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EXISTENCE AND EXPONENTIAL STABILITY OF ALMOST PERIODIC SOLUTION FOR SHUNTING INHIBITORY CELLULAR NEURAL NETWORKS WITH DISTRIBUTED DELAYS AND LARGE IMPULSES

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Abstract. This paper considers the problem of existence and exponential stability of almost periodic solution for shunting inhibitory cellular neural networks with distributed delays and large impulses. Based on the contraction principle and Gronwall-Bellman's inequality, some sufficient conditions are obtained. The results of this paper are new and they complement previously known results.

1. Introduction

Cellular neural networks, introduced by Chua and Yang [9, 10], have been extensively investigated due to their important applications in such fields as image processing and pattern recognition [3, 4, 5, 6, 7, 26]. It is known that time delays are inevitable in the interactions between neurons. Bouzerdout and Pinter [2] have introduced a new class of CNNs, namely the shunting inhibitory CNNs (SICNNs). SICNNs have been extensively applied in psychophysics, speech, perception, robotic, adaptive pattern recognition, vision, and image processing. Cosider a two-dimensional grid of processing cells, let C_{ij} denote the cell at the (i, j) position of the lattice, the r-neighborhood $N_r(i, j)$ of C_{ij} is

$$N_r(i,j) = \{C_{hl} : \max(|h-i|, |l-j|) \le r, 1 \le h \le m, 1 \le l \le n\}.$$

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In SICNNs, neighboring cells exert mutual inhibitory interactions of the shunting type. The dynamics of a cell C_{ij} is described by the following nonlinear ordinary differential equation:

$$\frac{dx_{ij}(t)}{dt} = -a_{ij}(t)x_{ij}(t) - \sum_{C_{hl}\in N_r(i,j)} C_{ij}^{hl}(t)f_{ij}(x_{hl}(t))x_{ij}(t) + L_{ij}(t),$$

where x_{ij} is the activity of the cell C_{ij}, $L_{ij}(t)$ is the external input to C_{ij}, a_{ij} represents the passive delay rate of the cell activity, $C_{ij}^{hl}(t)$ is the connection of coupling strength of postsynaptic activity of the cell transmitted to the cell C_{ij}, and the function $f_{ij}(\cdot)$ is a continuous function representing the output or firing rate of the cell C_{hl}. It is well known that studies on neural dynamic systems not only involve a discussion of stability property, but also involve many dynamic behaviors such as periodic oscillatory behavior, almost periodic oscillatory properties, chaos and bifurcation. In applications, if the various constituent components of the temporally nonuniform environment are with incommensurable (nonintegral multiples) periods, one has to consider the effects of the environmental factors, and the assumption of almost periodicity is more realistic, more important and more general. For significance of almost periodicity, one also can refer to [8, 11, 23, 24, 27]. Li [13] obtained several sufficient conditions to ensure the global exponential stability of almost periodic solutions or periodic solutions for the delayed SICNNs. The criteria for the stability of almost periodic solutions of the SICNNs with distributed delays were given in [17, 18, 30]. Recently, uniform asymptotical stability of almost periodic solutions of the SICNNs with both time-varying and distributed delays was studied in [19].

On the other hand, the theory of impulsive differential equations is now being recognized to be not only richer than the corresponding theory of differential equations without impulse, but also represents a more natural framework for mathematical modelling of many real-world phenomena, such as population dynamics and the neural networks. In recent years, the impulsive differential equations have been extensively studied (see the monographs [1, 12, 20] and the works [14, 15, 16, 21, 22, 28, 29]). Xia [25] investigated the exponential stability problem of almost periodic solutions for the SICNNs with impulses, but he did not take time delays into account.

To the author's best knowledge, there is no published paper considering the almost periodic solutions for SICNNs neural networks with both impulses and distributed delays. In this paper, we study the impulsive SICNNs neural networks with almost periodic coefficients and distributed delays

(1)
$$\begin{cases} \dfrac{\mathrm{d}x_{ij}(t)}{\mathrm{d}t} = -a_{ij}(t)x_{ij}(t) - \displaystyle\sum_{C_{hl}\in N_r(i,j)} C_{ij}^{hl}(t)\int_0^\infty k_{ij}(u)g(x_{hl}(t-u))\mathrm{d}u x_{ij}(t) + L_{ij}(t),\ t\neq t_k, \\ \Delta x_{ij}(t_k) = \alpha_{ij}^k x_{ij}(t_k) + I_{ij}^k(x_{ij}(t_k)) + L_{ij}^k,\ t = t_k,\ i=1,2,\dots,m,\ j=1,2,\dots,n, \end{cases}$$

where $\Delta x_{ij}(t_k) = x_{ij}(t_k^+) - x_{ij}(t_k^-)$ are impulses at moments t_k and $t_1 < t_2 < \cdots$ is a strictly increasing sequence such that $\lim_{k\to\infty} t_k = +\infty$.

We denote by $x(t) = x(t, t_0, z_0)$, $x = (x_{11}, \dots, x_{1n}, \dots, x_{i1}, \dots, x_{in}, \dots, x_{m1}, \dots, x_{mn})^{\mathrm{T}}$, $x_0 = (x_{011}, \dots, x_{01n}, \dots, x_{0i1}, \dots, x_{0in}, \dots, x_{0m1}, \dots, x_{0mn})^{\mathrm{T}} \in \Omega$, where Ω is a domain in $\mathbb{R}^{(m,n)}$, $\Omega \neq \phi$. The system (1) is supplemented with initial values problem given by

$$x(t_0 + 0, t_0, x_0) = x_0. \tag{2}$$

Denote by $PC(J, \mathbb{R}^{m\cdot n})$, $J \in \mathbb{R}$, the space of all piecewise continuous functions $x : J \to \mathbb{R}^{m\cdot n}$ with points of discontinuity of the first kind t_k, $k = \pm 1, \pm 2, \dots$ and which are continuous from the left, i.e., $x(t_k - 0) = x(t_k)$.

The rest of this paper is organized as follows. In the next section, we shall introduce some definitions and lemmas. Section 3 is devoted to establishing some criteria for the existence, uniqueness and exponential stability of almost periodic solution of system (1).

2. Definitions and lemmas

In this section, we shall introduce some known definitions and lemmas (to see [1, 12, 20]).

Since the solution of problem (1), (2) is a piecewise continuous function with points of discontinuity of the first kind $t = t_k$, $k \in \mathbb{Z}$ and we adopt the following definitions and Lemmas for almost periodicity.

Let $B = \{\{t_k\}_{k=-\infty}^{\infty} : t_k \in \mathbb{R}, t_k < t_{k+1}, k \in \mathbb{Z}, \lim_{k\to\pm\infty} t_k = \infty\}$ be the set of all sequence unbounded and strictly increasing. A matrix or vector $D \geq 0$ means that all entries of D are greater than or equal to zero. For matrices or vectors D and E, $D \geq E$ means $D - E \geq 0$.

Definition 1 ([20])**.** The set of sequences $\{t_k^j\}$, $t_k^j = t_{k+j} - t_k$, $k \in \mathbb{Z}$, $j \in \mathbb{Z}$, $\{t_k\} \in B$ is said to be uniformly almost periodic if for arbitrary $\varepsilon > 0$ there exists relatively dense set of ε-almost periods common for any sequences.

Definition 2 ([20])**.** A piecewise continuous function $\varphi : \mathbb{R} \to \mathbb{R}^{m\cdot n}$ with discontinuity of first kind at the points t_k is said to be almost periodic, if

- **(a):** the set of sequence $\{t_k^j\}$, $t_k^j = t_{k+j} - t_k$, $k \in \mathbb{Z}$, $j \in \mathbb{Z}$, $\{t_k\} \in B$ is uniformly almost periodic;
- **(b):** for any $\varepsilon > 0$ there exists a real number $\delta > 0$ such that if the point $t^{'}$ and $t^{''}$ belong to one and the same interval of continuity of $\varphi(t)$ and satisfy the inequality $|t^{'} - t^{''}| < \delta$, then $|\varphi(t^{'}) - \varphi(t^{''})| < \varepsilon$;
- **(c):** for any $\varepsilon > 0$ there exists a relatively dense set T such that if $\tau \in T$, then $|\varphi(t+\tau) - \varphi(t)| < \varepsilon$ for all $t \in \mathbb{R}$ satisfying the condition $|t - t_k| > \varepsilon$, $k \in \mathbb{Z}$.

Together with the system (1) we consider the linear system

$$\begin{cases} \dot{Z}(t) = P(t)Z(t), \ t \neq t_k, \\ \Delta Z(t) = P_k Z(t), \ t = t_k, \ k \in \mathbb{Z}. \end{cases} \tag{3}$$

Introduce the following conditions:

(i): $P(t) \in C(\mathbb{R}, \mathbb{R}^{m\cdot n})$ and it is almost periodic in the sense of Bohr.

(ii): $\det(E + P_k) \neq 0$ and the sequence $\{P_k\}$, $k \in \mathbb{Z}$ is almost periodic, $E \in \mathbb{R}^{(m\cdot n)\times(m\cdot n)}$.

(iii): The set of sequences $\{t_k^j\}$, $t_k^j = t_{k+j} - t_k$, $k \in \mathbb{Z}$, $j \in \mathbb{Z}$, $\{t_k\} \in B$ is uniformly almost periodic and there exists $\theta > 0$ such that $\inf_k t_k^1 = \theta > 0$.

Recall [1] that if $U_k(t, s)$ is the Cauchy matrix for the system

$$\dot{Z}(t) = P(t)Z(t),\ t_{k-1} < t \leq t_k,\ \{t_k\} \in B,$$

then the Cauchy matrix for the system (3) is in the form

$$W(t,s) = \begin{cases} U_k(t,s),\ t_{k-1} < t \leq t_k, \\ U_{k+1}(t, t_k + 0)(E + P_k)U_k(t,s),\ t_{k-1} < s \leq t_k < t \leq t_{k+1}, \\ U_{k+1}(t, t_k + 0)(E + P_k)U_k(t_k, t_k + 0) \cdots (E + P_i)U_i(t_i, s), \\ t_{i-1} < s \leq t_i \leq t_k < t \leq t_{k+1}. \end{cases}$$

Lemma 1 ([20])**.** *In addition to conditions* (i)-(iii) *are fulfilled.*

(iv): *For the Cauchy matrix $W(t,s)$ of the system* (3) *there exist positive constants K and λ such that*

$$|W(t,s)| \leq Ke^{-\lambda(t-s)},\ t \geq s,\ t, s \in \mathbb{R}.$$

Then for any $\varepsilon > 0$, $t \in \mathbb{R}$, $s \in \mathbb{R}$, $t \geq s$, $|t - t_k| > \varepsilon$, $|s - t_k| > \varepsilon$, $k \in \mathbb{Z}$ there exists a relatively dense set T of ε-almost periods of the matrix $P(t)$ and a positive constant Γ such that for $\tau \in T$ it follow:

$$|W(t + \tau, s + \tau) - W(t,s)| \leq \varepsilon\Gamma e^{\frac{\lambda}{2}(t-s)}.$$

Lemma 2 ([20])**.** *Let the condition* (iii) *be fulfilled. Then for any $p > 0$ there exists a positive integer N such that on each interval of length p no more than N elements of the sequence $\{t_k\}$, i.e.,*

$$i(t,s) \leq N(t-s) + N,$$

where $i(t,s)$ is the number of the points t_k lying in the interval (s,t).

Lemma 3 ([20])**.** *In addition to condition* (iii)*, let the following conditions be fulfilled.*

(v): *The function $\varphi \in PC(\mathbb{R}, \Omega)$, $\Omega \subset \mathbb{R}^{m\cdot n}$ and it is almost periodic.*

Then the sequence $\{\varphi(t_k)\}$ is almost periodic.

Lemma 4 ([20])**.** *In addition to condition* (iii) *and* (v)*, let the following conditions be fulfilled.*

(vi): *$F(y)$ is uniformly continuous defined in Ω.*

Then $F(\varphi(t))$ is an almost periodic function.

Lemma 5 ([20])**.** *Let $g(t)$, $g \in PC(\mathbb{R}, \Omega)$ and the sequence $\{g_k\}$, $k \in \mathbb{Z}$ is almost periodic. Then there exists a positive constant C_1 such that*

$$\max\left(\sup_{t\in\mathbb{R}}\|g(t)\|, \sup_{k=\pm1,\pm2,\dots}\|g_k\|\right) \le C_1.$$

Remark 1. Throughout this paper, we always assume $i = 1, \dots, m$; $j = 1, \dots, n$, unless otherwise stated.

In this paper, we introduce the following conditions:

(H_1): $a_{ij}(t)$, $C_{ij}^{hl}(t)$ and $L_{ij}(t)$ are almost periodic functions in the sense of Bohr, and denote

$$0 < \inf_{t\in R}\{a_{ij}(t)\} = a_{ij}^- < \infty,\ \tilde{C}_{ij}^{hl} = \sup_{t\in R}\{|C_{ij}^{hl}(t)|\},\ \tilde{L}_{ij} = \sup_{t\in R}\{|L_{ij}(t)|\}.$$

(H_2): The condition (iii) holds.

(H_3): There exists a nonnegative constant L_g, such that for $\forall x, y \in \mathbb{R}$, $|g(x) - g(y)| \le L_g|x - y|$.

(H_4): $\{\alpha_{ij}^k\}_{k\in\mathbb{Z}}$, and $\{L_{ij}^k\}_{k\in\mathbb{Z}}$ are almost periodic sequences and from Lemma 5, there exist strictly positive constants $\tilde{L}_{ij}$ such that $\sup_{t\in\mathbb{R}}\{|L_{ij}(t)|, \max_k |L_{ij}^k|\} \le \tilde{L}_{ij}$.

(H_5): The sequence of functions $I_{ij}^k(x_{ij}(t_k))$ is almost periodic uniformly with respect to $x \in \Omega$ and there exists $v_{ij} > 0$ such that $|I_{ij}^k(x) - I_{ij}^k(\bar{x})| \le v_{ij}|x - \bar{x}|$ for $k \in Z$, x, $\bar{x} \in \Omega$.

(H_6): $\lambda_{ij} := a_{ij}^- - N\ln(1 + \max_k |\alpha_{ij}^k|) > 0$.

(H_7): For $i, j = 1, 2, \dots, n$, $\int_0^\infty |k_{ij}(s)|\mathrm{d}s$ is existent, and there exist nonnegative constants k_{ij}^+ such that

$$\int_0^\infty |k_{ij}(s)|\mathrm{d}s \le k_{ij}^+.$$

Now from [20], we have:

Lemma 6. *Assume the conditions (H_1), (H_2), (H_4) hold. Then for each $\varepsilon > 0$ there exist ε_1, $0 < \varepsilon_1 < \varepsilon$ and relatively dense set T of real numbers and Q of whole numbers, such that the following relations are fulfilled.*

(a): $|P(t+\tau) - P(t)| < \varepsilon$, $t \in \mathbb{R}$, $\tau \in T$, $|t - t_k| > \varepsilon$;
(b): $|P_{k+q} - P_k| < \varepsilon$, $k \in \mathbb{Z}$, $q \in Q$;
(c): $|L_{ij}(t+\tau) - L_{ij}(t)| < \varepsilon$, $t \in \mathbb{R}$, $\tau \in T$, $|t - t_k| > \varepsilon$, $k \in \mathbb{Z}$;
(d): $|L_{ij}^{k+q} - L_{ij}^k| < \varepsilon$, $k \in \mathbb{Z}$, $q \in Q$;
(e): $|C_{ij}^{hl}(t+\tau) - C_{ij}^{hl}(t)| < \varepsilon$, $t \in \mathbb{R}$, $\tau \in T$, $|t - t_k| > \varepsilon$, $k \in \mathbb{Z}$;
(f): $|C_{ij}^{hl,k+q} - C_{ij}^{hl,k}| < \varepsilon$, $k \in \mathbb{Z}$, $q \in Q$;
(g): $|\bar{t}_k^q - \tau| < \varepsilon$, $q \in Q$, $\tau \in T$, $k \in \mathbb{Z}$.

Lemma 7. *Assume (H_1)-(H_2), (H_4) and (H_6) hold. Then*

1. *For Cauchy matrix $W(t,s)$ of the system* (3)*, there exist positive constants K and λ such that $W(t,s) < Ke^{-\lambda(t-s)}$, $t \geq s$, $t,s \in \mathbb{R}$.*
2. *For any $\varepsilon > 0$, $t \in \mathbb{R}$, $s \in \mathbb{R}$, $t \geq s$, $|t-t_k| > \varepsilon > 0$, $|s-t_k| > 0$, $k \in \mathbb{Z}$, there exist a relatively dense set T of ε-almost periods of matrix $P(t)$ and positive constant Γ such that for $\tau \in T$ it follows*

$$|W(t+\tau, s+\tau) - W(t,s)| < \varepsilon \Gamma e^{\frac{\lambda}{2}(t-s)}.$$

Proof. Recall [12] the matrix $W(t,s)$ for system (3) is in the form $W(t,s) = e^{P(t)(t-s)} \prod_{i(s,t)} (E+P_k)$. By Lemma 2, one has

$$\begin{aligned} W(t,s) &\leq e^{P^-(t-s)}(E+P_k^+)^{i(s,t)} \leq e^{P^-(t-s)}(E+P_k^+)^{N(t-s)+N} \\ &= \operatorname{diag}(\xi_{11}e^{-\lambda_{11}(t-s)}, \ldots, \xi_{ij}e^{-\lambda_{ij}(t-s)}, \ldots, \xi_{mn}e^{-\lambda_{mn}(t-s)})_{(m\cdot n)\times(m\cdot n)}, \end{aligned} \tag{4}$$

where

$$\begin{aligned} P(t) &= \operatorname{diag}(-a_{11}(t), \ldots, -a_{ij}(t), \ldots, -a_{mn}(t))_{(m\cdot n)\times(m\cdot n)}, \\ P^- &= \operatorname{diag}(-a_{11}^-, \ldots, -a_{ij}^-, \ldots, -a_{mn}^-)_{(m\cdot n)\times(m\cdot n)}, \\ P_k^+ &= \operatorname{diag}\left(\max_k |\alpha_{11}^k|, \ldots, \max_k |\alpha_{ij}^k|, \ldots, \max_k |\alpha_{mn}^k|\right)_{(m\cdot n)\times(m\cdot n)}, \\ \xi_{ij} &= \exp\left\{N \ln\left(1+\max_k |\alpha_{ij}^k|\right)\right\},\ \lambda_{ij} = a_{ij}^- - N\ln\left(1+\max_k |\alpha_{ij}^k|\right), \\ &\quad i = 1,2,\ldots,m,\ j = 1,2,\ldots,n. \end{aligned}$$

Take

$$K = \max_{1\leq i\leq m, 1\leq j\leq n}\{\xi_{ij}\},\ \lambda = \min_{1\leq i\leq m, 1\leq j\leq n}\{\lambda_{ij}\}. \tag{5}$$

It follows from (4) and condition (H_6) that

$$|W(t,s)| \leq e^{-\lambda(t-s)},\ t \geq s,\ t,s \in \mathbb{R}.$$

This completes the Proof of Assertion 1. From Lemma 1, the Assertion 2 is immediately proved. ☐

3. Main results

Theorem 1. *In addition to (H_1)-(H_7) hold, then the following hold:*

1. *If $r < 1$, $\delta = \delta_1 + \delta_2 \leq 1$ and $\frac{\bar{K}}{1-\delta} \leq 1$, system* (1) *has a unique almost periodic solution $y(t)$. Here*

$$\begin{aligned} \delta_1 &= \max_{(i,j)}\left\{\xi_{ij}\frac{1}{\lambda_{ij}}\sum_{C_{hl}\in N_r(i,j)} \tilde{C}_{ij}^{hl} k_{ij}^+ L_g\right\},\ \delta_2 = \max_{(i,j)}\left\{\frac{v_{ij}N}{1-e^{-\lambda_{ij}}}\xi_{ij}\right\}, \\ \bar{K} &= \max_{(i,j)}\left\{\left(\frac{1}{\lambda_{ij}} + \frac{N}{1-e^{-\lambda_{ij}}}\right)\xi_{ij}\tilde{L}_{ij}\right\},\ r = \frac{2\delta_1\bar{K}}{1-\delta} + \delta_2. \end{aligned}$$

2. *If*

$$\lambda - \max_{(i,j)} \left\{ \xi_{ij} \sum_{C_{hl} \in N_r(i,j)} \tilde{C}_{ij}^{hl} k_{ij}^{+} L_g \frac{2\bar{K}}{1-\delta} \right\} + \ln(1 + \max_{(i,j)}\{\xi_{ij} v_{ij}\}) > 0,$$

then the unique solution $y(t)$ is exponentially stable.

Proof of Assertion 1. Let $D = \{\varphi(t) \mid \varphi(t) = (\varphi_{11}(t), \ldots, \varphi_{ij}(t), \ldots, \varphi_{mn}(t))^{\mathrm{T}}\} \in PC(\mathbb{R}, \mathbb{R}^{m\cdot n})$ be the almost periodic with $\|\varphi\| < \bar{K}$, where $\|\varphi\| = \sup_{t\in R} \max_{(i,j)} |\varphi_{ij}(t)|$ and $\bar{K} = \max_{(i,j)} \left\{ \left(\frac{1}{\lambda_{ij}} + \frac{N}{1-e^{-\lambda_{ij}}} \right) \xi_{ij} \tilde{L}_{ij} \right\}$. Obviously $D \supset PC(\mathbb{R}, \mathbb{R}^{m\cdot n})$. Set $\mathcal{F}(t, x(t)) = (F_{11}(t, x(t)), \ldots, F_{ij}(t, x(t)), \ldots, F_{mn}(t, x(t)))^{\mathrm{T}}$, where

$$\begin{aligned}
F_{ij}(t, x(t)) &= \sum_{C_{hl} \in N_r(i,j)} C_{ij}^{hl}(t) \int_0^{\infty} k_{ij}(u) g(x_{hl}(t-u)) \mathrm{d}u x_{ij}(t), \\
\mathcal{F}_k(x(t_k)) &= (I_{11}^k(x_{11}(t_k)), \ldots, I_{ij}^k(x_{ij}(t_k)), \ldots, I_{mn}^k(x_{mn}(t_k)))^{\mathrm{T}}, \\
H(t) &= (L_{11}(t), \ldots, L_{ij}(t), \ldots, L_{mn}(t))^{\mathrm{T}}, \\
H_k &= (L_{11}^k, \ldots, L_{ij}^k, \ldots, L_{mn}^k)^{\mathrm{T}}.
\end{aligned}$$

Define an operator S in D

$$S\varphi = \int_{-\infty}^{t} W(t,s)[\mathcal{F}(s, \varphi(s)) + H(s)]\mathrm{d}s + \sum_{t_k < t} W(t, t_k)[\mathcal{F}_k(\varphi(t_k)) + H_k]. \tag{6}$$

Obviously, it is easy to check that $S\varphi$ is a solution of (1).

Take subset $D^* \subset D$,

$$D^* = \left\{ \varphi \in D \Big| \|\varphi - \varphi_0\| \le \frac{\delta \bar{K}}{1-\delta} \right\}, \tag{7}$$

where

$$\varphi_0 = \int_{-\infty}^{t} W(t,s)H(s)\mathrm{d}s + \sum_{t_k<t} W(t,t_k)H_k. \tag{8}$$

From (8), it follows Lemma 7 that

(9)

$$\begin{aligned}
\|\varphi_0\| &= \sup_{t\in R} \left\{ \max_{(i,j)} \int_{-\infty}^{t} |W(t,s)||L_{ij}(s)|\mathrm{d}s + \max_{(i,j)} \sum_{t_k<t} |W(t,t_k)| \cdot \max_k |L_{ij}^k| \right\} \\
&\le \sup_{t\in R} \left\{ \max_{(i,j)} \int_{-\infty}^{t} \xi_{ij} e^{-\lambda_{ij}(t-s)} |L_{ij}(s)|\mathrm{d}s + \max_{(i,j)} \sum_{t_k<t} \xi_{ij} e^{-\lambda_{ij}(t-t_k)} \cdot \max_k |L_{ij}^k| \right\} \\
&\le \max_{(i,j)} \left\{ \left(\frac{1}{\lambda_{ij}} + \frac{N}{1-e^{-\lambda_{ij}}} \right) \xi_{ij} \tilde{L}_{ij} \right\} := \bar{K}.
\end{aligned}$$

Then for arbitrary $\varphi \in D^*$, it follows from (6), (7) and (9) that

$$\|\varphi\| \le \|\varphi - \varphi_0\| + \|\varphi_0\| \le \frac{\delta}{1-\delta}\bar{K} + \bar{K} = \frac{1}{1-\delta}\bar{K}. \tag{10}$$

Now we prove that S is self-mapping from D^* to D^*.

Firstly, we shall show that $S\varphi \in D^*$ for arbitrary $\varphi \in D^*$. In fact,

$$S\varphi - \varphi_0 = \int_{-\infty}^{t} W(t,s)\mathcal{F}(s,\varphi(s))\mathrm{d}s + \sum_{t_k<t} W(t,t_k)\mathcal{F}_k(\varphi(t_k)). \tag{11}$$

This, together with (4) and $\|\varphi\| \le \frac{1}{1-\delta}\bar{K} \le 1$, we have

(12)

$$\begin{aligned}
&\|S\varphi - \varphi_0\| \\
= &\sup_{t\in R}\Bigg\{\max_{(i,j)}\int_{-\infty}^{t}|W(t,s)|\cdot\Bigg|\sum_{C_{hl}\in N_r(i,j)} C_{ij}^{hl}(s)\int_0^{\infty}k_{ij}(u)g(\varphi_{hl}(s-u))\mathrm{d}u\varphi_{ij}(s)\Bigg|\mathrm{d}s \\
&\quad + \max_{(i,j)}\sum_{t_k<t}|W(t,t_k)|\cdot\max_k|I_{ij}^k(\varphi_{ij}(t_k))|\Bigg\} \\
\le &\sup_{t\in R}\Bigg\{\max_{(i,j)}\int_{-\infty}^{t}\xi_{ij}e^{-\lambda_{ij}(t-s)}\sum_{C_{hl}\in N_r(i,j)}\tilde{C}_{ij}^{hl}L_g k_{ij}^{+}\|\varphi\|^2\mathrm{d}s \\
&\quad + \max_{(i,j)}\sum_{t_k<t}\xi_{ij}e^{-\lambda_{ij}(t-t_k)}v_{ij}|\varphi_{ij}(t_k)|\Bigg\} \\
\le &\max_{(i,j)}\left\{\left(\frac{1}{\lambda_{ij}}\sum_{C_{hl}\in N_r(i,j)}\tilde{C}_{ij}^{hl}k_{ij}^{+}L_g\|\varphi\|^2 + \frac{v_{ij}N}{1-e^{-\lambda_{ij}}}\|\varphi\|\right)\xi_{ij}\right\} \\
= &\max_{(i,j)}\left\{\left(\frac{1}{\lambda_{ij}}\sum_{C_{hl}\in N_r(i,j)}\tilde{C}_{ij}^{hl}k_{ij}^{+}L_g\|\varphi\| + \frac{v_{ij}N}{1-e^{-\lambda_{ij}}}\right)\xi_{ij}\right\}\|\varphi\| \\
\le &\max_{(i,j)}\left\{\left(\frac{1}{\lambda_{ij}}\sum_{C_{hl}\in N_r(i,j)}\tilde{C}_{ij}^{hl}k_{ij}^{+}L_g + \frac{v_{ij}N}{1-e^{-\lambda_{ij}}}\right)\xi_{ij}\right\}\|\varphi\| := \delta\|\varphi\| \le \frac{\delta}{1-\delta}\bar{K}.
\end{aligned}$$

Secondly, we shall prove that $S\varphi$ is almost periodic. In fact, let $\tau \in T$, $q \in Q$, where the sets T and Q are determined in Lemma 6. By Lemmas 6 and 7, we have

(13)

$$\begin{aligned}
&\|S\varphi(t+\tau) - S\varphi(t)\| \\
\le &\sup_{t\in R}\Bigg\{\int_{-\infty}^{t}|W(t+\tau,s+\tau) - W(t,s)|
\end{aligned}$$

$$\times \max_{(i,j)} \Bigg[\Bigg| \sum_{C_{hl} \in N_r(i,j)} C_{ij}^{hl}(s+\tau) \int_0^\infty k_{ij}(u) g(\varphi_{hl}(s+\tau-u)) \mathrm{d}u \varphi_{ij}(s+\tau) + L_{ij}(s+\tau) \Bigg| \Bigg] \mathrm{d}s$$

$$+ \int_{-\infty}^t |W(t,s)| \max_{(i,j)} \Bigg[\Bigg| \sum_{C_{hl} \in N_r(i,j)} C_{ij}^{hl}(s+\tau) \int_0^\infty k_{ij}(u) g(\varphi_{hl}(s+\tau-u)) \mathrm{d}u \varphi_{ij}(s+\tau)$$

$$- \sum_{C_{hl} \in N_r(i,j)} C_{ij}^{hl}(s) \int_0^\infty k_{ij}(u) g(\varphi_{hl}(s-u)) \mathrm{d}u \varphi_{ij}(s) \Bigg| + |L_{ij}(s+\tau) - L_{ij}(s)| \Bigg] \mathrm{d}s$$

$$+ \sum_{t_k < t} |W(t+\tau, t_{k+q}) - W(t, t_k)| \max_{(i,j)} \Big[|I_{ij}^{k+q}(\varphi_{ij}(t_{k+q})) + L_{ij}^{k+q}| \Big]$$

$$+ \sum_{t_k < t} |W(t, t_k)| \max_{(i,j)} \Big[|I_{ij}^{k+q}(\varphi_{ij}(t_{k+q})) - I_{ij}^{k}(\varphi_{ij}(t_k))| + |L_{ij}^{k+q} - L_{ij}^{k}| \Big] \Bigg\}$$

$$\leq \varepsilon \bar{M},$$

where

$$\bar{M} = \frac{1}{\lambda} \Bigg[\max_{(i,j)} \Bigg\{ \sum_{C_{hl} \in N_r(i,j)} (\Gamma + K) \tilde{C}_{ij}^{hl} k_{ij}^{+} L_g \frac{2\bar{K}}{1-\delta} + \tilde{L}_{ij} + K \Bigg\} \Bigg] + \frac{\Gamma N}{1-e^{-\lambda}} \Big[\max_{ij} v_{ij} + 1 \Big] + \frac{\Gamma N}{1-e^{-\frac{\lambda}{2}}} \Big[\max_{ij} v_{ij} \frac{\bar{K}}{1-\delta} + \tilde{L}_{ij} \Big].$$

It follows from (12) and (13) that $S\varphi \in D^*$. For arbitrary φ, $\psi \in D^*$,

$$S\varphi - S\psi = \int_{-\infty}^t W(t,s)[\mathcal{F}(s,\varphi(s)) - \mathcal{F}(s,\psi(s))]\mathrm{d}s + \sum_{t_k<t} W(t,t_k)[\mathcal{F}_k(\varphi(t_k)) - \mathcal{F}_k(\psi(t_k))]. \tag{14}$$

It follows from (4) and (14) that

(15)

$$\|S\varphi - S\psi\|$$

$$\leq \sup_{i \in \mathbb{R}} \Bigg\{ \max_{(i,j)} \int_{-\infty}^t |W(t,s) \Bigg| \sum_{C_{hl} \in N_r(i,j)} C_{ij}^{hl}(s) \int_0^\infty k_{ij}(u) g(\varphi_{hl}(s-u)) \mathrm{d}u \varphi_{ij}(s)$$

$$- \sum_{C_{hl} \in N_r(i,j)} C_{ij}^{hl}(s) \int_0^\infty k_{ij}(u) g(\psi_{hl}(s-u)) \mathrm{d}u \psi_{ij}(s) \Bigg| \mathrm{d}s$$

$$+ \sum_{t_k<t} |W(t,t_k)| \big[\max_{(i,j)} |I_{ij}^{k+q}(\varphi_{ij}(t_k)) - I_{ij}^{k}(\psi_{ij}(t_k))| \big] \Bigg\}$$

$$
\begin{aligned}
&\le \sup_{i\in\mathbb{R}}\max_{(i,j)}\Big\{\int_{-\infty}^{t}\xi_{ij}e^{-\lambda_{ij}(t-s)}\Big[\sum_{C_{hl}\in N_r(i,j)}\tilde{C}_{ij}^{hl}\Big|\int_0^\infty k_{ij}(u)g(\psi_{hl}(s-u))\mathrm{d}u\Big||\varphi_{ij}(s)-\psi_{ij}(s)|\\
&\quad+\sum_{C_{hl}\in N_r(i,j)}\tilde{C}_{ij}^{hl}\Big|\int_0^\infty k_{ij}(u)g(\varphi_{hl}(s-u))\mathrm{d}u-\int_0^\infty k_{ij}(u)g(\psi_{hl}(s-u))\mathrm{d}u\Big||\varphi_{ij}(s)|\Big]\\
&\quad+\sum_{t_k<t}\xi_{ij}e^{-\lambda_{ij}(t-t_k)}|I_{ij}^k(\varphi_{ij}(t_k)-I_{ij}^k(\psi_{ij}(t_k))|\}\Big\}\\
&\le \sup_{i\in\mathbb{R}}\max_{(i,j)}\Big\{\int_{-\infty}^{t}\xi_{ij}e^{-\lambda_{ij}(t-s)}\Big[\sum_{C_{hl}\in N_r(i,j)}\tilde{C}_{ij}^{hl}L_gk_{ij}^{+}\|\psi\|\big|\varphi_{ij}(s)-\psi_{ij}(s)\big|\\
&\quad+\sum_{C_{hl}\in N_r(i,j)}\tilde{C}_{ij}^{hl}L_gk_{ij}^{+}\|\varphi-\psi\|\big|\varphi_{ij}(s)\big|\Big]+\sum_{t_k<t}\xi_{ij}e^{-\lambda_{ij}(t-t_k)}v_{ij}|\varphi_{ij}(t_k)-\psi_{ij}(t_k)|\Big\}\\
&\le \sup_{i\in\mathbb{R}}\max_{(i,j)}\Big\{\int_{-\infty}^{t}\xi_{ij}e^{-\lambda_{ij}(t-s)}\sum_{C_{hl}\in N_r(i,j)}\tilde{C}_{ij}^{hl}L_gk_{ij}^{+}\big(\|\varphi\|+\|\psi\|\big)\|\varphi-\psi\|\mathrm{d}s\\
&\quad+\sum_{t_k<t}\xi_{ij}e^{-\lambda_{ij}(t-t_k)}v_{ij}\|\varphi-\psi\|\Big\}\\
&\le \max_{(i,j)}\left\{\left(\frac{1}{\lambda_{ij}}\sum_{C_{hl}\in N_r(i,j)}\tilde{C}_{ij}^{hl}L_gk_{ij}^{+}\frac{2\bar{K}}{1-\delta}+\frac{v_{ij}N}{1-e^{-\lambda_{ij}}}\right)\xi_{ij}\right\}\|\varphi-\psi\|\\
&:=\frac{2\delta_1\bar{K}}{1-\delta}+\delta_2=r\|\varphi-\psi\|.
\end{aligned}
$$

From (15) and condition (H_6), it follows that S is contraction mapping in D^*. Therefore, there exists a unique $y\in D^*$ such that $Sy=y$, and then there exists a unique almost periodic solution $y(t)$ of (1). □

Proof of Assertion 2. Let $x(t)$ be arbitrary solution of (1) with the initial condition (2), and $y(t)=(y_{11}(t),\dots,\ y_{ij}(t),\dots,y_{mn}(t))^{\mathrm{T}}$ be the unique almost periodic solution of (1) with the initial condition $y(t_0+0,t_0,y_0)=y_0$. From (6), we have

$$
\begin{aligned}
(16)\qquad x(t)-y(t)&=W(t,t_0)(x_0-y_0)+\int_{t_0}^{t}W(t,s)[\mathcal{F}(s,x(s))-\mathcal{F}(s,y(s))]\\
&\quad+\sum_{t_0<t_k<t}W(t,t_k)[\mathcal{F}_k(x(t_k))-\mathcal{F}_k(y(t_k))].
\end{aligned}
$$

It follows from Lemma 7, (4), (16) and the derivation of (15) that

$$
\begin{aligned}
&\|x(t)-y(t)\|\\
&\le Ke^{-\lambda(t-t_0)}\|x_0-y_0\|
\end{aligned}
$$

$$
\begin{aligned}
&+\sup_{i\in\mathbb{R}}\max_{(i,j)}\Bigg\{\int_{t_0}^{t}|W(t,s)|\Big|\sum_{C_{hl}\in N_r(i,j)}C_{ij}^{hl}(s)\int_0^{\infty}k_{ij}(u)g(x_{hl}(s-u))\mathrm{d}ux_{ij}(s)\\
&-\sum_{C_{hl}\in N_r(i,j)}C_{ij}^{hl}(s)\int_0^{\infty}k_{ij}(u)g(y_{hl}(s-u))\mathrm{d}uy_{ij}(s)\Big|\mathrm{d}s\\
&+\sum_{t_0\le t_k<t}|W(t,t_k)|\Big[\big|I_{ij}^{k+q}(x_{ij}(t_k))-I_{ij}^{k}(y_{ij}(t_k))\big|\Big]\Bigg\}\\
\le{}& Ke^{-\lambda(t-t_0)}\|x_0-y_0\|\\
&+\sup_{i\in\mathbb{R}}\max_{(i,j)}\Bigg\{\int_{t_0}^{t}\xi_{ij}e^{-\lambda_{ij}(t-s)}\Big[\sum_{C_{hl}\in N_r(i,j)}\tilde{C}_{ij}^{hl}L_gk_{ij}^{+}\|y(t)\|\big|x_{ij}(s)-y_{ij}(s)\big|\\
&+\sum_{C_{hl}\in N_r(i,j)}\tilde{C}_{ij}^{hl}L_gk_{ij}^{+}\|x(t)-y(t)\|\big|x_{ij}(s)\big|\Big]\mathrm{d}s\\
&+\sum_{t_k<t}\xi_{ij}e^{-\lambda_{ij}(t-t_k)}v_{ij}|x_{ij}(t_k)-y_{ij}(t_k)|\Bigg\}\\
\le{}& Ke^{-\lambda(t-t_0)}\|x_0-y_0\|\\
&+\max_{(i,j)}\Bigg\{\xi_{ij}\sum_{C_{hl}\in N_r(i,j)}\tilde{C}_{ij}^{hl}L_gk_{ij}^{+}\frac{2\bar{K}}{1-\delta}\Bigg\}\int_{t_0}^{t}e^{-\lambda(t-s)}\|x(t)-y(t)\|\mathrm{d}s\\
&+\max_{(i,j)}\{\xi_{ij}v_{ij}\}\sum_{t_k<t}e^{-\lambda(t-t_k)}\|x(t_k)-y(t_k)\|.
\end{aligned}
$$

Set $z(t)=\|x(t)-y(t)\|e^{-\lambda t}$. From Gronwall-Bellman's Lemma [20], we have

$$
\begin{aligned}
\|x(t)-y(t)\|\le K\|x_0-y_0\|&\left(1+\max_{(i,j)}\{\xi_{ij}v_{ij}\}\right)^{i(t_0,t)}\\
&\times\exp\left\{\left[-\lambda+\max_{(i,j)}\left\{\xi_{ij}\sum_{C_{hl}\in N_r(i,j)}\tilde{C}_{ij}^{hl}L_gk_{ij}^{+}\frac{2\bar{K}}{1-\delta}\right\}\right](t-t_0)\right\}.
\end{aligned}
$$

By Lemma 2, one has

$$
\begin{aligned}
&\|x(t)-y(t)\|\\
\le{}& K\exp\left\{N\ln\left(1+\max_{(i,j)}\{\xi_{ij}v_{ij}\}\right)\right\}\|x_0-y_0\|\\
&\times\exp\left\{-\Big[\lambda-\max_{(i,j)}\Big\{\xi_{ij}\sum_{C_{hl}\in N_r(i,j)}\tilde{C}_{ij}^{hl}L_gk_{ij}^{+}\frac{2\bar{K}}{1-\delta}\Big\}+\ln\Big(1+\max_{(i,j)}\{\xi_{ij}v_{ij}\}\Big)\Big](t-t_0)\right\}.
\end{aligned}
$$

From the assumption of Assertion 2, the solution $y(t)$ is exponentially stable. □

Remark 2. When system (1) is without impulse, the conditions in Theorem 1 reduces to $r < 1$, $\delta_1 \le 1$ and $\frac{K_0}{1-\delta} \le 1$, where $K_0 = \max_{(i,j)}\{\frac{1}{\lambda_{ij}}\xi_{ij}\tilde{L}_{ij}\}$.

Remark 3. References [18, 17, 19, 30] studied the stability of almost periodic solutions for the SICNNs with distributed delays. However, they did not consider the impulse influence. Xia [25] obtained several sufficient conditions to guarantee the exponential stability of almost periodic solutions for the SICNNs with impulses, but he did not take time delays into account. To our best knowledge, there is no published paper considering the almost periodic solutions for SICNNs with both distributed delays and impulses. Therefore, our results are essentially new and complement previously known results.

Remark 4. It should be pointed out that the precision of Theorem 1 is very high due to the high precision computation of each ξ_{ij} and λ_{ij} in the proof of Lemma 7.

4. An example

In this section, we present a numerical example to verify the Theorem 1 we have given in the previous section.

Example 1. Consider the shunting inhibitory cellular neural networks with distributed delays and large impulses (1) with

$$(a_{ij})_{3\times 3} = \begin{pmatrix} 1.4103+\sin(t) & 1.3529+\cos(t) & 1.1389-\sin(t) \\ 1.8936+\sin(2t) & 1.8132-\sin(t) & 1.2028+\sin(t) \\ 1.0579+\cos(t) & 1.0099-\cos(t) & 1.1987+\sin(t) \end{pmatrix},$$

$$(C_{ij})_{3\times 3} = \begin{pmatrix} 0.6038\sin(3t) & 0.0153\cos(t) & 0.9318\sin(t) \\ 0.2722\sin(2t) & 0.7468\sin(t) & 0.4660\sin(t) \\ 0.1988\cos(t) & 0.4451\cos(2t) & 0.4186\sin(t) \end{pmatrix},$$

$$(L_{ij})_{3\times 3} = \begin{pmatrix} 0.8462\sin(t) & 0.6721\cos(t) & 0.6813\sin(2t) \\ 0.5252\sin(2t) & 0.8381\sin(t) & 0.3795\sin(t) \\ 0.2026\cos(t) & 0.0196\cos(3t) & 0.8318\sin(t) \end{pmatrix}.$$

Let the r-neighborhood $N_r(i,j)$ $(i,j = 1,2,3)$ of C_{ij} be

$$\begin{aligned}
&N_r(1,1) = \{C_{11}, C_{12}, C_{21}, C_{22}\}, && N_r(2,1) = \{C_{21}, C_{11}, C_{22}, C_{31}\},\\
&N_r(3,1) = \{C_{31}, C_{21}, C_{22}, C_{32}\}, && N_r(1,2) = \{C_{12}, C_{11}, C_{22}, C_{13}\},\\
&N_r(2,2) = \{C_{22}, C_{21}, C_{12}, C_{23}, C_{32}\}, && N_r(3,2) = \{C_{32}, C_{22}, C_{31}, C_{33}\},\\
&N_r(1,3) = \{C_{13}, C_{12}, C_{23}\}, && N_r(2,3) = \{C_{23}, C_{13}, C_{22}, C_{33}\},\\
&N_r(3,3) = \{C_{33}, C_{32}, C_{23}, C_{22}\}.
\end{aligned}$$

Set $\alpha_{ij}^k = 0.05 + \sin k\pi$, $L_{ij}^k = 0.15 + \cos k\pi$, $I_{ij}^k(x_{ij}(t_k)) = 0.4x_{ij}(t_k)$, $k \in \mathbb{Z}^+$, $g(\cdot) = \frac{1}{10}(|x+1| - |x-1|)$, $k_{ij}(t) = \exp(-t)$, so $k_{ij}^+ = 1$, $\tilde{C}_{ij}^{hl} = 1$, $L_g = \frac{1}{10}$. Then conditions $H_1 - H_7$ hold, and we have

$$\delta_1 = \max_{(i,j)} \left\{ \xi_{ij} \frac{1}{\lambda_{ij}} \sum_{C_{hl} \in N_r(i,j)} \tilde{C}_{ij}^{hl} k_{ij}^+ L_g \right\} \leq 0.2, \ \delta_2 = \max_{(i,j)} \left\{ \frac{v_{ij} N}{1 - e^{-\lambda_{ij}}} \xi_{ij} \right\} \leq 0.45,$$

$$\bar{K} = \max_{(i,j)} \left\{ \left(\frac{1}{\lambda_{ij}} + \frac{N}{1 - e^{-\lambda_{ij}}} \right) \xi_{ij} \tilde{L}_{ij} \right\} \leq 0.116, \ r = \frac{2\delta_1 \bar{K}}{1 - \delta} + \delta_2 \leq 1,$$

$$\lambda - \max_{(i,j)} \left\{ \xi_{ij} \sum_{C_{hl} \in N_r(i,j)} \tilde{C}_{ij}^{hl} k_{ij}^+ L_g \frac{2\bar{K}}{1 - \delta} \right\} + \ln(1 + \max_{(i,j)} \{\xi_{ij} v_{ij}\}) > 0.$$

This implies that all of the conditions of Theorem 1 satisfied, Thus, by Theorem 1, the system (1) exist a unique almost periodic solution, and all solutions of the system (1) converge exponentially to its a unique almost periodic solution.

5. Conclusions

In this paper, the shunting inhibitory cellular neural networks with continuously distributed delays and large impulses have been studied. Some sufficient conditions insuring the existence and exponential stability of the almost periodic solutions have been proposed using the contraction principle and Gronwall-Bellman's inequality techniques. These obtained results are new and they complement previously known results. Moreover, an example is given to illustrate the effectiveness of our method.

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