# Adaptive Mode Switching in Correlated Multiple Antenna Cellular Networks

Chulhan Lee, Chan-Byoung Chae, Sriram Vishwanath, and Robert W. Heath, Jr.

*Abstract:* **This paper proposes an adaptive mode switching algorithm between two strategies in multiple antenna cellular networks: A single-user mode and a multi-user mode for the broadcast channel. If full channel state information is available at the base-station, it is known that a multi-user transmission strategy would outperform all single-user transmission strategies. In the absence of full side information, it is unclear what the capacity achieving method is, and thus there are few criteria to decide which of the myriad possible methods performs best given a system configuration. We compare a single-user transmission and a multi-user transmission with linear receivers in this paper where the transmitter and the receivers have multiple antennas, and find that neither strategy dominates the other. There is instead a transition point between the two strategies. Then, the mode switching point is determined both analytically and numerically for a multiple antenna cellular downlink with correlation between transmit antennas.**

*Index Terms:* **Multiple-input multiple-output (MIMO) broadcast channel, partial channel state information, single-user transmission and multi-user transmission.**

## I. INTRODUCTION

Multiple-input multiple-output (MIMO) technology has been one of the most intriguing technologies in emerging wireless standards due to demands for high data rate services. Hence, understanding the fundamental capacity limits on MIMO communication systems is invaluable. Researchers have been actively uncovering these limits with a large body of work studying the capacity of MIMO communication channels under perfect channel state information (CSI). These limits for both single-user and multi-user systems are well summarized in [2]–[6]. Although a body of work exists on throughput without perfect side information [7], [8], little is known in terms of the optimality of any one scheme.

In this paper we study the performance of downlink transmission with imperfect feedback information at the transmitter. In the multiple antenna systems, transmission strategies can be mostly categorized to be either single-user or multi-user in nature. A single-user transmission strategy implies that a transmitter chooses a single-user in the system and all data streams are allocated to that user, while a multi-user transmission strategy focuses on distributing data streams across multiple users for its transmission.

It is well known that the optimal multi-user transmission strategy, dirty paper coding (DPC), outperforms the optimal single-user transmission strategies with time division multiple access (TDMA) from the point of view of sum rate with perfect CSI at the transmitter [9]. Perfect CSI, however, is impractical thus single-user transmission strategies can show better performance than multi-user transmission strategies under different situations. Specifically, under some partial CSI assumptions, the throughput of single-user transmission is higher than that of multi-user transmission when the number of users is below a threshold [10] and simulation results were presented on the potential benefits of considering both single-user and multi-user MIMO transmissions in [11]. All prior work, however, does not consider spatial correlation that is inevitable in practical communication systems.

In this paper, we consider a MIMO communication system where the number of receive antennas at the user terminal is greater than or equal to the number of transmit antennas at the base-station. Based on this system model, we present an analytical framework for the transmission mode switching point and investigate the impact of transmit side spatial correlation where the linear receivers are considered to find the minimum number of users to justify the use of multi-user MIMO over single-user MIMO.[1] In this paper, we do not consider aspects of the problem such as near-far effect and user fairness. These issues are left for our future research. We consider, however, that this paper takes an important step in the direction of providing an analytical framework for this problem while ensuring its tractability.

This paper has the following progression: Section II describes the system model, while Sections III and IV present the performance analysis and simulation results, respectively. We conclude with Section V.

## II. SYSTEM MODEL

Before explaining the system model, we introduce the notation used in this paper. The lower case boldface (e.g., $\mathbf{a}$) and the upper case boldface (e.g., $\mathbf{A}$) indicate vectors and matrices, respectively. Assuming $\mathbf{A}$ denotes a complex matrix, then $\mathbf{A}^T$, $\mathbf{A}^H$, $\mathbf{A}^{-1}$, and $[\mathbf{A}]_{i,j}$ denote the transpose, Hermitian, inverse and the $(i,j)$th element of $\mathbf{A}$, respectively. $\mathbb{E}$ represents expectation operator and $\mathbf{I}_N$ indicates the $N\times N$ identity matrix. Con-

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[1]The authors in [12] proposed an adaptive transmission mode switching between diversity and spatial multiplexing but did not consider multi-user MIMO environments.

vergence in probability and the derivative of $F(x)$ are denoted as $\xrightarrow{\mathrm{p}}$ and $F'(x)$, respectively.

### A. Channel Model

Since the base-station does not have perfect CSI to allocate optimal power to each antenna, we assume equal power allocation through all antennas. Then the received signal of user $k$ in the downlink channel is expressed as

$$\mathbf{y}_k = \sqrt{\frac{\rho}{N_t}}\mathbf{H}_k\mathbf{x} + \mathbf{n}_k, \ \ k = 1, \cdots, K \tag{1}$$

where $\mathbf{y}_k$ is an $N_r \times 1$ receive signal vector at the $k$th user, $\mathbf{x}$ is an $N_t \times 1$ transmit signal vector, $\mathbf{H}_k$ is an $N_r \times N_t$ channel matrix and $\mathbf{n}_k$ is an $N_r \times 1$ additive Gaussian noise vector. In (1), $N_t$ and $N_r$ are the number of transmit and receive antennas, respectively. Without loss of generality, we assume that $\mathbf{n}_k$ has its covariance matrix as the identity matrix $\mathbf{I}_{N_r}$ and $\rho$ is the received signal to noise ratio (SNR) and $\mathbb{E}[\mathbf{x}^H\mathbf{x}] = N_t$.

We assume that each receiver can estimate the channel matrix perfectly, while the transmitter requires feedback information from the receivers as partial CSI similar to that in [10]. This information available at the transmitter is detailed in Section III.

We consider correlation at the transmit antennas while the receive antennas are uncorrelated. This assumption is reasonable since the transmitter in the broadcast channel is usually placed in an environment with limited scattering, while the receivers are usually located in a rich scattering environment [13], [14]. Thus, we have the channel matrix

$$\mathbf{H}_k = \mathbf{H}_w\mathbf{R}_t^{\frac{1}{2}} \tag{2}$$

where $\mathbf{H}_w$ has its elements as independent and identically distributed (i.i.d.) unit variance zero-mean complex Gaussian random variables and $\mathbf{R}_t$ is the transmit correlation matrix with correlation factor $0\leq\alpha<1$, i.e., $[\mathbf{R}_t]_{i,j} = \alpha^{|i-j|}$.[2] This channel model was also considered to analyze transmit correlation effects on the MIMO broadcast channel in [15]. Note that transmit correlation modeling in this paper does not represent all aspects of correlation observed in MIMO systems. The correlation model assumed provides us with insight on the tradeoffs between transmission schemes and is thus a useful starting point. More sophisticated channel models will be considered in our future work.

### B. Single-User and Multi-User MIMO Transmission

We assume a linear reception policy for both single-user and multi-user transmission strategies. In addition, as in conventional open-loop spatial multiplexing techniques, we assume $N_r \geq N_t$. This is so that we achieve the multiplexing gain of $N_t$ in the single-user transmission setting. To have a comparable system configuration between single-user and multi-user MIMO transmission strategies, we also apply this assumption to multi-user transmission. Note that, if $N_r < N_t$, the multiplexing gain of $N_t$ cannot be realized in single-user transmission, and it makes the comparison biased towards multi-user modes of operation.[3] Also, based on practicality restrictions, we do not utilize a beamformer/precoder at the transmitter since perfect CSI is not available at the transmitter.

In the single-user transmission setting, one chosen user receives multiple parallel streams from the transmitter and we assume that the receiver can employ a successive interference cancellation (SIC) scheme to determine each of these streams. Using the combination of a linear receiver and a successive interference canceler, the receiver operates close to optimal performance at the high SNR regime [10], [16]. In a multi-user transmission, we use a relatively simple multi-user MIMO transmission scheme called *independent stream scheduling* [8], [14]. As only one stream is assigned per user, this receiver, though suboptimal, is a natural choice for this setting.

## III. PERFORMANCE ANALYSIS

We assume that the received SNR is high enough to utilize the spatial multiplexing effect and the number of transmit antennas does not exceed the number of receive antennas in the system as explained in Section II-B. Based on these assumptions, we derive simple mathematical expressions for the system throughput. In this section, we focus on asymptotic analysis based on large number of users and antennas since finite user analysis leads to intractable expressions. Later, we show using numerical results that the solution obtained is also applicable to system with a finite number of users and antennas in Section IV.

### A. Single-User MIMO Transmission

The achievable rate of user $k$ for single-user MIMO transmission with equal power allocation is given by

$$R_k = \log_2\left|\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{H}_w\mathbf{R}_t\mathbf{H}_w^H\right|. \tag{3}$$

This may not always be the optimal choice, but the loss in system throughput by equal power allocation compared with optimal power allocation is small if SNR ($\rho$) is high [16], [17]. The transmitter obtains $K$ achievable rates as partial CSI and serves the user with maximum achievable rate in each time slot. Thus, the maximum rate $R_{SU}$ of single-user transmission is

$$R_{SU} = \max_{k=1,\cdots,K} R_k. \tag{4}$$

Since the correlation matrix $\mathbf{R}_t$ has full rank by assumption, the rate of single-user transmission (3) under a high SNR assumption can be rewritten as [17]

$$R_k \approx \min(N_r, N_t)\log_2\frac{\rho}{N_t} + \log_2\left|\mathbf{H}_w^H\mathbf{H}_w\right| + \log_2\left|\mathbf{R}_t\right|. \tag{5}$$

In [18], it was shown that as the number of antennas increases, $R_k$ converges to the following Gaussian random variable[4]

$$R_k \sim \mathcal{N}\left(\mu, \sigma^2\right)$$

[2] $\mathbf{R}_t = \mathbf{I}_{N_t}$ if $\alpha = 0$. In this paper, we assume $\mathbf{R}_t$ is identical for all users for simplicity.

[3] If $N_t > N_r$, additional feedback is required from the receiver for rank adaptation and it increases system complexity. In this paper, we assume $N_r \geq N_t$ to minimize the feedback overhead.

[4] This convergence to a Gaussian random variable is reasonably rapid enough to make this approximation valuable in practical situations.

where

$$\mu = n\log_2\frac{\rho}{N_t} + n\log_2 e\left(\sum_{i=1}^{N-n}\frac{1}{i} - 0.5772\right) + \log_2|\mathbf{R}_t| + \log_2 e\sum_{i=1}^{n-1}\frac{i}{N-i}, \tag{6}$$

$$\sigma^2 = (\log_2 e)^2\left(\sum_{i=1}^{n-1}\frac{i}{(N-n+i)^2} + \frac{n\pi^2}{6} - \sum_{i=1}^{N-1}\frac{n}{i^2}\right)$$

and $n = \min(N_r, N_t), N = \max(N_r, N_t)$. To analyze $R_{SU}$ asymptotically, we introduce the following proposition.

**Proposition 1:** Let $R_k$ and $R_{SU}$ have the relationship in (4). Note that $\mu$ and $\sigma^2$ are expressed in (6). Then as $K \to \infty$,

$$\frac{R_{SU}}{\mu + \sqrt{2\sigma^2 \ln K}} \xrightarrow{\mathrm{P}} 1.$$

Proposition 1 helps us characterize the maximum rate $R_{SU}$ for the single-user transmission setting, showing that it has an effective growth rate of $\sqrt{2\sigma^2 \ln K}$ as $K$ approaches infinity. Before proving Proposition 1, we present the two lemmas that directly help us in its proof.

**Lemma 2:** For a twice differentiable cumulative distribution function (CDF) $F(x)$, if

$$\lim_{x\to\infty}\frac{F''(x)(1-F(x))}{(F'(x))^2} = -1,$$

then $F$ is a Von Mises function.

*Proof:* See [19, p. 40]. □

**Lemma 3:** Denote $K$ i.i.d. random variables as $X_1, \cdots, X_K$. Let $M = \max_{k=1,\cdots,K} X_k$. If the CDF $F(x)$ of $X_k$ is a Von Mises function, then there exist $a_K$ and $b_K$ satisfying that as $K$ increases

$$\mathbb{P}\left(\frac{M-b_K}{a_K} \le x\right) \to \exp(-e^{-x}) \tag{7}$$

where $1 - F(b_K) = K^{-1}$ and $a_K = \frac{1-F(b_K)}{F'(b_K)}$. In addition, if $\lim_{K\to\infty}\frac{b_K}{a_K} \to \infty$, then as $K$ increases,

$$M/b_K \xrightarrow{\mathrm{P}} 1. \tag{8}$$

*Proof:* From the properties of Von Mises functions, $\mathbb{P}\left(\frac{M-b_K}{a_K} \le x\right) \to \exp(-e^{-x})$ where $1 - F(b_K) = K^{-1}$ and $a_K = \frac{1-F(b_K)}{F'(b_K)}$ [19, Ch. 1]. Let us show that $M/b_K \xrightarrow{\mathrm{P}} 1$ as $K \to \infty$. For any fixed $\epsilon > 0$, using (7) as $K$ increases

$$\begin{aligned}\mathbb{P}&\left(\left|\frac{M}{b_K} - 1\right| > \epsilon\right)\\ &= \mathbb{P}(M - b_K > \epsilon b_K) + \mathbb{P}(M - b_K < -\epsilon b_K)\\ &= 1 - \exp(-e^{-\epsilon\frac{b_K}{a_K}}) + \exp(-e^{\epsilon\frac{b_K}{a_K}})\\ &\to 0 \quad \left(\because \lim_{K\to\infty}\frac{b_K}{a_K} \to \infty\right).\end{aligned}$$

This means that $\frac{M}{b_K} \xrightarrow{\mathrm{P}} 1$. □

Now, using Lemmas 2 and 3, we complete the proof of Proposition 1.

*Proof:* [Proof of Proposition 1] Let the CDF of $R_k$ be $F(x)$. We have

$$\begin{aligned}F'(x) &= \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}},\\ F''(x) &= -\frac{1}{\sqrt{2\pi\sigma^2}}\frac{x-\mu}{\sigma^2}e^{-\frac{(x-\mu)^2}{2\sigma^2}} = -\frac{x-\mu}{\sigma^2}F'(x),\\ 1 - F(x) &\approx \frac{\sigma^2}{x-\mu}F'(x), \quad \text{(by Mills' ratio [19])}.\end{aligned} \tag{9}$$

Therefore,

$$\lim_{x\to\infty}\frac{1-F(x)}{F'(x)}\frac{F''(x)}{F'(x)} = \frac{\sigma^2}{x-\mu}\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) = -1.$$

This means that $F(x)$ is a Von Mises function by Lemma 2. Using Lemma 3, there exist $a_K$ and $b_K$ satisfying

$$\mathbb{P}\left(\frac{R_{SU} - b_K}{a_K} \le x\right) = \exp(e^{-x}).$$

Now, let us derive $a_K$ and $b_K$. By Mills' ratio (9),

$$\begin{aligned}1 - F(b_K) &\approx \frac{\sigma^2}{b_K - \mu}F'(b_K)\\ &= \frac{\sigma}{\sqrt{2\pi}(b_K-\mu)}\exp\left(\frac{-(b_K-\mu)^2}{2\sigma^2}\right) = \frac{1}{K}.\end{aligned} \tag{10}$$

By taking $-\ln(\cdot)$ to both sides of (10),

$$\frac{(b_K-\mu)^2}{2\sigma^2} + \ln\frac{(b_K-\mu)}{\sigma} + \frac{1}{2}\ln(2\pi) = \ln K. \tag{11}$$

Dividing by $\frac{(b_K-\mu)^2}{2\sigma^2}$, the second and the third terms on the left hand side of (11) go to arbitrarily small values as $K \to \infty$ since $b_K \to \infty$. Then, we have

$$b_K \approx \mu + \sqrt{2\sigma^2 \ln K}.$$

In addition, using $a_K = \frac{1-F(b_K)}{F'b_K} \approx \frac{\sigma^2}{b_K-\mu}$, $a_K \approx \sqrt{\frac{\sigma^2}{2\ln K}}$. This leads us to $\lim_{K\to\infty}\frac{b_K}{a_K} \to \infty$. Therefore, by Lemma 3,

$$\frac{R_{SU}}{b_K} \xrightarrow{\mathrm{P}} 1$$

where $b_K \approx \mu + \sqrt{2\sigma^2 \ln K}$. □

In Proposition 1, we can find the next order approximation of $b_K$. Let us rewrite $b_K$ as

$$b_K = \mu + \sqrt{2\sigma^2 \ln K} + r_K \tag{12}$$

where $r_K = o(\sqrt{\ln K})$. By substituting (12) into (11), we have

$$\frac{r_K^2}{2\sigma^2} + \frac{r_K}{\sigma}\sqrt{2\ln K} + \frac{1}{2}\ln\ln K + \frac{1}{2}\ln 4\pi + \ln\left(1 + \frac{r_K}{\sqrt{2\sigma^2 \ln K}}\right) = 0. \tag{13}$$

Dividing (13) by $r_K\sqrt{2\ln K}$, we have

$$\begin{aligned}&\frac{r_K}{2\sigma^2\sqrt{2\ln K}}+\frac{1}{\sigma}+\frac{1}{2r_K\sqrt{2\ln K}}\ln\ln K\\&+\frac{1}{2r_K\sqrt{2\ln K}}\ln 4\pi+\frac{\ln\left(1+\frac{r_K}{\sqrt{2\sigma^2\ln K}}\right)}{r_K\sqrt{2\ln K}}\\&\approx o(1)+\frac{1}{\sigma}+\frac{\ln\ln K+4\pi}{2r_K\sqrt{2\ln K}}=0.\end{aligned}\tag{14}$$

Therefore from (14), we can derive $r_K$ approximately by solving $\frac{1}{\sigma}+\frac{\ln\ln K+4\pi}{2r_K\sqrt{2\ln K}}=0$ as

$$r_K\approx-\frac{\sigma}{2}\frac{\ln\ln K+\ln 4\pi}{\sqrt{2\ln K}}\tag{15}$$

and finally, we have $b_K$ from (12) and (15)

$$b_K\approx\mu+\sqrt{2\sigma^2\ln K}-\frac{\sigma}{2}\frac{\ln\ln K+\ln 4\pi}{\sqrt{2\ln K}}.$$

Now, we know that Proposition 1 represents that as $K$ increases

$$\begin{aligned}R_{SU}&\xrightarrow{\mathrm{P}} o\left(\sqrt{\ln K}\right)+b_K\\&=o\left(\sqrt{\ln K}\right)+\mu+\sqrt{2\sigma^2\ln K}-\frac{\sigma}{2}\frac{\ln\ln K+\ln 4\pi}{\sqrt{2\ln K}}.\end{aligned}\tag{16}$$

Therefore from (16), as $K$ increases, the ergodic (sum) rate of single-user transmission is given by

$$\begin{aligned}C_{SU}&=\mathbb{E}[R_{SU}]\\&\to o\left(\sqrt{\ln K}\right)+\mu+\sqrt{2\sigma^2\ln K}-\frac{\sigma}{2}\frac{\ln\ln K+\ln 4\pi}{\sqrt{2\ln K}}.\end{aligned}\tag{17}$$

### *B. Multi-User MIMO Transmission*

As mentioned in Section II-B, we consider the independent stream scheduler to minimize feedback overhead [8]. With zero-forcing (ZF) receivers, the post-processing signal to interference and noise ratio (SINR) of the $i$th stream at user $k$ is given by

$$S_{i,k}=\frac{\rho}{N_t}\frac{1}{\left[\left(\mathbf{H}_k^H\mathbf{H}_k\right)^{-1}\right]_{i,i}}$$

where $i=1,\cdots,N_t$ and $k=1,\cdots,K$. The transmitter obtains $KN_t$ post-processing SINRs from all users as partial CSI. Then the maximum achievable rate of the $i$th stream amongst $K$ users is

$$R_{MU,i}=\max_{k=1,\cdots,K}\log_2(1+S_{i,k}),\tag{18}$$

thus the maximum sum rate of multi-user transmission is the summation of the rates for all streams, i.e.,

$$R_{MU}=\sum_{i=1}^{N_t}R_{MU,i}.\tag{19}$$

In [14] and [20], it was shown that the post-processing SINR of a ZF receiver behaves as a weighted chi-squared distributed random variable with $2(N_r-N_t+1)$ degrees of freedom. This can be written in equation form as,

$$S_{i,k}=\frac{\rho}{N_t}\frac{1}{\left[\left(\mathbf{H}_k^H\mathbf{H}_k\right)^{-1}\right]_{i,i}}=\frac{\rho}{N_t}V_{i,k}\tag{20}$$

where the p.d.f. $f_{V_{i,k}}(v)$ of $V_{i,k}$ is given by

$$\begin{aligned}f_{V_{i,k}}(v)&=\frac{\lambda_i\exp(-v\lambda_i)(v\lambda_i)^{N_r-N_t}}{(N_r-N_t)!},\\\lambda_i&=\left[\mathbf{R}_t^{-1}\right]_{i,i}.\end{aligned}\tag{21}$$

By combining (18) and (20), we have

$$\begin{aligned}R_{MU,i}&=\max_{k=1,\cdots,K}\log_2\left(1+\frac{\rho}{N_t}V_{i,k}\right)\\&=\log_2\left(1+\frac{\rho}{N_t}\max_{k=1,\cdots,K}V_{i,k}\right)\\&=\log_2\left(1+\frac{\rho}{N_t}M_i\right)\end{aligned}\tag{22}$$

where $M_i$=$\max_k V_{i,k}$. Note that max moves into $\log_2$ since $\log_2$ is a non-decreasing function. Now let us see the relationship between $M_i$ and $V_{i,.k}$ in the following proposition.

**Proposition 4:** Let $M_i$ and $V_{i,k}$ be expressed in (20) and (22). Then as $K\to\infty$,

$$\frac{M_i}{\ln K/\lambda_i}\xrightarrow{\mathrm{P}}1.$$

*Proof:* Let CDF of $V_{i,k}$ be $F(x)=\int_{-\infty}^{x}f_{V_{i,k}}(v)dv$. Then we have

$$F''(x)=F'(x)(-\lambda_i+(N_r-N_t)/x).\tag{23}$$

Then from (23) and using L'Hospital's rule,

$$\begin{aligned}\lim_{x\to\infty}\frac{1-F(x)}{F'(x)}&=\lim_{x\to\infty}\frac{-F'(x)}{F''(x)}\\&=\lim_{x\to\infty}\frac{-F'(x)}{F'(x)(-\lambda_i+(N_r-N_t)/x)}\\&=\frac{1}{\lambda_i}.\end{aligned}\tag{24}$$

Therefore

$$\begin{aligned}\lim_{x\to\infty}\frac{F''(x)(1-F(x))}{(F'(x))^2}&=\lim_{x\to\infty}\frac{F''(x)}{F'(x)}\frac{1-F(x)}{F'(x)}\\&=-\lambda_i\frac{1}{\lambda_i}\\&=-1\end{aligned}$$

which implies $F(x)$ is a Von Mises function by Lemma 2. Then from Lemma 3 we can derive $a_K$ and $b_K$ satisfying that

$$\mathbb{P}\left(\frac{M_i-b_K}{a_K}\le x\right)\to\exp(-e^{-x})\quad\text{as } K \text{ increases.}$$

Note that $b_K$ increases as $K$ increases. From (24), we know that $a_K = \frac{1-F(b_K)}{F'(b_K)} \approx \frac{1}{\lambda_i}$ since $b_K \to \infty$.

Now let us derive $b_K$. From (24), we know that

$$1 - F(x) \sim \frac{F'(x)}{\lambda_i}.$$

Thus, using (21)

$$\begin{aligned}\frac{F'(b_K)}{\lambda_i} &= \frac{f_{V_{i,k}}(b_K)}{\lambda_i} \\ &= \frac{\exp(-b_K\lambda_i)(b_K\lambda_i)^{N_r-N_t}}{(N_r-N_t)!} \\ &= \frac{1}{K}.\end{aligned} \quad (25)$$

Applying ln on both sides of (25), we have

$$(N_r - N_t)\ln(b_K\lambda_i) - b_K\lambda_i - \ln(N_r - N_t)! = -\ln K. \quad (26)$$

Dividing (26) by $b_K$, we see

$$\begin{aligned}(N_r - N_t)\frac{\ln(b_K\lambda_i)}{b_K} - \lambda_i - \frac{\ln(N_r-N_t)!}{b_K} &\approx 0 - \lambda_i - 0 \\ &= -\frac{\ln K}{b_K}.\end{aligned}$$

Therefore,

$$b_K \approx \frac{\ln K}{\lambda_i}. \quad (27)$$

Since $\frac{b_K}{a_K} \to \infty$ as $K$ increases, by Lemma 3 we have

$$\frac{M_i}{b_K} \approx \frac{M_i}{\ln K/\lambda_i} \xrightarrow{\text{p}} 1.$$

□

In a similar manner to the single-user strategy, we find the next order approximation of $b_K$. Rewrite $b_K$ as

$$b_K = \frac{\ln K}{\lambda_i} + r_K \quad (28)$$

where $r_K = o(\ln K)$. To derive $r_K$, substituting (28) into (26)

$$\lambda_i r_K + \ln(N_r - N_t)! = (N_r - N_t)\ln(\ln K + \lambda_i r_K).$$

Therefore,

$$\begin{aligned}r_K = &\frac{-\ln(N_r-N_t)! + (N_r-N_t)\ln\ln K}{\lambda_i} \\ &+ \frac{(N_r-N_t)}{\lambda_i}\ln\left(1 + \frac{\lambda_i r_K}{\ln K}\right).\end{aligned}$$

Since $\frac{\lambda_i r_K}{\ln K} \to 0$ as $K \to \infty$, we get $r_K$ as

$$r_K \approx \frac{-\ln(N_r-N_t)! + (N_r-N_t)\ln\ln K}{\lambda_i}.$$

Therefore,

$$b_K = \frac{\ln K - \ln(N_r-N_t)! + (N_r-N_t)\ln\ln K}{\lambda_i}. \quad (29)$$

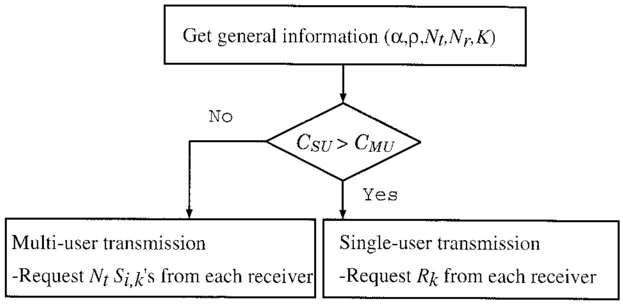


Fig. 1. Adaptive mode switching algorithm at the transmitter.

Now we know that Proposition 4 says that as $K$ increases

$$M_i \xrightarrow{\text{p}} o(\ln K) + b_K. \quad (30)$$

From (19), the ergodic sum rate of the multi-user transmission in the MIMO broadcast channel is given by

$$\begin{aligned}C_{MU} &= \mathbb{E}[R_{MU}] = \sum_{i=1}^{N_t}\mathbb{E}[R_{MU,i}] \\ &= \sum_{i=1}^{N_t}\mathbb{E}\left[\log_2\left(1 + \max_{k=1,\cdots,K}\frac{\rho}{N_t}V_{i,k}\right)\right] \\ &= \sum_{i=1}^{N_t}\mathbb{E}\left[\log_2\left(1 + \frac{\rho}{N_t}M_i\right)\right].\end{aligned} \quad (31)$$

Therefore, as $K$ increases

$$C_{MU} \to \sum_{i=1}^{N_t}\log_2\left(1 + \frac{\rho}{N_t}(o(\ln K) + b_K)\right) \quad (32)$$

where $b_K = \frac{\ln K - \ln(N_r-N_t)! + (N_r-N_t)\ln\ln K}{\lambda_i}$ from (29).

### C. Adaptive Mode Switching Algorithm

To obtain an analytical expression for the number of users at the point where the throughput resulting from one scheme exceeds the other, we compare the expressions given in (17) and (32). We refer to the point where the transition happens as the mode switching point. In essence, the mode switching point corresponds to a comparison between the achievable rate expressions:

$$C_{SU}(\alpha, N_t, N_r, K) \gtrless C_{MU}(\alpha, N_t, N_r, K)$$

where $\alpha, N_t, N_r$, and $K$ are correlation factor, the number of antennas at the transmitter and the receiver and the number of users, respectively. If the transmitter acquires system parameters such as SNR, the number of antennas, the number of users and correlation factors, it can determine the (approximate) ergodic rate achieved by the two strategies using (17) and (32). If $C_{SU} > C_{MU}$, the transmitter requests receivers to send their achievable rates as feedback information and decides to serve the receiver that has maximum rate $R_k$. If $C_{SU} < C_{MU}$, the transmitter decides to operate in the multi-user transmission strategy.[5] This algorithm is illustrated in Fig. 1.

[5]In this paper, we propose the mode switching point using ergodic sum rates of single-user and multi-user MIMO strategies. More practical mode switching

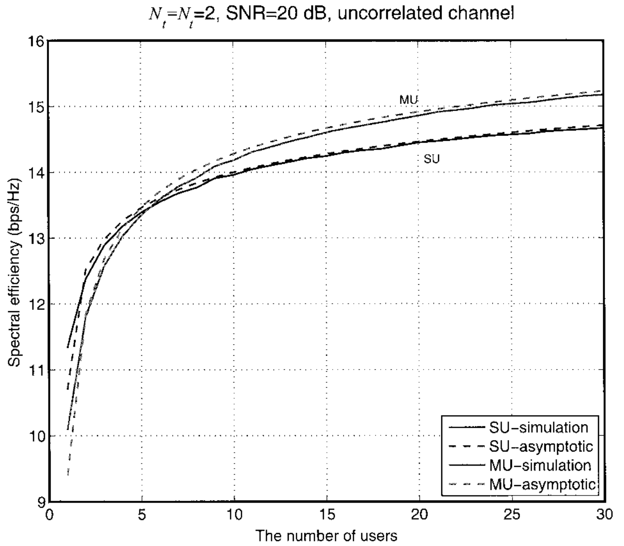


Fig. 2. Ergodic (sum) rates for single-user and multi-user transmissions.

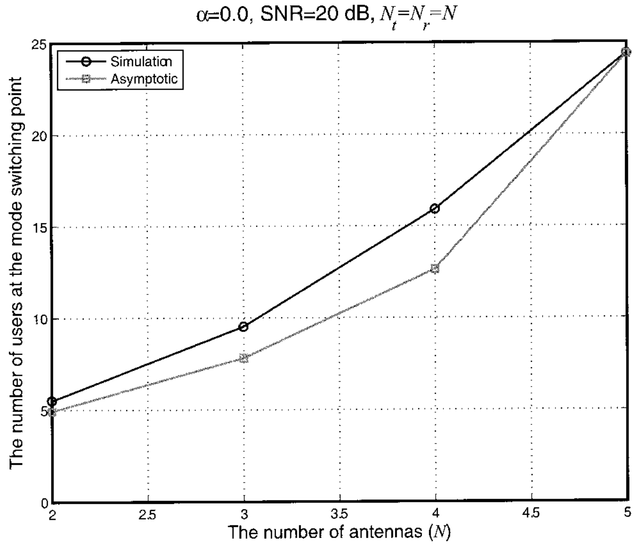


Fig. 4. The relationship between the number of users at the mode switching point and the number of antennas.

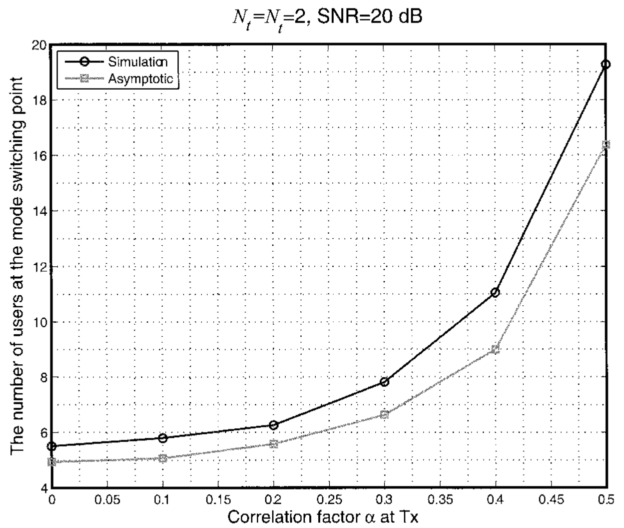


Fig. 3. The relationship between the number of users at the mode switching point and the correlation factor $\alpha$.

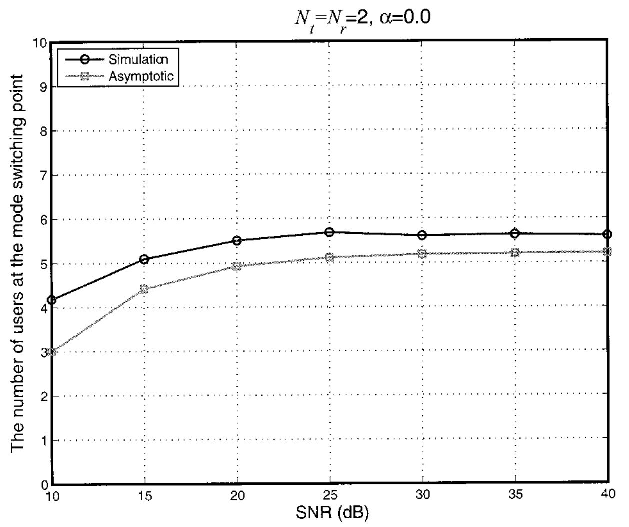


Fig. 5. The relationship between the number of users at the mode switching point and SNR.

## IV. SIMULATION RESULTS

In Fig. 2, we illustrate the achievable sum rates of single-user and multi-user transmissions and determine the mode switching point between the two strategies. When there are only a few users (less than five where $N_t$=$N_r$=2, uncorrelated channel), single-user transmission has a larger (sum) rate than multi-user transmission as the former uses a more sophisticated receiver. The growth rate of multi-user transmission, however, is larger than that of single-user transmission thanks to multi-user diversity.

Figs. 3–5 illustrate the number of users at the mode switching point according to three different parameters: The transmit side correlation factor $\alpha$, the number of antennas (assuming $N_t$=$N_r$=$N$) and SNR $\rho$. From these results, we confirm that our analytic expressions match simulation results fairly closely. In addition, we are able to determine which transmission mode is better from these results. For example, multi-user transmission has higher throughput than single-user transmission in upper regions of curves and vice versa.[6]

From Fig. 3, it can be observed that the single-user transmission scheme results in larger gains than the multi-user transmission scheme as the correlation factor $\alpha$ increases. This tendency, however, is in the reverse direction that observed in the numerical results of [11], where a multi-user precoding is shown to attain greater benefits from correlated channels than single-user precoding schemes. This discrepancy can be explained as follows: If the transmitter uses a precoding that is matched to the channel characteristics, then multi-user transmission indeed performs better than single-user transmission schemes. It is noticeable, however, that this requires that the transmitter use strate-

based on instantaneous channel information will be considered in our future work.

[6]The proposed approach can be extended to the case of MIMO-OFDM and random access feedback channels along the lines of [21].

gies that possess a level of sophistication greater than what is assumed in this paper.

## V. CONCLUSION

In this paper, we investigated two transmission strategies termed single-user transmission and multi-user transmission in the MIMO broadcast channel with partial channel state information at the transmitter. In a multi-user MIMO transmission, multiple users are served at a time where a ZF receiver is assumed at the user. The single-user MIMO strategy transmits to one user, while utilizing a receiver that incorporates inter-stream interference mitigation by successive interference cancellation. Thus, there is an inherent tradeoff between inter-stream interference mitigation and the performance gains obtained by serving multiple users at once. This translates into a mode switching making single-user transmissions better in some regimes and multi-user ones better in others. The key contribution of this paper is an analytical framework for determining this mode switching point, and to study the impact of correlation and the number of antennas on the switching point.

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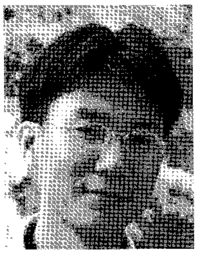


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