}^{2} + v_{,x}^{2} + w_{,x}^{2} \right) \sigma_{x 0} + \dots + \left(u_{,x} u_{,z} + v_{,x} v_{,z} + w_{,x} w_{,z} \right) \tau_{zx 0} \right) dV$$

$$= \int_{V} \{ \varepsilon \}^{T} \{ \sigma_{0} \} = \frac{1}{2} \{ u \}^{T} [K_{0}] \{ u \}$$
(8)

where

$$\{\varepsilon\}^T = \{\varepsilon_x \ \varepsilon_y \cdots \gamma_{zx}\}, \{\sigma_0\}^T = \{\sigma_{x\,0} \ \sigma_{y\,0} \cdots \tau_{zx\,0}\}$$

To design a stiffening structure under a given loading, mean compliance is chosen as the objective function, which is defined as the least amount of displacement and the minimum mean compliance. Thus, optimization involves not only minimizing the mean compliance or elastic strain energy of the structure, but also minimizing the effect of external forces. The compliance of a structure with stress stiffness under a given loading can be written as

$$U = \frac{1}{2} \{u\}^T ([K] + [K_0]) \{u\}.$$
(9)

Taking the derivatives of eq. (9) with respect to the design parameter gives,

$$\frac{dU}{d\rho} = U' = \frac{1}{2} \left\{ \frac{du}{d\rho} \right\}^{T} \left(\left[K \right] + \left[K_{0} \right] \right) \left\{ u \right\}$$

$$+ \frac{1}{2} \left\{ u \right\}^{T} \left(\left[\frac{dK}{d\rho} \right] + \left[\frac{dK_{0}}{d\rho} \right] \right) \left\{ u \right\} + \frac{1}{2} \left\{ u \right\}^{T} \left(\left[K \right] + \left[K_{0} \right] \right) \left\{ \frac{du}{d\rho} \right\}$$

$$= \frac{1}{2} \left\{ u' \right\}^{T} \left(\left[K \right] + \left[K_{0} \right] \right) \left\{ u \right\} + \frac{1}{2} \left\{ u \right\}^{T} \left(\left[K' \right] + \left[K_{0} \right] \right) \left\{ u \right\}$$

$$+ \frac{1}{2} \left\{ u \right\}^{T} \left(\left[K \right] + \left[K_{0} \right] \right) \left\{ u \right\} + \frac{1}{2} \left\{ u \right\}^{T} \left(\left[K' \right] + \left[K_{0} \right] \right) \left\{ u \right\}$$

$$= \left\{ u' \right\}^{T} \left(\left[K \right] + \left[K_{0} \right] \right) \left\{ u \right\} + \frac{1}{2} \left\{ u \right\}^{T} \left(\left[K' \right] + \left[K_{0} \right] \right) \left\{ u \right\}$$

$$(10)$$

Introducing the derivative of eq. (3), eq. (10) can be rewritten by

$$U' = \{u'\}^T \{F\} + \frac{1}{2} \{u\}^T \left([K'] + [K_0'] \right) \{u\}.$$
(11)

Assuming that the coordinates under a given loading are only considered in the work done by the external forces and that the coordinates in the free domain are not under loading, the following may be defined:

$$\{u'\}^T \{F\} = \left[\frac{du_f}{d\rho}^T \frac{du_e}{d\rho}^T \right] \left\{ \begin{array}{l} 0\\ F_e \end{array} \right\} = \left\{ \frac{du_e}{d\rho} \right\}^T \{F_e\}$$
$$= \left\{ \{u_e\}^T \{F_e\} \right\}'$$
$$= 2 U'$$
(12)

where u_f is the displacement field that is not under a given loading and u_e is the displacement field under a given loading.

Using eq. (12), the sensitivity of compliance from eq. (11) is given by the following expression

$$U' = -\frac{1}{2} \{u\}^{T} \left([K'] + [K_{0}'] \right) \{u\}$$

= $U_{e}' + U_{0}'$ (13)

where U_e' and U_0' are the strain energy sensitivities due to external forces and preloading. [K'] and $\begin{bmatrix} K_0' \end{bmatrix}$ defined on an element level can be given by the material densities of elements relative to the stress ratio. The full stress scaling is used for the preloading. The derivative of the stress stiffness matrix depends on the initial stress $\{\sigma_0\}$. If these stresses remain constant, $\begin{bmatrix} K_0' \end{bmatrix}$ is zero. But the topological distribution or geometric dimensions of structure under preloading may change on the design domain and in structural rigidities.

$$\left[K_{0}'\right] = \frac{d}{d\rho} \sum_{e=1}^{r} \int_{V_{e}} [G]^{T} [S][G] dV_{e} = \sum_{e=1}^{r} \int_{V_{e}} [G]^{T} [S'][G] dV_{e}$$
(14)

where [S'] is the derivatives of initial stress with respect to the topological distribution and geometric dimensions of the structure.

2.2 Static response correlation

In the static analysis, the response error vector $\langle \Delta R^n \rangle$ consists of the n-th normalized displacement of configuration and DAC (Displacement Assurance Criterion), Φ .

$$\left\{\!\Delta R^n\right\}\!=\!\left\{\!\!\begin{array}{c}\!\Delta x^n\\\!\Delta \Phi^n\end{array}\!\right\} \tag{15}$$

where

$$\Delta x^{n} = \frac{\Delta x^{n}}{x_{test}^{n}} = \frac{x_{test}^{n} - x_{analysis}^{n}}{x_{test}^{n}}$$
(16)

$$\Delta \Phi^{n} = 1 - \Phi^{n}$$

$$= 1 - \left(\frac{\left(\left\{ \phi_{test}^{n} \right\}^{T} \left\{ \phi_{analysis}^{n} \right\} \right)^{2}}{\left(\left\{ \phi_{test}^{n} \right\}^{T} \left\{ \phi_{test}^{n} \right\} \right) \left(\left\{ \phi_{analysis}^{n} \right\}^{T} \left\{ \phi_{analysis}^{n} \right\} \right)} \right)$$
(17)

 ϕ_{test}^{n} and $\phi_{analysis}^{n}$ represent the n-th normalized static displacement vector of the test and analysis. The sensitivity sub-matrices can be given by,

$$\left[\mathcal{Q}^{n}\right] = \begin{bmatrix} \left[\frac{\partial \mathbf{x}^{n}}{\partial P}\right] \\ \left[\frac{\partial \Phi^{n}}{\partial P}\right] \end{bmatrix}$$
(18)

2.3 Dynamic response correlation

In the dynamic response analysis, the response error vector $\{\Delta R^n\}$ can be given by,

$$\{\Delta R\} = \begin{cases} \left\{ \Delta R^{1} \\ \Delta R^{2} \\ \vdots \\ \left\{ \Delta R^{n} \right\} \end{cases} = \begin{cases} \Delta y^{1} \\ \Delta \Psi^{1} \\ \Delta y^{2} \\ \Delta \Psi^{2} \\ \vdots \\ \Delta Y^{n} \\ \Delta \Psi^{n} \end{cases}$$
(19)

where the error vector $\{\Delta R^n\}$ consists of the n-th independent variable and DDAC (Dynamic Displacement Assurance Criterion), Ψ under unit dynamic loading.

$$\Delta y^n = \frac{\Delta y^n}{y_{test}^n} = \frac{y_{test}^n - y_{analysis}^n}{y_{test}^n}$$
(20)

$$\Delta \Psi^{n} = 1 - \Psi^{n}$$

$$= 1 - \left(\frac{\left(\left\{ \Psi_{test}^{n} \right\}^{T} \left\{ \Psi_{analysis}^{n} \right\} \right)^{2}}{\left(\left\{ \Psi_{test}^{n} \right\}^{T} \left\{ \Psi_{test}^{n} \right\} \right) \left(\left\{ \Psi_{analysis}^{n} \right\}^{T} \left\{ \Psi_{analysis}^{n} \right\} \right)} \right)$$
(21)

 y_{test}^{n} and $y_{analysis}^{n}$ represent the m-th dynamic response time or frequency of behaviors under dynamic loadings. ψ_{test} and $\psi_{analysis}$ represent the normalized dynamic displacements of test and analysis. The sensitivity matrix consists of two parts: the sensitivity matrices of normalized response times of behaviors and DDAC with respect to design variables, $\{P\}$.

$$[Q] = \begin{bmatrix} Q^{1} \\ Q^{2} \\ \vdots \\ Q^{n} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \frac{\partial y^{1}}{\partial P} \\ \frac{\partial \Psi^{1}}{\partial P} \\ \frac{\partial \Psi^{2}}{\partial P} \\ \frac{\partial \Psi^{2}}{\partial P} \\ \vdots \\ \frac{\partial \psi^{n}}{\partial P} \\ \frac{\partial \Psi^{n}}{\partial P} \end{bmatrix}$$
(22)

3. Optimization scheme for the update of finite element model

The updated finite element model using sensitivity analysis minimizes the residual difference between test and analysis. The residual differences are defined by the differences of static deflections and modal vectors of each natural frequency. The residual differences of modal vectors are calculated from entries of diagonals and off-diagonals of MAC and transfer function differences. The changes to the model update can be represented by eq. (23). The design variables are simultaneously used in the static and dynamic response correlations.

$$\{R(P)\}^{i+1} = \{R(P)\}^{i} + [Q]^{i} \{\Delta P\}$$
(23)

where
$$Q_{i,j} = \frac{\partial R_i}{\partial P_j}$$

The design process for mechanical systems can be viewed as an optimization process to find parts that fulfill certain quality requirements toward their functionality, appearance and economy. It can be described as an iterative search process that uses the following steps:



Fig. 1 Design overall process

- ① Define an initial design $P^{(i=0)}$
- 2 Analyze the properties of the components
- ③ Compare the results of the analysis with the requirements, including allowable static and dynamic performance
- ④ If the requirements are not met, change the design such that $P^{(i+1)} = P^{(i)} + \Delta P$

The general formulation of an optimization problem appears as

 $\begin{array}{l} \textit{Objective: } \Delta R(P) \Rightarrow \min\\ \textit{Constraint } s: G_i(P) \leq 0\\ \textit{Design space: } P^L \leq P \leq P^U \end{array}$

Objective $\Delta R_i(P)$ can be approximated for each design $P^{(i)}$ using the series expansion,

$$\Delta R_i = \Delta R_i \left(P^{(i)} \right) + \sum_{j=1}^n \frac{\partial (\Delta R)_i}{\partial P_j} \Delta P_j \quad . \tag{24}$$

The gradient $\frac{\partial (\Delta R)_i}{\partial P_j}$ can be obtained directly from the results of finite element analysis. If the gradient is known, the search direction ΔP can be obtained from the solution of an approximate optimization problem.

4. Simulation

4.1 Static correlation

For the static response, the bending and torsion tests



(a) Aluminum Finite Element Frame model



(b) Manufactured model Fig. 2 B.I.W. model of electrical vehicle

are performed and using the same loading conditions, the static simulations are performed. In the bending test and simulation, each center point of front shock towers is constrained in the vertical direction (z) and each center point of rear shock towers is constrained in the three translational directions (x, y, z). The vertical force at the center point connected to seat hip points is acted by 3,332 N in the -z direction. In the torsion test and



(a) Results of static bending test and simulation







simulation, each center point of rear shock towers is constrained in three translational directions and the center point between two front shock towers is acted by torque of 1,356 Nm. The finite element model and manufactured model are shown in figure 2. The results of bending and torsion simulations compared with test through the updated process are shown in figure 3.

4.2 Dynamic correlation

For finding a feasible finite element model for dynamic response, it is necessary to choose a measuring point with deflection or kinetic energy in static or dynamic views, respectively. The most difficult correlation is performed in the dynamic views. So, this simulation represents the correlation in the dynamic views. The most important factor in the dynamic views is for the engineer to note that if the shape functions are chosen such that they satisfy some orthogonality relationships, the kinetic energy due to $i \neq j$ can be set equal to zero. In the case that the i-th elemental mass has the effect of n-th mode shape, the elemental kinetic energy in the frequency response analysis can be given by.

$$KE = \frac{1}{2}m_i V_{i,n}^2 = \frac{1}{2}m_i \psi_{i,n}^2 f_n^2, \qquad (25)$$

where m_i is the *i*-th elemental mass, $V_{i,n}$ and $\Psi_{i,n}$ are the velocity and shape function at f_n , and f_n is the frequency of the *n*-th mode.

For finding an arbitrary driving point under the restraints of a vehicle model, the density of point residue can be defined as follows. Taking into account the mode shape of the *i*-th point and the *j*-th point against the *n*-th mode, the frequency response functions can be given by

$$H_{i,j,n}(f) = \sum_{n=1}^{N} \left(\frac{\psi_{i,n} \, \psi_{j,n}}{k_n} \right) \left[\frac{1}{\left(1 - r_n^2 \right) + \hat{j}(2\,\zeta\,r_n)} \right]$$
(26)

where k_n is the modal stiffness of the n-th mode and the relationship with the mode shape and mass matrix is $k_n = f_n^2 \psi_{i,n} m \psi_{j,n}$.

 ϵ is the modal damping ratio $\left(=\frac{c}{2mf_n}\right)$, r_n is the frequency ratio $\left(=\frac{f}{f_n}\right)$.

Eq. (26) is frequently employed in determining the vibrational characteristics of a system experimentally. If i = j, then $\psi_{j,n} = \psi_{i,n}$. If $i \neq j$, $\psi_{i,n} \cdot \psi_{j,n} \approx 0$ because of the orthogonalities due to mode tracking among the mode shapes. However, the actual problem has no zero off-diagonal orthogonalities and thus the off-diagonal orthogonalities generally have a limit of $\psi_{i,n} \cdot \psi_{j,n} < 0.2$. The maximum driving point residue (DPR) of an arbitrary *n*-th mode can be given by



Fig. 4 Updating flowchart of the vehicle model for dynamic performance correlation

$$\left(\Theta_{i,n}\right)_{\max} = \frac{\{\psi_{i,n}\}^T \{\psi_{i,n}\}}{k_n} = \frac{\{\psi_{i,n}\}^T \{\psi_{i,n}\}}{m_i f_n^2}$$
(27)

From equations (25), (26) and (27), we know that the density of kinetic energy is related to the point residue. For investigating the global dynamic behavior of a structure, the point with the maximum point residue or the maximum density of kinetic energy has to be chosen and excited. Using the aluminum frame model, the update and correlation between test and simulation are shown. The dynamic responses are calculated by the commercial software and the optimization algorithm minimizing the residual response errors is performed on theresults. The boundary condition is set as the test mounting condition in order to estimate the feasible dynamic characteristics. The driving point is chosen as the point with maximum kinetic energy and driving residue on modal analysis. For performing the design target, the optimization process is shown in figure 4. For driving point selection, the sum contour of scaled kinetic energy from the 1st to the 5th modes is shown in figure 5. The MAC values of the chosen driving points



Fig. 5 Sum of scaled kinetic energy from 1st to 5th mode

Table 1 MAC matrix for candidate driving points

No.		1	2	3	4	5
Freq. (Hz)		190.94	235.32	290.45	325.26	377.90
1	190.94	1.000	0.000	0.001	0.000	0.000
2	235.32	0.000	1.000	0.000	0.000	0.002
3	290.45	0.001	0.000	1.000	0.002	0.001
4	325.26	0.000	0.000	0.002	1.000	0.001
5	377.90	0.000	0.003	0.001	0.001	1.000



Fig. 6 Scaled driving point residue for candidate exciting points

in figure 5 are given in table 1, and the maximum offdiagonal values must be less than 0.2% to determine the calculated modes from the finite element model. Figure 6 shows some driving points with the average driving residues in the frequency ranges. The maximum driving residue occurs at the front shock absorber and Reinf-tunnel UPR. The candidate driving points are compared with those of the test. The finite element model based on geometric dimensions has the problems of clearance among layers, welding material properties and uneven thickness in the panels. So, the MAC matrix and frequency errors for the initial model in figure 6 can generally be assumed as shown in table 2. Because the off-diagonal values in MAC and frequency errors are respectively over 0.6 and 5%, improvement in the finite element model is required. Figure 7 shows the sensitivity results of the initial finite

 Table 2 MAC matrix and frequencies error for initial finite element model

Modal Test							Freq.	
F E M	No. Freq. (Hz)		1	2	3	4	5	Error
			190.66	200.76	284.56	295.54	340.34	(%)
	1	190.94	0.497	0.637	0.015	0.004	0.001	23.42
	2	235.32	0.354	0.278	0.002	0.128	0.008	-4.89
	3	290.45	0.014	0.006	0.001	0.845	0.000	14.23
	4	325.26	0.012	0.006	0.895	0.004	0.001	-1.72
	5	377.90	0.012	0.005	0.000	0.000	0.900	11.04



Fig. 7 Sensitivity results of initial finite element model

element model. In comparing the finite element model and manufactured products, thickness and material properties vary within 10% on static and dynamic analysis. The flexural stiffness in finite element models is given by:

$$D = \frac{Et^3}{12(1-v^2)}$$
(28)

Table 3 Main design variables chosen for minimizing the dynamic response errors

Ty's	Design variables	Initial dimension Values	Product dimension variations (Allowable limit)	Modified dimension values
1	FRONT CROSS UPR	2.0		2.20
2	C/MBR-RADI. FRT	2.3		2.40
3	C/MBR-RADI. RR	2.3		2.35
4	LONGITUDINAL-FRT,FRT	2.3		2.20
5	LONGITUDINAL-FRT,RR	2.3		2.45
6	WHEELHOUSE-FRT, FRT	1.8		2.00
7	WHEELHOUSE-FRT, RR	1.8		2.00
8	LOCATOR-SHOCK ABSORBER	3.5		3.80
9	BRKT-ENGINE MTG,UPR	2.3		2.35
10	BRKT-ENGINE MTG,LWR	2.5		2.65
11	EXTEN-FRT LONGI.	2.0	$\pm 5 \sim 10\%$ in the product tolerance	2.00
12	BRACE-FRT LONGI	2.0		2.30
13	DASH TOP	2.3		2.40
14	DASH UPR	2.5		2.65
15	BULKHEAD-DASH UPR	2.0		2.30
16	C/MBR-DASH UPR	2.0		1.85
17	DASH LWR	2.3		2.50
18	TUNNEL	2.6		2.85
19	PANEL-FLOOR, FRT	1.8		2.00
20	PANEL-FLOOR, RR	2.3		2.35
21	C/MBR-RR SEAT,FRT	2.5		2.45
22	C/MBR-RR SEAT,RR	3.0		2.80
23	BRKT-RR AXLE MTG	2.3		2.25
24	LONGITUDINAL-RR	2.3		2.35
25	Young's modulus	65GPa	±10% in test	62.8
26	Welding stiffness (heat-affected zone)	55.2GPa	±10% in test	52.3

Modal Test							Freq.	
F E M	No. Freq. (Hz)		1	2	3	4	5	Error
			190.66	200.76	284.56	295.54	340.34	(%)
	1	191.25	0.823	0.034	0.006	0.006	0.001	0.31
	2	202.22	0.013	0.805	0.003	0.001	0.002	0.73
	3	286.47	0.000	0.003	0.883	0.037	0.006	0.67
	4	294.03	0.003	0.002	0.044	0.848	0.017	-0.51
	5	335.92	0.002	0.003	0.002	0.037	0.878	-1.30

Table 4 MAC matrix and frequency errors for final updated finite element model

where E is Young's modulus, t is the panel thickness and v is Poisson's ratio.

The dynamic response error vector is shown by eq. (19) and for minimizing it, the design variables in figure 7 are selected on kinetic energy distributions, geometric dimensions and welding lines as shown in table 3. The critical design variables sensitive to structural performance are chosen on the base of sensitivity results shown in figure 7, with consideration of allowable product variations. Table 4 shows the results of the updated finite element model which has a larger diagonal value of MAC than 0.8.

5. Conclusions

This paper presents the analytical model update with respect to the MAC correlations between test and finite element models. It has a sensitivity-based model configuration and the optimization design for achieving the target through the automotive body. The feasible measuring points are set using the modal kinetic energy and, through the driving point residue, the effective driving points may be defined as follows.

- The model update is effectively made on the basis of natural frequency errors and cross-orthogonalities between mode shapes.
- The physical dimensions and material properties can simultaneously be used as design parameters for updating the finite element model.
- 3. Mechanical structures are considered under a preloading

environment for the preloading configuration must be included in the optimization analysis as well as in the sensitivity analysis.

- 4. The preloading effect has two contributions. One is the initial deformation level. The other is the initial stress level corresponding to the initial loading.
- For nonlinear characteristics where the residual deformation, strain and stress are important, it is necessary to perform structural optimization considering the preloading condition.

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