韓國開發硏究
제31권 제1호(통권 제104호)

Estimation the Natural Output Korea: A Bayesian DSGE Approach

Youngjin Hwang

(Hankuk Univeirsity of Foreign Studies,
Department of International Economics and Law, Full-Time Lecturer)

한국의 자연 산출량 추정: 베이지안 DSGE 접근법

황 영 진

(한국외국어대학교 국제통상학과 전임강사)

* Youngjin Hwang: (e-mail) youngjin@hufs.ac.kr, (address) Imun-dong 270, Dongdaemun-gu, Seoul, Korea

• Key Word: Natural output(자연 산출량(GDP)), Natural rate of interest(자연 이자율), DSGE model(동태적 확률 일반 균형 모형), Bayesian estimation(베이지안 추정)
• JEL code: D11, H21, H26, H31
• Received: 2009. 1. 15 • Referee Process Started: 2009. 1. 16
• Referee Reports Completed: 2009. 6. 4

ABSTRACT

This paper attempts to estimate the natural rates of output and interest of Korea in a simple DSGE set-up with a few stylized New Keynesian features using Bayesian methods. The major findings of this paper are as follows. First, the estimates of output gaps are less volatile than the measures from conventional approaches, although they exhibit non-negligible variations depending on the model specification. Another key finding is that the hybrid type Phillips curve with a backward-looking component and/or habit formation in consumption may play an important role in characterizing the macroeconomic dynamics of Korea.

본 논문은 일반적인 뉴 케인지언 이론의 주요 특징을 바탕으로 하는 동태적 확률 일반 균형 모형을 설정하고 이를 바탕으로 하여, 베이지안 추정법을 통해 한국의 자연 산출량과 자연 이자율의 추정을 시도하였다.

본 논문의 주요 결과는 다음과 같다. 첫째, 이러한 이론 모형에 의해 추정된 산출량 갭은 기존의 일반적인 접근법에 의한 추정치보다 변동 폭이 훨씬 작은 것으로 나타났다. 둘째, 다양한 모형 설정을 통해 결과의 민감도를 살펴본 경우, 필립스 커브에서의 과거 지향적 요인 및 소비 행태에서의 습관 형성 등이 한국 거시경제의 동태적 양상을 설명하는 데 중요한 요인일 수 있는 것으로 나타났다.

Ⅰ. Introduction

Central banks in most countries operate under a dual mandate to achieve both price stability and maximum sustainable employment. In that context, the potential output, the level of output at which demand and supply in the aggregate economy are balanced so that, all else being equal, inflation tends to move around its long-run expected value, can be naturally thought of as the level of output consistent with the above notion.[1]

The idea of potential output level and obtaining its appropriate estimates is critical in that they help policymakers assess (current) overall economic conditions and achieve such level consistent with the objective of dual mandates, and in that output gap, the difference between actual and potential output, may also play a key role in inflation dynamics. Hence, it does seem natural that the question of how we estimate the path of potential output, or how we determine whether the economy is operating above or below its maximum sustainable level, has been one of the key issues of practical importance and interest for policymakers as well as the academia.

Traditionally, there are two approaches in estimating the level of potential values of key macro economic variables such as the gross domestic product (GDP) and unemployment: (i) aggregate time-series approaches, which estimate the natural values of such variables, based on a few economic relationships such as the Phillips curve or Okun's law, through some reasonable statistical model specifications, and (ii) production function, or growth-accounting approaches, in which one generates estimates of potential variables, building on some nested functional form of aggregate production, from underlying factors of productions such as capital stock, labor input, and technological progress.[2]

It has become a recent standard practice in dynamic macroeconomic research to analyze a wide range of issues and policy agendas using models that feature optimizing agents with a variety of market imperfections. Such representative examples are the class of New Keynesian dynamic stochastic general equilibrium (DSGE) models. They contain many features of the earlier real business cycle literature but allow for rigidities and imperfections in the markets. As such, they are often referred to as the New Keynesian models. The New Keynesian DSGE models provide more realistic yet still theoretically elegant, representations of the economy, and their development has been an active area in macroeconomic research in recent years.

---

[1] The notion of the natural values of key macro variables such as output, unemployment rates, and interest rates dates back from Knut Wicksell, to Friedman (1968), and to the more recent research on monetary policy by Woodford (2003). Generally and simply put (at the risk of oversimplification), the natural levels/rates of macro variables are the market equilibrium values consistent with price and output stability (Amato, 2005). See also footnote 3).

[2] A detailed discussion on the individual approach is beyond the scope of this paper. See Mishkin (2007) for a brief yet comprehensive survey.

Building on this line of research, there have been a few recent attempts to estimate the potential/natural level of output in the DSGE framework. The DSGE models provide a somewhat different but complementary perspective on the definition of potential output than the one measured through conventional approaches mentioned above. In particular, the DSGE approaches think of potential output as the level of output that an economy could attain if the inefficiencies resulting from nominal wage and price rigidities were removed, that is, if wages and prices were fully flexible. The definition of potential output as a flexible price equilibrium has much in common with the more conventional definition because over time prices (and wages) do gravitate towards their equilibrium levels. As a result, the DSGE definition is in accordance with the idea that potential output is the level of output at which inflation tends neither to rise nor fall.[3]

With that said, it is worth emphasizing that the DSGE view of potential output also has important differences with the earlier approaches to estimating potential output. The DSGE measures of potential output are far more model-dependent than the conventional measures because they depend on the estimated parameters of the model and on the model's estimates of the structural shocks hitting the economy.[4]

What is more attractive is that the DSGE approaches, through appropriate general equilibrium model specification, allow us to jointly estimate natural interest rates as well as output level through a general equilibrium framework. The estimates of natural real interest rate are another key variable for the conduct of monetary policy, on which practically little emphasis has been given and which has been often generated in an ad-hoc manner, without much theoretical basis.

Motivated by the recent development on this line of research and with the advances of computing technique, this paper attempts to estimate the potential/natural level of output of Korea in a simple DSGE set-up. The baseline model can be characterized by a New Keynesian DSGE framework featuring nominal rigidities (in price) and market imperfection (monopolistically competitive producers), with a monetary policy - a la standard Taylor interest rate feedback rule. I then estimated the structural parameters of the model using Bayesian methods and, based on the estimation result, a series of output gaps, the key variable of interest, is generated. Despite its simplicity, the model allows us to

---

[3] However, it seems that this idea/definition of potential output is not universally accepted in DSGE literature. Some even argue that the notions of potential and natural output are conceptually distinct. For example, Justiano and Primericeri (2008) argued that the former is the level of output that would prevail if products and labor markets were perfectly competitive, while the latter is the level of output that would prevail under imperfectly competitive markets, but with flexible prices and wages. The latter is more closely related to the one in the model presented in this paper. See also McCallum (2001).

[4] Interestingly, as Mishkin (2007) pointed out, a few research have found that the properties of potential output and output gap fluctuations can be quite different from conventional measures. (See, Neiss and Nelson, 2005; Edge, Kiley, and Laforte, 2007; Justiniano and Primiceri, 2008). For example, in many DSGE models, potential output can undergo large swings over the business cycle, which are, in some sense, natural results given the equilibrium nature in which the business cycles are the primarily efficient responses to shocks. On the other hand, production function/growth-accounting approaches to estimating potential output, generally assuming that such shocks have no important effects on the potential output at business-cycle frequencies, typically yield smaller fluctuations than the measures of potential output derived from the DSGE models.

extract not only the potential level of output but also the natural nominal/real rates of interest and implicit intermediate inflation target for monetary authority.

The major findings of this paper are as follows. First, the estimates of output gaps are less volatile than the measures from conventional approaches, although they exhibit non-negligible variations depending on the model specification; models with no habit term in consumption or no lagged inflation term in Phillips curve result in more volatile output gap measures. Another key finding is that a hybrid type Phillips curve with a backward-looking component and/or habit formation in consumption may play an important role in characterizing the macroeconomic dynamics of Korea.

To the best of the author's knowledge, there has been no research that estimates the potential level of output, let alone the natural level of interest rate for the Korean economy from the perspective of the DSGE framework. Most of the previous research employed either a multivariate aggregate time-series or production function approach.[5] Although it is practically impossible to obtain perfectly reliable estimates and there is no guarantee that estimates from a particular DSGE model are clearly better or are more accurate, this work may shed some lights on this line of research emerging in Korea.

The rest of the paper is organized as follows. Section 2 outlines the baseline model in a linearized form, and then the estimation results of the model's parameters and key latent variables are presented in Section 3. Building on these results, Section 4 extends the baseline model in a few directions and investigates the robustness of the results. Finally, Section 5 concludes the study. The presentation of the original non-linear model and all the technical details are contained in the Appendix.

Ⅱ. A Model

The baseline model I considered is a simple version of New Keynesian DSGE model.[6] For expositional purposes, this section presents the log-linearized system of the model, while the original underlying non-linear model and related discussion are given in the Appendix.

1. Aggregate Demand Block

The demand side of the model is characterized by the following expectational "IS" equation, relating (log) real output, y_t, and real interest rate, $r_t - E_t[\pi_{t+1}]$, (nominal interest rate less expected inflation):

[5] See surveys by Park and Hur (2004) and Kim and Noh (2007).

[6] The baseline model is built on the works of Rotemberg and Woodford (1997), Furher (2000), Andres, Lopez-Salido, and Nelson (2005), and Bjornland, Leitemo, and Maih (2007).

$$\Delta y_t = \frac{\sigma}{h_1(\sigma-1)} E_t[\Delta y_{t+1}] - \frac{1}{h_1(\sigma-1)}(r_t - E_t[\pi_{t+1}] - \delta) + \frac{1}{h_1}(u_{p,t} - E_t[u_{p,t+1}]) \quad (1)$$

where Δ denotes the first difference operator, and E_t is the conditional expectation operator at time t. The Greek letters are the model parameters, σ is (constant relative) the risk aversion in the consumer's preference, δ is the time discount rate, and the degree of habit in consumption is represented by h_1, which reflects the utility gains from last period's consumption. Finally, the variable $u_{p,t}$ is the demand/preference shock, and its law of motion is described below.

2. Aggregate Supply Block

The model's supply block is represented by the familiar Phillips curve:

$$\pi_t = \mu E_t[\pi_{t+1}] + (1-\mu)\pi_{t-1} + \eta mc_t + u_{s,t} \quad (2)$$

where μ and $1-\mu$ are, respectively, the weights on the forward- and backward-looking components in the inflation dynamics and mc_t is the (log) real marginal cost. The magnitude of the response of inflation to the marginal cost is governed by the parameter η. As with the IS equation, every period the inflation rate is subject to a structural (cost-push) shock $u_{s,t}$.

While the "standard" Phillips curve takes the form without the lagged inflation term, the literature typically reports that standard Phillips curves empirically fail to describe actual inflation dynamics (Gali and Gertler, 1999). The extension section will later discuss more general versions of the Phillips curve and their implications.

I further assumed that (log) marginal costs are linear in the log deviation of output from the natural rate of output, that is, $mc_t = \zeta(y_t - y_t^n)$. By defining the deviation of output from the natural rate as the output gap, $x_t = y_t - y_t^n$, one can write the Phillips curve as:

$$\pi_t = \mu E_t[\pi_{t+1}] + (1-\mu)\pi_{t-1} + \kappa x_t + u_{s,t} \quad (3)$$

where $\kappa = \eta\zeta$.[7]

[7] As addressed in the Appendix, one may derive the Phillips curve (2) from a model set-up as in Calvo (1983). Literature typically makes assumptions on technology, preferences, and the structure of labor markets to justify the proportionate relation between the real marginal cost and the output gap (see Walsh, 2003, for example). In this context, the disturbance term can be interpreted as reflecting the deviations from the condition $mc_t = \zeta(y_t - y_t^n)$: The deviation from this proportionality condition can be caused, for example, by movements in nominal wages that push real wages away from their equilibrium values due to friction in the wage contracting process. Another interpretation of the disturbance is that it could reflect a shock to the natural and potential levels of output. More on

As noted in the Appendix, I do not endogenize the input of production factors and specify technology, but instead assume that the natural rate of output is given exogenously by the process:

$$\Delta y_t^n = g + u_{g,t} \tag{4}$$

where g is the unconditional expected growth rate of output and $u_{g,t}$ is an AR(1) shock (to be specified below) to the growth rate.

3. Natural Level of Variables

I will now describe the determination of the natural output and interest rate, and related processes. First, the relationship between natural output level and natural (nominal) interest rate can be determined by replacing y_t with y_t^n in (1):

$$\Delta y_t^n = \frac{\sigma}{h_1(\sigma-1)} E_t[\Delta y_{t+1}^n] - \frac{1}{h_1(\sigma-1)}(r_t^n - E_t[\pi_{t+1}] - \delta) + \frac{1}{h_1}(u_{p,t} - E_t[u_{p,t+1}]) \tag{5}$$

Solving (5) for the natural real interest rate yields:

$$r_t^n - E_t[\pi_{t+1}] = \delta + \sigma E_t[\Delta y_{t+1}^n] - h_1(\sigma-1)\Delta y_t^n + (\sigma-1)(u_{p,t} - E_t[u_{p,t+1}]) \tag{6}$$

To see the relationship between natural output and natural rate of interest from expression (6) more clearly, consider the standard case where there is no habit formation in consumption ($h_1 = 0$). In this case, the natural real interest rate is the sum of the expected future natural output growth times the reciprocal of intertemporal elasticity of substitution and discount rate. This is a standard relationship one may encounter in simple dynamic macro models.

Finally, using (1) and (5), the output gap process can be expressed as follows:

$$x_t = \frac{\sigma}{\sigma + h_1(\sigma-1)} E_t[x_{t+1}] + \frac{h_1(\sigma-1)}{\sigma + h_1(\sigma-1)} x_{t-1} - \frac{1}{\sigma + h_1(\sigma-1)}(r_t - r_t^n) \tag{7}$$

4. Monetary Policy

The monetary authority sets the nominal interest rate a la standard dynamic

---

this will be discussed in the estimation section. See also Clarida, Gali, and Gertler (1999) and Erceg, Henderson, and Levin (2000).

Taylor rule:

$$r_t = \psi r_{t-1} + (1-\psi)[r_t^n + \theta_\pi(\pi_t - \pi_t^T) + \theta_x x_t] + \epsilon_{mp,t}, \tag{8}$$

where Ψ measures the smoothing in the interest rate setting; r_t^n is the nominal natural interest rate; and $\epsilon_{mp,t}$ is the mean-zero interest rate policy shock. I assume the (unobserved) intermediate inflation target, π_t^T evolves as follows:

$$\pi_t^T = \rho_T \pi_{t-1}^T + u_{T,t} \tag{9}$$

The model is closed with the expressions for law of motion for stochastic shocks. I assumed that monetary policy shock is a white noise, with remaining shocks following the first-order autoregressive processes. More specifically,

$$u_{p,t} = \rho_p u_{p,t-1} + \epsilon_{p,t} \tag{10}$$

$$u_{s,t} = \rho_s u_{s,t-1} + \epsilon_{s,t} \tag{11}$$

$$u_{g,t} = \rho_g u_{g,t-1} + \epsilon_{g,t} \tag{12}$$

$$u_{T,t} = \rho_T u_{T,t-1} + \epsilon_{T,t} \tag{13}$$

The equilibrium of this economy consists of 10 endogenous variables, $\pi_t, \pi_t^T, x_t, y_t, r_t^n, r_t, u_{p,t}, u_{s,t}, u_{g,t},$ and $u_{T,t}$ (note that three variables of our interest, $\pi_t^T, x_t,$ and r_t^n are unobserved) whose dynamics satisfy the system characterized by (1), (3), (6)-(9), and (10)-(13). The model can then be solve by the standard methods developed by Blanchard and Khan (1980), Klein (2000), and Sims (2002).

The solution to this system takes the form of a state-space econometric model relating a vector of state variables, S_t, to a vector of flow variables, f_t:

$$S_{t+1} = \Phi_s S_t + \Phi_\varepsilon \epsilon_{t+1} \tag{14}$$

and

$$f_t = \Phi_f S_t \tag{15}$$

with $\epsilon_{t+1} = \{\epsilon_{p,t+1}, \epsilon_{s,t+1}, \epsilon_{g,t+1}, \epsilon_{T,t+1}, \epsilon_{mp,t+1}\}$ as the vector of error terms or innovations to shocks.

Equation (14) is the transition equation, while (15) is the measurement equation. In this solution, the vector of state variables includes all the predetermined variables $\{\pi_t, \pi_t^T, x_t, r_t\}$ and the exogenous variables $\{u_{p,t}, u_{s,t}, u_{g,t}, u_{T,t}\}$; the vector of flow variables contains $\{y_t, r_t^n\}$. Finally the elements in matrices $\Phi_s, \Phi_\epsilon,$ and Φ_f are the nonlinear functions of the (deep) structural parameters of the model and cannot be expressed in closed form.

Ⅲ. Estimation

The data I used are quarterly real output (gross domestic product), consumer price index (CPI), and call rate. All data were obtained from the Bank of Korea, Economic Statistical System (ECOS), and the sample period was from 1991:I -- 2008:III.[8]

I estimated the model using Bayesian methods. For the state-space representation of the model, (13) and (14), the log-likelihood function is constructed using the Kalman filter algorithm, as outlined in Hamilton (1994), and combined to some prior information to arrive at the posterior distribution of the parameters.

Table 1 lays out the assumptions for the prior distribution of the estimated parameters and structural shocks, and the parameter estimates for our baseline model. In the first four columns, the list of structural parameters with distribution type, their associated prior mean, and standard deviation are shown.[9] Following standard conventions, I used Beta distributions for the parameters that fall between 0 and 1; Gamma distributions for parameters that need to be constrained to be positive; and Normal distributions for other cases. The priors for the variances of structural shock variance are all inverted Gamma distributions. The next three columns report the posterior mean along with the 5th and 95th percentiles of the distribution.[10]

Starting with the expectational IS curve, I set the prior on the risk aversion parameter at $\sigma = 1.5$, which is well within the range of the estimates in the literature. The posterior has increased from the prior, although not too significantly (posterior mean equals 2.5185). In addition, the habit parameter, h_1, restricted to lie between zero and one, turns out to be fairly close to 0.7, which is similar to the estimate found in the literature (Fuhrer, 1995). Moreover, the demand/preference shocks display a high degree of persistence, with a coefficient of $\rho_p = 0.7871$ and large volatility of $\sigma_p = 0.0732$.[11]

---

8 All the log changes in the data (compared with the same quarter of last year) are measured at an annual rate. I treated inflation, nominal interest rate, and output growth as nonstationary - prior to estimation, the first two variables were Hodrick-Prescott filtered with a smoothing parameter of 1600, and GDP growth rates were demeaned. I calibrated the equilibrium steady-state inflation rate π^* to the (detrended) sample mean.

9 The Bayesian estimation technique allows us to use prior information from previous micro- and macro- based studies in a formal way, and the procedure of Bayesian inference starts out from a prior distribution of the model's non-calibrated parameters. This prior distribution describes the available information prior to observing the data used in the estimation. The observed data are then used to update the prior via Bayes theorem to the posterior distribution of the model's parameters. This distribution may then be summarized in terms of the usual measures of location (e.g., mode and mean) and spread (e.g. standard deviation and probability intervals). For the technical details of the computation, refer to the Appendix and references therein.

10 The discount rate δ is set to 0.025, reflecting a quarterly discount factor of 0.9756, and the value is fixed throughout all the cases.

<Table 1> **Parameter Estimates**

	prior distribution			posterior distribution
	type	mean	st. dev.	mean (90% confidence interval)
h_1	Beta	0.7	0.2	0.6873 (0.2656, 0.8746)
σ	Gamma	1.5	0.8	2.5185 (2.0122, 3.8779)
g	Normal	0.0	0.001	0.0000 (-0.0018, 0.0016)
μ	Beta	0.5	0.2	0.7206 (0.7097, 0.7621)
κ	Beta	0.005	0.005	0.0022 (0.0000, 0.0055)
Ψ	Beta	0.5	0.2	0.4510 (0.3779, 0.6154)
θ_π	Beta	0.7	0.2	0.7057 (0.6962, 0.7095)
θ_x	Beta	0.7	0.2	0.7193 (0.7086, 0.7147)
ρ_p	Beta	0.5	0.2	0.7871 (0.6471, 0.8659)
ρ_g	Beta	0.8	0.15	0.8424 (0.7674, 0.9207)
ρ_s	Beta	0.5	0.2	0.4581 (0.4339, 0.6837)
ρ_T	Beta	0.5	0.2	0.4803 (0.1470, 0.7619)
σ_p	Inv. Gamma	0.02	infinity	0.0732 (0.0234, 0.0854)
σ_g	Inv. Gamma	0.02	infinity	0.0209 (0.0177, 0.0239)
σ_s	Inv. Gamma	0.01	infinity	0.0039 (0.0023, 0.0043)
σ_T	Inv. Gamma	0.01	infinity	0.0060 (0.0027, 0.0098)
σ_{mp}	Inv. Gamma	0.01	infinity	0.0046 (0.0028, 0.0069)
log density				570.435305

<Table 2> **Shock Variance Decomposition**

variables & shocks	demand/ preference ($\epsilon_{p,t}$)	cost-push ($\epsilon_{s,t}$)	natural rate ($\epsilon_{g,t}$)	inflation target ($\epsilon_{mp,t}$)	monetary policy ($\epsilon_{mp,t}$)
output growth	1.74	1.20	96.06	0.58	0.41
output gap	24.00	49.59	0.79	20.80	4.83
inflation	0.01	99.99	0.00	0.00	0.00
inflation target	0.00	0.00	0.00	100.00	0.00
interest rate	84.63	10.79	3.59	0.36	0.63
natural interest	88.22	7.82	3.48	0.30	0.19

11 Given that the baseline model is a closed economy, it is not surprising that demand shock exhibits rather volatile behavior; confronted with actual data, part of this structural shock is supposed to reflect foreign (export/demand) shock as well.

<Table 3> Parameter Estimates - Consumption habit posterior mean (90% confidence interval)

	no habit		additional lags	
h_1			0.8865	(0.7420, 0.9889)
h_2			0.0316	(0.0000, 0.1294)
σ	1.9790	(1.9680, 2.0405)	1.9009	(1.6379, 2.0290)
g	-0.0002	(-0.0013, 0.0016)	-0.0003	(-0.0016, 0.0004)
μ	0.6732	(0.6686, 0.7033)	0.6162	(0.6152, 0.6177)
κ	0.0010	(0.0009, 0.0023)	0.0001	(0.0000, 0.0001)
Ψ	0.6012	(0.4756, 0.6385)	0.4414	(0.3874, 0.5947)
θ_π	0.7008	(0.6955, 0.7020)	0.6629	(0.6555, 0.6709)
θ_x	0.6751	(0.3937, 0.9989)	0.7496	(0.5841, 0.8539)
ρ_p	0.7826	(0.7383, 0.8765)	0.7363	(0.6193, 0.8136)
ρ_g	0.5736	(0.4059, 0.6550)	0.7580	(0.7027, 0.7630)
ρ_s	0.4520	(0.3275, 0.5014)	0.2759	(0.2210, 0.3081)
ρ_T	0.5374	(0.2298, 0.8631)	0.5486	(0.2242, 0.9046)
σ_p	0.1273	(0.1254, 0.1967)	0.0899	(0.0650, 0.1476)
σ_g	0.0217	(0.0193, 0.0265)	0.0214	(0.0197, 0.0256)
σ_s	0.0036	(0.0031, 0.0046)	0.0045	(0.0040, 0.0049)
σ_s	0.0096	(0.0027, 0.0158)	0.0057	(0.0028, 0.0083)
σ_{mp}	0.0094	(0.0030, 0.0105)	0.0047	(0.0033,0.0069)
log density	515.243692		555.317094	

Regarding the Phillips curve, I provided a prior for $\mu = 0.5$ that put equal weight on the forward-looking and backward-looking components with a large standard deviation providing a rather diffuse prior. The literature using foreign data suggested estimates in the whole zero-unity interval depending on the sample and specification, and there is no widely agreed value for domestic data along with relatively a small number of studies. I found that the Phillips curve is primarily forward looking: the estimate for μ is around 0.7. It has nevertheless a non-negligible weight on the backward-looking component, with $1-\mu$ just below 0.3. The response of inflation to the output gap, measured by the parameter κ, seems to be rather small, and is quite similar to the structural estimates by Cho (2007).[12]

[12] Literature reports mixed results for this parameter. Although the estimates are significantly positive (Gali, Gertler, and Lopez, 2001; Kim and Subramanian, 2008) in some cases. it is not uncommon that the estimates are very small and/or insignificant (see Furher and Moore, 1995, and Ireland, 2001, for example). Ideally, one may use a better proxy such as labor's share of income for

The prior for the (demeaned) equilibrium natural output growth rate is set to 0, and the posterior mean is estimated at virtually zero. This may provide some insight for the recent debate about whether there has been a decline in the growth rate of potential output of Korea for the past a few decades. The estimation result indicates that the decline in natural output growth, if any, is not solely associated with the natural output dynamics, but also with that of the actual GDP, so the observed (possibly declining) trend in actual output growth may not be interpreted as a strong or direct evidence to the decrease in natural output growth.

Turning to the monetary policy reaction function, the data seem to support a dynamic Taylor-type policy rule specification reasonably well. The monetary policy shock has a standard deviation of 0.46 percent, which is rather small compared with the values reported in the literature (e.g., Cho, 2007), and there is a pronounced gradual adjustment of the interest rate with $\Psi = 0.451$. Moreover, the weight on the inflation and output gap does not deviate much from the priors, which is set to what Taylor (1993) suggested as the likely coefficients. Overall, the estimates for policy reaction function are in good accordance with those in Cho (2007), who employed essentially identical functional form except for the consideration of the natural values of variables.

Finally, the results seem to indicate mildly persistent movements in the medium-run inflation target, $\rho_T = 0.4803$, with small shocks to the process, $\sigma_T = 0.0060$. The latter suggests that movements in the medium-run inflation target are done rather gradually over time. As is well known, prior to the foreign exchange crisis, the monetary policy target of the Bank of Korea was monetary aggregate, and only since 1998 the Bank of Korea adapted inflation targeting with announcement of official target value and used the Call rate as its primary policy instrument. Given that, the estimates must be interpreted as implicit value for former periods.

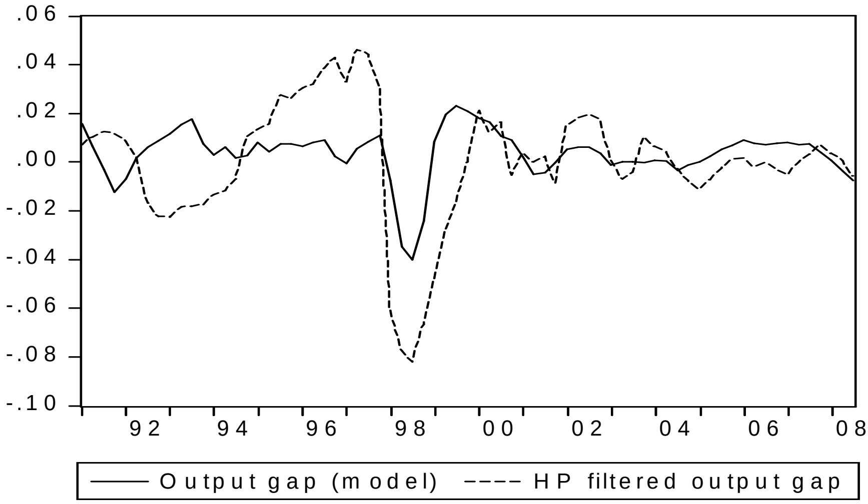
Turning to perhaps the most controversial and important variable for monetary policy, the output gap estimates are plotted in Figure 1 along with the Hodrick-Prescott filtered output series. There are quite smooth variations with one exception for the period of foreign exchange crisis around 1998, during which the output gap went down to around -0.04 percent. Before and after this period, the variations have been even considerably smaller, and the output gap has been mostly in the region between -0.01 percent and 0.01 percent. The resulting potential output growth is thus remarkably similar to the actual growth.[13]

The estimated smoothed natural nominal and real rates of interest are plotted in Figure 2. Roughly, the nominal interest rates show a slightly monotonically-declining pattern, while the estimated real interest rates exhibit a

marginal cost term in the Phillips curve. This series is not available at a quarterly frequency, and there are numerous complicated issues such as the treatment of income of the self-employed (See Yoo and Ahn, 2007; and Moon, Yun, and Lee, 2006, for example). Another related issue is the (poor/imprecise) measures of output gap, as there seems to be no consensus about the choice of filter (Hodrick-Prescott, linear, quadratic, and others), and all of them potentially contain some measurement errors.

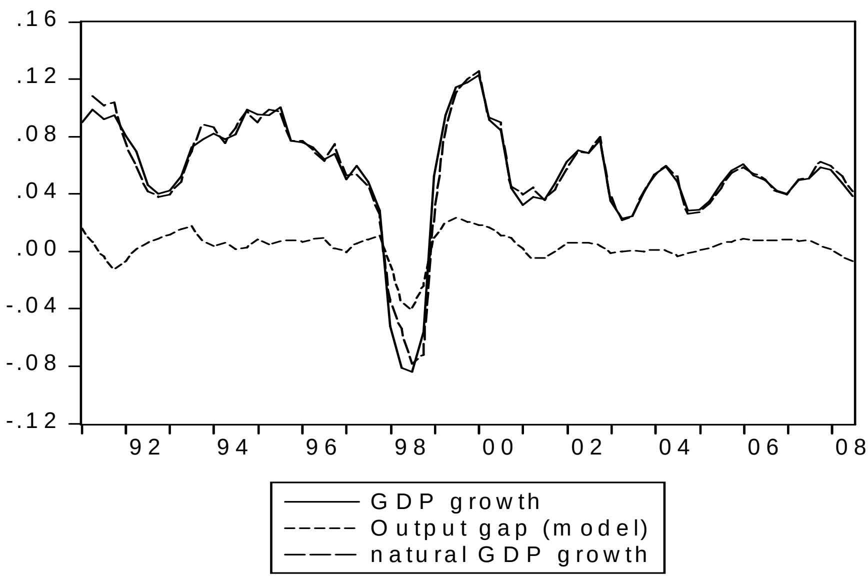
[13] This result is not too surprising given that the output gap measures are constructed from explicit general equilibrium consideration with a rational expectation behavior. See also footnote 3.

[Figure 1a] Estimated Output Gaps

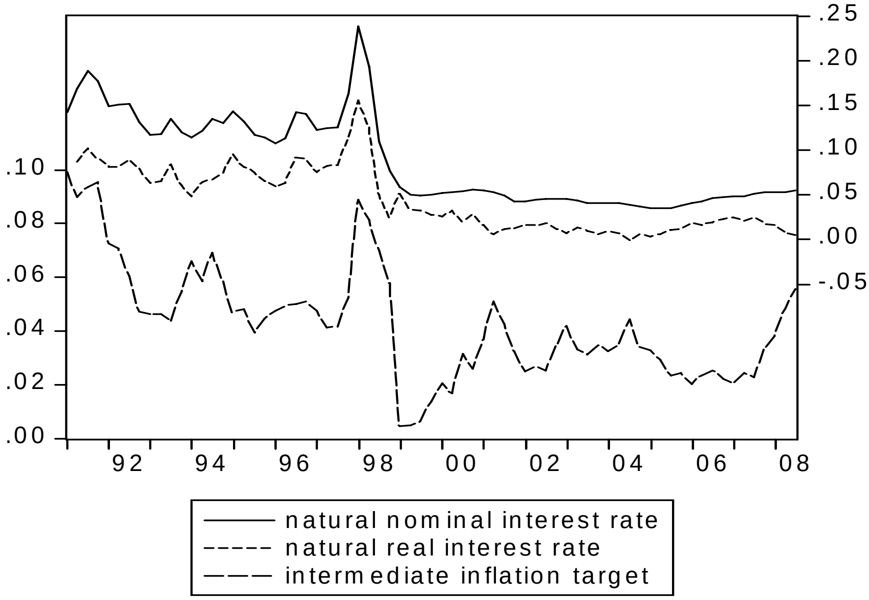


Notes: Hodrick-Prescott filtered output gaps are calculated with the smoothing parameter of 1600, and are multiplied by 0.01.

[Figure 1b] Estimated Output Gap and Natural Output Growth



[Figure 2] Estimated Natural Rates of Interest and Inflation Target



non-monotonic trending, with a rather steady value before 1998 and thereafter a fall in the trend to the present rate (with the exception of the crisis period for both variables). Note that the model-based measures of interest rate are remarkably stable in spite of the non-negligible fluctuations in real GDP growth over the post-crisis sample. Figure 2 also displays the smoothed estimate of the implicit inflation target. The implicit inflation target is rather volatile throughout the sample period and seems to be roughly consistent with the Bank of Korea's medium-run target ranges for the post crisis period.[14]

Finally, shock variance decomposition for endogenous variables is reported in Table 4. A few interesting observations are worth mentioning. First, note that cost-push shocks are not only the virtually exclusive source of inflation variation but also one of main drivers for the output gap (about 50 percent) with each of the demand and inflation target shocks responsible for the remaining half, whereas shocks to the natural rate of output explains surprisingly little for output gap. Second, the estimation ascribes about four-fifths of the variations in the interest rate to demand/preference shocks and the remainder to cost-push and natural

---

14 In the baseline estimation, I followed the recent literature and used the prior information reported in Table 1. To check if these priors are not responsible for the main estimation results, I re-estimated the model (i) with different priors and (ii) by a (classical) maximum likelihood method. Although the estimates for a few parameters, such as κ turn out with rather different values, the overall estimates for the parameters remain largely unchanged (the results are avalable on request); and the output gap estimates are remarkably similar to the baseline results; and the differences are almost not discernable.

<Table 4> Parameter Estimates: Phillips Curve posterior mean (90% confidence interval)

	no lag	additional 4 lags
h_1	0.4361 (0.3583, 0.6338)	0.6546 (0.3485, 0.9369)
σ	2.6233 (2.5906, 2.9607)	2.4965 (1.8821, 3.3991)
g	-0.0003 (-0.0014, 0.0011)	0.0000 (-0.0017, 0.0015)
μ	0.4039 (0.1421, 0.5825)	0.6668 (0.2531, 0.8885)
κ	0.0005 (0.0002, 0.0006)	0.0029 (0.0000, 0.0051)
ϕ_1		0.2395 (0.1065, 0.4139)
ϕ_2		0.2181 (0.0742, 0.3417)
ϕ_3		0.1696 (-0.0148, 0.2970)
ϕ_4		0.0523 (-0.1554, 0.1747)
Ψ	0.2353 (0.1351, 0.2964)	0.4153 (0.2754, 0.5443)
θ_π	0.9024 (0.7758, 0.9932)	0.5876 (0.2949, 0.9005)
θ_x	0.3824 (0.2793, 0.5980)	0.7544 (0.5982, 0.9951)
ρ_p	0.6602 (0.5927, 0.7444)	0.8018 (0.7266, 0.8845)
ρ_g	0.8030 (0.8005, 0.8201)	0.8400 (0.7486, 0.9138)
ρ_s	0.7467 (0.7339, 0.7563)	0.5614 (0.4321, 0.7042)
ρ_g	0.8130 (0.8005, 0.8201)	0.8400 (0.7486, 0.9138)
σ_p	0.0281 (0.0256, 0.0305)	0.0732 (0.0274, 0.1229)
σ_g	0.0071 (0.0049, 0.0085)	0.0209 (0.0182, 0.0247)
σ_s	0.0071 (0.0049, 0.0085)	0.0051 (0.0036, 0.0076)
σ_T	0.0057 (0.0031, 0.0103)	0.0068 (0.0029, 0.0142)
σ_{mp}	0.0056 (0.0043, 0.0069)	0.0047 (0.0027, 0.0072)
log density	540.154338	574.702562

output shocks over the entire period. Finally, movements in the natural real interest rate are driven mostly by demand shocks and less so by shocks to cost push and the natural rate of output.

Based on these observations, one may evaluate the performance of monetary policy with a Taylor rule as largely successful over the sample period, along with some reservations however. With a typical loss function for monetary policy authority that tries to minimize inflation and output gap variability in mind, the Bank of Korea has presumably tried to neutralize the effect of various shocks on both inflation and output gap. However, it seems that this attempt is not very successful. Although the demand shock explains more than 80 percent of the interest rate variations (the reliance on the two other shocks can be thought of as an attempt by the central bank to shield these shocks from having an impact on output gap and inflation), and inflation is shielded from this type of shock, demand shock is still responsible for the non-negligible portion of the output gap. The overall estimation results indicate that monetary policy has mainly focused on alleviating demand shock on output, putting relatively less weight on mitigating cost-push shock that is translated to variations in inflation.

Ⅳ. Extension

In this section, I will consider a few extensions of our baseline model and do a comparison to investigate the results from alternative model specifications. The purpose of this extension is twofold. First, it is an attempt to examine the robustness/sensitivity of the results (estimates of structural parameter as well as key variables of interest such as potential GDP and natural rate of interest) under several alternative specifications. Second, this exercise will help us figure out which ingredients are critical and play a key role in properly accounting for Korean economy when building dynamic macro models at the same time. Given the emerging state, it seems that which components and how they are specified and essential in DSGE model building are not widely agreed upon. The exercises in this section will then hopefully contribute to this line of emerging research, although this attempt is not meant to be complete or exhaustive.

Specifically, I will introduce a few variations in the specification of the Phillips curve and habit formation in the baseline model. In the first extension, I considered two alternative versions of the Phillips curve: a Phillips curve with no indexation to past inflation:

$$\pi_t = \mu E_t[\pi_{t+1}] + \kappa x_t + \epsilon_{s,t}$$

and a hybrid Phillips curve with more lags:

$$\pi_t = \mu E_t[\pi_{t+1}] + (1-\mu)(\Sigma_{j=1}^{4}\phi_i\pi_{t-j}) + \kappa x_t + \epsilon_{s,t}$$

As mentioned above, the motivation for the hybrid Phillips curve is largely empirical, and it allows for checking if the large degree of forward-looking behavior in the Phillips curve is (partly) due to neglected lagged dependence (Gali and Gertler, 1999).

In the second set of extension, I investigated the cases in which habit in consumption is augmented with additional lags or there is no habit formation. In each case, the demand block, or expectational IS equation becomes, respectively:

$$\Delta y_t = \frac{\sigma}{h_1(\sigma-1)} E_t[\Delta y_{t+1}] - \frac{h_2}{h_1}\Delta y_{t-1}$$

$$- \frac{1}{h_1(\sigma-1)}(r_t - E_t[\pi_{t+1}] - \delta) + \frac{1}{h_1}(u_{p,t} - E_t[u_{p,t+1}])$$

with additional lag (up to two periods) in the consumption habit (where h_2 is a parameter associated with the additional habit term), and

$$E_t[\Delta y_{t+1}] = \frac{1}{1-\sigma}(r_t - E_t[\pi_{t+1}] - \delta) - (u_{p,t} - E_t[u_{p,t+1}])$$

with no habit formation.[15]

The motivation for this extension is both empirical and theoretical.[16] In the empirical side, it is motivated by the stylized fact that reduced-form equations for aggregate demand typically depend on two lags of the output gap (King, Plosser, Stock, and Watson, 1991; Gali 1992). In the theoretical perspective, first note that the model's equilibrium constraint (output equals consumption) abstracts from investment, which is partly a simplifying device, but it is also on the basis that the long-run effects of capital formation can be put aside to analyze short-run macroeconomic fluctuations. However, several factors can account for the differences between the output-based and the consumption-based estimates. Perhaps the most obvious candidate is investment dynamics, which is potentially an important mechanism through which shocks are propagated is omitted. These omitted mechanisms may have important implications for understanding aggregate fluctuations and the design and implementation of an optimal monetary policy.[17]

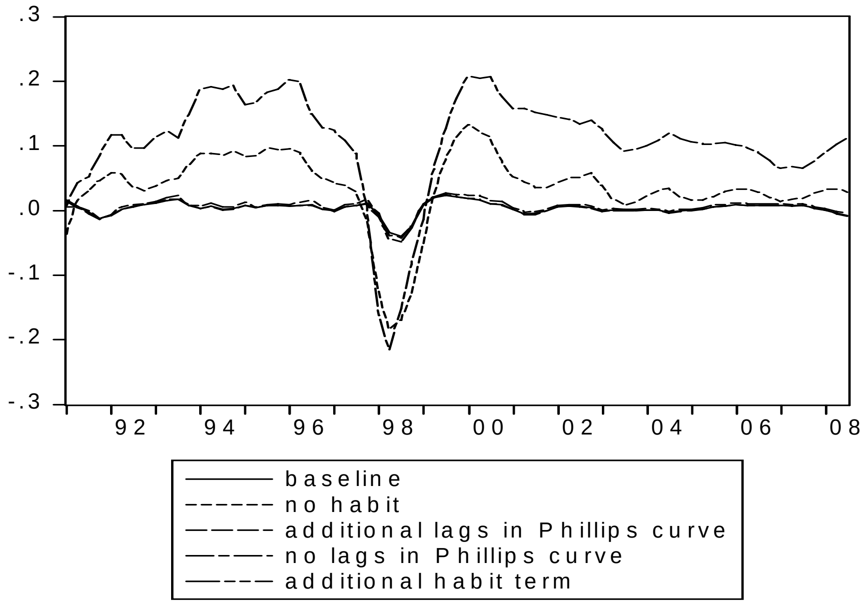
Parameter estimates for these specifications are presented in Tables 3 and 4. With two alternative specifications of habit in consumption, the estimates for most of the parameters are largely similar to the baseline estimates. There are a few noticeable changes, however. When consumption habit is removed from the model, there is a rather large decline in the estimate of risk aversion parameter. In addition, demand shock has become significantly volatile. One possible account is that this shock is forced to play quantitatively a bigger role in accounting for output variability in the absence of lagged consumption (i.e., smoothing effect or real inertia). The result is largely similar to the baseline case when an additional lag term in habits is added. The additional habit parameter is estimated close to 0

15 Of course, natural rate process (6) must be modified, whose derivation is straightforward. I also considered the cases with additional lags in the consumption habits; it turns out that the habit parameters for more than two lags are essentially 0, resulting in efficiency loss only.

16 Note that in comparing with the first extension in which we may examine the quantitative effect of nominal (price) rigidities in explaining aggregate dynamics and the resulting natural values of variables, this case allows us to explore the consequences of deviation from a standard model with real variables.

17 In addition, at a slightly more technical level, adding additional lag allows for habit formation with respect to the changes as well as the level of consumption.

[Figure 3] Alternative Output Gap Measures



with rather a smaller estimate for risk version parameter.

For the Phillips curve, some interesting results emerged as well. First, in the standard Phillips curve, that is, with a purely forward-looking term only, the estimates of habit parameter and forward-looking parameter in the Phillips curve were reduced to about two-thirds of that of the baseline model, both values of which are not common in the literature. Next, in the monetary policy reaction function, the decrease of similar magnitude in interest smoothing is observed, along with more weight on the inflation gap and less weight on the output gap. In addition, the volatilities and persistence of shocks have changed somewhat. For example, demand/preference shocks have become less volatile and persistent.

When the extended hybrid Phillips curve specification is considered, additional lagged terms in inflation dynamics are largely significant, implying that a potential trend component (as in Christiano, Eichenbaum, and Evans, 2005) may be also impotent. Likewise, note that the estimates for most of the other parameters are remarkably similar to the baseline case. Overall findings from the alternative Phillips curves illustrate that hybrid Phillips curves (with one or further lags) may have better fit in characterizing the inflation dynamics of Korea (as noted with the slightly increased log-likelihood values for these specifications).

Figure 3 plots the estimated output gap measures under these alternative specifications along with estimates from the baseline model specification. Observe that the output gap measures turn out to be noticeably more volatile when there is no lagged term in the Phillips curve or consumption habit, hence, the resulting potential output growth is less volatile than the actual GDP growth, while output gap estimates are quite similar in the other three cases.

One possible explanation for this is that these alternative models, in the absence of lagged terms that presumably generate persistence and inertia in output dynamics, allow for loosely linked dynamics between potential and actual output with more degrees of freedom. Likewise, this may further suggest that the omitted lagged terms in inflation dynamics and output growth may play a key role in producing smooth estimates of potential growth. This argument is further supported by log-likelihood values which have significantly decreased for the specifications of standard Phillips curve and/or with no habit.

Ⅴ. Concluding Remarks

This paper attempts to estimate the natural rate of interest and output gap using a DSGE framework with a few stylized New Keynesian features. The major results are summarized as follows. First, output gap estimates are far less volatile than the conventional measures, which is in a way, a natural outcome as the former is generated from an explicit general equilibrium model specification. Second, the hybrid type Phillips curve with backward-looking components and/or habit formation (in consumption) may play a potentially important role in explaining the aggregate dynamics of Korea. Although a variety of research on several directions of the DSGE models are ongoing and promising, measures of potential/natural output and other associated estimates from these models in Korea potentially remain controversial given its infancy. Thus, the results from this paper should be taken with some caution.

Some directions for future research are worth exploring. First, the model presented in this paper is a simple closed economy. As the Korean economy has been subject to a variety of foreign economic shocks as equally as, if not more than, domestic factors, it is fair to say that this paper may have not properly addressed these potentially important factors, at least quantitatively. Thus, it will then be an important and interesting direction to take to extend the model to estimate the natural rate of variables and evaluate the quantitative importance of foreign shocks.

Second, it is generally believed that the Korean economy has undergone non-trivial structural change in the past few decades (especially, during the financial crisis), and the nature of this change and its implications have been one of the active research areas recently. In this context, investigating the role of monetary policy in controlling output and inflation gaps in the face of these challenges and its effectiveness appears to be another critical task. As the conduct of monetary policy is admittedly too simply addressed, not considering this aspect, interpreting the overall results accordingly deserves some caution and future research on this issue will again be a particularly fruitful area.

References

Amato, Jeffrey, "The Role of the Natural Rate of Interest Rate in Monetary Policy," BIS Working Papers No.171, 2005.

An, Sungbae and Frank Schorfheide, "Bayesian Analysis of DSGE Models," *Econometric Reviews*, 2007, pp.113~172.

Andrés, Javier, David Lopez-Salido, and Edward Nelson, "Sticky-Price Models and the Natural Rate Hypothesis," *Journal of Monetary Economics*, 2005, pp.1025~1053.

Bjornland, Hilde, Kai Leitemo, and Junior Maih, "Estimating the Natural Rates in a Simple New Keynesian Framework," Norges Bank Working Paper 2007/10, 2007.

Blanchard, Olivier and Charles Kahn, "The Solution of Linear Difference Models under Rational Expectations," *Econometrica*, 1980, pp.1305~1311.

Brooks, Stephen and Andrew Gelman, "General Methods for Monitoring Convergence of Iterative Simulations," *Journal of Computational and Graphical Statistics*, 1998 pp.434~455.

Calvo, Guillermo, "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics*, 1983, pp.383~398.

Clarida, Richard, Jordi Gali, and Mark Gertler, "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, 1999, pp.1661~1707.

Cho, Seonghoon, "An Empirical Assessment of the Korean Monetary Policy Since the Foreign Exchange Crisis," *The Korean Economic Review*, 2007, pp.329~352.

Christiano, Lawrence, Martin Eichenbaum, and Charles Evans, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 2005, pp.1~45.

Edge, Rochelle, Michael Kiley, and Jean-Philippe Laforte, "Natural Rate Measures in an Estimated DSGE Model of the U.S. Economy," *Journal of Economic Dynamics and Control*, 2008, pp.2512~2535.

Erceg, Christopher, Dale Henderson, Andrew Levin, "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, 2000, pp.281~313.

Friedman, Milton, "The Role of Monetary Policy," *American Economic Review*, 1968, pp.1~17.

Fuhrer, Jeffrey, "Habit Formation in Consumption and Its Implications for Monetary-Policy Models," *American Economic Review*, 2000, pp.367~390.

Fuhrer, Jeffrey, and George Moore, "Inflation Persistence," *Quarterly Journal of Economics*, 1995 pp.127~159.

Gali, Jordi, "How Well Does the IS/LM Model fir Post-War U.S. Data," *Quarterly Journal of Economics*, 1992, pp.709~728.

Gali, Jordi and Mark Gertler, "Inflation Dynamics: A Structural Econometric Analysis," *Journal of Monetary Economics*, 1999, pp.195~222.

Gali, Jordi, Mark Gertler, and David Lopez-Salido, "European inflation dynamics," *European Economic Review*, 2001, pp.1237~1270.

Ireland, Peter, "Sticky Price Models of the Business Cycle: Specification and Stability," *Journal*

of Monetary Economics, 2001, pp.3~18.
Justiniano, Alejandro and Giorgio Primiceri, "Potential and Natural Output," manuscript, 2008.
Kim, Hwagyun and Chetan Subramanian, "Velocity of Money and Inflation Dynamics", Applied Economics Letters, 2008.
Kim, Ki-Ho and Young-Hwan Noh, "Estimation of Potential GDP in Korea with Nonstationary Time-Series Data," *Journal of Economic Studies*, 2007. [in Korean]
King, Robert, Charles Plosser, James Stock, and Mark Watson, "Stochastic Trends and Economic Fluctuations," *American Economic Review*, 1991, pp.189~840.
Klein, Paul, "Using the generalized Schur form to solve a multivariate linear rational expectations model," *Journal of Economic Dynamics and Control*, 2000, pp.1405~1423.
Mishkin, Frederick, "Estimating Potential Output," Speech at the Conference on Price Measurement for Monetary Policy at Federal Reserve Bank of Dallas, May 24th, 2007.
Neiss, Katherine and Edward Nelson, "Inflation Dynamics, Marginal Cost, and the Output Gap: Evidence from Three Countries," *Journal of Money, Credit, and Banking*, 2005, pp.1019~1045.
McCallum, Bennett, "Should Monetary Policy Respond Strongly to Output Gaps?," *American Economic Review Papers and Proceedings* 91, 2001, pp.258~262..
Moon, Sung-woo, Tack Yun, and Mi-Hye Lee, "An Empirical Analysis on Korean Inflation Dynamics Based on the New Keynesian Phillips Curve," *Journal of Economic Theory and Econometrics*, 2004, pp.59~87 [in Korean]
Park, Wonam and Hur Chankook, "Potential Growth Rate of Korea: Estimates and Prospect," Korea Economic Research Institute monograph, 2004. [in Korean]
Rotemberg, Julio and Michael Woodford, "An Optimization-Based Econometric Model for the Evaluation of Monetary Policy", *NBER Macroeconomic Annual*, 1997, pp.297~346.
Sims, Chris, "Solving Linear Rational Expectations Models," *Computational Economics*, 2002 pp.1~20.
Smets, Frank and Rafael Wouters, "An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area," *Journal of the European Economic Association*, 2003 pp.1123~1175.
Taylor, John, "Discretion versus Policy Rules in Practice," *Carnegie Rochester Conference Series on Public Policy*, 1993 pp.195~214.
Walsh, Carl, *Monetary Theory and Policy*, 2nd edition, MIT Press, 2003.
Woodford, Michael, "Control of the Public Debt: A Requirement for Price Stability?," NBER Working Paper No.W5684, 1996.
Woodford, Michael, *Interest and Prices: Foundations of A Theory of Monetary Policy*, Princeton University Press, 2003.
Yoo, Jinhyuk and Heejoo Ahn, "Inflation Dynamics of Korea," manuscript, 2007. [in Korean]

Appendix

Original Non-linear Model[18]

On the demand side, it is assumed that the economy is populated by households whose objective is to maximize the expected utility, which is derived from consumption, C_t, and from providing labor, N_t:

$$U = E_0 \Sigma_{t=0}^{\infty} \beta^t [U(C_t, C_{t-1}; V_t) - U(N_t)]$$

with

$$U(C_t, C_{t-1}; V_t) = \frac{1}{1-\sigma}\left(\frac{C_t V_t}{C_{t-1}^{h_1}}\right)^{1-\sigma}$$

subject to where β is the discount rate $(\beta = 1/(1+\delta), \delta$ being discount rate), σ is the constant relative risk aversion coefficient (note that $1/\sigma$ is not the elasticity of intertemporal substitution of consumption as the preference is not time-separable here), and V_t is preference or demand shock.

Households enter period t with money holdings, M_{t-1}, and maturing one-period risk-free bond holdings, B_{t-1}. At the beginning of each period, they also receive labor income, $W_t N_t$, (the product of nominal wage and labor supply) and real dividends, D_t. They use these resources either for consumption C_t, or for purchasing new one-period bond B_t, or nominal money, M_t, which are carried into t+1 periods.[19] Thus, the interpemporal budget constraint is given by:

$$C_t + \frac{B_t + M_t}{P_t} = \frac{W_t N_t}{P_t} + \frac{(1+R_{t-1})B_{t-1} + M_{t-1}}{P_t} + D_t$$

The first-order conditions for households are:

---

[18] A more detailed textbook-version treatment of the class of similar models and related mathematical derivations can be found in Walsh (2003). Much of the notations in this section anticipate a symmetric equilibrium in which all individuals behave similarly. Distinguishing household-level and firm-level variables from the aggregate variables would add little to the exposition, but significantly complicate the notation.

[19] Note that money does not directly enter in the utility function. The model may be seen as a limiting case where real money balance provides utility, but in the limit these liquidity services are arbitrarily small. Or, the model can be understood as one where utility is additively separable in real money balances. Then, the additional first-order condition associated with real money balances simply determines the nominal level of money balance and plays no role in determining the inflation, output, or interest rate, thus that we can ignore it for our purpose. See Rotemberg and Woodford (1997), and Woodford (1996).

$$\frac{\Lambda_t}{P_t} = \beta(1+R_t)\frac{\Lambda_{t+1}}{P_{t+1}} \quad \text{and} \quad \Omega_t = \frac{W_t}{P_t}\Lambda_t$$

where Λ_t and Ω_t are the marginal utility gain/loss from consumption and labor supply, that is,

$$\Lambda_t = \frac{\partial U_t}{\partial C_t} + E_t\left[\beta\left(\frac{\partial U_{t+1}}{\partial Z_{t+1}}\right)\left(\frac{\partial Z_{t+1}}{\partial C_t}\right)\right] \quad \text{and} \quad \Omega_t = \frac{\partial U_t}{\partial N_t}$$

Intuitions are standard: the optimal path of consumption should be such that a marginal change in consumption from one period to the next produces no change in utility. In other words, the decline in utility in period t, $-\partial U(C_t, Z_t)/\partial C_t$ must be equal to the discounted increase in utility in period t+1, $\beta(\partial U(C_{t+1}, Z_{t+1})/\partial C_{t+1})$, and the real interest that would accrue on the income saved until period t+1 at rate R_t. For simplicity, labor supply is assumed to be constant.

Using the functional form, we can express the Euler equation as follows:

$$\frac{(C_t C_{t-1}^{-h_1} V_t)^{1-\sigma}}{C_t} = \beta(1+R_t)E_t\left[\frac{(C_{t+1} C_t^{-h_1} V_{t+1})^{1-\sigma}}{C_{t+1}}\frac{P_t}{P_{t+1}}\right].$$

Log-linear approximation yields

$$-\sigma c_t + (1-\sigma)v_t - h_1(1-\sigma)c_{t-1} =$$

$$-\delta + r_t + E_t[-\sigma c_{t+1} + (1-\sigma)v_{t+1} - \pi_{t+1}] - h_1(1-\sigma)c_t$$

Next, for the supply side, I assumed that there are infinitesimally many monopolistically competitive firms. Firms maximize the discounted value of expected future profits, pricing along their demand curve to set prices as a fixed mark-up over marginal costs. Following Calvo (1983), each period a fixed proportion of firms, $1-\theta_P$ receive a signal to re-optimize their price and set P_t^*; firms that do not re-optimize index their price change to last period's inflation rate (Christiano, Eichenbaum, and Evans, 2005). Thus, the average price in period t satisfies

$$P_t^{1-\epsilon} = (1-\theta_P)P_t^{*\,1-\epsilon} + \theta_P P_{t-1}^{1-\epsilon}$$

where ε is the price elasticity of the individual intermediate goods. This is the weighted average of the optimal price-setting combined and price indexation by non-optimizing firms. When log-linearized about the economy's nonstochastic steady state,[20] one can obtain dynamics for aggregate inflation given by:

20 Christiano, Eichenbaum, and Evans (2005) showed that this approximation is valid for a

$$\pi_t = \mu E_t[\pi_{t+1}] + (1-\mu)\pi_{t-1} + \kappa mc_t$$

where $\mu = \beta/(1+\beta)$ and $\eta = (1-\theta_P)(1-\beta\theta_P)/((1+\beta)\theta_P)$, and mc_t represents (log) real marginal costs, which, because there is no capital in production, simply equals the real wage divided by the marginal product of labor. Any profits that firms earn are remitted to the households (the shareholders) in the form of a lump sum dividend payment.

The monetary authority sets a monetary policy interest rate feedback rule a la Taylor (1993). Policymakers smoothly adjust the actual interest rate to its (moving) target level, $\overline{R_t}$:

$$R_t = R_{t-1}^{\psi}\,\overline{R_t}^{1-\psi} e^{u_{mp,t}}$$

where the parameter Ψ represents the degree of interest rate smoothing, while $u_{mp,t}$ is the monetary policy shock. $\overline{R_t}$ depends on the inflation rate relative to its target, Π_t/Π_t^T, and the output level relative to its natural level, Y_t/Y_t^n:

$$\overline{R_t} = R_t^n(\Pi_t/\Pi_t^T)^{\theta_\pi}(Y_t/Y_t^n)^{\theta_x}$$

where θ_π and θ_x denote the weights in the interest rate rule, and R_t^n denotes natural nominal interest rate. Log-linearizing the policy rule above yields the expression in the main text.

Finally, letting $Y_t = [\int_0^1 Y_t(j)^{(\epsilon-1)/\epsilon}dj]^{\epsilon/(\epsilon-1)}$ denote the aggregate output and imposing the market clearing condition, $Y_t = C_t$, one obtains:

$$y_t = \frac{\sigma}{h_1(\sigma-1)+\sigma}E_t[y_{t+1}] + \frac{h_1(\sigma-1)}{h_1(\sigma-1)+\sigma}y_{t-1}$$

$$+\frac{1}{h_1(\sigma-1)+\sigma}(r_t - E_t[\pi_{t+1}] - \delta) + \frac{\sigma-1}{h_1(\sigma-1)+\sigma}(v_t - E_t[v_{t+1}])$$

From here, it is a straightforward exercise to obtain the expression in log differences and in the case with additional lags, respectively.

Technical Details of the Bayesian Estimation

The joint posterior distribution of all estimated parameters is obtained in two steps. First, the posterior mode and Hessian matrix evaluated at the mode is computed by standard numerical optimization routines. The likelihood is computed by first solving the model and then using the Kalman filter. Second, draws from

nonstochastic steady state in which inflation is not necessarily equal to zero.

the joint posterior are generated using the Metropolis-Hastings algorithm. The proposal distribution is taken to be the multivariate normal density centered at the previous draw with a covariance matrix proportional to the inverse Hessian at the posterior mode. This paper uses 50,000 runs of five parallel chains (a total of 250,000), with the first half of the initial draws discarded as burn-in to ensure the effect of the starting value vanishes. Convergence of the Markov Chain is assessed based on the multivariate potential scale reduction factor (Brooks and Gelman, 1998). See An and Schorfheide (2007) and Smets and Wouters (2003) for surveys on further technical details.