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Investigating Forms of Understandings in the Context of Trigonometry[1]

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This study reports a research which was conducted on how frequently and where the students use the unit circle method while dealing with trigonometric functions in solving the trigonometry questions. Moreover, the reasons behind the choice of the methods, which could be the unit circle method, the ratio method, or the use of trigonometric identities, are also investigated to get an insight about their understanding. In this study, the relationship between the students' choices of methods in solving questions is examined in terms of instrumental or relational understanding. This is a multi-method research which involves a range of research strategies. The research techniques used in this study are test, verbal protocol (think aloud), and interview. The test has been applied to ten tenth grade students of a public school to get students' solution processes on the paper. Later on, verbal protocol has been performed with three students of these ten who were of the upper, middle and lower sets in terms of their performance in the test. The aim was to get much deeper data on the students' thinking and reasoning. Finally, interview questions have been asked both these three students and other three from the initial ten students to question the reasons behind their answers to the trigonometry questions. Findings in general suggest that students voluntarily choose to learn instrumentally whose reasons include teachers' and students' preference for the easier option and the anxiety resulting from the external exam pressure.

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1. INTRODUCTION

Trigonometry is generally subject that poses difficulties for an ordinary mathematics learner as it necessitates a grasp of algebra and geometry altogether. A good example to observe this bridging role of trigonometry is the teaching of trigonometric functions. In many countries mainly two different methods are used to teach trigonometric functions: the ratio method and unit circle method (Delice, 2003; Kendal & Stacey, 1996). In the ratio method trigonometric functions are defined as ratios of the sides of right triangles, whereas in the unit circle method trigonometric functions get their definitions via the unit circle.

In their research Kendal & Stacey (1996) have lectured two groups of students with these two different methods, and later, with a simple question (finding the lengths of the sides of a single right triangle), they have tried to see which group was quantitatively more successful. In this research, the understanding processes of the students have not been analyzed, no deep analyses has been made, but instead, a decision on which method should be used was tried to be made based on the results of a single trigonometric question. According to this research, students were able to find the lengths of the sides of the right triangle more easily having been taught with the ratio method. However, this quantitative result does not necessarily imply the needfulness of preferring the ratio method.

The ratio method is a method which is already based on the sides of a right triangle, and thus the only difficulty the students would have to deal with would be the algebraic equations that come along in the solution process. Apart from this, preferring the ratio method will lead the students to instrumental understanding, and keep them farther away from the unit circle method which is more suitable to relational understanding. Not only in the field of mathematics but in all kinds of information being learnt, in order to remember the concepts for a longer time, and to "fully understand" the concepts, it is necessary for one to understand the new concepts relationally (Skemp, 1978). Also, the student who learns the unit circle method may realize that the ratio method is not an independently different method, but rather "a derivative of the unit circle method", derived from a right triangle drawn similar to any right triangle formed in the first region of the unit circle.

Another research on this topic is about the use of trigonometric identities in

simplifying trigonometric expressions, and about the apprehension of the solutions of trigonometric problems (Delice, 2003). In this study of Delice, the focus of the research is not the comparison of the unit circle method and the ratio method, and rather than examining the understanding processes of the students, their performances are stressed instead.

The aim of this research however, is to observe where and how frequently the students use or think about the unit circle, and to be able to spot the reasons behind these choices. In this study, the concern is not shown directly to the students' success or process in problem solving, rather a deeper analyses has been made by taking the students' mental processes into consideration, and by trying to associate the use of unit circle with relational or instrumental understanding. Furthermore, the educational processes of the students have not been interfered, rather an interpretative paradigm has been used (Ozden, 2005).

2. THE RESEARCH TECHNIQUES

In order to determine in the best way on how frequently and where the students use the unit circle, and the underlying reasons, three research techniques have been used. As the multi method approach (Cohen, Manion & Morrison, 2000) was used in collecting the data, the research techniques are the test technique (in which data is collected with written exams), the verbal protocol and interview. To be able to observe the students' problem solving processes a test has been given (Schofield, 1972). Furthermore, the reason behind using the test technique before the others is that information obtained from that can be used to choose among the students the ones to whom interview and verbal protocol will be given. After the test, to be able to bring the students' problem solving processes and the reasons behind their mistakes to light, and to be able to observe their thoughts better, verbal protocol has been used (Ericsson & Simon, 1993).

The verbal protocol technique is important for giving the students the opportunity to be able to express the unit circle in their minds not only by means of hand writing but also verbally. While in this technique there is no interference from outside, and the mental processes of the students are analyzed in a deeper sense, in the interview technique through a conversational environment a shallower opinion is obtained about the mistakes and processes of the students. The reason behind using the interview as the last technique is that the questions of the interview are planned according to the findings of the other two techniques, and that there is the opportunity to freely communicate with the student and thus to be able to get information through this environment (Cohen *et al.*, 2000.; Dyer, 1995).

The test consists of questions where the student can use the unit circle but the use of unit circle is not explicitly suggested in any of the questions. Via these questions where the answers were written down by the students, the intention is to observe how frequently and in which kinds of questions the students use the unit circle. Students' use of unit circle is taken as an indication of whether they have understood the concept instrumentally or relationally.

Before the test, a pilot test has been administered. The aim of the pilot test is to see if the questions are consistent with the aim of the study, and to determine the questions of the "real" test, in other words, to be able to provide validity and the reliability to the test. The pilot test was given to eight tenth grade students of a private university preparation course, and to three tenth grade students, and one eleventh grade students from a private high school. Of the ten questions in the pilot test eight of them have been chosen for the "real" test, and the rest have been used in the verbal protocol.

The sample for the test technique has been chosen among tenth grade students since they had learnt the essential concepts of trigonometry. The test has been given to ten tenth grade Anatolian high school (a type of school in Turkey that follow an advanced curriculum in mathematics, science and foreign languages) students. In order to maintain certain stability with the data of each three techniques, and for the data of each technique to complete each other, the same sample has been used for the verbal protocol and interview techniques. In order to ascertain the content and the construct validity of the instrument, defined as the agreement of professionals about the acceptability of the test items for the purpose of measurement (Munby, 1997) the test was given to three expert teachers (teachers with more than 10 years of experience) (Berliner, 1988) before the test administration.

Verbal protocol has been conducted with three students among the ten, one week after the test. In order to have the nearest best representation of the whole sample, these three students have been chosen according to their use of the unit circle in the test, as one successful, one mediocre, and one poor performance. The verbal protocol consists of four questions where, again, the use of unit circle is possible. While the first three questions do not mention the unit circle, the last verbal protocol question clearly asks the student to solve the question using the unit circle method. With this way, it can to a certain extend, be seen what the students, who had not used the unit circle in the first three questions, know about the unit circle and how well they have comprehended it, and a comparison between these finding and their not-using the unit circle can be made. The verbal protocol data is taken by a voice recorder, and then later put down on paper. In order to train the students for the verbal protocol, a training question has been used.

The three students who had done the verbal protocol and three other students from the sample have been interviewed with eight verbal questions. To the students who have done

the verbal protocol, the interview has been conducted immediately after the verbal protocol. The other three have been interviewed separately in the following week. The interview begins with general questions concerning relational and instrumental learning, and finishes with questions about trigonometry and the unit circle in particular. With these questions, the intention is to understand whether there is a relationship between the students' general point of view about mathematics and their perspective of the unit circle as well as the kind of schema the unit circle accommodates in inside their minds. On the other hand, the purpose of the particular questions about trigonometry and the unit circle is to try to learn the students' direct thoughts about the concepts as well as how their teacher uses the unit circle. The data obtained from the interview is also recorded on a voice recorder, and then put down on paper.

When we look at these three techniques that have been used in this research, we can see that they reveal mostly qualitative data, and that their order of use is designed specially to give the students more opportunity to express themselves in each coming technique. Furthermore, the questions and sample of each technique is based on the findings obtained from the previous technique/s.

3. FINDINGS

In reporting the research findings the techniques used in this study have been analyzed separately. The findings from the qualitative analyses of the findings from the test and the verbal protocols are given in this section but the answers given to the interview questions are used in the discussion section.

3.1. The Test

The data obtained from the test have been analyzed in two phases (Driver & Erickson, 1983; Miles & Huberman, 1984). In each analyses the answers to the questions have been repeatedly read and later have been categorized in a specific way (Hammersley & Atkinson, 1983). Then the percentage of each category has been calculated. In the first phase of the analyses, in order to have a general view of the students' performances, the questions have been categorized as correct, wrong, empty, partial answer (neither right nor wrong). In the second phase the students' processes, their use of the unit circle, and their type of understanding have been deeply analyzed.

In order not to lose any data, in itself this phase has been divided into two sub-phases. In the first sub-phase the questions have been categorized as "questions where in the answer the unit circle method is used", "questions the ratio method is used", "questions where methods other than the unit circle and the ratio are used", and "the questions that

are empty"; and in the second sub-phase as "questions where the student has understood the concepts of the question relationally", "questions where the student has understood the concepts of the question instrumentally", "questions where it could not be determined in which way the student has understood the concepts of the question", and the "questions that are empty".

Since the success of the students is not in direct relation with the aim of this research, the first phase of the analyses has yielded findings that give information only about the students' performances. The information gotten according to this categorization (Figure 1) indicates that the most frequent answers are those that were categorized as "partial correct" and the correct answers are the least frequent are the correct answers.

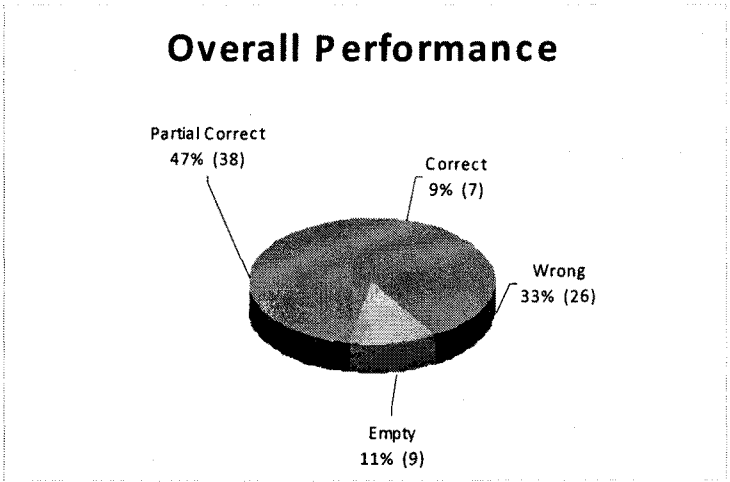

Figure 1. Students' overall performances

According to the results about the used methods in solving questions (Table 1) while the unit circle method is used in only one quarter of the questions, the frequency of the use of the unit circle (UC) method is (out of 80 cases) 19 which is less than the frequency of the use of ratio method (24 cases). Furthermore, in more than one third of the questions methods other than the unit circle and ratio methods were used. These methods specified as "other methods" mostly consist of trigonometric identities. Table 1 also indicates that the unit circle method (which is used in questions 8, 6, 4 and 1 in the order of frequency) is used less frequently than the ratio methpresented od (which is used mainly in questions 1 and 7).

Table 1. Students' use of methods in each of eight questions (S = student, Q= question)

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
S1	R	O	E	UC	O	UC	R	E
S2	R	O	O	O	R	R	R	UC
S3	R	O	O	UC	O	UC	R	UC
S4	UC	O	O	E	O	O	R	UC
S5	UC	O	O	R	O	R	R	R
S6	R	O	O	UC	O	UC	R	UC
S7	R	O	O	UC	O	UC	R	UC
S8	R	O	O	E	O	E	R	O
S9	R	O	UC	UC	R	UC	R	UC
S10	R	E	E	E	O	E	R	E

Note. Unit Circle R = Ratio, O = Other, E= Empty

Table 2 present findings of an analysis in which forms of understanding were investigated on question basis as revealed from their performances. The findings obtained from the second sub-phase of the second phase reveals that there is a big difference between relational and instrumental understanding (5 as opposed to 44 cases). Moreover, relational understanding is observed only in the answers of four students, *i.e.* in question 8 (in 4 cases) and in question 4 (only one case). Instrumental understanding, on the other hand, is the dominant form of understanding in questions 1, 2, 3, 5 and 7 (total of 44 cases).

Table 2. Students' forms of understanding of trigonometric identities as revealed from their performances

	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
S1	IU	IU	IU	UD	IU	UD	IU	RU
S2	IU	IU	IU	E	IU	E	IU	IU
S3	IU	E	E	E	IU	E	IU	E
S4	IU	IU	UD	UD	UD	UD	IU	UD
S5	IU	IU	IU	UD	IU	UD	IU	UD
S6	UD	UD	UD	IU	IU	IU	IU	IU
S7	UD	UD	IU	E	IU	IU	IU	RU
S8	IU	IU	IU	RU	IU	UD	IU	RU
S9	IU	UD	UD	IU	UD	IU	IU	RU
S10	IU	IU	E	UD	IU	UD	IU	E

Note. RU=Relational Understanding, IU=Instrumental Understanding, E=Empty, UD=Undefined

The following example is one of such questions:

Question 4: Please show the $\cos\alpha$ lenght for any $\pi/2 < \alpha < \pi$ interval.

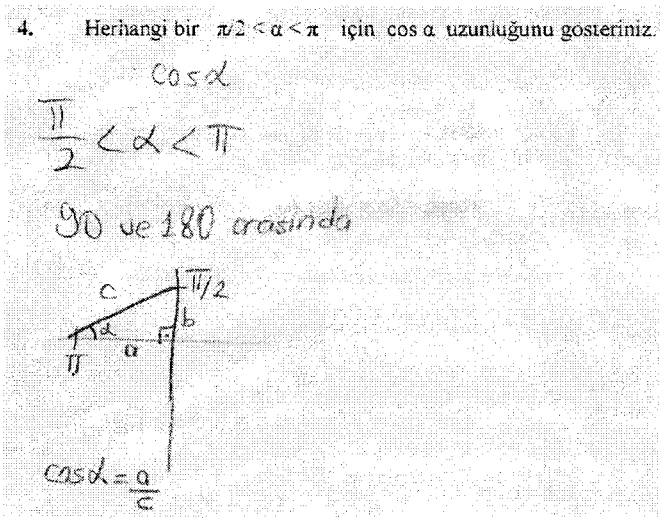

Figure 2. Question 4

The comparison of the results from these two sub-phases suggests that questions where the ratio method is used and the ones where instrumental understanding is present (first and seventh questions) coincide. Moreover, the questions where the unit circle method is used and the ones where relational understanding is present (fourth and eighth questions) also coincide. This result is summarized in Table 3.

Table 3. Students' understanding of trigonometric identities vs. their use of the unit circle

	UC	R	RU	IU
Q1		✓		✓
Q4	✓		✓	
Q7		✓		✓
Q8	✓		✓	

UC = Unit Circle **R** = Ratio **RU** = Relational Understanding
IU = Instrumental Understanding

Below examples for each questions are given.

Question 1: Explain what are $\sin\theta$ and $\cos\theta$ for a any θ. (Ratio method, Instrumental understanding)

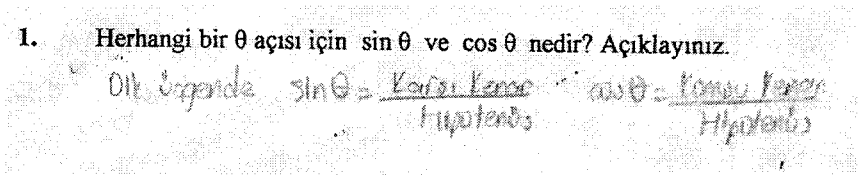

Figure 3. Question 1

Question 4: Please show the $\cos\alpha$ length for any $\pi/2 < \alpha < \pi$ interval. (Unit circle method, Relational understanding)

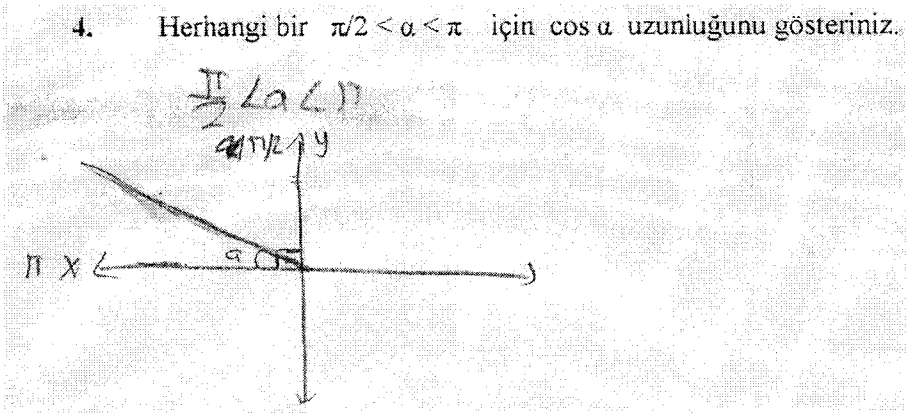

Figure 4. Question 1

Question 7: For the given right angled triangle (a) how is $\cot\alpha$ value found? (b) why is this value equal to the $\cot\alpha$ value? (Ratio method, Instrumental understanding)

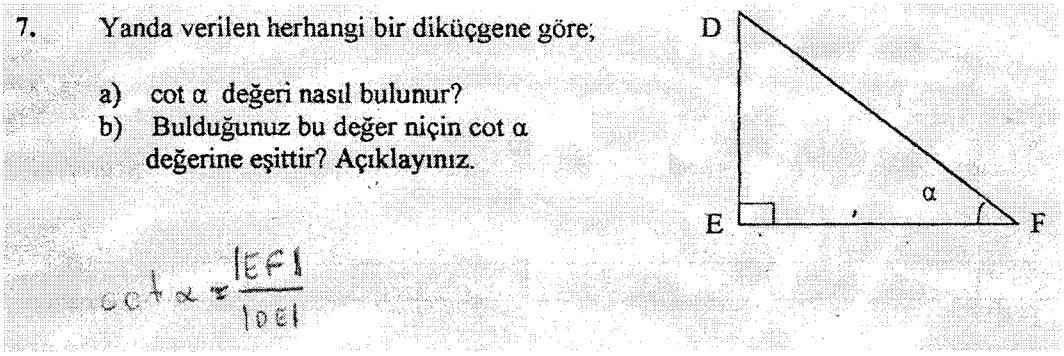

Figure 5. Question 7

Question 8: For any interval $3\pi/2 < \varphi < 2\pi$ please explain the $\sin\varphi < 0$ inequality. (Unit circle method, Relational understanding)

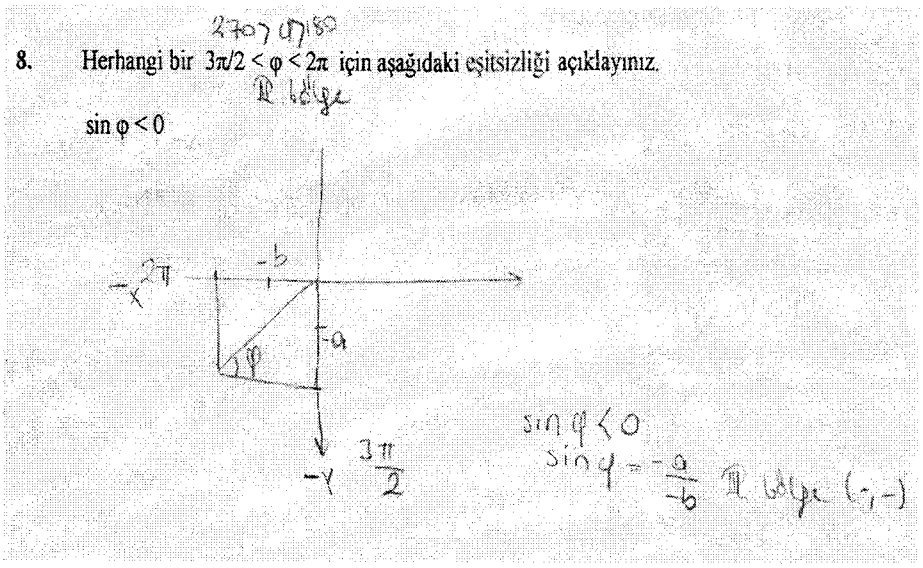

Figure 6. Question 8

3.2. The Verbal Protocol

Data obtained from the *verbal protocol* was first written down and then segment analysis was carried out (Ericsson and Simon, ibid.). In this analysis, firstly, the sentences that the students have said during the protocol have been firstly categorized in such a way that a general information about the students' problem solving process can be obtained.

These categories are as follows: reading (reading aloud or silently), comprehending (starting to solve the problem by using data given in the question), calling information (calling information with the help of what is being reads), using (using the called information in the solving process), forming mathematics (writing down mathematical expressions, like equations, using the called information), operation (performing algebraic/arithmetic operations), and result (coming to a solution). After the sentences are categorized, the percentage of each category is calculated whose results are presented in Figure 7.

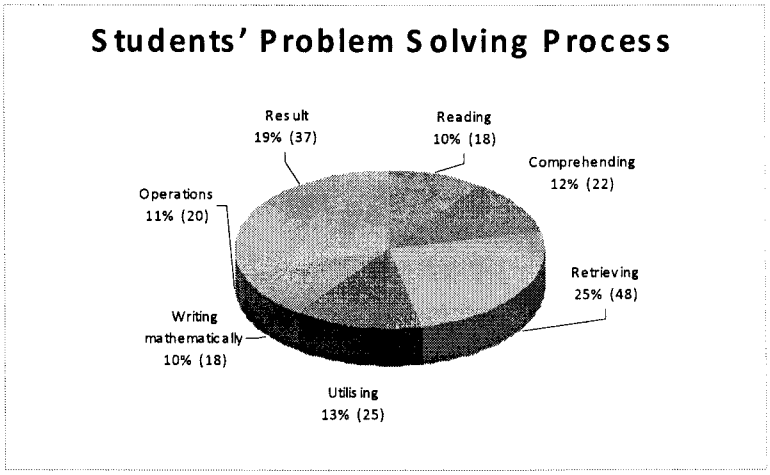

Figure 7. General information about the students' problem solving processes

According to these results, students spend most of their time on calling information. The categories calling information and using are important since they have the ability to show whether the student have understood the concepts of the question relationally or instrumentally. Thus, for these two categories, segment analysis (Ericsson and Simon, ibid) was conducted. In this analysis; the called information is categorized as rule and non-rule, while the used information is categorized as using the rule in the wrong way, using the rule in the correct way, using the non-rule in the wrong way, using the non-rule in the right way. The results are summarized in Figure 8.

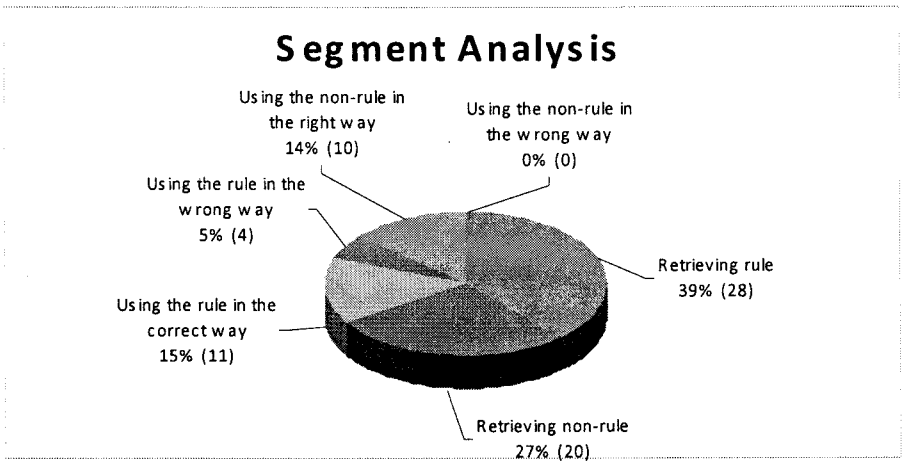

Figure 8. Segment analysis conducted on the two categories: calling Information and using

According to these results, most of the information that is being called a rule. Furthermore, while the non-rules are always used correctly, one quarter of the rules are used wrongly. The places where the student says the word "unit circle" and where the student draws the unit circle are also examined. As a result, data reveals that the successful students are the ones who used the unit circle method, and that they use it in every question they are asked. It is also seen that the students who have not or could not have used the unit circle in the first three questions where the unit circle is not mentioned do not seem to have a clear understanding of the unit circle.

4. DISCUSSION

In this study firstly a research has been done on *how frequently* the students use the unit circle. According to the findings, in only one quarter of the questions the students have used the unit circle. In the rest of the questions they have either used the ratio method or some other methods like memorized trigonometric identities. The search for the *underlying reasons* for the students' frequent use of the ratio method or some other methods like identities reveals certain patterns. According to the data obtained from the interviews, while the unit circle can be *hard to understand*, it can also be possible that the students *get bored* or *lose enthusiasm*.

> "I never have understood these subjects .... I feel uneasy when one mentions the unit circle. I like to use the formulas but I don't know why, when it comes with a figure or something I feel sick. I never get used to the unit circle thing."

Could the reason why the students don't prefer the unit circle method be the fact that the unit circle method requires more relational understanding than instrumental? Matching with the data from one of the interviews, the students prefer to learn and use *formulas*. As it has been stressed in the test analyses, just as the questions where the student uses the ratio method and the questions where instrumental understanding is present coincide, the questions where the student uses the unit circle method and the questions where relational understanding is present also coincide. In this case it is possible to conclude that while the ratio method is predisposed mostly to instrumental understanding, the unit circle method is more predisposed to relational understanding. According to Skemp (1978), there are three main reasons for the preference for the instrumental understanding:

(1) it is easier to learn the instrumental way;
(2) the reward is instantaneous; and
(3) the answer is easier and faster to reach.

It seems likely that students' general preference for the instrumental understanding in the whole mathematics could be affecting their choice in trigonometric methods:

"I always like the shortest way in math because it is easier and practical"

Due to verbal protocol analyses, the students have called information mostly in the form of rules. This supports the claim on the students' preference of instrumental understanding. Furthermore, according to the same analyses, if the information was a non-rule the students always used it correctly, while in the case of a rule, one quarter of the rules could not be used correctly. Thus it can be said that information in the form of a rule mostly stays in short-term memory and hence it may be remembered wrongly, while on the other hand non-rule information tends to stay longer in memory and are mostly used correctly. Hence, it can be concluded that although information learnt relationally are more lasting and used correctly, the students prefer to understand concepts instrumentally.

The students prefer the unit circle method only in specific questions. When the questions of the test and the verbal protocol are examined, we see that the students have used the unit circle mostly in questions were about the signs of trigonometric functions, namely in questions concerning the regions of the unit circle. The students' interview responses highlight where they use or prefer to use the unit circle:

"Let me give an easy example. Let's say sine for example. When the teacher asks what the sign for sine 155 is, I always think about the region in the unit circle, where it is plus, where it is minus."

"It is asked, for example, is sine 280 plus or minus in the third region."

This tendency of the students can also be traced in the findings of the test analyses. In question 4 of the test which is about the angles in specific regions of the unit circle (Please show the $\cos\alpha$ length for any $\pi/2 < \alpha < \pi$ interval), in question 6 (For any $180^\circ < \alpha < 270^\circ$, please show the $\tan\alpha$ and $\cot\alpha$ lengths") and especially in question 8 in which the signs of the trigonometric functions are asked (For a given $3\pi/2 < \varphi < 2\pi$ interval, please explain the $\sin\varphi < 0$ inequality), most of the students have preferred to use the unit circle.

In the cases where students prefer to use the unit circle method, relational understanding does not always seem to occur. The fact that the students tend to use the unit circle in specific questions can lead to the conclusion that they have not understood the unit circle "totally". As underlined before, the questions of both the test and the verbal protocol can also be solved using the unit circle. However, the students have used the unit circle method in only one quarter of the questions. Apart from the questions dealing with the *signs of trigonometric functions*, or with *angles in specific regions of the unit circle*, the students have not use the unit circle so frequently. However, this is not the only role

of the unit circle. The fact that only 17% of the students who were interviewed have uttered the word "unit circle" when they were asked what came first to their minds when they heard the word "trigonometry" supports the assertion that students have not been able to associate the unit circle well enough with the trigonometric functions. Furthermore, in the verbal protocol, one of the students who have not used the unit circle until the last question *(Please explain the equality* $\cos(-65^\circ) = \sin 155^\circ$ *using the unit circle)* where it is demanded to use the unit circle, has in this question just drawn the regions of the unit circle and put the plus and minus signs in the regions "like a parrot" (Hejni, 2004):

Question: Please explain the equality $\cos(-65^\circ) = \sin 155^\circ$ using the unit circle.

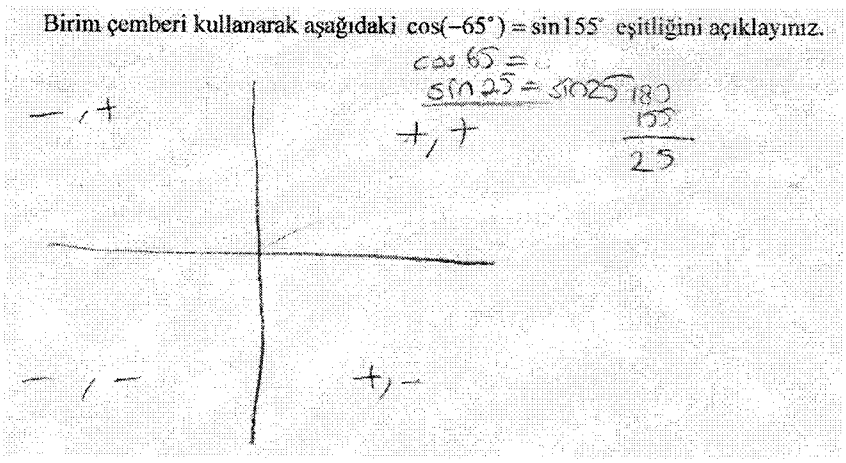

Figure 9. Question

According to Hejni (2004), information which is not understood can be qualified as *parrot knowledge.* The one who cannot express what he knows in his own words but can only repeat the information like a parrot, is considered as *not have understood* the information. Another indication to the students' instrumental understanding of the unit circle is that in the questions of the test where in section (a) only an information, and in section (b) an explanation to that information is asked, while most of the students have answered section (a) correctly, they have either left section (b) empty or they have simply written down the answer *"rule"*:

Question 5: For any ω angle (a) $\sin^2\theta + \cos^2\theta = ?$ and (b) How can you reached this equation? Students' answer: (b) the rule says so

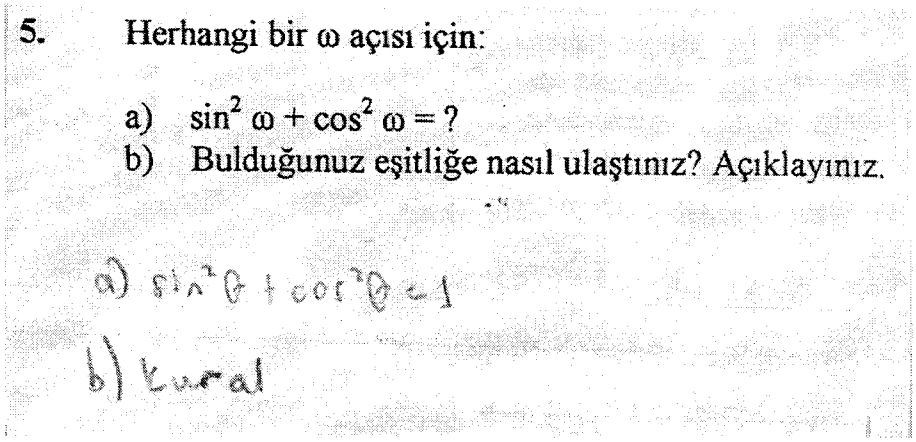

Figure 10. Question 5

From here it can be said that the equality $\sin^2 \omega + \cos^2 \omega = 1$ is an *object* in the student's mind where as the *process* of this object is not known (Gray & Tall, 1994).

A last example supporting the claim that the students have learnt the unit circle mostly instrumentally is that most of the students who have used the unit circle in the test questions could not place the angles correctly in the unit circle and thus could not answer the questions correctly.

Question 6: For any $180° < 270°$ please show the $\tan \alpha$ and $\cos \alpha$ lengths.

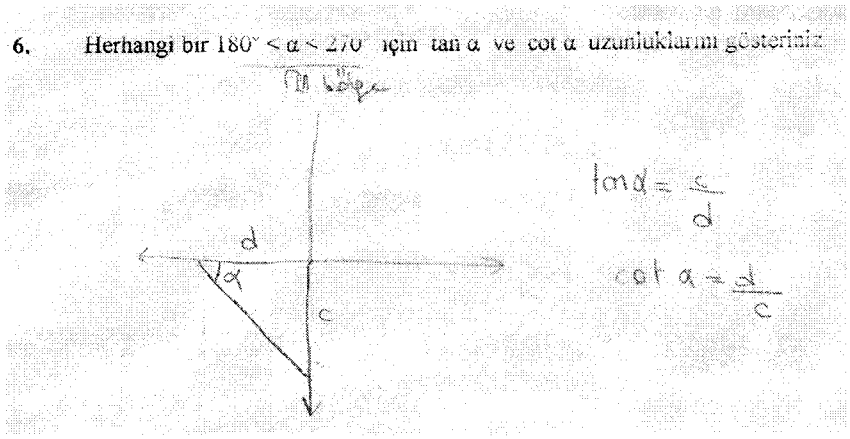

Figure 11. Question 6

It seems that the students are not the only ones to blame for having mostly learnt instrumentally and for having the tendency to prefer the ratio method. At this point, the issues that have to be discussed can be the roles of the teachers, the sources, and the

system of high school and university entrance exams in Turkey.

Understanding is to feel what is to be understood and being able to use this feeling in other situations (Mrozek, 1998). Understanding mathematics is realized in three main levels:

(1) understanding what is the symbol to be understood,
(2) how, and
(3) why is this symbol been used (Mrozek, 1998).

When these three steps are examined, it can be seen that they go in a certain order and getting deeper in each step. Going from memorized information to deep understanding, this arrangement resembles the six levels of the Bloom Taxonomy (Bloom, 1956). Given that understanding mathematics is a level that comes after the grasp of certain prerequisite knowledge, the teachers have an important role in the formation of this cognitive process. According to today's modern perception of *student centered education*, the teachers should abandon the *traditional behaviorist approach*, and adopt the *constructivist approach* instead. The role of a constructivist teacher according to Kamii and Lewis (2001) is to guide and support the students in their discovery the application of new mathematical ideas, not transmitting the "correct" and "mature" ways of thinking. Students learn in a constructivist environment, having discovered the essential concepts by themselves, will be able to learn the *hows and whys* behind the concepts, and thus will be able to understand the concepts relationally.

During the interview the students have been asked whether their mathematics teacher had mentioned the unit circle in the trigonometry unit or not and if so, what their teacher had told them. In response the students have said that their teacher has mentioned the unit circle and drawn it on the blackboard in the beginning of the trigonometry unit, has repeated this act for some lectures, and that after some time has stopped doing so. The fact that *only in specific questions* the students have used the unit circle method can be a possible indicator that the teacher has not given the students the opportunity to discover the concepts involving the unit circle by themselves.

The students' tendency to choose instrumental understanding is not the only reason why they prefer to use the ratio method rather than the unit circle method. Students are under the effect of the books that their school, their teacher, or the Ministry of Education determines as source books, or of other books that they themselves find. These source books influence the students in many ways. First of all, in the Turkish curriculum trigonometric functions are taught for the first time in eighth grade and with the ratio method as stated in the "measurement" learning area, "measurement in triangles" sub-area of the mathematics teaching program document as "The student uses the trigonometric ratios of acute angles in right angled triangles in solving problems" (MEB,

2005a). Following the eighth grade, the students do not work with trigonometry until they re-encounter trigonometric functions, this time in tenth grade and with the unit circle method. as stated in the second article of the "trigonometry learning area", "directional angles" sub-area of the mathematics teaching program document as "the student identifies the unit circle and writes the equation".

Hence, the students meet with trigonometric concepts with the ratio method and embrace this method for two years. This can lead the students to associate trigonometric concepts firstly and only with the right angled triangle. This might resemble the mother tongue of a person which is firstly heard and learnt. The ratio method that is being taught firstly and two years prior to the unit circle method can, just like the firstly learnt mother tongue, solidly settle in the students' minds and can, as the findings of this research also supports, not be easily bent. This is why we can say that the order in which the two methods are given in a source book can negatively influence the students' comprehension and adoption of the unit circle.

Moreover, the teaching methods that the source books embrace also have an important role. If the concepts in the sources are told with *the lecture method*, constructing the new information could get harder for the students, and it might be the case that they would have to learn the concepts instrumentally. The case might be the same for the unit circle, which is also a mathematical concept.

A final possible underlying reason for the students to prefer instrumental understanding might be the *examination system in Turkey* being pedagogically inappropriate. Both for the high school entrance and the university entrance exams, the students are directed to solving the most number of questions in the least amount of time, and their possible success in this three-hours-long exam, which more or less determines their lives, becomes their only aim in life.

While having the possibility to cause psychological and medical problems, this examination system can also damage the student educationally. It is possible that the students wouldn't have the ability to separate these exams from their main education in school, and thus tend to aim solving problems in every course in the shortest way. This might finally end up causing the students prefer instrumental understanding which requires less time and effort. (Skemp, 1978):

> "I believe that in general short methods are more of a rote learning type of thing, but because of the OSS (external) exam, time is very important for us. So we do not have time to learn things deeply."

4. CONCLUSION

With this study about the unit circle, it is concluded that students in Turkey do not prefer the unit circle method so much when dealing with trigonometric concepts. The students use the ratio method or trigonometric identities at least as much as they use the unit circle method. They use the unit circle method mostly in specific questions such as those concerning the signs of the trigonometric functions. One important reason for this tendency seems to be that they have learnt the unit circle method mostly instrumentally, and that as a result they haven't been able to associate the unit circle well enough with the trigonometric concepts.

Students have generally preferred instrumental understanding which consumed less time, less effort, and which leads to quicker solution. Furthermore, the fact that their teacher has not taught the unit circle method in the constructivist approach and that he has not spent enough time on it, the fact that the ratio method is shown prior to the unit circle method in the Turkish mathematics curriculum, and the corrupt system of the high school and university entrance exams in Turkey have resulted in directing the students to learn the unit circle method instrumentally, make them go for the ratio method or memorize trigonometric identities instead.

In order to give the students the opportunity to better comprehend and more frequently use the unit circle, it seems better that trigonometric functions is taught firstly with the unit circle method. The ratio method may be explained after the unit circle method as 'a derivative of the unit circle method', derived from a right triangle drawn similar to any right triangle formed in the first region of the unit circle, and the students can understand the ratio method also relationally. The important thing, it seems, is to understand both methods relationally, and to realize that *actually there is only one method.* When the student understands the unit circle method relationally, they could possibly use this method more frequently and more consciously.

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