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# Component Outsourcing Contracts in a Two-Component Assembly System

**Eungab Kim**†

Associate Professor, College of Business Administration, Ewha Womans University

# 두 가지 부품으로 구성된 조립시스템에서 부품 아웃소싱 계약에 대한 고찰

**김은갑**

이화여자대학교 경영대학

This paper considers a two-component assembly system that makes different types of purchasing contracts by component type and studies the issue of coordinating those contracts. Acquisition of type 1 component is based on the long-term contract. In contrast, type 2 component is intermittently purchased under the sort-term contract. We identify the structural properties of the optimal short-term contract and investigate how the changes in system parameters affect the optimal performance. To provide managerial insights, we compare the short-term and long-term contracts for type 2 component and discuss the conditions that make the short-term contract preferable to the long-term contract. We also present a result which shows that coordinating the contracts of type 1 and type 2 components can be significantly profitable over uncoordinating them.

*Keyword:* supply contract, coordination of contracts, purchasing, multi-component assembly

## 1. Introduction

This paper considers a firm that produces a single product by assembling two types of components (referred to as type 1 component and type 2 component) and makes different types of purchasing contracts by component type. Acquisition of type 1 component from the market is very competitive while type 2 component can be easily acquired from many suppliers in the market. The firm wants to receive type 1 component from supplier 1 based on the long-term contract which is appealing to her/him because it results in high guaranteed capacity utilization. In contrast, there are many suppliers in the market that can provide type 2 component and the firm prefers to intermittently purchasing it under the sort-term contract, depending on available stock level of type 2 component as well as market demand on the assembled products.

From the model presented in this paper, we raise the following strategic issues : (1) when should the firm purchase type 2 component to maximize its profit? This issue is closely related to effectively managing the firm's assembly capacity. (2) Under what conditions, the short-term contract is preferable to the long-term contract for type 2 component? This issue is important because the short-term contract is characterized by typically higher purchasing cost, including supplier searching cost, than the long-term

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†Corresponding author : PAssociate Professor, College of Business Administration, Ewha Womans University, 11-1 Daehyun-Dong, Seodaemun-Gu, Seoul, 120-750 Korea, Fax : +82-2-3277-2835, E-mail : evanston@ewha.ac.kr
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contract. (3) How do the changes and the randomness in arrival and assembly processes affect the firm's purchasing contracts and its profit? (4) What are the benefits of coordinating the purchasing contracts of type 1 and type 2 components over uncoordinating them?

The main contribution of this paper to the literature with multi-component assemble systems is to present an appropriate model that considers different types of purchasing contracts by component type according as their acquisition in the market is competitive or non-competitive and to investigate the structural properties and benefits of the coordinated purchasing contracts. Even though there is a rich literature on inventory management in multi-component assembly systems, it has mainly focused on determining order timing and lot sizing of components with EOQ-type models.

Kumar (1989) investigated a component purchasing policy in a multi-component assembly system when lot size of each component is one and its lead times are random. Yano (1987) determined when to procure components and when to begin the assembly process in a two-component assembly system that both component lead times and assembly times are random. Employing an ordering policy with a common order quantity for all the component and different reorder points for each component, Fujiwara and Sedarage (1997) studied the issue of finding these decision variables which minimize the system costs.

The issue of random yields in component procurement and assembly processes is covered in Yao (1988), Gerchak *et al*. (1994), and Gurnani *et al*. (1996) (2000). Yao (1988) addressed the problem of finding the run quantities of components in an assembly system consisting of multiple assembly lines each of which produces a certain type of component with yields loss. Gerchak *et al*. (1994) investigated the problem of determining components' lot size and assembly lot size when the yield at the component and possibly at the assembly stage is random.

Gurnani *et al*. (1996) considered an assembly system with two components that can be ordered from individual suppliers or in a set from the joint supplier, and investigated the optimal component order policy and derived conditions under which diversification is optimal. Gurnani *et al*. (2000) addressed a two-component assembly system that the firm agrees to accept a random fraction of the order quantity from the suppliers due to assembly yield losses. They found the target level of finished products to assemble and the order quantity of the components from the suppliers.

The rest of the paper is organized as follows. Section 2 presents model assumptions and problem formulation. In Section 3, we examine the effects of system parameters on the optimal average profit. Section 4 characterizes an optimal short-term contract. In Section 5, we compare long-term contract and short-term contract for type 2 component. Section 6 presents a sensitivity analysis and the last section contains conclusions.

## 2. Problem formulation

Type 1 component is supplied from supplier 1 under a general random process with mean lead time $\lambda_1^{-1}$ which is contractually specified. Supply contracts between companies have been modeled using a probability distribution in the literature. For example, in Carr and Duenyas (2000), a long-term contract between an OEM company and a subcontractor is modeled using a Poisson process with contractually specified mean rate. Type 2 component is intermittently purchased depending on stock level of type 2 component as well as market demand of the assembled products. If the firm decides to purchase type 2 component, one of the suppliers is selected and type 2 component is delivered after a random amount of time $L$ with mean $\lambda_2^{-1}$. Since the acquisition of type 1 component from the market is very competitive, the benefit that the long-term contract exercises can be limited. That is, it can happen that the assembly process stops due to stockout of type 1 component even though type 2 component is available.

Without loss of generality, purchasing batch size of each type component is equal to one and each assembled product requires one unit of each type component. The results obtained in this paper can be extended to the case that purchasing batch sizes are larger than one and the assembly requires different units of each type component. Assembly time $A$ is a random variable with mean $\mu^{-1}$. In many applications with multi-component assembly operation, final assembly time is significantly shorter than component procurement/production times and, even further, most of the literature on assembly systems assumes instantaneous assembly (see Benjaafar and Elhafsi (2006)).

Let $n_1$ and $n_2$ respectively represent the inventory level of type 1 and type 2 components, respectively. Denote a state by the vector $(n_1, n_2)$. Each assembled product generates a revenue of $R_A$ (This assumption is relevant in a situation that the firm provides the assembled products for its OEM company). Inventory holding costs are incurred at the rate of $h_1$ ($h_2$) per unit time for each unit of type 1 (type 2) component in inventory. Whenever a contract for type 2 component is made, a lump-sum cost of $P_2$ occurs.

A control policy, $\pi$, specifies when the firm should enter into a contract with supplier 2. The long run average profit $g^\pi$ over an infinite horizon under $\pi$ can be written as

$$g^\pi = \lim_{T\to\infty} \frac{1}{T} E\ [P(n_1,\ n_2,\ T,\ \pi)] \tag{1}$$

where $P(n_1, n_2, T, \pi)$ is the accumulated profit obtained up to time $T$ given the initial state $(n_1, n_2)$ using a policy $\pi$. Denote by $\pi^*$ the optimal short-term contract which maximizes $g^\pi$ and by $g$ the optimal average profit under $\pi^*$.

## 3. Effects of system parameters on the optimal average profit

The next two theorems present the sensitivity analysis on $g$ with respect to assembly time and purchase lead time, respectively. A random variable $X$ is said to be stochastically larger than a random variable $Y$, denoted by $X \geq_{st} Y$, if $\Pr(X > t) \geq \Pr(Y > t)$ for all $t$ and, if $X \geq_{st} Y$, $E(X) \geq E(Y)$ (Ch9 of Ross [14]).

**Theorem 1 :** *If the assembly time decreases from $A$ to $A'$ such that $A \geq_{st} A'$, the optimal average profit will be non-decreasing.*

**Proof :** The proof is based on a coupling argument. Let $\pi^*$ be an optimal policy for the original system and $F_A$ $(F'_A)$ be the cumulative distribution function of $A(A')$. Along any sample path $\omega$ of the stochastic process, assume that the inter-arrival time of type 1 component and the purchase lead time of type 2 component take the same realization in original system as in the new.

Denote the time realization of the $n^{th}$ assembly in the original and new system by $A(n, w)$ and $A'(n, w)$, respectively. Let $\{\mathrm{U(n)} : \mathrm{n} \in \mathrm{N}\}$ denote an *i.i.d.* sequence of uniform distribution random variables on [0, 1]. For $n \in N$, let $A(n, w) = F_A^{-1}(U(n, w))$ and $A'(n, w) = F_{A'}^{-1}(U(n, w))$ where $F^{-1}(y) \equiv \min \mathrm{x} \in \mathrm{R} : \mathrm{F(x)} \geq \mathrm{y}$. Then, $A(n, w)$ and $A'(n, w)$ respectively have distribution $F_A$ and $F_{A'}$ from Lemma 9.2.1 of Ross [14] and $A'(n, w) \leq A(n, w)$ with probability one from Proposition 9.2.2 of Ross [14].

To show that there exists a policy $\pi'$ for the new system which performs at least as well as $\pi^*$, we employ idling along each sample path $\omega$. Define the $n^{th}$ idle period by $I(n, w) = A(n, w) - A'(n, w)$. Suppose that right after the $n^{th}$ assembly, the new system becomes idle during $I(n, w)$. Then, the corresponding assembly is coupled to the realization of the original system, and along any sample path $\omega$, $\pi'$ can reconstruct the state of the original system under $\pi^*$ and generate equal performance. Because $\pi'$ may not necessary be optimal for the new system, the result follows. □

**Theorem 2 :** *If the purchasing lead time of type 2 component decreases from L to L' such that $L \geq_{st} L'$, the optimal average profit will be non-decreasing.*

**Proof :** As in Theorem 1, we employ a coupling argument and present a sketch of the proof. Denote the time realization of the time epoch of $n^{th}$ purchase order in the original and new system by $L(n, w)$ and $L'(n, w)$, respectively. Then, $L'(n, w) \leq L(n, w)$. To couple the behavior of the original system in the new system, we employ delaying of purchase order along each sample path $\omega$. Let $\tau_n$ be the time epoch that the $n^{th}$ purchase order is placed under $\pi^*$ in the original system. If we consider $\pi'$ for the new system which places a purchase order at $\tau_n + L(n, w) - L'(n, w)$, then both systems receive the $n^{th}$ order at $\tau_n + L(n, w)$. Because $\pi'$ may not necessarily be optimal for the new system, the result follows. □

## 4. Structure of the optimal short-term contract for type 2 component

It is not tractable to characterize the optimal purchasing policy, $\pi^*$, under general probability distribution. To provide some insights into the nature of $\pi^*$, we address this issue in the context of Poisson component arrival and exponential assembly processes. Despite restricted assumptions in practice, this assumption is found in most of the literature that examines the optimal structure of scheduling and inventory control policy (Ha, 1997, Carr and Duenyas, 2000, Benjaafar and Elhafsi (2006)), since the insights gained are useful in addressing real systems for which exponential distributions are inappropriate.

Let $v(n_1, n_2)$ be the optimal value function over an infinite horizon when the initial state is given by $(n_1, n_2)$. Denote $A(n_1, n_2) = (n_1 - 1, n_2 - 1)$ if $n_1 > 0$ and $n_2 > 0$; $(n_1, n_2)$ otherwise. Operator $A$ corresponds to an assembly completion. Define the value iteration operator $T$ on $v(n_1, n_2)$ by

$$\begin{aligned} Tv(n_1, n_2) = & \frac{1}{\gamma}[-(h_1 n_1 + h_2 n_2) \\ & + \mu\{R_A 1(n_1 > 0, n_2 > 0) + v(A(n_1, n_2)\} \\ & + \lambda_1\{v(n_1+1, n_2) - P_1\} \\ & + \lambda_2 \max\{v(n_1, n_2+1) - P_2, v(n_1, n_2)\}] \end{aligned} \tag{2}$$

where $\gamma = \lambda_1 + \lambda_2 + \mu$, and $1(a) = 1$ if $a$ is true, otherwise, 0.

Since $\gamma$ is the sum of all transition rates, $1/\gamma$ in (2) is the expected state transition time. The terms multiplied by $\mu$ represent the assembly revenue and transition generated with assembly completion, the term multiplied by $\lambda_1$ imply transitions associated with an arrival of type 1 component, the terms multiplied by $\lambda_2$ imply transitions associated with an arrival of type 2 component. Then, the optimality equation of the average profit MDP can be rewritten as (see Equation (6.2.8) in Puterman (2005))

$$g + v(n_1, n_2) = Tv(n_1, n_2) \tag{3}$$

It is known that the optimal average profit $g$ in (3) is independent of the initial state $(n_1, n_2)$ (see Puterman (2005)). Next, consider the value iteration algorithm to solve for (3):

$$v_{k+1}(n_1, n_2) = Tv_k(n_1, n_2),\ k = 0, 1, \cdots \tag{4}$$

where $v_0(n_1, n_2) = 0$ for every state $(n_1, n_2)$. Here $v_{k+1}(n_1, n_2)$ is the value function in state $(n_1, n_2)$ when the problem is terminated after $k$ iterations.

Applying the approach of Carr and Duenyas (2000), it can be shown that there exists an integer $N$, a constant $g$ and a function $v$ such that $v_{kN+t}(n_1, n_2) - (kN+t)g$ converges to $v(n_1, n_2)$ for all $t = 0, \cdots, N-1$ as $k \to \infty$, which implies that (3) has a well-defined solution, and $v_k(n_1, n_2)$ in (4) converges to $v(n_1, n_2)$ in (3).

<Figure 1> graphically illustrates the optimal short-term contract for type 2 component which is found using the example in <Table 1>. In this example, type 1 component is more expensive to hold and to purchase than type 2 component and final assembly time is significantly shorter than component procurement times. We note that if the firm makes the long-term contract for type 2 component, the purchasing cost $P_2$ can be smaller. <Figure 1> indicates that the optimal short-term contract can be characterized by a switching curve $S(n_1)$ such that

*(i) in state $(n_1, n_2)$, if $n_2 \le S(n_1)$, it is optimal to make a short-term contract to purchase type 2 component* and

*(ii) the curve $S(n_1)$ is increasing in $n_1$.*

The switching curve $S(n_1)$ is derived by the following equation:

$$S(n_1) \equiv \max\{n_2 : v(n_1, n_2+1) - P_2 \ge v(n_1, n_2)\}.$$

In other words, $S(n_1)$ is the largest integer that making the short-term contract is more profitable than not. Even though the structure of the optimal short-term contract ((i) and (ii)) is independent of system parameter values, the slope of $S(n_1)$ can be affected if some parameter value is changed. This issue is covered in Section 6.

The switching curve $S(n_1)$ separates the state space into two regions: 1) Purchase type 2 component 2) Do not purchase type 2 component. For example, in state (0, 5), the firm should not purchase type 2 component while in state (10, 5), it should purchase type 2 component. The structure of the switching curve also indicates that (i) the marginal profit of holding one type of component will be increased as inventory of other component increases and (ii) the incremental profit of holding one type of component will be decreased in its inventory. The first indication can be explained from the fact that both types of components are complementary goods in that an assembly operation requires one unit by each type of component. The second indication comes

Table 1. A case for finding an optimal short-term contract

| $\lambda_1$ | $\lambda_2$ | $\mu$ | $h_1$ | $h_2$ | $R_A$ | $P_1$ | $P_2$ |
|---|---|---|---|---|---|---|---|
| 0.2 | 0.4 | 2 | 3 | 1 | 100 | 10 | 5 |

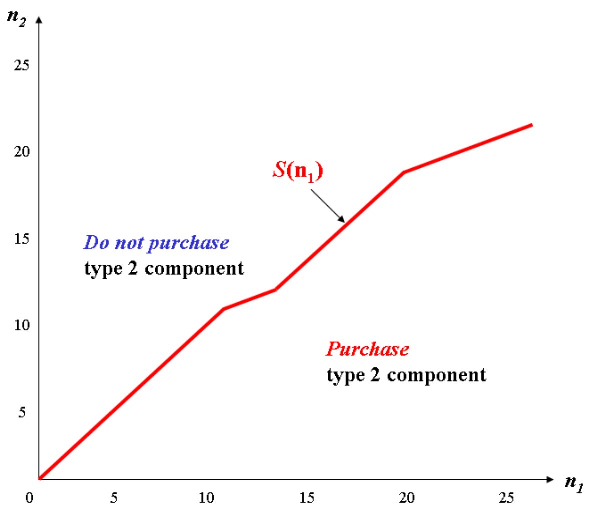


Figure 1. A graphical representation of $S(n_1)$

from the law of diminishing marginal utility.

## 5. Numerical study

In this section, we focus on the following numerical investigation. First, we consider two types of contracts that the firm can make in purchasing type 2 component: *long-term contract* and *short-term contract*, and examine the impacts of these contracts on the firm's profit. Second, we examine the benefit of the coordinated contracts for type 1 and type 2 components over the uncoordinated ones. The reference example is $R_A = 100,\ h_1 = 2,\ h_2 = 1,\ P_1 = 10,\ \mu = 1$ and $P_2 = 5$ under long-term contract and $P_2 = 10$ under short-term contract. Besides this reference example, we extensively tested more than 100 examples with varying the values of system parameters and observed that findings and insights derived by those examples are very similar to the ones in this section. Test scenarios are summarized in <Table 1>.

### 5.1 Comparison of long-term and short-term contracts for type 2 component

Under the long-term contract, type 2 component is continuously supplied from a supplier with contractually specified mean rate. This comparison is done based on the following two scenarios:

Scenario 1 : *The firm receives type 1 component on the long-term contract and type 2 component on the long-term contract*

Under this scenario, the firm does not have to decide when to purchase type 2 component. We assume that type 1 and type 2 contracts are coordinated. Two dimensional search along with value iteration algorithm shows that the coordinated optimal lead time processes for type 1 and type 2 components become $\lambda_1^* = 0.85$ and $\lambda_2^* = 0.85$, respectively. In other words, the firm can expect the maximum profit when it enters into the long-term contracts with supplier 1 and supplier 2 which specify that $\lambda_1^* = 0.85$ and $\lambda_2^* = 0.85$. With these contracts, the optimal average profit is given by 25.99.

Scenario 2 : *The firm receives type 1 component on the long-term contract and type 2 component on the short-term contract*

Under this scenario, the firm must decide when to purchase type 2 component. The optimal average profit under the short-term contract is derived with the same example as the long-term contract except for $P_2$. For this short-term contract, we consider four types of purchasing price $P_2$ : 10, 15, 20, and 25. Compared to the long-term contract, the purchasing price is set higher because the short-term contract can be typically characterized by a higher purchasing setup cost than the long-term contract. To compare Scenario 1 and Scenario 2, we take the following steps for each $P_2$.

- Step 1 : The long-term contract with supplier 1 is set to $\lambda_1 = 0.85$ and then the short-term with supplier 2 contract is set to $\lambda_2 \in [0.85,\ 1.75]$.
- Step 2 : Given $\lambda_1 = 0.85$ and $\lambda_2$, we compute the optimal average profit using value iteration algorithm.
- Step 3 : Find $\lambda_2$ that gives the maximum of the optimal average profit.

<Figure 1> illustrates the performance comparison between long-term and short-term contracts as a function of $\lambda_2$. The results in <Figure 1> suggests

several strategical insights for the management.

- As purchasing price of the short-term contract becomes much larger than that of the long-term contract, the short-term contract should have much faster lead time process than that of the long-term contract. In fact, when $\lambda_1 = \lambda_2 = 0.85$, Scenario 2 performs much worse than Scenario 1.
- If purchasing price is not too high compared to the long-term contract, the firm can expect the huge profit gains even with a short-term contract whose lead time process is slightly faster than the long-term contract. For example, if $\lambda_2 = 0.95$ when $P_2 = 10$, the optimal average profit increases 49%, compared to Scenario 1.
- Even though the optimal average profit increases in $\lambda_2$, the marginal improvement is not large if $\lambda_2$ is beyond some point. This observation indicates that the firm will face a trade-off between the value of $\lambda_2$ and the costs due to purchase setup and other contract-related activities, since a short-term contract with fast lead time process can be typically much more expensive than that of slow lead time process.

### 5.2 Comparison of coordinated and uncoordinated contracts

In this subsection, we present an example which shows that coordinating the contracts of type 1 and 2 components generates a larger profit than uncoordinating them. This comparison is done based on the following two scenarios, assuming that the firm makes a short-term contract for type 2 component:

Scenario 3 : C*ontracts for type 1 and type 2 components are not coordinated*

We assume that the long-term contract for type 1 component is performed first and the short-term contract for type 2 component is subsequently considered. In this case, the firm faces with a problem of deciding the optimal level of $\lambda_1$ without considering the existence of type 2 component. From one dimensional search, the optimal level of $\lambda_1$, $\lambda_1^*$, is found 1.0. After $\lambda_1^* = 1.0$ is chosen, the short-term contract with $\lambda_2 \in [0.85,\ 2.25]$ is implemented and we have $\lambda_2^* = 1.9$ and $g = 27.27$.

Scenario 4 : C*ontracts for type 1 and type 2 components are coordinated*

In Scenario 4, $\lambda_1$ and $\lambda_2$ are simultaneously determined. From a two-dimensional search, $\lambda_1^* = 0.8$ and $\lambda_2^* = 2.25$ are found and the optimal average profit is given by 52.81.

<Figure 2> shows the performance comparison between coordinated and uncoordinated contracts as a function of $\lambda_2$. The results in <Figure 2> shows the performance degradation of uncoordinated decision. Even when Scenario 3 is optimally implemented ($\lambda_1^* = 1.0$ and $\lambda_2^* = 2.25$), the uncoordinated behav-

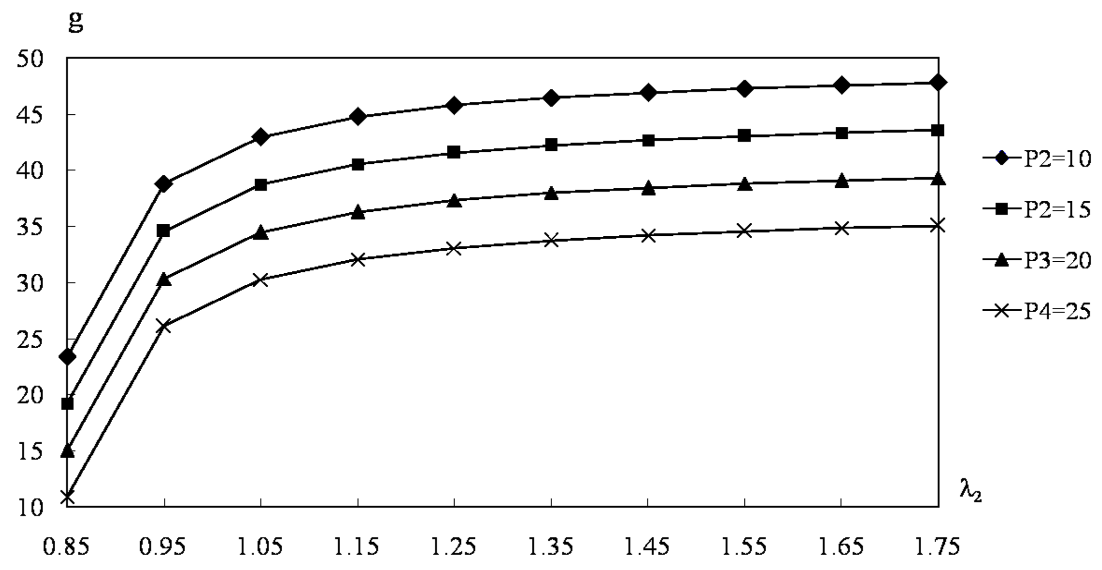


**Figure 2.** Comparison of the optimal profit between long-term and short-term contracts as a function of $\lambda_2$

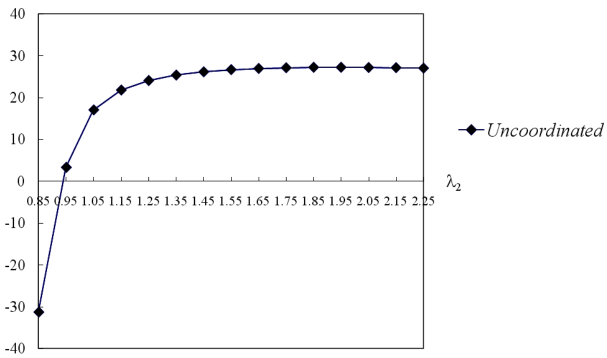


**Figure 3.** Comparison of the optimal profit between coordinated and uncoordinated contracts as a function of $\lambda_2$

ior would result in a 49% decrease in the firm's profits. We tested more than 100 cases and, without any exception, observed that the coordinated contract outperforms the uncoordinated contract. This result is meaningful because in most of the literature on inventory management in multi-component assembly systems, order timing and lot sizing is decided by component type, that is, it is not coordinated.

## 6. Sensitivity analysis on the optimal short-term contract and optimal profit

In this section, we numerically implement a sensitivity analysis of the optimal short-term contract identified in Section 4. Besides the reference example in <Table 3>, we extensively tested cases with varying the values of system parameters. It is observed that test results with those examples are very similar to the ones obtained using the reference example. For this reason, we only report the observations and insights that are derived by the reference example.

Based on the computational results, we observe the following monotonic behavior of the optimal short-term contract, $S(n_1)$, and the optimal average profit, $g$:

**Table 3.** Reference example for the sensitivity analysis

| $\lambda_1$ | $\lambda_2$ | $\mu$ | $h_1$ | $h_2$ | $R_A$ | $P_1$ | $P_2$ |
|---|---|---|---|---|---|---|---|
| 0.3 | 0.8 | 1 | 2 | 1 | 100 | 15 | 5 |

- *$S(n_1)$ is decreasing and $g$ is increasing as $\lambda_2$ increases.*

If the lead time process becomes faster, it is reasonable to expect that the firm will delay a purchasing decision for type 2 component because the inventory level between type 1 and type 2 components should be balanced. In addition, the delay of purchasing type 2 component will reduce the number of purchasing type 2 component per unit time. Hence, the average profit per unit time will be increased,

- *$S(n_1)$ and $g$ are increasing as $\mu$ increases.*

The increase in the assembly rate will reduce inventory of type 2 component more rapidly and thus the firm has a good motivation to purchase more type 2 component. Therefore, the system states where the optimal policy purchases type 2 component will be enlarged. Since the increase in the assembly rate will also enhance sales revenue rate ($\mu R_A$) and reduce the inventory cost of type 1 component, the firm can expect the increased average profit despite the increased purchasing cost of type 2 component.

- *$S(n_1)$ is non-decreasing as $\lambda_1$ increases (see <Figure 4>).*

This phenomenon states that if the arrival rate of type 1 component increases, the firm will make the short-term purchasing contract of type 2 component in more enhanced states. This marginal behavior of

**Table 2.** Summaryof test scenarios

| Scenario | Contract for type 2 component | Contract coordination of type 1 and 2 components | Optimal contract | Optimal profit |
|---|---|---|---|---|
| 1 | Long-term | Coordinated | $\lambda_1^* = 0.85$ $\lambda_2^* = 0.85$ | 25.99 |
| $2^a$ | Short-term | Coordinated | | |
| 3 | Short-term | Uncoordinated | $\lambda_1^* = 1.0$ $\lambda_2^* = 1.9$ | 27.27 |
| 4 | Short-term | Coordinated | $\lambda_1^* = 0.8$ $\lambda_2^* = 2.25$ | 52.81 |

$a$: Scenario 2 is implemented with $\lambda_1 = 0.85$ and $\lambda_2$ varied.

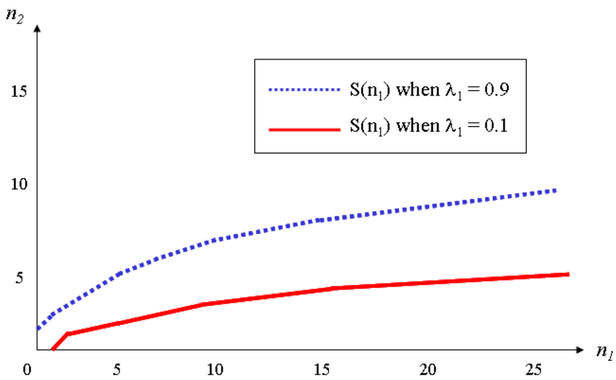


Figure 4. Effects of $\lambda_1$ on $S(n_1)$

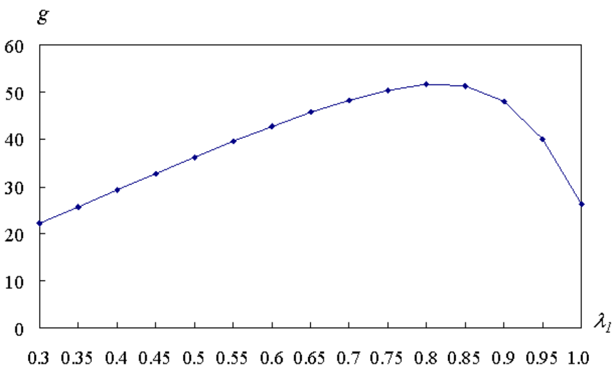


Figure 5. Effects of $\lambda_1$ on the optimal average profit $g$

$S(n_1)$ will contribute to balancing the inventory level between type 1 and type 2 components.

- It is interesting to see that the optimal average profit does not necessarily increase in the arrival rate of type 1 component (see <Figure 5>). Further, it is conjectured that the firm's profit would be concave with respect to $\lambda_1$. We can explain this phenomenon in the context of the assembly capacity utilization. When $\lambda_1/\mu$ is low, having faster lead time process of type 1 component contributes to enhancing the sales revenue because stockout of type 1 component can be avoided. In contrast, if lead time process of type 1 component becomes faster when $\lambda_1/\mu$ is high, the sales revenue rate $\mu R$ will be hardly affected by it but the inventory cost of type 1 component will be increased.

## 7. Conclusion

While most of multi-component assembly models in the literature deal with the issue of purchasing order timing and lot sizing by component type, this paper considered a two-component assembly system in which purchasing contracts of each component can be different according to the availability of their acquisition in the market, and focused on the issues of what are the impacts of the short-term contract on the firm's profit compared to the long-term contract and what are the benefits of coordinating the contracts of both types of components over uncoordinating them.

Assuming that a firm receives type 1 component based on long-term contract and type 2 component based on short-term contract, we analytically showed that the optimal profit has the monotonic behavior with respect to some system parameters using stochastic coupling arguments. Since it is not tractable to characterize the optimal short-term contract of type 2 component under general probability distribution, we numerically identified its structure under the Markov model that can be characterized as a monotonic threshold function.

Numerical study shows that the short-term contract can be more effective than the long-term contract if the purchasing cost of type 2 component is not too high. It also shows that when it is very expensive, the firm should make a short-term contract with much faster lead time process than the long-term contract. We presented an example which shows that even when optimally implemented, the uncoordinated contracts would result in a 49% decrease in the firm's profits compared to the coordinated contracts.

We also implemented a sensitivity analysis on the optimal short-term contract and the optimal profit with respect to lead time and assembly processes and observed many meaningful monotonic properties that can be very useful in developing effective heuristic policies that can work in more realistic problems.

## References


Benjaafar, S. and Elhafsi, M. (2006), Production and inventory control of a single product assemble-to-order system with multiple customer classes, *Management Science*, **52**, 1896-1912.

Carr, S. and Duenyas, I. (2000), Optimal admission control and sequencing in a Make-to-stock and Make-to-order production system, *Operations Research*, **48**, 709-720.

Fujiwara, O. and Sedarage, D. (1997), An optimal (Q, r) policy for a multicomponent assembly system under stochastic component procurement lead times, *European Journal of Operational Research*, **100**, 550-556.

Gerchak, Y., Yunzeng W., and Yano, C. (1994), Lot sizing in assembly systems with random component yields, *IIE Transactions*, **26**, 19-24.

Gurnani, H., Akella, R., and Lehoczky, J. (1996), Optimal order policies in assembly systems with random demand and supply delivery, *IIE Transactions*, **28**, 865-878.

Gurnani, H., Akella, R., and Lehoczky, J. (2000), Supply managment in assembly systems with random yield and random demand, *IIE Transactions*, **32**, 701-714.

Ha, A. Y. (1997), Optimal dynamic scheduling policy for a make-to-stock assembly system, *Operations Research*, **45**, 42-53.

Kumar, A. (1989), Component inventory costs in an assembly problem with uncertain supplier lead-times, *IIE Transactions*, **21**, 112-121.

Puterman, M. (2005), *Markov Decision processes*, John Wiely and Sons.

Ross, S. (1996), *Stochastic processes*, $2^{th}$ Edition, Wiley, New York.

Yano, C. A. (1987), Stochastic leadtimes in two-level assembly systems, *IIE Transactions*, **19**, 371-378.

Yao, D. D. (1988), Optimal run quantities for an assembly system with random yields, *IIE Transactions*, **20,** 399-403.

Eungab Kim

He received B. S. and M. S in Department of Industrial Engineering in Seoul National University in 1987 and 1989, respectively, and Ph.D. in Department of Industrial Engineering and Management Sciences in Northwestern University in 1996.

During 1996～1997, he was a Postdoctoral Fellow in Rotman School of Management in University of Toronto. During 1998-2003, he was a principal consultant in Samsung SDS. Now he is an associate professor in College of Business Administration, Ewha Womans University.

His research interests are supply chain management, stochastic control, and optimization of queueing networks