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트럭-트레일러 타입의 모바일로봇을 위한 귀환 제어기 설계

(Digital Implementation of Backing up control of Truck-trailer type Mobile Robots)

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요 약

본 논문은 실제적인 제약, 컴퓨팅 시간 지연, 양자화를 고려하여 퍼지 모델을 기초로 한 제어기를 트럭-트레일러 타입의 모바일 로봇의 귀환 제어기에 적용하여 설계하였다. 퍼지 귀환 제어기의 출력은 단위 샘플링 시간동안 지연되므로 이를 예측하여 설계하였다. 시간 지연을 고려한 해석 및 디자인 문제는 제안된 제어기가 샘플링 시간과 동기되어 있기 때문에 쉽게 해결된다. 또한 퍼지 제어기 구조 개발 시 양자화가 이루어지기 때문에 안정성 있는 해석이 가능하고 양자화 이외에 발생하는 사소한 문제도 역시 안정함을 보여주므로, 양자화한 시스템은 일반적으로는 극단적인 수렴을 한다. 실험결과에서 제안된 시스템의 효율성이 증명됨을 볼 수 있다.

Abstract

In this paper, the implementation of the backward movement control of a truck-trailer type mobile robot using fuzzy model based control scheme considering the practical constraints, computing time-delay and quantization is presented. We propose the fuzzy feedback controller whose output is delayed with unit sampling period and predicted. The analysis and the design problem considering the computing time-delay become very easy because the proposed controller is syncronized with the sampling time. Also, the stability analysis is made when the quantization exists in the implementation of the fuzzy control architectures and it is shown that if the trivial solution of the fuzzy control system without quantization is asymptotically stable, then the solutions of the fuzzy control system with quantization are uniformly ultimately bounded. The experimental results are shown to verify the effectiveness of the proposed scheme.

Keywords : Computing time-delay, quantization, fuzzy model, fuzzy control, truck-trailer type vehicle

I. Introduction

The control of backward movement of articulated vehicles such as truck-trailer type vehicles has been adopted as a testbed for a variety of control-design methods[1]. Backward movement controls of computer simulated articulated vehicles have been realized via intelligent controls such as fuzzy control, neural control and both of them. However, stability of the control systems has not been analyzed in the research. Recently, experimental results for the backward movement control of truck-trailer have been reported in [1~2]. In the researches, intelligent modeling and control of truck-trailer has been accomplished via vision sensing for the state feedback of the system. In [1~2], CCD camera has been utilized for the state sensing and the concept of PDC (Parallel Distributed Compensation) has been adopted to design a fuzzy controller from the

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TS(Takagi-Sugeno) fuzzy model of it. In [2], the multiobjective control design of the truck-trailer system from the practical point of view has been presented. The practical constraint such as a avoidance of steering angle saturation, measurement noise rejection, jackknife phenomenon, and so on. The LMI(Linear matrix inequality) based fuzzy control has been presented for the achievement of a multiobjective control design satisfying these practical constraints. However, the approaches have not considered the computing time-delay and the quantization effect due to the vision based state sensing processing. Because of the long control processing time, in [2], the vehicle cannot even move continuously and be stopped during the state sensing at every sampling. In this paper, the additive practical constraints as computing time-delay due to the control processing time and quantization effect from the digital implementation of the control architecture are considered and the control design which guarantees the stability under the existence of the delay and quantization effect is proposed.

Digital fuzzy control systems can be defined as hybrid dynamical systems which usually consist of an interconnection of a continuous-time plant and discrete time fuzzy controller. The analysis and design of such fuzzy systems have been of continuing interest for several decades. Since Takagi-Sugeno(TS) fuzzy model was presented, various kinds of TS fuzzy model based controllers have been suggested and systematic design of the fuzzy controller can be possible. The stability of the overall fuzzy systems could be determined by the Lyapunov stability analysis and recently linear matrix inequality(LMI) based approaches have been used to determine the existence of a common positive definite matrix[3~4]. However, these results do not take into account the computing time-delay and quantization effect in digital implementation of the fuzzy control systems. In this paper, for the backing up control of a truck-trailer type vehicle, we propose the design method ofa fuzzy feedback controller which guarantees the stability of the system in the presence of the computing time-delay and investigate the qualitative stability analysis of the digital fuzzy control systems with quantization in both the controller and the interconnection elements.

We first study the design method of digital fuzzy controller(DFC) considering the practical constraint : computing time-delay. If the system has a considerable computing time-delay, the analysis and the design of the controller are very difficult since it makes the output of the controller not synchronized with the sampling time. We propose the fuzzy feedback controller whose output is delayed with unit sampling period and predicted using current states and the control input to the plant at previous sampling time. The analysis and the design of the controller become very easy because the output of the proposed controller is synchronized with the sampling time. Therefore, the proposed control system can be designed using the conventional methods such as parallel distributed compensation(PDC) and LMI based analysis.

Next, we study the qualitative effects of quantization of the proposed digital fuzzy control system. It is shown that if the trivial solution of the fuzzy control system without quantization is asymptotically stable, then the solutions of the digital fuzzy control system with quantization are uniformly ultimately bounded.

To verify the validity and the effectiveness of the scheme, the proposed fuzzy feedback controller is applied to backing up control of a truck-trailer type vehicle considering the constraints as the computing time-delay and quantization.

Ⅱ. Discrete TS Fuzzy Model Based Control

In the discrete time TS fuzzy systems without control input, the dynamic properties of each subspace can be expressed as the following fuzzy IF-THEN rules[5].

Rule i : If $x_1(k)$ is $M_{i1}\cdots$ and $x_n(k)$ is M_{in}
THEN $\mathbf{x}(k+1)=\mathbf{G}_i\mathbf{x}(k)$

$$i=1,2,\cdots,r \tag{1}$$

where $\mathbf{x}(k)=[x_1(k)\ \ x_2(k)\ \ \cdots\ \ x_n(k)]^T\in\Re^n$ denotes the state vector of the fuzzy system, r is the number of the IF-THEN rules, and Mij is fuzzy set.

If the state x(k) is given, the output of the fuzzy system expressed as the fuzzy rules of Eq. (1) can be inferred as follows.

$$w_i(k)=\prod_{j=1}^{n}\overline{M}_{ij}(x_j(k)) \tag{2}$$

where, $\overline{M}_{ij}(x_j(k))$ is the grade of membership of $x_j(t)$ in M_{ij} and $h_i(k)=\frac{w_i(k)}{\sum_{i=1}^{r}w_i(k)}$

A sufficient condition for ensuring the stability of the fuzzy system(2) is given in Theorem 1.

Theorem 1 : The equilibrium point for the discrete time fuzzy system (2) is asymptotically stable in the large if there exists a common positive definite matrix P satisfying the following inequalities.[5]

$$\mathbf{G}_i^T\mathbf{P}\mathbf{G}_i-\mathbf{P}<\mathbf{0},\ \ i=1,2,\cdots,r \tag{3}$$

In the discrete time fuzzy system with control input to the plant, the dynamic properties of each subspace can be expressed as the following fuzzy IF-THEN rules.

Rule i : If $x_1(k)$ is $M_{i1}\cdots$ and $x_n(k)$ is M_{in}
THEN $\mathbf{x}(k+1)=\mathbf{A}_i\mathbf{x}(k)+\mathbf{B}_i\mathbf{u}(k)$

$$i=1,2,\cdots,r \tag{4}$$

where, $\mathbf{u}(k)=[u_1(k)\ \ u_2(k)\ \ \cdots\ \ u_m(k)]^T\in\Re^m$ denotes the input of the fuzzy system.

If the set of $(\mathbf{x}(k),\mathbf{u}(k))$ is given the output of the fuzzy system (4) can be obtained as follows.

$$\mathbf{x}(k+1)=\frac{\sum_{i=1}^{r}w_i(k)\{\mathbf{A}_i\mathbf{x}(k)+\mathbf{B}_i\mathbf{u}(k)\}}{\sum_{i=1}^{r}w_i(k)}$$

$$=\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i\mathbf{x}(k)+\mathbf{B}_i\mathbf{u}(k)\} \tag{5}$$

where $w_i(k)=\prod_{j=1}^{n}\overline{M}_{ij}(x_j(k))$, $\overline{M}_{ij}(x_j(k))$ is the grade of membership of $x_j(t)$ in M_{ij} and $h_i(k)=\frac{w_i(k)}{\sum_{i=1}^{r}w_i(k)}$.

In PDC concept, the fuzzy controller is designed distributively according to the corresponding rule of the plant[12]. Therefore, the PDC for the plant (4) can be expressed as follows.

Rule j : If $x_1(k)$ is $M_{j1}\cdots$ and $x_n(k)$ is M_{jn}
THEN $\mathbf{u}(k)=-\mathbf{F}_j\mathbf{x}(k)$

$$j=1,2,\cdots,r \tag{6}$$

The fuzzy controller output of Eq. (6) can be inferred as follows.

$$\mathbf{u}(k)=-\frac{\sum_{i=1}^{r}w_i(k)\mathbf{F}_i\mathbf{x}(k)}{\sum_{i=1}^{r}w_i(k)}=-\sum_{j=1}^{r}h_j(k)\mathbf{F}_j\mathbf{x}(k) \tag{7}$$

where $h_j(k)$ is the same function in Eq. (5).

Substituting Eq. (7) into Eq. (5) gives the following closed loop discrete time fuzzy system.

$$\mathbf{x}(k+1)=\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i\mathbf{x}(k)-\mathbf{B}_i\sum_{j=1}^{r}h_j(k)\mathbf{F}_j\mathbf{x}(k)\}$$

$$=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(k)h_j(k)\{\mathbf{A}_i-\mathbf{B}_i\mathbf{F}_j\}\mathbf{x}(k) \tag{8}$$

Defining $\mathbf{G}_{ij}=\mathbf{A}_i-\mathbf{B}_i\mathbf{F}_j$, the following equation is obtained.

$$\mathbf{x}(k+1)=\sum_{i=1}^{r}h_i(k)h_i(k)\mathbf{G}_{ii}\mathbf{x}(k)+2\sum_{i<j}^{r}h_i(k)h_j(k)\{\frac{\mathbf{G}_{ij}+\mathbf{G}_{ji}}{2}\}\mathbf{x}(k) \tag{9}$$

Applying Theorem 1 to analyze the stability of the discrete time fuzzy system (9), the stability condition of Theorem 2 can be obtained.

Theorem 2 [5]: The equilibrium point of the closed loop discrete time fuzzy system (9) is asymptotically stable in the large if there exists a common positive definite matrix P which satisfies the following inequalities for all i and j except the set (i,j) satisfying $h_i(k)\cdot h_j(k)=0$.

$$\mathbf{G}_{ii}^T\mathbf{P}\mathbf{G}_{ii}-\mathbf{P}<\mathbf{0} \tag{10a}$$

$$\left(\frac{\mathbf{G}_{ij}+\mathbf{G}_{ji}}{2}\right)^T\mathbf{P}\left(\frac{\mathbf{G}_{ij}+\mathbf{G}_{ji}}{2}\right)-\mathbf{P}\le\mathbf{0},\ i<j \tag{10b}$$

If $\mathbf{B}=\mathbf{B}_1=\mathbf{B}_2=\cdots=\mathbf{B}_r$ in the plant (5) is satisfied, the closed loop system (8) can be obtained as follows.

$$\mathbf{x}(k+1)=\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i\mathbf{x}(k)-\mathbf{B}\sum_{j=1}^{r}h_j(k)\mathbf{F}_j\mathbf{x}(k)\}$$

$$=\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i-\mathbf{B}\mathbf{F}_i\}\mathbf{x}(k) \tag{11}$$

where $\mathbf{G}_i=\mathbf{A}_i-\mathbf{B}\mathbf{F}_i$

Hence, Theorem 1 can be applied to the stability analysis of the closed loop system (11).

To prove the stability of the discrete time fuzzy control system by Theorem 1 and Theorem 2, the common positive definite matrix P must be solved. LMI theory can be applied to solving P. LMI theory is one of the numerical optimization techniques. Many of the control problems can be transformed into LMI problems and the recently developed Interior-point method can be applied to solving numerically the optimal solution of these LMI problems.

The stability condition of Theorem 1 can be transformed into the LMI feasibility problem as follows.

LMI feasibility problem about the stability condition of Theorem 1 [5]: The problem of finding $\mathbf{P}$ which satisfies the LMI's, $\mathbf{P}>\mathbf{0}$ and $\mathbf{G}_i^T\mathbf{P}\mathbf{G}_i-\mathbf{P}<\mathbf{0}$, $i=1,2,\cdots,r$ or proving the unfeasibility in the case that $\mathbf{A}_i\in\Re^{n\times n}$, $i=1,2,\cdots,r$ are given. If the design object of a controller is to guarantee the stability ofthe closed loop system (5), the design of the PDC fuzzy controller(7) is equivalent to solving the following LMI feasibility problem using Schur complements.

LMI feasibility problem equivalent to the PDC design problem (Case I)[5]: The problem of finding $\mathbf{X}>\mathbf{0}$ and $\mathbf{M}_1,\mathbf{M}_2,\cdots,\mathbf{M}_r$ which satisfy the following inequalities.A

$$\begin{bmatrix}\mathbf{X} & \{\mathbf{A}_i\mathbf{X}-\mathbf{B}_i\mathbf{M}_i\}^T\\ \mathbf{A}_i\mathbf{X}-\mathbf{B}_i\mathbf{M}_i & \mathbf{X}\end{bmatrix}>\mathbf{0},\qquad i=1,2,\cdots,r$$

$$\begin{bmatrix}\mathbf{X} & 1/2\{\mathbf{A}_i\mathbf{X}+\mathbf{A}_j\mathbf{X}-\mathbf{B}_i\mathbf{M}_i-\mathbf{B}_j\mathbf{M}_j\}^T\\ 1/2\{\mathbf{A}_i\mathbf{X}+\mathbf{A}_j\mathbf{X}-\mathbf{B}_i\mathbf{M}_i-\mathbf{B}_j\mathbf{M}_j\} & \mathbf{X}\end{bmatrix}>\mathbf{0},$$

$i<j$

where $\mathbf{X}=\mathbf{P}^{-1}$, $\mathbf{M}_1=\mathbf{F}_1\mathbf{X}$, $\mathbf{M}_2=\mathbf{F}_2\mathbf{X}$, and $\mathbf{M}_r=\mathbf{F}_r\mathbf{X}$.

If $\mathbf{B}=\mathbf{B}_1=\mathbf{B}_2=\cdots=\mathbf{B}_r$ is satisfied, the design of the PDC fuzzy controller(7) is equivalent to solving the following LMI feasibility problem.

LMI feasibility problem equivalent to the PDC design problem (Case II) [5]: The problem of finding $\mathbf{X}>\mathbf{0}$ and $\mathbf{M}_1,\mathbf{M}_2,\cdots,\mathbf{M}_r$ which satisfy the following equations.

$$\begin{bmatrix}\mathbf{X} & \{\mathbf{A}_i\mathbf{X}-\mathbf{B}_i\mathbf{M}_i\}^T\\ \mathbf{A}_i\mathbf{X}-\mathbf{B}_i\mathbf{M}_i & \mathbf{X}\end{bmatrix}>\mathbf{0}\quad i=1,2,\cdots,r$$

where $\mathbf{X}=\mathbf{P}^{-1}$, $\mathbf{M}_1=\mathbf{F}_1\mathbf{X}$, $\mathbf{M}_2=\mathbf{F}_2\mathbf{X}$, and $\mathbf{M}_r=\mathbf{F}_r\mathbf{X}$.

The feedback gain matrices $\mathbf{F}_1,\mathbf{F}_2,\cdots,\mathbf{F}_r$ and the common positive definite matrix $\mathbf{P}$ can be given by the LMI solutions, $\mathbf{X}$ and $\mathbf{M}_1,\mathbf{M}_2,\cdots,\mathbf{M}_r$, as follows.

$$\mathbf{P}=\mathbf{X}^{-1},\ \mathbf{F}_1=\mathbf{M}_1\mathbf{X}^{-1},\ \mathbf{F}_2=\mathbf{M}_2\mathbf{X}^{-1},\ \mathbf{F}_r=\mathbf{M}_r\mathbf{X}^{-1} \tag{12}$$

Ⅲ. Practical Constraint : Computing Time–Delay

In real control systems, a considerable computing time-delay can occur due to the computing processing in sensor and controller part. Let τ be defined as the sum of all this time-delay. In the case of the real system, the ideal fuzzy controller of Eq. (6) can be described as (13) due to computing the time-delay, τ.

$$\begin{aligned} &\text{Rule } j: \text{ If } x_1(kT) \text{ is } M_{j1} \cdots \text{and } x_n(kT) \text{ is } M_{jn} \\ &\qquad \text{THEN } \mathbf{u}(kT+\tau) = -\mathbf{F}_j \mathbf{x}(kT) \\ &j = 1, 2, \cdots, r \end{aligned} \tag{13}$$

Becausethe time-delay makes the output of the controller not synchronized with the sampling time, Theorem 1 can not be applied to this system In this paper, DFC which has the following fuzzy rules (14) is proposed to consider the time-delay of the fuzzy plant (4). In this scheme, the output of the fuzzy controller is delayed with unit sampling period and predicted. Hence the analysis and the design of the controller are very easy because the output of the proposed controller is synchronized with the sampling time.

$$\begin{aligned} &\text{Rule } j: \text{ If } x_1(k) \text{ is } M_{j1} \cdots \text{and } x_n(k) \text{ is } M_{jn} \\ &\qquad \text{THEN } \mathbf{u}(k+1) = \mathbf{D}_j \mathbf{u}(k) + \mathbf{E}_j \mathbf{x}(k) \\ &j = 1, 2, \cdots, r \end{aligned} \tag{14}$$

The output of DFC (14) is inferred as follows.

$$\begin{aligned} \mathbf{u}(k+1) &= \frac{\sum_{j=1}^{r} w_j(k) \{\mathbf{D}_j \mathbf{u}(k) + \mathbf{E}_j \mathbf{x}(k)\}}{\sum_{j=1}^{r} w_j(k)} \\ &= \sum_{j=1}^{r} h_j(k) \{\mathbf{D}_j \mathbf{u}(k) + \mathbf{E}_j \mathbf{x}(k)\} \end{aligned} \tag{15}$$

The general timing diagram of fuzzy control loop is shown in Fig. 1. T is the sampling period of the control loop, τ_v and τ_c are the delay made by sensor system and fuzzy controller respectively. Therefore the output of the controller is applied to the plant after overall dealy $\tau = \tau_v + \tau_c$.

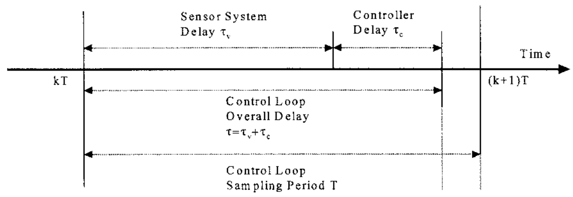

그림 1. 퍼지 제어기 루프의 타이밍도
Fig. 1. Timing Diagram of the Fuzzy Control Loop.

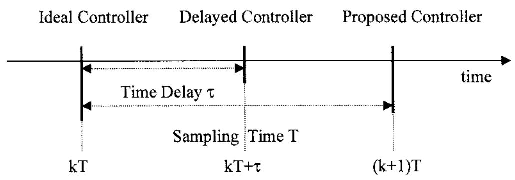

그림 2. 제어기의 출력 타이밍도(세가지 경우)
Fig. 2. Output Timing of the Controllers (three cases).

The output timing of a ideal controller, a delayed controller, and the proposed controller is shown in the Fig. 2. In the ideal controller, it is assumed that there is no computing time-delay. If this controller is implemented in real systems, the time-delay τ is added like (13). The analysis and the design of this system with delayed controller are very difficult since the output of controller is not syncronized with the sampling time.

On the other hand, the analysis and the design of the proposed controller can be very easy because the controller output is syncronized with the sampling time delayed with unit sampling period. Using this proposed controller, we can realize a control algorithm during the time interval $T - \tau_v$ in Fig. 1. In this time interval, a complex algorithm such as not only fuzzy algorithm but also nonlinear control algorithm can be sufficiently realized in real time.

Combining the fuzzy plant (5) with the DFC (15), the closed loop system is given as follows.

$$\begin{bmatrix}\mathbf{x}(k+1)\\ \mathbf{u}(k+1)\end{bmatrix}=\sum_{i=1}^{r}h_i(k)\begin{bmatrix}\mathbf{A}_i & \mathbf{B}_i\\ \mathbf{E}_i & \mathbf{D}_i\end{bmatrix}\begin{bmatrix}\mathbf{x}(k)\\ \mathbf{u}(k)\end{bmatrix} \quad (16)$$

Defining the new state vector as $\mathbf{w}(k)=\begin{bmatrix}\mathbf{x}(k)\\ \mathbf{u}(k)\end{bmatrix}$, the closed loop system (16) can be modified as

$$\mathbf{w}(k+1)=\sum_{i=1}^{r}h_i(k)\mathbf{G}_i\mathbf{w}(k) \quad (17)$$

where $\mathbf{G}_i=\begin{bmatrix}\mathbf{A}_i & \mathbf{B}_i\\ \mathbf{E}_i & \mathbf{D}_i\end{bmatrix}$

Hence, the stability condition of the closed loop system (17) becomes the same as the sufficient condition of Theorem 1 and the stability can be determined by solving LMI feasibility problem about the stability condition of Theorem 1. Also, the design problem of the DFC guaranteeing the stability of the closed loop system can be transformed into LMI feasibility problem. To do this, the design problem of the DFC is transformed into the design problem of the PDC fuzzy controller.

PDC design problem equivalent to DFC design problem :

The problem of designing the PDC fuzzy controller $\mathbf{v}(k)=-\sum_{j=1}^{r}h_j(k)\overline{\mathbf{F}}_j\mathbf{w}(k)$ in the case that the fuzzy plant $\mathbf{w}(k+1)=\sum_{i=1}^{r}h_i(k)\{\overline{\mathbf{A}}_i\mathbf{w}(k)+\overline{\mathbf{B}}\mathbf{v}(k)\}$ is given.

where $\overline{\mathbf{A}}_i=\begin{bmatrix}\mathbf{A}_i & \mathbf{B}_i\\ \mathbf{0} & \mathbf{0}\end{bmatrix}$, $\overline{\mathbf{B}}=\begin{bmatrix}\mathbf{0}\\ \mathbf{I}\end{bmatrix}$, and $\overline{\mathbf{F}}_j=-[\mathbf{E}_j \quad \mathbf{D}_j]$

Therefore, using the same notation in section 2, the design problem of the DFC can be equivalent to the following LMI feasibility problem.

LMI feasibility problem equivalent to DFC design problem :

The problem of finding $\mathbf{X}>\mathbf{0}$ and $\mathbf{M}_1,\mathbf{M}_2,\cdots,\mathbf{M}_r$ which satisfy following equation.

$$\begin{bmatrix}\mathbf{X} & \{\overline{\mathbf{A}}_i\mathbf{X}-\overline{\mathbf{B}}\ \mathbf{M}_i\}^T\\ \overline{\mathbf{A}}_i\mathbf{X}-\overline{\mathbf{B}}\ \mathbf{M}_i & \mathbf{X}\end{bmatrix}>\mathbf{0}, \quad i=1,2,\cdots,r$$

where $\mathbf{X}=\mathbf{P}^{-1}$, $\mathbf{M}_1=\overline{\mathbf{F}}_1\mathbf{X}$, $\mathbf{M}_2=\overline{\mathbf{F}}_2\mathbf{X}$, , and $\mathbf{M}_r=\overline{\mathbf{F}}_r\mathbf{X}$

The feedback gain matrices $\overline{\mathbf{F}}_1,\overline{\mathbf{F}}_2,\cdots,\overline{\mathbf{F}}_r$ and the common positive definite matrix $\mathbf{P}$ can be given by the LMI solutions, $\mathbf{X}$ and $\mathbf{M}_1,\mathbf{M}_2,\cdots,\mathbf{M}_r$, as follows.

$$\mathbf{P}=\mathbf{X}^{-1},\ \overline{\mathbf{F}}_1=\mathbf{M}_1\mathbf{X}^{-1},\ \overline{\mathbf{F}}_2=\mathbf{M}_2\mathbf{X}^{-1} \quad (18)$$

Therefore, the control gain matrices $\mathbf{D}_1,\cdots,\mathbf{D}_r,\mathbf{E}_1,\cdots,\mathbf{E}_r$ of the proposed DFC can be obtained from the feedback gain matrices $\overline{\mathbf{F}}_1,\overline{\mathbf{F}}_2,\cdots,\overline{\mathbf{F}}_r$.

Ⅳ. Practical Constraint : Quantization

In the implementation of digital fuzzy controllers, the quantization is unavoidable. This is due to the fact that computers store numbers with finite bits. In this present section, we investigate the nonlinear effects caused by quantization.

If $x\in\Re$ is the input to a quantizer and $Q(x)$ is the output of a quanizer, the quantization processing can be described as follows.

$$Q(x)=x+p(x) \quad (19)$$

where $p(x)$ describes the quantization nonlinearities determined by the several method of quatization such as round off, value truncation, magnitude truncation, etc. There are many types of quantization. Presenstly, we will concern ourselves primarily with the most commonly used fixed-point quantization which can be characterized by the relation

$$|p(x)|<\varepsilon \quad (20)$$

where positive constants, ϵ in Eq. (20) is quantization error determined by the charateristics of a quatizer.

Therefore, the quantized state $Q_x(\mathbf{x}(k))$ with respect to the system state $\mathbf{x}\in\Re^n$ can be defined as the following form.

$$\mathbf{x_q}(k)=Q_\mathbf{x}(\mathbf{x}(k))=\begin{bmatrix}Q_x(x_1(k))\\Q_x(x_2(k))\\\vdots\\Q_x(x_n(k))\end{bmatrix}=\begin{bmatrix}x_1(k)\\x_2(k)\\\vdots\\x_n(k)\end{bmatrix}+\begin{bmatrix}p_x(x_1(k))\\p_x(x_2(k))\\\vdots\\p_x(x_n(k))\end{bmatrix}$$

$$=\mathbf{x}(k)+\mathbf{p_x}(\mathbf{x}(k)) \quad (21)$$

where $\|\mathbf{p_x}(\mathbf{x}(k))\|\le\varepsilon_\mathbf{x}$.

A Similar definition can be obtained for the quatizated controller $Q_\mathbf{u}(\mathbf{u}(k))$ with respect to the control input $\mathbf{u}\in\Re^m$ as

$$\mathbf{u_q}(k)=Q_\mathbf{u}(\mathbf{u}(k))=\begin{bmatrix}Q_u(u_1(k))\\Q_u(u_2(k))\\\vdots\\Q_u(u_m(k))\end{bmatrix}=\begin{bmatrix}u_1(k)\\u_2(k)\\\vdots\\u_m(k)\end{bmatrix}+\begin{bmatrix}p_u(u_1(k))\\p_u(u_2(k))\\\vdots\\p_u(u_m(k))\end{bmatrix}$$

$$=\mathbf{u}(k)+\mathbf{p_u}(\mathbf{u}(k)) \quad (22)$$

where $\|\mathbf{p_u}(\mathbf{u}(k))\|\le\varepsilon_\mathbf{u}$.

In real digital control systems, the TS fuzzy plant model includes the quantized input term as

$$\mathbf{x}(k+1)=\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i\mathbf{x}(k)+\mathbf{B}_i\mathbf{u_q}(k)\} \quad (23)$$

In order to control this fuzzy plant model with quantized input, the proposed digital fuzzy controller(15) can be transformed into the Eq. (24)

$$\mathbf{u_q}(k+1)=Q_\mathbf{u}(\sum_{i=1}^{r}h_i(k)\{\mathbf{D}_i\mathbf{u_q}(k)+\mathbf{E}_i\mathbf{x_q}(k)\})$$

$$=\sum_{i=1}^{r}h_i(k)\{\mathbf{D}_i\mathbf{u_q}(k)+\mathbf{E}_i\mathbf{x_q}(k)\}+\varDelta_\mathbf{u}(k) \quad (24)$$

where,

$$\varDelta_\mathbf{u}(k)=Q_\mathbf{u}(\sum_{i=1}^{r}h_i(k)\{\mathbf{D}_i\mathbf{u_q}(k)+\mathbf{E}_i\mathbf{x_q}(k)\})-\sum_{i=1}^{r}h_i(k)\{\mathbf{D}_i\mathbf{u_q}(k)+\mathbf{E}_i\mathbf{x_q}(k)\}$$

$$=\mathbf{p_u}(\sum_{i=1}^{r}h_i(k)\{\mathbf{D}_i\mathbf{u_q}(k)+\mathbf{E}_i\mathbf{x_q}(k)\})$$

The state $\mathbf{x}(k)$ in the fuzzy plant model(23) and the state $\mathbf{x}_q(k)$ in the fuzzy controller(24) need to be unified to derive the closed loop equation. Therefore, we apply the quantization operator to the equation of the fuzzy plant model(23).

$$\mathbf{x_q}(k+1)=Q_\mathbf{x}(\mathbf{x}(k+1))=Q_\mathbf{x}(\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i\mathbf{x}(k)+\mathbf{B}_i\mathbf{u_q}(k)\})$$

$$=\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i\mathbf{x_q}(k)+\mathbf{B}_i\mathbf{u_q}(k)\}+\varDelta_\mathbf{x}(k) \quad (25)$$

where,

$$\varDelta_\mathbf{x}(k)=Q_\mathbf{x}(\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i\mathbf{x}(k)+\mathbf{B}_i\mathbf{u_q}(k)\})-\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i\mathbf{x_q}(k)+\mathbf{B}_i\mathbf{u_q}(k)\}$$
$$=Q_\mathbf{x}(\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i\mathbf{x}(k)+\mathbf{B}_i\mathbf{u_q}(k)\})-\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_iQ_\mathbf{x}(\mathbf{x}(k))+\mathbf{B}_i\mathbf{u_q}(k)\}$$
$$=Q_\mathbf{x}(\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i\mathbf{x}(k)+\mathbf{B}_i\mathbf{u_q}(k)\})-\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i(\mathbf{x}(k)+\mathbf{p_x}(\mathbf{x}(k)))+\mathbf{B}_i\mathbf{u_q}(k)\}$$
$$=Q_\mathbf{x}(\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i\mathbf{x}(k)+\mathbf{B}_i\mathbf{u_q}(k)\})-\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i\mathbf{x}(k)+\mathbf{B}_i\mathbf{u_q}(k)\}-\sum_{i=1}^{r}h_i(k)\mathbf{A}_i\mathbf{p_x}(\mathbf{x}(k))$$
$$=\mathbf{p_x}(\sum_{i=1}^{r}h_i(k)\{\mathbf{A}_i\mathbf{x}(k)+\mathbf{B}_i\mathbf{u_q}(k)\})-\sum_{i=1}^{r}h_i(k)\mathbf{A}_i\mathbf{p_x}(\mathbf{x}(k))$$

Hence, the state space model of the quatized closed loop system can be obtained from Eq. (25) and (24) as

$$\mathbf{w}(k+1)=\sum_{i=1}^{r}h_i(k)\mathbf{G}_i\mathbf{w}(k)+\varDelta(k) \quad (26)$$

where $\mathbf{w}(k)=\begin{bmatrix}\mathbf{x_q}(k)\\\mathbf{u_q}(k)\end{bmatrix}$ is the augmented state and $\mathbf{G}_i=\begin{bmatrix}\mathbf{A}_i & \mathbf{B}_i\\\mathbf{E}_i & \mathbf{D}_i\end{bmatrix}$, $\varDelta(k)=\begin{bmatrix}\varDelta_\mathbf{x}(k)\\\varDelta_\mathbf{u}(k)\end{bmatrix}$

If there exists any reference signal or noise, the state space model (26) can be

$$\mathbf{w}(k+1)=\sum_{i=1}^{r}h_i(k)\mathbf{G}_i\mathbf{w}(k)+\varDelta(k)+\mathbf{r}(k) \quad (27)$$

where $\mathbf{r}(k)$ is due to the reference signal or noise.

Now, we analyze the stability of the digital fuzzy systems considering quantization effects. Let us define the norm $\|\bullet\|_\mathbf{P}$ in $\Re^n$ to obtain the stability condition for the closed loop system (27) as follows.

$$\|\mathbf{w}(k)\|_\mathbf{P}=(\mathbf{w}^T(k)\mathbf{P}\mathbf{w}(k))^{\frac{1}{2}} \quad (28)$$

where $\mathbf{P}\in\Re^{n\times n}$ is symmetric positive definite matrix.

Definition 2 : The digital fuzzy system (27) is said to be uniformly ultimately bounded with bound α if and only if for any β>0 there exists $T(\beta)$>0, independent of $K \geq 0$, such that whenever $|\mathbf{w}_K| \leq \beta$ and $k \geq T(\beta)$, one has $|\mathbf{w}_{k+K}| \leq \alpha$.

Remark 1 : Uniform ultimate boundness is similarto uniform asymptotic stability, except that the attracting point $x=0$ is now replaced by an attracting set given by $\{x \in \Re^n : |x| \leq \alpha\}$

Theorem 3 : If the following two conditions are satisfied, there exist a very small positive constant δ such that $\|\Delta(k)\|_{\mathbf{P}} < \delta$ for all integers k and positive constant J such that the closed loop system (27) is uniformly ultimately bounded by bound $J\delta$.

I) There exists a common positive definite matrix P for the system

$$\mathbf{w}(k+1) = \sum_{i=1}^{r} h_i(k)\mathbf{G}_i\mathbf{w}(k) \tag{29}$$

which satisfies the sufficient condition (3) in Theorem 1.

II) There exists $\bar{\delta}$〉0 such that $r(k) \in B_{\bar{\delta}}\{x \mid \|x\| \langle \bar{\delta}\}$ for k>0.

Proof :

If there exists a commom positive definite matrix P satisfying the sufficient condition(3) in Theorem 1, $V(\mathbf{w}(k)) = \|\mathbf{w}(k)\|_{\mathbf{P}} = (\mathbf{w}^T(k)\mathbf{P}\mathbf{w}(k))^{\frac{1}{2}}$ can be a norm Lyapunov function for the system (29).

Since the system(29) satisfies the asymptotical stability by assumption, there exists a constant c such that

$$\Delta V_{(29)}(\mathbf{w}(k)) = V(\mathbf{w}(k+1)) - V(\mathbf{w}(k))$$

$$= \|\sum_{i=1}^{r} h_i(k)\mathbf{G}_i\mathbf{w}(k)\|_{\mathbf{P}} - \|\mathbf{w}(k)\|_{\mathbf{P}} \leq (c-1)\|\mathbf{w}(k)\|_{\mathbf{P}} = (c-1)V(\mathbf{w}(k)),$$

$$0 < c < 1$$

where $\Delta V_{(29)}(\mathbf{w}(k))$ denotes the first forward difference along the solution of the system(29).

The first forward difference for the closed loop system (27) can be given as

$$\Delta V_{(27)}(\mathbf{w}(k),k) = \|\sum_{i=1}^{r} h_i(k)\mathbf{G}_i\mathbf{w}(k) + \Delta(k) + \mathbf{r}(k)\|_{\mathbf{P}} - \|\mathbf{w}(k)\|_{\mathbf{P}}$$

$$\leq \|\sum_{i=1}^{r} h_i(k)\mathbf{G}_i\mathbf{w}(k)\|_{\mathbf{P}} - \|\mathbf{w}(k)\|_{\mathbf{P}} + \|\Delta(k)\|_{\mathbf{P}} + \|\mathbf{r}(k)\|_{\mathbf{P}}$$

$$\leq (c-1)V(\mathbf{w}(k)) + \|\Delta(k)\|_{\mathbf{P}} + \|\mathbf{r}(k)\|_{\mathbf{P}}$$

$$\leq (c-1)V(\mathbf{w}(k)) + \delta + \|\mathbf{r}(k)\|_{\mathbf{P}}$$

Therefore, whenever $\mathbf{w}(K)$ is picked so that $V(\mathbf{w}(K)) = \|\mathbf{w}(K)\|_{\mathbf{P}} \leq$, then $V(\mathbf{w}(K))$ must be less than the solution of the following comparison equation.

$$X_{k+1} - X_k = (c-1)X_k + \delta + \|\mathbf{r}(k)\|_{\mathbf{P}}, \quad \text{or}$$

$$X_{k+1} = c\,X_k + \delta + \|\mathbf{r}(k)\|_{\mathbf{P}}, \text{ for all integers } k \geq K \tag{30}$$

The solution of the comparison equation (30) can be obtained as

$$X_{k+K} = c^k\beta + \frac{1-c^k}{1-c}\delta + \sum_{j=0}^{k} c^{k-j}\|\mathbf{r}(j+K)\|_{\mathbf{P}} \tag{31}$$

Because $0 < c < 1$ in Eq. (31), $c^k \to 0$ as $k \to \infty$. And

$$\gamma_k(K) \equiv \sum_{j=0}^{k} c^{k-j}\|\mathbf{r}(j+K)\|_{\mathbf{P}} \langle \bar{\delta}\sum_{j=0}^{k} c^{k-j} = \bar{\delta}\frac{1-c^{k+1}}{1-c}.$$

Now we can say that V_{k+K} converges uniformly to $\frac{1}{1-c}(\delta+\bar{\delta})$ for $K \geq 0$ as $k \to \infty$ from the comparison equation (30) and $J = \frac{1}{1-c}(1+\frac{\bar{\delta}}{\delta})$ will do because $V(\mathbf{w}_{k+K}) \leq X_{k+K}$.

If the closed loop system(26) is uniformly ultimately bounded by bound $J\delta$, the system converges to the attractiong set $\{\mathbf{w} \in \Re^{m+n} : \|\mathbf{w}\|_{\mathbf{P}} \leq J\delta\}$, not to the equlibrium point $\mathbf{w} = \mathbf{0}$. If $\Delta(k) = 0$ in closed loop system(26), that is, quatization error does not exist, the constant δ is zero and the attracting set becomes to $\mathbf{w} = \mathbf{0}$. In this case, the closed loop system is asymptotically stable.

Theorem 3 shows the analysis of qualitative characteristics of the fuzzy control systems considering the quantization. The positive constant J is related to the equation $\mathbf{w}(k+1)=\sum_{i=1}^{r}h_i(k)\mathbf{G}_i\mathbf{w}(k)$, not to $\Delta(k)$. Therefore, once the digital fuzzy control system is stably designed, it does not diverge although quantization error exists and the smaller the quatization errors become, the more the asymptotic stability is guaranteed.

V. Backing up Control of a Truck-Trailer type Vehicles

In this section, we apply the proposed controller to backing up control of a truck-trailer system with the constraints as computing time-delay and investigate the quantization effect to the control performance.

1. Models of a Truck-Trailer

M. Tokunaga derived the following model about the truck-trailer system. Figure 3 shows the schematic diagram of this system.

$$x_0(k+1)=x_0(k)+vT/l\tan[u(k)]$$
$$x_1(k)=x_0(k)-x_2(k)$$
$$x_2(k+1)=x_2(k)+vT/L\sin[x_1(k)]$$
$$x_3(k+1)=x_3(k)+vT\cos[x_1(k)]\sin[\{x_2(k+1)+x_2(k)\}/2]$$
$$x_4(k+1)=x_4(k)+vT\cos[x_1(k)]\cos[\{x_2(k+1)+x_2(k)\}/2] \quad (32)$$

where $u(k)$: The steering angle of the truck

l: The length of the truck, L: The length of the trailer

T: Sampling time, v: The constant backward speed

K. Tanaka defined the state vector as in the truck-trailer model (32) and expressed the plant as two following fuzzy rules.

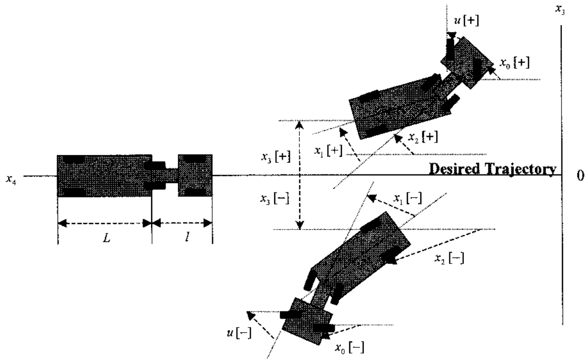

그림 3. 트럭 트레일러 모델과 등가의 시스템
Fig. 3. Truck Trailer Model and Its Coordinate System.

Rule 1 : If $x_2(k)+vT/\{2L\}\,x_1(k)$ is M_1
THEN $\mathbf{x}(k+1)=\mathbf{A}_1\mathbf{x}(k)+\mathbf{B}_1u(k)$

Rule 2 : If $x_2(k)+vT/\{2L\}\,x_1(k)$ is M_2
THEN $\mathbf{x}(k+1)=\mathbf{A}_2\mathbf{x}(k)+\mathbf{B}_2u(k)$ (33)

where

$$\mathbf{A}_1=\begin{bmatrix}1-\frac{vT}{L} & 0 & 0\\ \frac{vT}{L} & 1 & 0\\ \frac{v^2T^2}{2L} & vT & 1\end{bmatrix},\quad \mathbf{A}_2=\begin{bmatrix}1-\frac{vT}{L} & 0 & 0\\ \frac{vT}{L} & 1 & 0\\ \frac{dv^2T^2}{2L} & dvT & 1\end{bmatrix},$$

$$\mathbf{B}=\mathbf{B}_1=\mathbf{B}_2=\begin{bmatrix}\frac{vT}{l}\\ 0\\ 0\end{bmatrix}$$

$$l=0.15\,[\mathrm{m}],\ L=0.38\,[\mathrm{m}],\ v=-1.0\,[\mathrm{m/s}],$$
$$T=2.0\,[\mathrm{s}],\ d=10^{-2}/\pi$$

Fig. 4 shows the membership function of the premise part in the fuzzy system (33).

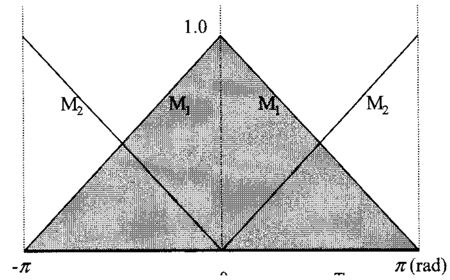

그림 4. 퍼지 시스템에서 전제로 한 부분의 관련 기능
Fig. 4. Membership function of premise part in fussy system.

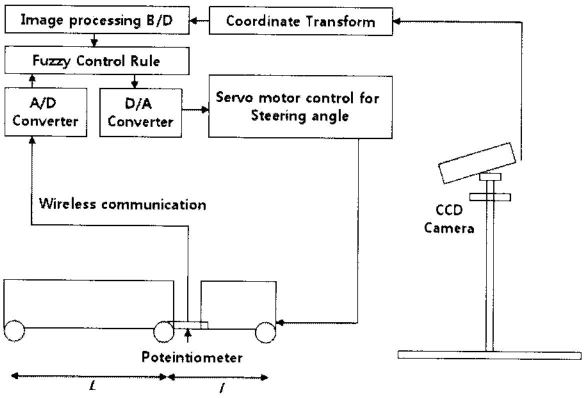

그림 5. 트럭-트레일러 타입의 모바일 로봇을 위한 귀환 제어기의 실험적인 셋업

Fig. 5. Experimental setup for the backing-up control of truck-trailer type mobil-robot.

The experimental setup is shown in Fig. 5, The states of the controlled system are obeserved by the angle detection sensor, potentiometer and CCD camera similar to [1, 3]. The computing time-delay inevitably arises in the generation of control law because of the image processing and the fuzzy control rule computing. The quantization is also from the A/D converting processing and the image processing for the state sensing. The computing time-delay and quantization degrade the backing-up control performance if one design the controller without considering them, of which effect to the control performance will be shown in the experimental results.

2. Backing-up control of truck-trailer type vehicle using conventional discrete time fuzzy control

In this subsection, the experiments of backing up control of a truck-trailer type vehicle is made by the conventional discrete time fuzzy controller without considering the computing time-delay and quantization effect.

To solve the backward parking problem of (33), the PDC fuzzy controller can be designed as follows.

$$\begin{aligned}&\text{Rule 1: If } x_2(k)+vT/\{2L\}\cdot x_1(k) \text{ is } M_1\\ &\qquad \text{THEN } u(k)=\mathbf{F}_1^T\mathbf{x}(k)\\ &\text{Rule 2: If } x_2(k)+vT/\{2L\}\cdot x_1(k) \text{ is } M_2\\ &\qquad \text{THEN } u(k)=\mathbf{F}_2^T\mathbf{x}(k)\end{aligned} \tag{34}$$

where $\mathbf{F}_1=\begin{bmatrix}1.2837\\-0.4139\\0.0201\end{bmatrix}$ and $\mathbf{F}_2=\begin{bmatrix}0.9773\\-0.0709\\0.0005\end{bmatrix}$

Ricatti equation for linear discrete systems was used to determine these feedback gains. The detailed derivation of these feedback gains was given in.

Substituting Eq. (34) into Eq. (33) yields the following closed loop system due to $\mathbf{B}=\mathbf{B}_1=\mathbf{B}_2$.

$$\mathbf{x}(k+1)=\sum_{i=1}^{2}h_i(k)\mathbf{G}_i\mathbf{x}(k) \tag{35}$$

where

$$\mathbf{G}_1=\begin{bmatrix}0.448 & 0.296 & -0.014\\-0.364 & 1 & 0\\0.364 & -2 & 1\end{bmatrix}$$

and $\mathbf{G}_2=\begin{bmatrix}0.448 & 0.296 & -0.014\\-0.364 & 1 & 0\\0.116\times10^{-2} & -0.637\times10^{-2} & 1\end{bmatrix}$

Since there exists the common positive matrix P which satisfies the stability sufficient condition (3), the closed loop system is asymptotically stable in the large. That is, the backward parking can be accomplished for all initial conditions if the computing time-delay does not exist.

Common positive definite matrix :

$$\mathbf{P}=\begin{bmatrix}113.9 & -92.61 & 2.540\\-92.61 & 110.7 & -3.038\\2.540 & -3.038 & 0.5503\end{bmatrix}$$

However, in the case of the real system, the ideal fuzzy controller of Eq. (34) can be described as (36) due to the computing time-delay, τ.

$$\begin{aligned}&\text{Rule 1: If } x_2(kT)+vT/\{2L\}\cdot x_1(kT) \text{ is } M_1\\ &\qquad \text{THEN } u(kT+\tau)=\mathbf{F}_1^T\mathbf{x}(kT)\\ &\text{Rule 2: If } x_2(kT)+vT/\{2L\}\cdot x_1(kT) \text{ is } M_2\\ &\qquad \text{THEN } u(kT+\tau)=\mathbf{F}_2^T\mathbf{x}(kT)\end{aligned} \tag{36}$$

표 1. 트럭-트레일러 시스템의 초기 조건
Table 1. The initial conditions of the truck-trailer system.

CASE	$x_1(0)$[deg]	$x_2(0)$[deg]	$x_3(0)$[m]
CASE I	*0*	*0*	*1*
CASE II	*-90*	*135*	*-0.5*

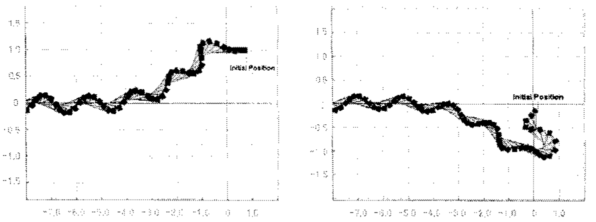

그림 6. 전통적인 이산 시간 퍼지 제어기의 실험결과
Fig. 6. Experimental results by conventional discrete time fuzzy control.

Two initial conditions used for the experiments of the truck-trailer system are given in Table 1.

The experiments are executed in the case that the maximum time-dealy τ is a half of the sampling time (τT=1 [sec]). Figure 6 shows that the truck-trailer system is oscillating and the fuzzy controller can not accomplish the backing up control effectively due to the computing time-delay. Note that the experimental results shown in [1,3] could be accomplished by the following the contol procedure that the vehicle is stopped during the state sensing

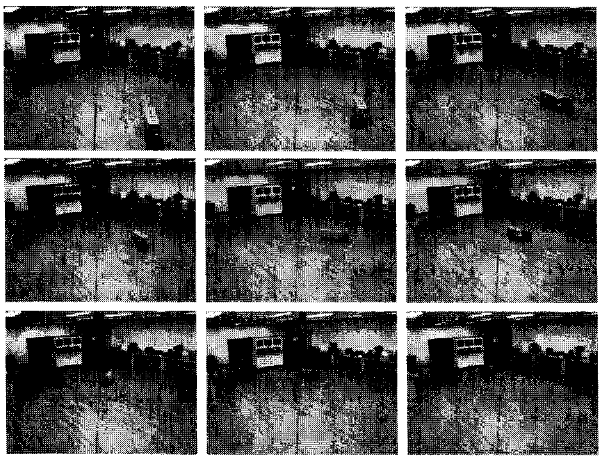

그림 7. 전통적인 퍼지 제어기의 실험결과 사진
Fig. 7. The photograph of experimental results by conventional fuzzy control.

and adjusting wheels because of the computing time-delay. However, it can be found that in this experiments, the successive continuous control results cannot be achieved like [*] due to the computing time-delay. The photographs presented in Fig. 7 show some selective experimental results for CASE I.

In the present subsection, we design the DFC considering the computing time-delay and analyze the quantization effects to the control system. Following the design technique of DFC in section 3, we can construct the DFC for the backing up control problem as follows.

$$\begin{aligned}&\text{Rule 1: If } x_2(k)+vT/\{2L\}\cdot x_1(k) \text{ is } M_1\\ &\qquad \text{THEN } u(k+1)=\mathbf{D}_1u(k)+\mathbf{E}_1\mathbf{x}(k)\\ &\text{Rule 2: If } x_2(k)+vT/\{2L\}\cdot x_1(k) \text{ is } M_2\\ &\qquad \text{THEN } u(k+1)=\mathbf{D}_2u(k)+\mathbf{E}_2\mathbf{x}(k)\end{aligned} \tag{37}$$

Combining Eq. (33) with Eq. (37), the augmented closed loop system is given as follows.

$$\mathbf{w}(k+1)=\sum_{i=1}^{2}h_i(k)\mathbf{G}_i\mathbf{w}(k) \tag{38}$$

where $\mathbf{G}_1=\begin{bmatrix}\mathbf{A}_1 & \mathbf{B}_1\\ \mathbf{E}_1 & \mathbf{D}_1\end{bmatrix}$, $\mathbf{G}_2=\begin{bmatrix}\mathbf{A}_2 & \mathbf{B}_2\\ \mathbf{E}_2 & \mathbf{D}_2\end{bmatrix}$

To obtain the control gain matrices $\mathbf{D}_1,\mathbf{D}_2,\mathbf{E}_1,\mathbf{E}_2$ guaranteeing the stability of the closed loop system (38), we solve the LMI feasibility problem equivalent to DFC design problem as follows.

The problem of finding $\mathbf{X}>\mathbf{0}$ and $\mathbf{M}_1,\mathbf{M}_2$ which satisfy the following inequalities :

$$\begin{bmatrix}\mathbf{X} & \{\overline{\mathbf{A}}_i\mathbf{X}-\overline{\mathbf{B}}\,\mathbf{M}_i\}^T\\ \overline{\mathbf{A}}_i\mathbf{X}-\overline{\mathbf{B}}\,\mathbf{M}_i & \mathbf{X}\end{bmatrix}>\mathbf{0}$$

where $\overline{\mathbf{A}}_i=\begin{bmatrix}\mathbf{A}_i & \mathbf{B}_i\\ \mathbf{0} & \mathbf{0}\end{bmatrix}$ and $\overline{\mathbf{B}}=\begin{bmatrix}\mathbf{0}\\ \mathbf{I}\end{bmatrix}$, $i=1,2$

The matrices X and $\mathbf{M}_1,\mathbf{M}_2$ in LMI's are determined using a convex optimization technique offered by [14].

$$\mathbf{X} = \begin{bmatrix} 157.0056 & 61.9680 & -1.6565 & 220.727 \\ 61.9680 & 50.4822 & 69.8423 & 53.4329 \\ -1.6565 & 69.8423 & 489.4416 & -2.3866 \\ 220.7727 & 53.4329 & -2.3866 & 442.6866 \end{bmatrix},$$

$$\mathbf{M}_1 = [-96.3672 \quad -43.1521 \quad 41.8056 \quad -5.8356],$$

$$\mathbf{M}_2 = [-116.3143 \quad -66.0021 \quad 1.3065 \quad -22.9842]$$

The feedback gains and a common positive definite matrix, Pare determined by the relationship (18) as follows.

$$\mathbf{P} = \mathbf{X}^{-1}$$

$$= \begin{bmatrix} 0.0995 & -0.1036 & 0.0149 & -0.0370 \\ -0.1036 & 0.1373 & -0.0198 & 0.0350 \\ 0.0149 & -0.0198 & 0.0049 & -0.0050 \\ -0.0370 & 0.0350 & -0.0050 & 0.0165 \end{bmatrix}, \quad (39)$$

$$\overline{\mathbf{F}}_1 = \mathbf{M}_1\mathbf{X}^{-1}$$

$$= -[\mathbf{E}_1 \quad \mathbf{D}_1] = [-3.9047 \quad 2.6765 \quad -0.3020 \quad 1.5869],$$

$$\overline{\mathbf{F}}_2 = \mathbf{M}_2\mathbf{X}^{-1}$$

$$= -[\mathbf{E}_2 \quad \mathbf{D}_2] = [-3.8624 \quad 2.1564 \quad -0.3102 \quad 1.6123]$$

Therefore, the closed loop system is asymptotically stable in the large and the control gain matrices are given as follows by PDC design problem equivalent to DFC design problem.

$$\mathbf{D}_1 = -1.5869, \ \mathbf{D}_2 = -1.6123,$$

$$\mathbf{E}_1 = [3.9047 \quad -2.6765 \quad 0.3020],$$

$$\mathbf{E}_2 = [3.8624 \quad -2.1564 \quad 0.3102]$$

Next, we analyze the stability of the fuzzy control system with the consideration of quantization. The quantization problem is unavoidable because digital sensors such as vision sensors and A/D converting processing from potentiometers in order to control the truck-trailer system.

There exists a common positive definite matrix P (39) for the closed loop system (38) and $\mathbf{r}(k) = \mathbf{0}$ since it is a regulation problem. Hence, all the sufficient conditions of Theorem 3are satisfied. Therefore, we can say that the closed loop fuzzy system is uniformly ultimately bounded and does not diverge.

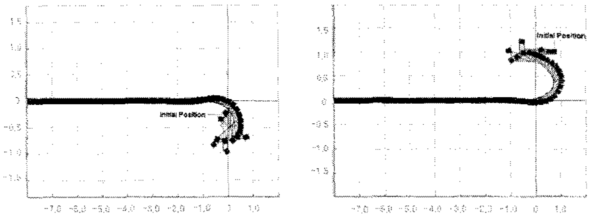

그림 8. 제안된 제어기의 실험 결과
Fig. 8. Experimental results by proposed control.

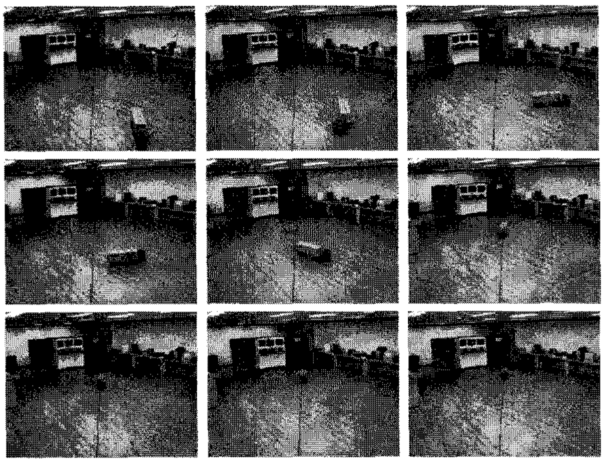

그림 9. 개발된 퍼지 제어기의 실험결과 사진
Fig. 9. The photograph of experimental results by proposed fuzzy control.

Figure 8 shows the experimental results of the designed DFC with the computing time-delay($\tau = 1$ [sec]) and quantization effect with the precision, $\varepsilon = 10^{-2}$ and the photograph presented in Fig. 9 shows the some selective experimental results for CASE I. As can be seen in the figures, the backward parking is accomplished successfully compared with Fig. 6 although a considerable computing time-delay exists. However, due to the quantization effects, the solution of the present feedback control system seems to have oscillation with small amplitude. Thus, we can say that the closed loop system converges to some small neighborhood of origin.

VI. Conclusions

In this paper, we have developed a backward movement control of a truck-trailer type vehicle using fuzzy model based control. The practical constraints as computing time-delay due to the control processing time and the quantization from the digital implementation of the control architecture were considered and the control design which guarantees the stability under the existence of them is proposed. The stability of the system was guaranteed in the existence of the computing time-delay and the real-time control processing could be possible. Furthermore, we have proved that quantization has the effect of replacing convergence of solutions to the origin by convergence to some small neighborhood of the origin. Experimental results have shown that the designed fuzzy controller effectively achieves the backward movement control of a truck-trailer type vehicle although considerable computing time-delay exists and it was also shown that in the presence of quantization, the control system was uniformly ultimately bounded.

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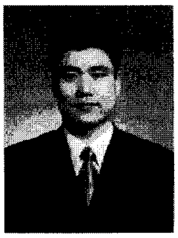

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