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DERIVATIONS OF PRIME AND SEMIPRIME RINGS

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Abstract. Let R be a prime ring, I a nonzero ideal of R, d a derivation of R and n a fixed positive integer. (i) If $(d(x)y+xd(y)+d(y)x+yd(x))^n = xy + yx$ for all $x, y \in I$, then R is commutative. (ii) If $\mathrm{char}R \neq 2$ and $(d(x)y + xd(y) + d(y)x + yd(x))^n - (xy + yx)$ is central for all $x, y \in I$, then R is commutative. We also examine the case where R is a semiprime ring.

1. Introduction

Throughout the paper R will represent an associative ring with center $Z(R)$. For any $x, y \in R$, the symbol $[x, y]$ stands for the commutator $xy - yx$. Recall that a ring R is prime if $xRy = 0$ implies either $x = 0$ or $y = 0$, and R is semiprime if $xRx = 0$ implies $x = 0$. An additive mapping $d : R \to R$ is called a derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$.

In [1], M. Ashraf and N. Rehman proved that if R is a prime ring, I is a nonzero ideal of R and d is a derivation of R such that $d(x)y + xd(y) + d(y)x + yd(x) = xy + yx$ for all $x, y \in I$, then R is commutative. In this paper we shall generalize this result, assuming that n is a fixed positive integer and $(d(xy + yx))^n - (xy + yx)$ is 0 for all $x, y \in I$ or is central for all $x, y \in I$. We obtain some analogous results for semiprime rings in the case $I = R$.

2. Derivations in prime rings

In all that follows, unless stated otherwise, R will be a prime ring, I a nonzero ideal of R. For any ring S, $Z(S)$ will denote its center.

We will also make frequent use of the following result due to Kharchenko [9] (see also [12]):

Let R be a prime ring, d a nonzero derivation of R and I a nonzero two-sided ideal of R. Let $f(x_1, \ldots, x_n, d(x_1), \ldots, d(x_n))$ be a differential identity in I, that is,

$$f(r_1, \ldots, r_n, d(r_1), \ldots, d(r_n)) = 0 \quad \text{for all } r_1, \ldots, r_n \in I.$$

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One of the following holds:

1) Either d is an inner derivation in Q, the Martindale quotient ring of R, in the sense that there exists $q \in Q$ such that $d(x) = [q, x]$ for all $x \in R$, and I satisfies the generalized polynomial identity

$$f(r_1, \ldots, r_n, [q, r_1], \ldots, [q, r_n]) = 0;$$

or

2) I satisfies the generalized polynomial identity

$$f(x_1, \ldots, x_n, y_1, \ldots, y_n) = 0.$$

Theorem 1. *Let R be a prime ring and I a nonzero ideal of R, and let n be a fixed positive integer. If R admits a derivation d such that $(d(x)y + xd(y) + d(y)x + yd(x))^n = xy + yx$ for all $x, y \in I$, then R is commutative.*

Proof. If $d = 0$, then $xy + yx = 0$ for all $x, y \in I$. Replacing y by yz and using the fact that $xy = -yx$, we find that $y[x, z] = 0$ for all $x, y, z \in I$ and hence $IR[x, z] = 0$ for all $x, z \in I$. Since $I \neq 0$ and R is prime, we get $[x, z] = 0$ for all $x, z \in I$, hence R is commutative.

Now we assume that $d \neq 0$ and $(d(x)y + xd(y) + d(y)x + yd(x))^n = xy + yx$ for all $x, y \in I$. This condition is a differential identity satisfied by I. By using Kharchenko's theorem [9], either $d = ad(A)$ is the inner derivation induced by an element $A \in Q$, the Martindale quotient ring R, or I satisfies the polynomial identity

$$(zy + xw + wx + yz)^n = xy + yx \quad \text{for all } x, y \in I.$$

In the latter case set $z = w = 0$ to obtain the identity $xy + yx = 0$ for all $x, y \in I$. Then R is commutative as we have just seen. Assume now that $d = ad(A)$. Then $([A, x]y + x[A, y] + [A, y]x + y[A, x])^n = xy + yx$ for any $x, y \in I$. Since by [3] I and Q satisfy the same generalized polynomial identities, we have $([A, x]y + x[A, y] + [A, y]x + y[A, x])^n = xy + yx$ for any $x, y \in Q$. Moreover, since Q remains prime by the primeness of R, replacing R by Q we may assume that $A \in R$ and C is the just the center of R. Note that R is a centrally closed prime C-algebra in the present situation [5], i.e., $RC = R$. By Martindale's theorem in [13], RC (and so R) is a primitive ring. Since R is primitive, there exists a vector space V and a division ring D such that R is a dense ring of D-linear transformations over V.

Assume first that $\dim_D V \geq 3$.

Our aim is to show that for any $v \in V$, v and Av are linearly D-dependent. If $Av = 0$, then $\{v, Av\}$ is D-dependent. So we may suppose that $Av \neq 0$. If v and Av are D-independent, since $\dim_D V \geq 3$, there exists $w \in V$ such that v, Av, w are also linearly independent. By the density of R, there exist $x, y \in R$ such that:

$$xv = 0, \quad xAv = w, \quad yv = 0, \quad yAv = 0, \quad yw = v.$$

Hence we get $(-1)^n v = ([A, x]y + x[A, y] + [A, y]x + y[A, x])^n v = (xy + yx)v = 0$, a contradiction. So v and Av are linearly D-dependent for all $v \in V$. Now we want to show that there exists $\lambda \in D$ such that $Av = \lambda v$ for any $v \in V$. Now choose $v, w \in V$ linearly independent. Since $\dim_D V \geq 3$, there exists $u \in V$ such that v, w, u are linearly independent. Then $\lambda_v, \lambda_w, \lambda_u \in D$ such that

$$Av = \lambda_v v, \quad Aw = \lambda_w w, \quad Au = \lambda_u u,$$

that is,

$$A(v + w + u) = v\lambda_v + w\lambda_w + u\lambda_u.$$

Moreover $A(v+w+u) = \lambda_{v+w+u}(v+w+u)$ for a suitable $\lambda_{v+w+u} \in D$. Then we have $0 = (\lambda_{v+w+u} - \lambda_v)v + (\lambda_{v+w+u} - \lambda_w)w + (\lambda_{v+w+u} - \lambda_u)u$. Since v, w, u are linearly independent we get $\lambda_v = \lambda_w = \lambda_u = \lambda_{v+w+u}$, that is, λ does not depend on the choice of v. So there exists $\lambda \in D$ such that $Av = \lambda v$ for all $v \in V$.

Now for any $r \in R$, $v \in V$ we get $Av = v\lambda, r(Av) = r(v\lambda)$ and also $A(rv) = (rv)\lambda$. Thus $0 = [A, r]v$ for any $v \in V$, that is, $[A, r]V = 0$. Since V is a left faithful irreducible R-module, $[A, r] = 0$ for all $r \in R$, i.e., $A \in Z(R)$ and $d = 0$, which contradicts our hypothesis.

Therefore $\dim_D V$ must be ≤ 2. In this case R is a simple GPI ring with 1, and so it is a central simple algebra finite dimensional over its center. From Lemma 2 in [11] it is clear that there exists a suitable field F such that $R \subseteq M_k(F)$, the ring of all $k \times k$ matrices over F, and moreover $M_k(F)$ satisfies the same generalized polynomial identity as R.

If we assume $k \geq 3$, by the same argument as in the above, we get a contradiction.

If $k = 1$, then it is clear that R is commutative. Thus we may assume $R \subseteq M_2(F)$, where $M_2(F)$ satisfies the generalized polynomial identity $([A, x]y + x[A, y] + [A, y]x + y[A, x])^n = xy + yx$.

Let us denote $[A, x]y + x[A, y] + [A, y]x + y[A, x]$ by K. If we choose $x = e_{12}, y = e_{21}$, then $K = 0$. Hence we get $0 = K^n = xy + yx = e_{11} + e_{22} \neq 0$, a contradiction. Therefore $k = 1$, i.e., R is commutative. □

Lemma 1. *Let $R = M_s(F)$, the ring of $s \times s$ matrices over a field F of characteristic $\neq 2$, n a fixed positive integer. If there exists a nonzero matrix A in R such that $([A, x]y + x[A, y] + [A, y]x + y[A, x])^n - (xy + yx) \in F$ for any $x, y \in R$, then A is central.*

Proof. Assume that $s \geq 3$. Let i, j, r be distinct indices and $A = \sum a_{mn}e_{mn}$, with $a_{mn} \in F$. Suppose that A is not diagonal. Let $a_{ij} \neq 0$ for fixed $i \neq j$. If we choose $x = e_{jr}, y = e_{ri}$ with i, j, r distinct indices, then $xy + yx = e_{ji}$. Let us denote $[A, x]y + x[A, y] + [A, y]x + y[A, x] = [A, xy + yx]$ by K. Then

$$K = Ae_{ji} - e_{ji}A$$

and

$$K^n = \sum_{l+t=n} (-1)^l (e_{ji}A)^l (Ae_{ji})^t.$$

By the hypothesis we have

$$(M) \qquad \sum_{l+t=n}(-1)^l(e_{ji}A)^l(Ae_{ji})^t-(e_{ji})\in F.$$

All the entries of this matrix are

- (j,i) entries and we don't care about them;
- the entries from the terms $(Ae_{ji})^n$ and $(e_{ji}A)^n$.

In particular from $(Ae_{ji})^n$, consider all the entries $a_{hj}a_{ij}^{n-1}e_{hi}$ for any $h\neq j,i$. First of all notice that these entries don't occur in $(e_{ji}A)^n$. Since $h\neq i$ and (M) must be central, it follows that $a_{hj}a_{ij}^{n-1}=0$ for any $h\neq j,i$.

Since $a_{ij}\neq 0$, we obtain $a_{hj}=0$ for all $h\neq i,j$. Now, if we choose $xy+yx=e_{jk}$ for a fixed $k\neq i,j$, we have that

$$(M') \qquad \sum_{l+t=n}(-1)^l(e_{jk}A)^l(Ae_{jk})^t-(e_{jk})\in F.$$

But we know that $a_{kj}=0$ for all $k\neq i,j$, that is, the matrix (M') is reduced to $e_{jk}\in F$, a contradiction. Therefore $a_{ij}=0$ for all $i\neq j$, i.e., A is diagonal.

Now we suppose that $s=2$. Then we have

$$[A,xy+yx]^n-(xy+yx)\in F \quad\text{and}\quad R=M_2(F).$$

If $n=1$, then $K-(xy+yx)\in F$. Choose $x=e_{11},y=e_{12}$ so that $xy+yx=e_{12}$. Hence $B=[A,e_{12}]-e_{12}\in F$. In the matrix $B=\sum_{i,j}b_{ij}e_{ij}$ we must have $b_{11}=b_{22}$. Moreover one can see that $b_{11}=-a_{21}$ and $b_{22}=a_{21}$. Then we have $2a_{21}=0$. Since $\mathrm{char}R\neq 2$, we get $a_{21}=0$. Similarly we can see that $a_{12}=0$. Therefore A is a diagonal matrix.

Let $n=2$ and $K=\left(\begin{smallmatrix}k_{11}&k_{12}\\k_{21}&k_{22}\end{smallmatrix}\right)$. So we get

$$K^2=\begin{pmatrix}k_{11}^2+k_{12}k_{21} & k_{11}k_{12}+k_{12}k_{22}\\ k_{21}k_{11}+k_{22}k_{21} & k_{21}k_{12}+k_{22}^2\end{pmatrix}.$$

Moreover since $K=[A,xy+yx]$ we get $\mathrm{trace}(K)=0$, that is, $k_{11}+k_{22}=0$. By using the this fact, we have

$$K^2=\begin{pmatrix}k_{11}^2+k_{12}k_{21} & 0\\ 0 & k_{21}k_{12}+k_{11}^2\end{pmatrix} \quad\text{and hence}\quad K^2\in F.$$

Therefore when n is even, we have $K^n\in F$. So we get $xy+yx\in F$. Choose $x=e_{11},y=e_{12}$, so that $xy+yx=e_{12}\in F$, a contradiction.

If $n=2t+1$ is odd, then for any $x,y\in R$, there exists $\gamma\in F$ such that

$$[A,xy+yx]^{2t}[A,xy+yx]=(xy+yx)+\gamma.$$

Moreover there exists $\beta\in F$ such that $[A,xy+yx]^{2t}=\beta$, say

$$\beta[A,xy+yx]=(xy+yx)+\gamma.$$

In particular pick $x=e_{11},y=e_{12}$, then there exist $\beta,\gamma\in F$ such that

$$\beta[A,e_{12}]=e_{12}+\gamma.$$

If $\beta = 0$, then $e_{12} \in F$-a contradiction. Hence it follows that $\beta \neq 0$ and we have

$$B = \beta[A, e_{12}] - e_{12} \in F.$$

In the matrix $B = \sum_{i,j} b_{ij}e_{ij}$ we must have $b_{11} = b_{22}$. Moreover one can see that:

$$-\beta a_{21} = b_{11} \text{ and } b_{22} = \beta a_{21}.$$

Then we have $2a_{21} = 0$. Since $\mathrm{char}F \neq 2$, we get $a_{21} = 0$. Similarly we can see that $a_{12} = 0$. Therefore A is a diagonal matrix in any case, unless s=1 and R is commutative.

For any φ the inner automorphism on $M_k(F)$, we have $[\varphi(A), \varphi(x)\varphi(y) + \varphi(y)\varphi(x)]^n - (\varphi(x)\varphi(y) + \varphi(y)\varphi(x)) \in F$ for all $x, y \in F$ and so, by the previous case, $\varphi(A)$ must be a diagonal matrix in $M_2(F)$. In particular, if $\varphi(x) = (1 - e_{ij})x(1 + e_{ij})$ for $i \neq j$, then $\varphi(A) = \sum_t a_{tt}e_{tt} + (a_{ii} - a_{jj})e_{ij}$ must be diagonal, that is $a_{ii} = a_{jj}$ for $i \neq j$. Hence A is a central matrix. □

Theorem 2. *Let R be a prime ring with* $\mathrm{char}R \neq 2$, *I a nonzero ideal of R and n a fixed positive integer. If R admits a derivation d such that $(d(x)y + xd(y) + d(y)x + yd(x))^n - (xy + yx) \in Z(R)$ for all $x, y \in I$, then R is commutative.*

Proof. If $d = 0$, then $xy + yx \in Z(R)$ for all $x, y \in I$ and R satisfies the same identities. In this case the identity is polynomial so that there exists a field F such that R and F_n satisfy the same identities. Thus pick $x = e_{12}$, and $y = e_{22}$ and $xy + yx = e_{12} \notin Z(R)$, a contradiction. Therefore $n = 1$ and R is commutative. We may assume that $d \neq 0$.

If $(d(x)y + xd(y)) + d(y)x + yd(x))^n = xy + yx$ for all $x, y \in I$, then R is commutative by Theorem 1. Otherwise we have $I \cap Z(R) \neq 0$ by our assumptions. Let now J be a nonzero two-sided ideal of R_Z, the ring of the central quotients of R. Since $J \cap R$ is an ideal of R, then $J \cap R \cap Z(R) \neq 0$. Hence that is J contains an invertible element in R_Z, and so R_Z is simple with 1. By the hypothesis for any $x, y \in I$ and $r \in R$, thus I satisfies the differential identity

$$[(d(x)y + xd(y) + d(y)x + yd(x))^n - (xy + yx), r] = 0.$$

If d is not inner, then I satisfies the polynomial identity

$$[(zy + xw + yz + wx)^n - (xy + yx), r] = 0$$

by Kharchenko's theorem; and setting $z = w = 0$ yields the identity $[xy + yx, r] = 0$. In this case it is well known that there exists a field F such that R and F_m satisfy the same polynomial identities. Thus $xy + yx$ is central in F_m. Suppose $m \geq 2$ and choose $x = e_{12}, y = e_{22}$. Then $xy + yx = e_{12} \notin Z(R)$ contrary to our assumptions. This forces $m \leq 1$, i.e., R is commutative.

Now let d be an inner derivation induced by an element $A \in Q$. Since $d \neq 0$ we may assume that $0 \neq A$. By localizing R at $Z(R)$ it is easy to see that $([A, x]y + x[A, y] + [A, y]x + y[A, x])^n - (xy + yx) \in Z(R_Z)$ for any $x, y \in R_Z$. Since R and R_Z satisfy the same polynomial identities, in order to prove that R satisfies $s_4(x_1, x_2, x_3, x_4)$, we may assume that R is simple with 1. In this case, $([A,x]y + x[A,y] + [A,y]x + y[A,x])^n - (xy + yx) \in Z(R)$ for all $x, y \in R$. Therefore R satisfies a generalized polynomial identity and it is simple with 1, which implies that $Q = RC = R$ and R has a minimal right ideal. Thus $A \in R = Q$ and R is simple Artinian; that is, $R = D_k$, where D is a division ring finite dimensional over $Z(R)$ by [13]. Then it follows that there exists a suitable field F such that $R \subseteq M_k(F)$, the ring of all $k \times k$ matrices over F, and moreover $M_k(F)$ satisfies the generalized polynomial identity $[([A,x]y + x[A,y] + [A,y]x + y[A,x])^n - (xy + yx), r] = 0$ by [11, Lemma 2]. By Lemma 1, R is commutative. □

Corollary 1. *Let R be a prime ring, I a nonzero ideal of R and d a derivation of R.*

(i) *If $d(x)x + xd(x) = x^2$ for all $x \in I$, then R is commutative.*

(ii) *If $\mathrm{char}R \neq 2$ and $d(x)x + xd(x) - x^2 \in Z(R)$ for all $x \in I$, then R is commutative.*

Proof. (i) Linearizing $d(x)x + xd(x) = x^2$ for all $x \in I$ we get $d(x)y + xd(y) + d(y)x + yd(x) = xy + yx$ for all $x, y \in I$. Now apply Theorem 1 for $n = 1$.

Similarly (ii) can be proved by using Theorem 2. □

The following examples show that we cannot omit the primeness condition on Theorem 1.

Example. Let S be any commutative ring.

(i) Let $R = \left\{ \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} : a, b \in S \right\}$ and $I = \left\{ \begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix} : a \in S \right\}$.

Define $d : R \to R$ by $d\left(\left(\begin{smallmatrix} a & b \\ 0 & 0 \end{smallmatrix}\right)\right) = \left(\begin{smallmatrix} 0 & a \\ 0 & 0 \end{smallmatrix}\right)$. Then R is a ring under the usual operations. It is easy to see that I is a nonzero ideal of R and d is a nonzero derivation of R such that for all positive integers n $(d(x)y + xd(y) + d(y)x + yd(x))^n = xy + yx$ for all $x, y \in I$ but R is not commutative.

(ii) Let $R = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, b, c \in S \right\}$ and $I = \left\{ \begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix} : a \in S \right\}$.

Define $d : R \to R$ by $d\left(\left(\begin{smallmatrix} a & b \\ 0 & c \end{smallmatrix}\right)\right) = \left(\begin{smallmatrix} 0 & a-b-c \\ 0 & 0 \end{smallmatrix}\right)$. Then R is a ring under the usual operations. It is easy to see that I is a nonzero ideal of R and d is a nonzero derivation of R such that $d(x)y + xd(y) + d(y)x + yd(x) = xy + yx$ for all $x, y \in I$ but R is not commutative.

3. Derivations in semiprime rings

In all that follows, R will be a semiprime ring. We will make use of the left Utumi quotient ring U of R. So we need to mention that the definition, the axiomatic formulation and the properties of this quotient ring can be found in [2], [6], [10].

In order to prove that the same results are also valid for a semiprime ring R rather than any nonzero ideal of R, we will make use of the following facts:

Claim 1 ([2, Proposition 2.5.1])**.** Any derivation of a semiprime ring R can be uniquely extended to a derivation of its left Utumi quotient ring U and so any derivation of R can be defined on the whole U.

Claim 2 ([2, p. 38])**.** If R is a semiprime ring, then so is its left Utumi quotient ring. The extended centroid C of a semiprime ring coincides with the center of its left Utumi quotient ring.

Claim 3 ([2, p. 42])**.** Let B be the set of all the idempotents in C, the extended centroid of R. Assume R is a B-algebra which is orthogonal complete. For any maximal ideal P of B, PR forms a minimal prime ideal of R, which is invariant under any derivation of R.

Theorem 3. *Let R be a semiprime ring and n a fixed positive integer. If R admits a derivation d such that $(d(x)y + xd(y) + d(y)x + yd(x))^n = xy + yx$ for all $x, y \in R$, then R is a commutative ring.*

Proof. Since R is semiprime, by Claim 2, $Z(U) = C$, the extended centroid of R, and, by Claim 1, derivation d can be uniquely extended on U. Since U and R satisfy the same differential identities (see [12]), then $(d(x)y + xd(y) + d(y)x + yd(x))^n = xy + yx$ for all $x, y \in U$. Let B be the complete boolean algebra of idempotents in C and M be any maximal ideal of B.

Since U is a B-algebra which is orthogonal complete (see [12], p. 42, (2) of Fact 1), by Claim 3, MU is a prime ideal of U, which is d-invariant. Denote $\overline{U} = U/MU$ and $\overline{d}$ the derivation induced by d on U. Therefore $\overline{d}$ satisfies in $\overline{U}$ the same property of d on U. In particular $\overline{U}$ is a prime ring and so, by Theorem 1, for all maximal ideals M of B we obtain that $[U, U] \subseteq MU$. Therefore $[U, U] \subseteq \bigcap_M MU = 0$. In particular R is commutative. □

Theorem 4. *Let R be a 2-torsion free semiprime ring and n a fixed positive integer. If R admits a derivation d such that $(d(x)y+xd(y)+d(y)x+yd(x))^n - (xy + yx) \in Z(R)$ for all $x, y \in R$, then R is commutative.*

Proof. By Claim 2, $Z(U) = C$, and by Claim 1, d can be uniquely defined on whole U. Since U and R satisfies the same differential identities $(d(x)y+xd(y)+d(y)x+yd(x))^n-(xy+yx) \in Z(U)$ for all $x, y \in U$. Let B the complete boolean algebra of idempotents in C and M any maximal ideal of B. As already pointed out in the proof of Theorem 3, U is a B-algebra which is orthogonal complete and by Claim 3, MU is a prime ideal of U, which is d-invariant. Let $\overline{d}$ be the derivation induced by d on $\overline{U} = U/MU$. Since $Z(\overline{U}) = (C + MU)/MU = C/MU$, then $(\overline{d}(x)y + x\overline{d}(y) + \overline{d}(y)x + y\overline{d}(x))^n - (xy + yx) \in (C + MU)/MU$ for any $x, y \in \overline{U}$. Moreover $\overline{U}$ is a prime ring, hence we may conclude $\overline{U}$ is commutative by Theorem 2. This implies that, for any maximal ideal M of B, we have $[U, U] \subseteq MU$. Hence $[U, U] \subseteq \bigcap_M MU = 0$. In particular R is commutative. □

Corollary 2. *Let R be a semiprime ring and d a nonzero derivation of R.*

(i) *If $d(x)x + xd(x) = x^2$ for all $x \in R$, then R is commutative.*

(ii) *If R is 2-torsion free and $d(x)x + xd(x) - x^2 \in Z(R)$ for all $x \in R$, then R is commutative.*

Proof. (i) Linearizing $d(x)x + xd(x) = x^2$ for all $x \in R$, we get $(d(x)y + xd(y) + d(y)x + yd(x))^n - (xy + yx) = 0$ for all $x, y \in R$. Then R is commutative by Theorem 3.

Similarly (ii) can be proved by using Theorem 4. □

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