

# A Multi-stage Multi-criteria Transshipment Model for Optimal Selection of Transshipment Nodes - Case of Train Ferry-

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*Abstract : A strategic decision making on location selection for product transportation includes many tangible and untangible factors. To choose the best locations is a difficult job in the sense that objectives usually conflict with each other. In this paper, we consider a multi stage multi criteria transshipment problem with different types of items to be transported from the sources to the destination points. For the optimization of the problem, a goal programming formulation will be presented in which the location selection for each product type will be determined under the multi objective criteria. In the study, we generalize the transshipment model with a variety of product types and finite number of different intermediate nodes between origins and destinations. For the selection of the criteria we selected the costs(fixed cost and transportation cost), location numbers, and unsatisfied demand for each type of products in multi stage transportation, which are the main goals in transshipment modelling problems. The related conditions are also modelled through linear formats.*

Key words : Goal programming, Multi-criteria, Multi-stage, Location selection

## 1. Introduction

Traditionally, the transportation and transshipment problems have been solved with one objective of minimization of operating cost through different mathematical programming techniques. However, in modern transport industries the decision makers should consider more aspects than just the costs involved in the product transportation. In this regards, the selection for the best locations requires the trade off among quantitative and qualitative factors some of which may conflict with each other.

This paper considers a multi stage multi criteria problem for location selections from which multi-typed items will be transported through intermediate transshipment points(nodes) on to the destinations. The multiple criteria are chosen considering the transportation cost, fixed cost for each location, limitation on the location number and the amount of unsatisfied demand at the destinations. The priority level is identified for each criterion. A goal programming along with other constraints will be constructed and a numerical example with a train ferry transport will be presented as a simple application to the model.

Yang and Choi(2007) used the goal programming method for Rice processing industry with cost and managerial performance as the two criteria. Gunneç and Salman(2006)

presented a two stage stochastic programming model for the selection of location of emergency centers to prepare for the earthquake. Badri(1999) combined the analytic hierarchy process(AHP) method with the goal programming model for international production location allocation problem, where the AHP weights were used for the qualitative factors. Ghodsypour and Brien(1998) considered AHP and linear programming for supplier selection problem. In the study, we generalize the transshipment model with a variety of product types and finite number of different intermediate nodes between origins and destinations In section II, the goal programming model is developed considering several goals with corresponding priority levels, section III presents an example of the model with inter modal transportation system, and the concluding remarks and future research will be summarized in section IV.

## 2. Mathematical model

### 2.1. Purpose of goal programming

The location selection for multi typed items is formulated as a multi stage multi criteria goal programming model. For the first stage, the decision is whether to open a location for the transport of items of each type. If so, we determine how many of the items will be transshipped through the possible

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intermediate nodes to the destinations. For this purpose, we minimize the overall deviations in the objective function given the several goals with priority levels and various corresponding constraints. The goals and notation for the study are defined as follows:

## 2.2 Goals

Goal 1: The sum of the fixed (opening) cost for all types of items(products) should not exceed the targeted amount ( $F$ )

Goal 2: The total transportation costs for all types items should not exceed the targeted amount( $G$ )

Goal 3: The sum of Goal 1 and Goal 2 should not exceed the targeted amount( $H$ )

Goal 4: The total number of the selected locations for all typed items should not exceed the targeted number( $L$ )

Goal 5: The total number of unsatisfied demand by the destinations should not exceed the targeted number( $U$ )

## 2.3. Notation

The following notation is used for the formulation of the model .

1) parameters to be chosen

The parameters include item(product) types, the number of destinations, the number of intermediate stages and number of nodes for each stage for each type, and the target value for each goal.

$C$  : Set of product(item) types

$N_c$  : Number of transshipment stages for the product type  $c$  to be transported

$K$  : Number of destinations

$I_{nc}$  : Set of possible points(nodes) for product  $c$  in stage  $n$

$f_i^c$  : Fixed cost of opening location(source)  $i$  for type  $c$

$F_c$  : Total fixed cost for type  $c$

$F$  : Total fixed cost for all types of products

$L_c$  : Number of locations to be opened for type  $c$

$L$  : Total number of locations to be opened for all types

$G_c$  : Total transportation cost for type  $c$

$G$  : Total transportation cost for all types

$H_c$  : Total operating cost(fixed cost + transportation cost) for type  $c$

$H$  : Total operating cost for all types

$D_k^c$  : Demand of destination  $k$  for type  $c$

$U$  : Total amount of unsatisfied product of all types

$M$  : A big number

$C_{ij}^{nc}$  : Unit transportation cost from node  $i$  in stage  $n$  to node  $j$  in stage  $(n+1)$

2) decision variables to be decided

The model includes three types of decision variables. The first set of 0-1 integer variable denoting the selection of locations for each type of items, the second set of decision variable to transport items for each type from one stage to the next, and the final set of decision variable representing the amount of unsatisfied items for each type by the destinations.

$Y_i^c$  : 1, if location  $i$  for type  $c$  is open and 0, otherwise

$X_{ij}^{nc}$  : Number of units transported from node  $i$  in stage  $n$  to node  $j$  in stage  $(n+1)$

$X_k^{(N_c-1)c}$  : Amount of unsatisfied type  $c$  product by destination  $k$

## 2.4. Goal programming model

Along with the goals and notation described, the goal programming model is constructed as follows. The objective is to minimize the total positive deviations, which stem from the prescribed goals and the target values. For the selection of the criteria we selected the costs(fixed cost and transportation cost), location numbers, and unsatisfied demand for each type of products in multi stage transportation, which are the main goals in transshipment modelling problems. The related conditions are also modelled through linear formats as follows.

$$\text{Objective Min } P_1(d_1^+), P_2(d_2^+), P_3(d_3^+), P_4(d_4^+), P_5(d_5^+) \quad (1)$$

subject to

$$\sum_{i \in I_{1c}} f_i^c Y_i^c \leq F_c, \text{ for all } c \in C \quad (2)$$

$$\sum_{c \in C} \sum_{i \in I_{1c}} f_i^c Y_i^c + d_1^- - d_1^+ = F \quad (3)$$

$$\sum_{i \in I_{nc}} \sum_{j \in I_{(n+1)c}} C_{ij}^{nc} X_{ij}^{nc} \leq G_c, \text{ for all } c \in C, \quad (4)$$

$$n \in N_c$$

$$\sum_{c \in C} \sum_{i \in I_{nc}} \sum_{j \in I_{(n+1)c}} C_{ij}^{nc} X_{ij}^{nc} + d_2^- - d_2^+ = G, \quad (5)$$

$$\sum_{i \in I_{1c}} f_i^c Y_i^c + \sum_{i \in I_{nc}} \sum_{j \in I_{(n+1)c}} C_{ij}^{nc} X_{ij}^{nc} \leq H_c, \text{ for all } c \in C \quad (6)$$

$$\sum_{c \in C} \left( \sum_{i \in I_{1c}} f_i^c Y_i^c + \sum_{i \in I_{nc}} \sum_{j \in I_{(n+1)c}} C_{ij}^{nc} X_{ij}^{nc} \right) + d_3^- - d_3^+ = H, \quad (7)$$

$$\sum_{i \in I_{1c}} Y_i^c \geq L_c \text{ for all } c \in C \quad (8)$$

$$\sum_{c \in C} \sum_{i \in I_{1c}} Y_i^c + d_4^- - d_4^+ = L \quad (9)$$

$$\sum_{i \in I_{(N_c-1)c}} X_{ik}^{(N_c-1)c} + X_k^{(N_c-1)c} = D_k^c, \text{ for all } k \in K, c \in C \quad (10)$$

$$\sum_{c \in C} \sum_{k \in K} X_k^{(N_c-1)c} + d_5^- - d_5^+ = U \quad (11)$$

$$\sum_{i \in I_{nc}} X_{ij}^{nc} = \sum_{k \in I_{(n+2)c}} X_{jk}^{(n+1)c}, \text{ for all } j \in I_{(n+1)c}, c \in C, n \in N_c \quad (12)$$

$$\sum_{j \in I_{2c}} X_{ij}^{1c} \leq M Y_i^c, \text{ for all } c \in C, i \in I_{1c}, \quad (13)$$

$$X_{ij}^{nc}, X_k^{(N_c-1)c} \geq 0, \text{ for all } i \in I_{nc}, j \in I_{(n+1)c}, n \in N_c, c \in C, k \in K$$

$$d_i^+, d_i^- \geq 0, \text{ for all } i = 1, 2, 3, 4, 5$$

The objective function (1) represents the 5 goals each of which is assigned the priority level by  $P_i$ . Constraint (2) is the total fixed cost for type  $c$  items, and (3) is the goal equation for all types. Constraint (4) is the total transportation costs for type  $c$  and (5) is the goal equation for all types of items. Constraint (6) is the sum of operating cost(fixed cost + transportation costs) for type  $c$  and (7) is the goal equation for all types. Constraint (8) is the lower

limit on the number of locations for type  $c$  items and (9) is the goal equation for all types. Constraint (10) is the unsatisfied amount of items for type  $c$  by each destination and (11) is the goal equation for the sum of all types. Constraint (12) is the balance equation for each stage and node of each type. Constraint (13) is the amount of type  $c$  items leaving the location  $i$  if it is opened.

The priority level and the number of each goal can be determined at the decision maker's disposal depending on the relative importance of the criteria in the problem construction.

### 3. Numerical example

With the goal programming model developed for transport of different types of items through intermediate points, a simple numerical example with several objectives will be presented in this section. For the example, we apply the transportation model and cost data from Chu(2008). Assume that a decision maker of a company wishes to transport products from the production sites in Korea to ports in China through the train-ferry mode of transportation. Also, suppose that the manufacturer has 3 production sites, from which the products are carried by trains to the 3 transshipment ports through which the train ferry transports them on to the 4 destination ports. The following <table 1> and <table 2> represent the expected(hypothesized) transportation costs. Since the train ferry transportation between two countries are still not available, these costs are estimated according to the distance to travel and ferry fare(See Chu(2008) for cost estimation).

Table 1 Transportation cost from a location to a port (train transport per TEU, unit: won)

from-to	4. In-Cheon	5. Pyung-Taek	6. Gwang-Yang
1. Seoul	19,800	37,070	199,540
2. Dae-Jeon	89,870	44,990	121,010
3. Chang-Won	216,810	172,370	68,640

Table 2 Cost from each port to a destination port in China (train-ferry, Unit: won)

from-to	7. Lianyungang	8. Dalian	9. Shanghai	10. Chingdao
4. In-Cheon	866,000	649,000	1,097,000	750,000
5. Pyung-Taek	879,000	700,000	1,115,000	772,000
6. Gwang-Yang	1,050,000	1,198,300	972,000	1,023,000

The multiple criteria goal programming model is formulated and solved with the aid of QM for Windows software program. The goals are total transportation( $d_1^+$ ) cost, the unsatisfied amount by Shanghai port( $d_2^+$ ), and the total amount of unsatisfied items required by all ports( $d_3^+$ ). The right hand side of each constraint is hypothesized value.

$$\text{Objective Min } P_1(d_1^+), P_2(d_2^+), P_3(d_3^+),$$

subject to

$$19,800X_{14} + 37,070X_{15} + 199,540X_{16} + 89,870X_{24} + 44,990X_{25} + 121,010X_{26} + 216,810X_{34} + 172,370X_{35} + 68,640X_{36} + 866,000X_{47} + 649,000X_{48} + 1,097,000X_{49} + 750,000X_{410} + 879,000X_{57} + 700,000X_{58} + 1,115,000X_{59} + 772,000X_{510} + 1,050,000X_{67} + 1,198,300X_{68} + 972,000X_{69} + 1,023,000X_{610} + d_1^- - d_1^+ = 100,000,000$$

$$X_9 + d_2^- - d_2^+ = 10$$

$$X_7 + X_8 + X_9 + X_{10} + d_3^- - d_3^+ = 35$$

$$X_{14} + X_{15} + X_{16} \geq 1$$

$$X_{24} + X_{25} + X_{26} \geq 1$$

$$X_{34} + X_{35} + X_{36} \geq 1$$

$$X_{14} + X_{24} + X_{34} - X_{47} - X_{48} - X_{49} - X_{410} = 0$$

$$X_{15} + X_{25} + X_{35} - X_{57} - X_{58} - X_{59} - X_{510} = 0$$

$$X_{16} + X_{26} + X_{36} - X_{67} - X_{68} - X_{69} - X_{610} = 0$$

$$X_{47} + X_{57} + X_{67} + X_7 = 2150$$

$$X_{48} + X_{58} + X_{68} + X_8 = 300$$

$$X_{49} + X_{59} + X_{69} + X_9 = 370$$

$$X_{410} + X_{510} + X_{610} + X_{10} = 160$$

$$X_{ij}, X_i, d_i^-, d_i^+ \geq 0$$

The result is presented in the <table 3> below showing different amount to be delivered from origins to destinations with different priority levels for objective functions. For the original priority level, the first and second goals have been met whereas the third goal exceeds the limit by 7.34. With the priority levels changed, the third goal on the transportation cost is 6,499,600 above the predetermined limit.

Table 3 Result of goal programming with 3 criteria

P1(d1+), P2(d2+), P3(d3+)			Goal priority	P1(d3+), P2(d2+), P3(d1+)		
Decision variable analysis						
110.66			X14	118		
1			X25	1		
1			X35	1		
110.66			X47	118		
2			X57	2		
37.34			X7	30		
Priority analysis						
0			Priority 1	0		
0			Priority 2	0		
7.34			Priority 3	6499600		
RHS	d+	d-	Constraint analysis	RHS	d+	d-
100000000	0	0		100000000	6499600	0
10	0	10		10	0	10
30	7.34	0		30	0	0

#### 4. Conclusion

In this paper, we constructed the location selection model having multi stage, multi criteria and multi type items with goal programming and various constraints. The model can be applied to production(manufacturing) system, computer networking and other network systems as well as (multi modal) transportation system. For more practical applications of the model, however, stochastic demand at destinations(or origins) for items of each type and sojourn times between any two intermediate points should be considered as additional goals to reflect on the real life situations.

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