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INTERVAL-VALUED FUZZY m-SEMIOPEN SETS AND INTERVAL-VALUED FUZZY m-PREOPEN SETS ON INTERVAL-VALUED FUZZY MINIMAL SPACES

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Abstract. We introduce the concepts of IVF m-semiopen sets, IVF m-preopen sets, IVF m-semicontinuous mappings and IVF m-precontinuous mappings on interval-valued fuzzy minimal spaces. We investigate characterizations of IVF m-semicontinuous mappings and IVF m-precontinuous mappings and study properties of IVF m-semiopen sets and IVF m-preopen sets.

1. Introduction and preliminaries

Zadeh [9] introduced the concept of fuzzy set and several researchers were concerned about the generalizations of the concept of fuzzy sets, intuitionistic fuzzy sets [1] and interval-valued fuzzy sets [3]. Alimohammady and Roohi [2] introduced fuzzy minimal structures and fuzzy minimal spaces and some results are given. In [6], Min introduced the concepts of IVF minimal structures and IVF m-continuous mappings which are a generalization of IVF topologies and IVF continuous mappings [8], respectively. Min and Yoo [7] introduced the concepts of IVF $m\alpha$-open sets and IVF $m\alpha$-continuous mappings defined on interval-valued fuzzy minimal spaces. In this paper, we introduce the concepts of IVF m-semiopen sets, IVF m-preopen sets, IVF m-semicontinuous mappings and IVF m-precontinuous mappings on interval-valued fuzzy minimal spaces. We investigate basic properties of IVF m-semiopen sets and IVF m-preopen sets. The concepts of IVF m-semicontinuous mappings and IVF m-precontinuous mappings are generalizations of IVF

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m-continuous mappings and IVF $m\alpha$-continuous mappings in interval-valued fuzzy minimal spaces. We investigate characterizations and relationships among IVF m-open sets, IVF $m\alpha$-open sets, IVF m-semiopen sets and IVF m-preopen sets.

Let $D[0,1]$ be the set of all closed subintervals of the interval $[0,1]$. The elements of $D[0,1]$ are generally denoted by capital letters $M, N, \cdots$ and note that $M = [M^L, M^U]$, where M^L and M^U are the lower and the upper end points respectively. Especially, we denote $\mathbf{0} = [0,0], \mathbf{1} = [1,1]$, and $\mathbf{a} = [a,a]$ for $a \in (0,1)$. We also note that

(1) $(\forall M, N \in D[0,1])(M = N \Leftrightarrow M^L = N^L, M^U = N^U)$.

(2) $(\forall M, N \in D[0,1])(M \leq N \Leftrightarrow M^L \leq N^L, M^U \leq N^U)$.

For every $M \in D[0,1]$, the complement of M, denoted by M^c, is defined by $M^c = 1 - M = [1 - M^U, 1 - M^L]$.

Let X be a nonempty set. A mapping $A : X \to D[0,1]$ is called an interval-valued fuzzy set (simply, IVF set) in X. For each $x \in X$, $A(x)$ is a closed interval whose lower and upper end points are denoted by $A(x)^L$ and $A(x)^U$, respectively. For any $[a,b] \in D[0,1]$, the IVF set whose value is the interval $[a,b]$ for all $x \in X$ is denoted by $\widetilde{[a,b]}$. In particular, for any $a \in [a,b]$, the IVF set whose value is $\mathbf{a} = [a,a]$ for all x X is denoted by simply $\widetilde{a}$. For a point $p \in X$ and for $[a,b] \in D[0,1]$ with $b > 0$, the IVF set which takes the value $[a,b]$ at p and $\mathbf{0}$ elsewhere in X is called an interval-valued fuzzy point (simply, IVF point) and is denoted by $[a,b]_p$. In particular, if $b = a$, then it is also denoted by a_p. Denoted by IVF(X) the set of all IVF sets in X. An IVF point M_x, where $M \in D[0,1]$, is said to belong to an IVF set A in X, denoted by $M_x \widetilde{\in} A$, if $A(x)^L \geq M^L$ and $A(x)^U \geq M^U$. In [8], it has been shown that $A = \cup\{M_x : M_x \widetilde{\in} A\}$.

For every $A, B \in IVF(X)$, we define

$$A = B \Leftrightarrow (\forall x \in X)([A(x)]^L = [B(x)]^L \text{ and } [A(x)]^U = [B(x)]^U),$$

$$A \subseteq B \Leftrightarrow (\forall x \in X)([A(x)]^L \subseteq [B(x)]^L \text{ and } [A(x)]^U \subseteq [B(x)]^U).$$

The complement A^c of A is defined by

$$[A^c(x)]^L = 1 - [A(x)]^U \text{ and } [A^c(x)]^U = 1 - [A(x)]^L$$

for all $x \in X$.

For a family of IVF sets $\{A_i : i \in J\}$ where J is an index set, the union $G = \cup_{i \in J} A_i$ and $F = \cap_{i \in J} A_i$ are defined by

$$(\forall x \in X)([G(x)]^L = \sup_{i \in J}[A_i(x)]^L, [G(x)]^U = \sup_{i \in J}[A_i(x)]^U),$$

$$(\forall x \in X)([F(x)]^L = \inf_{i \in J}[A_i(x)]^L, [F(x)]^U = \inf_{i \in J}[A_i(x)]^U),$$

respectively.

Let $f : X \to Y$ be a mapping and let A be an IVF set in X. Then the image of A under f, denoted by $f(A)$, is defined as follows

$$[f(A)(y)]^L = \begin{cases} \sup\limits_{f(x)=y} [A(x)]^L, & \text{if } f^{-1}(y) \neq \emptyset,\ y \in Y \\ 0, & \text{otherwise}, \end{cases}$$

$$[f(A)(y)]^U = \begin{cases} \sup\limits_{f(x)=y} [A(x)]^U, & \text{if } f^{-1}(y) \neq \emptyset,\ y \in Y \\ 0, & \text{otherwise}, \end{cases}$$

for all $y \in Y$.

Let B be an IVF set in Y. Then the inverse image of B under f, denoted by $f^{-1}(B)$, is defined as follows

$$(\forall x \in X)([f^{-1}(B)(x)]^L = [B(f(x))]^L, [f^{-1}(B)(x)]^U = [B(f(x))]^U).$$

Definition 1.1 ([7]). A family τ of IVF sets in X is called an *interval-valued fuzzy topology* on X if it satisfies:

(1) $\mathbf{0}, \mathbf{1} \in \tau$.
(2) $A, B \in \tau \Rightarrow A \cap B \in \tau$.
(3) For $i \in J$, $A_i \in \tau \Rightarrow \cup_{i \in J} A_i \in \tau$.

Every member of τ is called an IVF open set. An IVF set A is called an IVF closed set if the complement of A is an IVF open set. And (X, τ) is called an *interval-valued fuzzy topological space*.

Definition 1.2 ([4, 5]). An IVF set A in an IVF topological space (X, τ) is called

(1) an IVF semiopen set in X if $(\exists B \in \tau)(B \subseteq A \subseteq Cl(B))$;
(2) an IVF preopen set in X if $A \subseteq Int(Cl(A))$;
(3) an IVF α-open set in X if $A \subseteq Int(Cl(Int(A))))$.

And an IVF set A is called an IVF semiclosed (resp., IVF preclosed, IVF α-closed) set if the complement of A is an IVF semiopen (resp., IVF preopen, IVF α-open) set.

Definition 1.3 ([6]). A family $\mathfrak{M}$ of interval-valued fuzzy sets in X is called an *interval-valued fuzzy minimal structure* on X if

$$\mathbf{0}, \mathbf{1} \in \mathfrak{M}.$$

In this case, $(X, \mathfrak{M})$ is called an *interval-valued fuzzy minimal space* (simply, *IVF minimal space*). Every member of $\mathfrak{M}$ is called an IVF m-open set. An IVF set A is called an IVF m-closed set if the complement of A (simply, A^c) is an IVF m-open set.

Let $(X, \mathfrak{M})$ be an IVF minimal space and A in IVF(X). The IVF minimal-closure and the IVF minimal-interior of A [6], denoted by $mC(A)$ and $mI(A)$, respectively, are defined as

$$mC(A) = \cap\{B \in IVF(X) : B^c \in \mathfrak{M} \text{ and } A \subseteq B\},$$

$$mI(A) = \cup\{B \in IVF(X) : B \in \mathfrak{M} \text{ and } B \subseteq A\},$$

respectively.

Theorem 1.4 ([6]). *Let $(X, \mathfrak{M})$ be an IVF minimal space and A, B in IVF(X).*

(1) $mI(A) \subseteq A$ and if A is an IVF m-open set, then $mI(A) = A$.
(2) $A \subseteq mC(A)$ and if A is an IVF m-closed set, then $mC(A) = A$.
(3) If $A \subseteq B$, then $mI(A) \subseteq mI(B)$ and $mC(A) \subseteq mC(B)$.
(4) $mI(A) \cap mI(B) \supseteq mI(A \cap B)$ and $mC(A) \cup mC(B) \subseteq mC(A \cup B)$.
(5) $mI(mI(A)) = mI(A)$ and $mC(mC(A)) = mC(A)$.
(6) $\mathbf{1} - mC(A) = mI(\mathbf{1} - A)$ and $\mathbf{1} - mI(A) = mC(\mathbf{1} - A)$.

An IVF set A in an IVF minimal space $(X, \mathfrak{M})$ is called an *IVF $m\alpha$-open* [7] set in X if $A \subseteq mI(mC(mI(A)))$.

And an IVF set A is called an *IVF $m\alpha$-closed* set if the complement of A is an IVF $m\alpha$-open set.

Let $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$ be two IVF minimal spaces. Then a mapping $f : X \to Y$ is said to be

(1) *IVF m-continuous* [6] if for every $A \in \mathcal{M}_Y$, $f^{-1}(A)$ is in $\mathcal{M}_X$;

(2) *IVF $m\alpha$-continuous* [7] if for each IVF point M_x and each IVF m-open set V containing $f(M_x)$, there exists an IVF $m\alpha$-open set U containing M_x such that $f(U) \subseteq V$.

2. IVF m-semiopen sets and IVF m-semicontinuous mappings

Definition 2.1. Let $(X, \mathfrak{M})$ be an IVF minimal space and A in IVF(X). Then an IVF set A is called an IVF m-semiopen set in X if

$$A \subseteq mC(mI(A)).$$

An IVF set A is called an *IVF m-semiclosed* set if the complement of A is IVF m-semiopen.

Remark 2.2. Let $(X, \mathfrak{M})$ be an IVF minimal space and A in IVF(X). If the IVF minimal structure $\mathfrak{M}$ is an IVF topology, clearly an IVF m-semiopen set is IVF m-semiopen by Definition 1.2.

Every IVF $m\alpha$-open set is clearly IVF m-semiopen but the converse is not always true as shown in the next example.

Example 2.3. Let $X = \{a, b\}$, let A and B be IVF sets defined as follows

$$A(a) = [0.1, 0.6], A(b) = [0.3, 0.7]$$

and

$$B(a) = [0.2, 0.5], B(b) = [0.7, 0.3].$$

Consider $\mathfrak{M} = \{\mathbf{0}, A, B, \mathbf{1}\}$ as an IVF minimal structure on X. Let us consider an IVF set C defined as follows $C(a) = [0.3, 0.6]$ and $C(b) = [0.7, 0.3]$. Then C is an IVF m-semiopen set but it is not IVF $m\alpha$-open.

Lemma 2.4. *Let $(X, \mathfrak{M})$ be an IVF minimal space and $A \in IVF(X)$. Then*

A is an IVF m-semiclosed set if and only if $mI(mC(A)) \subseteq A$.

Proof. It is obtained from Theorem 1.4 and Definition 2.1. □

Theorem 2.5. *Let $(X, \mathfrak{M})$ be an IVF minimal space. Any union of IVF m-semiopen sets is IVF m-semiopen.*

Proof. Let A_i be an IVF m-semiopen set for $i \in J$. Then from Theorem 1.4,

$$A_i \subseteq mC(mI((A_i))) \subseteq mC(mI(\cup A_i)).$$

This implies $\cup A_i \subseteq mC(mI(\cup A_i))$. Hence $\cup A_i$ is an IVF m-semiopen set. □

Remark 2.6. Let $(X, \mathfrak{M})$ be an IVF minimal space. The intersection of any two IVF m-semiopen sets may not be an IVF m-semiopen set as shown in the next example.

Example 2.7. Let $X = \{a, b\}$, let A and B be IVF sets defined as follows

$$A(a) = [0.3, 0.7], A(b) = [0.4, 0.7]$$

and

$$B(a) = [0.2, 0.8], B(b) = [0.8, 0.4].$$

Consider $\mathfrak{M} = \{\mathbf{0}, A, B, \mathbf{1}\}$ as an IVF minimal structure on X. Then A, B are IVF m-semiopen sets but $C = A \cap B$ is not IVF m-semiopen.

Definition 2.8. Let $(X, \mathfrak{M})$ be an IVF minimal space. For $A \in IVF(X)$, the *semi-closure* and the *semi-interior* of A, denoted by $smC(A)$ and $smI(A)$, respectively, are defined as

$$smC(A) = \cap\{F \in IVF(X) : A \subseteq F, F \text{ is IVF } m\text{-semiclosed in } X\}$$

$$smI(A) = \cup\{U \in IVF(X) : U \subseteq A, U \text{ is IVF } m\text{-semiopen in } X\}.$$

Theorem 2.9. *Let $(X, \mathfrak{M})$ be an IVF minimal space and $A \in IVF(X)$. Then*
(1) $smI(A) \subseteq A$.
(2) If $A \subseteq B$, then $smI(A) \subseteq smI(B)$.
(3) A is IVF m-semiopen iff $smI(A) = A$.
(4) $smI(smI(A)) = smI(A)$.
(5) $smC(\mathbf{1} - A) = \mathbf{1} - smI(A)$ and $smI(\mathbf{1} - A) = \mathbf{1} - smC(A)$.

Proof. (1), (2) Obvious.
(3) It follows from Theorem 2.5.
(4) It follows from (3).
(5) For $A \in IVF(X)$,

$$\begin{aligned}\mathbf{1} - smI(A) &= \mathbf{1} - \cup\{U \in IVF(X) : U \subseteq A, U \text{ is IVF } m\text{-semiopen }\}\\ &= \cap\{\mathbf{1} - U : U \subseteq A, U \text{ is IVF } m\text{-semiopen }\}\\ &= \cap\{\mathbf{1} - U : \mathbf{1} - A \subseteq \mathbf{1} - U, U \text{ is IVF } m\text{-semiopen }\}\\ &= smC(\mathbf{1} - A).\end{aligned}$$

Similarly, we have $smI(\mathbf{1} - A) = \mathbf{1} - smC(A)$. □

Theorem 2.10. *Let $(X, \mathfrak{M})$ be an IVF minimal space and $A \in IVF(X)$. Then*
(1) $A \subseteq smC(A)$.
(2) If $A \subseteq B$, then $smC(A) \subseteq smC(B)$.
(3) F is IVF m-semiclosed iff $smC(F) = F$.
(4) $smC(smC(A)) = smC(A)$.

Proof. It is similar to the proof of Theorem 2.9. □

Theorem 2.11. *Let $(X, \mathfrak{M})$ be an IVF minimal space and $A \subseteq X$. Then*
(1) $M_x \widetilde{\in} smC(A)$ if and only if $A \cap V \neq \mathbf{0}$ for every IVF m-semiopen set V containing M_x.
(2) $M_x \widetilde{\in} smI(A)$ if and only if there exists an IVF m-semiopen set U such that $U \subseteq A$.

Proof. (1) Suppose there is an IVF m-semiopen set V containing M_x such that $A \cap V \widetilde{=} \mathbf{0}$. Then $\mathbf{1} - V$ is an IVF m-semiclosed set such that $A \subseteq \mathbf{1} - V$, $M_x \widetilde{\notin} \mathbf{1} - V$. This implies $M_x \widetilde{\notin} smC(A)$.

The other relation is obvious.

(2) Obvious. □

Definition 2.12. Let $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$ be IVF minimal spaces. Then $f : X \to Y$ is said to be *interval-valued fuzzy m-semicontinuous* (simply, IVF m-semicontinuous) if for each IVF point M_x and each IVF m-open set V containing $f(M_x)$, there exists an m-semiopen set U containing M_x such that $f(U) \subseteq V$.

Every IVF $m\alpha$-continuous mapping is IVF m-semicontinuous but the converse is not always true as shown in the next example.

Example 2.13. Let $X = \{a, b\}$, let A and B be IVF sets defined as in Example 2.3. Consider $\mathfrak{M} = \{\mathbf{0}, A, B, \mathbf{1}\}$ and $\mathfrak{N} = \{\mathbf{0}, A, B, C, \mathbf{1}\}$ as IVF minimal structures on X. Consider the identity mapping $f : (X, \mathfrak{M}) \to (X, \mathfrak{N})$. Then f is IVF m-semicontinuous but it is not IVF $m\alpha$-continuous.

Remark 2.14. Let $f : X \to Y$ be an IVF m-semicontinuous mapping between IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. If the IVF minimal structures $\mathcal{M}_X$ and $\mathcal{M}_Y$ are IVF topologies on X and Y, respectively, then f is IVF semicontinuous [5].

Theorem 2.15. *Let $f : X \to Y$ be a mapping on IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. Then the following statements are equivalent:*
(1) f is IVF m-semicontinuous.
(2) $f^{-1}(V)$ is an IVF m-semiopen set for each IVF m-open set V in Y.
(3) $f^{-1}(B)$ is an IVF m-semiclosed set for each IVF m-closed set B in Y.
(4) $f(smC(A)) \subseteq mC(f(A))$ for $A \subseteq X$.
(5) $smC(f^{-1}(B)) \subseteq f^{-1}(mC(B))$ for $B \in IVF(Y)$.
(6) $f^{-1}(mI(B)) \subseteq smI(f^{-1}(B))$ for $B \in IVF(Y)$.

Proof. (1) ⇒ (2) Let V be an IVF m-open set in Y and $M_x \widetilde{\in} f^{-1}(V)$. By hypothesis, there exists an IVF m-semiopen set U_{M_x} containing M_x such that $f(U_{M_x}) \subseteq V$. This implies $f^{-1}(V)$ is IVF m-semiopen.

(2) ⇒ (3) Obvious.

(3) $\Rightarrow$ (4) For $A \in IVF(X)$, we have

$f^{-1}(mC(f(A)))$
$= f^{-1}(\cap\{F \in IVF(Y) : f(A) \subseteq F$ and F is IVF m-closed$\})$
$= \cap\{f^{-1}(F) \in IVF(X) : A \subseteq f^{-1}(F)$ and F is IVF m-semiclosed$\}$
$\supseteq \cap\{K \in IVF(X) : A \subseteq K$ and K is IVF m-semiclosed$\}$
$= smC(A)$

Hence $f(smC(A)) \subseteq mC(f(A))$.

(4) $\Rightarrow$ (5)Obvious.

(5) $\Rightarrow$ (6) For $B \in IVF(Y)$, from Theorem 1.4, it follows

$$\begin{aligned} f^{-1}(mI(B)) &= f^{-1}(\mathbf{1} - mC(\mathbf{1} - B)) \\ &= \mathbf{1} - (f^{-1}(mC(\mathbf{1} - B))) \\ &\subseteq \mathbf{1} - smC(f^{-1}(\mathbf{1} - B)) \\ &= smI(f^{-1}(B)). \end{aligned}$$

Hence (6) is obtained.

(6) $\Rightarrow$ (1) Let M_x be an IVF point in X and V an IVF m-open set containing $f(M_x)$. Then from (6), it follows $M_x \widetilde{\in} f^{-1}(V) = f^{-1}(mI(V)) \subseteq smI(f^{-1}(V))$. So there exists an IVF m-semiopen set U containing M_x such that $M_x \widetilde{\in} U \subseteq f^{-1}(V)$. Hence f is IVF m-semicontinuous. $\square$

3. IVF m-preopen sets and IVF m-precontinuous mappings

In this section, we introduce the concepts of interval-valued fuzzy m-preopen sets and interval-valued fuzzy m-precontinuous mappings. And we study properties of such concepts.

Definition 3.1. Let $(X, \mathfrak{M})$ be an IVF minimal space and A in IVF(X). Then an IVF set A is called an IVF m-preopen set in X if

$$A \subseteq mI(mC(A)).$$

An IVF set A is called an *IVF m-preclosed* set if the complement of A is IVF m-preopen.

Remark 3.2. Let $(X, \mathfrak{M})$ be an IVF minimal space and A in IVF(X). If the IVF minimal structure $\mathfrak{M}$ is an IVF topology, obviously an IVF m-preopen set is IVF preopen.

Lemma 3.3. *Let $(X, \mathfrak{M})$ be an IVF minimal space. Then A is an IVF m-preclosed set if and only if $mC(mI(A)) \subseteq A$.*

Proof. It follows from Theorem 1.4. □

We have the following implications but the converses are not always true as shown in the next example.

$$\begin{array}{ccccc} & & & & \text{IVF } m\text{-semiopen} \\ & & & \nearrow & \\ \text{IVF } m\text{-open} & \longrightarrow & \text{IVF } m\alpha\text{-open} & & \\ & & & \searrow & \\ & & & & \text{IVF } m\text{-preopen} \end{array}$$

Example 3.4. Let $X = \{a, b\}$, let A and B be IVF sets defined as follows

$$A(a) = [0.1, 0.6], A(b) = [0.3, 0.7]$$

and

$$B(a) = [0.2, 0.5], B(b) = [0.7, 0.3].$$

Consider $\mathfrak{M} = \{\mathbf{0}, A, B, \mathbf{1}\}$ as an IVF minimal structure on X.

(1) Let us consider an IVF set E defined as follows: $E(a) = [0.1, 0.4]$ and $E(b) = [0.3, 0.6]$. Then E is IVF m-preopen but it is neither IVF m-semiopen nor IVF $m\alpha$-open.

(2) Let us consider an IVF set D defined as follows $D(a) = [0.3, 0.6]$ and $D(b) = [0.7, 0.3]$. Then $mI(D) = B$, $mC(B) = B^c$ and $mI(B^c) = B$. Therefore, D is IVF m-semiopen but it is not IVF m-preopen.

Theorem 3.5. *Let $(X, \mathfrak{M})$ be an IVF minimal space. Any union of IVF m-preopen sets is IVF m-preopen.*

Proof. Let A_i be an IVF m-preopen set for $i \in J$. Then

$$A_i \subseteq mI(mC(A_i)) \subseteq mI(mC(\cup A_i)).$$

This implies $\cup A_i \subseteq mI(mC(\cup A_i))$. Hence $\cup A_i$ is an IVF m-preopen set. □

Remark 3.6. Let $(X, \mathfrak{M})$ be an IVF minimal space. The intersection of any two IVF m-preopen sets may not be IVF m-preopen as shown in the next example.

Example 3.7. Let $X = \{a, b\}$, let A and B be IVF sets defined as follows

$$A(a) = [0.2, 0.5], A(b) = [0.6, 0.7]$$

and

$$B(a) = [0.3, 0.8], B(b) = [0.7, 0.6].$$

Consider $\mathfrak{M} = \{\mathbf{0}, A, B, \mathbf{1}\}$ as an IVF minimal structure on X. Then A, B are IVF m-peropen sets but $C = A \cap B$ is not IVF m-preopen.

Definition 3.8. Let $(X, \mathfrak{M})$ be an IVF minimal space. For $A \in IVF(X)$, the *pre-closure* and the *pre-interior* of A, denoted by $pmC(A)$ and $pmI(A)$, respectively, are defined as follows

$$pmC(A) = \cap\{F \in IVF(X) : A \subseteq F, F \text{ is IVF } m\text{-preclosed in } X\}$$

$$pmI(A) = \cup\{U \in IVF(X) : U \subseteq A, U \text{ is IVF } m\text{-preopen in } X\}.$$

Theorem 3.9. *Let $(X, \mathfrak{M})$ be an IVF minimal space and $A \in IVF(X)$. Then*

(1) $pmI(A) \subseteq A \subseteq pmC(A)$.

(2) If $A \subseteq B$, then $pmI(A) \subseteq pmI(B)$ and $pmC(A) \subseteq pmC(B)$.

(3) A is IVF m-preopen iff $pmI(A) = A$.

(4) F is IVF m-preclosed iff $pmC(F) = F$.

(6) $pmI(pmI(A)) = pmI(A)$ and $pmC(pmC(A)) = pmC(A)$.

(6) $pmC(\mathbf{1} - A) = \mathbf{1} - pmI(A)$ and $pmI(\mathbf{1} - A) = \mathbf{1} - pmC(A)$.

Proof. It is similar to the proof of Theorem 2.9. □

Theorem 3.10. *Let $(X, \mathfrak{M})$ be an IVF minimal space and $A \subseteq X$. Then*

(1) $M_x \widetilde{\in} pmC(A)$ if and only if $A \cap V \neq \mathbf{0}$ for every IVF m-preopen set V containing M_x.

(2) $M_x \widetilde{\in} pmI(A)$ if and only if there exists an IVF m-preopen set U such that $U \subseteq A$.

Proof. (1) Suppose there is an IVF m-preopen set V containing M_x such that $A \cap V = \mathbf{0}$. Then $X - V$ is an IVF m-preclosed set such that $A \subseteq \mathbf{1} - V$, $M_x \widetilde{\notin} \mathbf{1} - V$. This implies $M_x \widetilde{\notin} pmC(A)$.

The reverse relation is obvious.

(2) Obvious. □

Definition 3.11. Let $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$ be two IVF minimal spaces. Then $f : X \to Y$ is said to be *interval-valued fuzzy m-precontinuous* (simply, IVF m-precontinuous) if for each IVF point M_x and each IVF m-open set V containing $f(M_x)$, there exists an IVF m-preopen set U containing M_x such that $f(U) \subseteq V$.

We have the following implications but the converses are not always true as shown in the next example.

$$\text{IVF } m\text{-continuous} \longrightarrow \text{IVF } m\alpha\text{-continuous} \begin{array}{l} \nearrow \text{ IVF } m\text{-semicontinuous} \\ \searrow \text{ IVF } m\text{-precontinuous} \end{array}$$

Example 3.12. Let $X = \{a, b\}$, let A and B be IVF sets defined as follows

$$A(a) = [0.1, 0.6], A(b) = [0.2, 0.5],$$
$$B(a) = [0.2, 0.5], B(b) = [0.3, 0.4],$$
$$G(a) = [0.2, 0.7], G(b) = [0.3, 0.4].$$

(1) Consider $\mathfrak{M} = \{\mathbf{0}, A, B, \mathbf{1}\}$ and $\mathfrak{N} = \{\mathbf{0}, A, B, G, \mathbf{1}\}$ as IVF minimal structures on X. Let $f : (X, \mathfrak{M}) \to (X, \mathfrak{N})$ be the identity mapping. Then f is IVF m-semicontinuous. Note that:

$mC(G)$ as follows $mC(G)(a) = [0.4, 0.8], mC(G)(b) = [0.5, 0.7]$.

$mI(G)$ as follows $mI(G)(a) = [0.2, 0.6], mI(G)(b) = [0.3, 0.5]$.

Then G is IVF m-semiopen but it is not IVF m-preopen in $(X, \mathfrak{M})$. Thus f is not IVF m-precontinuous.

(2) Consider $\mathfrak{M} = \{\mathbf{0}, A, B, \mathbf{1}\}$ and $\mathfrak{N} = \{\mathbf{0}, A, B, A \cap B, \mathbf{1}\}$ as IVF minimal structures on X. Let $f : (X, \mathfrak{M}) \to (X, \mathfrak{N})$ be the identity mapping. Then f is IVF m-precontinuous. Note that $A \cap B$ is IVF m-pereopen but it is not IVF m-semiopen in $(X, \mathfrak{M})$. Thus f can not be IVF m-semicontinuous.

Recall that: Let (X, τ_1) and (Y, τ_2) be two IVFTS's. Then a mapping $f : X \to Y$ is said to be *IVF pre-continuous* [5] if for every IVF open set B in Y, $f^{-1}(B)$ is IVF preopen in X.

Remark 3.13. Let $f : X \to Y$ be an IVF m-precontinuous mapping between IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. If the IVF minimal structures $\mathcal{M}_X$ and $\mathcal{M}_Y$ are IVF topologies on X and Y, respectively, then f is IVF precontinuous.

Theorem 3.14. *Let $f : X \to Y$ be a mapping on IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. Then the following statements are equivalent:*

(1) f is IVF m-precontinuous.

(2) $f^{-1}(V)$ is an IVF m-preopen set for each IVF m-open set V in Y.

(3) $f^{-1}(F)$ is an IVF m-preclosed set for each IVF m-closed set F in Y.

(4) $f(pmC(A)) \subseteq mC(f(A))$ for $A \in IVF(X)$.

(5) $pmC(f^{-1}(B)) \subseteq f^{-1}(mC(B))$ for $B \in IVF(Y)$.

(6) $f^{-1}(mI(B)) \subseteq pmI(f^{-1}(B))$ for $B \in IVF(Y)$.

Proof. (1) $\Rightarrow$ (2) Let V be an IVF m-open set in Y and $M_x \widetilde{\in} f^{-1}(V)$. By (1), there exists an IVF m-preopen set U_{M_x} containing M_x such that $f(U_{M_x}) \subseteq V$. This implies $M_x \widetilde{\in} U_{M_x} \subseteq f^{-1}(V)$ for all $M_x \widetilde{\in} f^{-1}(V)$. Hence $f^{-1}(V)$ is IVF m-preopen.

(2) $\Rightarrow$ (3) It is obvious.

(3) $\Rightarrow$ (4) For $A \in IVF(X)$, we have the following:

$f^{-1}(mC(f(A)))$

$= f^{-1}(\cap\{F \in IVF(Y) : f(A) \subseteq F$ and F is IVF m-closed$\})$

$= \cap\{f^{-1}(F) \in IVF(X) : A \subseteq f^{-1}(F)$ and F is IVF m-preclosed$\}$

$\supseteq \cap\{K \in IVF(X) : A \subseteq K$ and K is IVF m-preclosed$\}$

$= pmC(A)$

Hence $f(pmC(A)) \subseteq mC(f(A))$.

(4) $\Rightarrow$ (5) For $B \in IVF(Y)$, from (4), it follows

$$f(pmC(f^{-1}(B))) \subseteq mC(f(f^{-1}(B))) \subseteq mC(B).$$

(5) $\Rightarrow$ (6) For $B \in IVF(Y)$, from Theorem 1.4, it follows

$$\begin{aligned} f^{-1}(mI(B)) &= f^{-1}(\mathbf{1} - mC(\mathbf{1} - B)) \\ &= \mathbf{1} - (f^{-1}(mC(\mathbf{1} - B))) \\ &\subseteq \mathbf{1} - pmC(f^{-1}(\mathbf{1} - B)) \\ &= pmI(f^{-1}(B)). \end{aligned}$$

Hence (6) is obtained.

(6) $\Rightarrow$ (1) Let M_x be an IVF point in X and V an IVF m-open set containing $f(M_x)$. Then $M_x \widetilde{\in} f^{-1}(V) = f^{-1}(mI(V)) \subseteq pmI(f^{-1}(V))$. So from Theorem 3.10, there exists an IVF m-preopen set U containing M_x such that $M_x \widetilde{\in} U \subseteq f^{-1}(V)$. Hence from definition of the IVF m-precontinuous mapping, f is IVF m-precontinuous. □

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