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Model Averaging Methods for Estimating Implied and Local Volatility Surfaces

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Abstract. In this paper, we review widely used methods to extract local volatility surfaces (LVSs) from implied volatility surfaces (IVSs) and suggest a model averaging method for constructing implied and local volatility surfaces weighted by trading volumes. It makes use of model averaging method by means of bandwidth priors, and then produces a robust LVS estimation. The method is shown to provide the information about the confidence interval of estimators as well as a rather less variable weighted mean value for the IVS and LVS. To show the merits of our proposed method, we conduct simulations on equity-linked warrants (ELWs) with reasonable and acceptable results.

Keywords: ELW, Local Volatility, Nonparametric Smoothing, Model Averaging

1. INTRODUCTION

In pricing options (or warrants), the Black-Scholes model has been widely used since the seminal paper by Black and Sholes (1973) has appeared. It has a closed form and the parameters in the model are unequivocally observable, except the volatility. The volatility is a crucial parameter in option pricing. When all other parameters have equal values, an option's theoretical value is a monotonic increasing function of volatility. Given an option price on the market, an implied volatility is the unique number that satisfies the B-S formula to match the market value and under the Black-Sholes model, the implied volatility does not depend on its strikes and time to maturities. However, after 1987 market crash, it has been observed that option markets behave increasingly self-governed by its own supply and demand and plain vanilla option price movements are no longer attributed to movements in the underlying. Now the implied volatility surface for traded options with different strikes and time to maturities, shows the smile effect and consistency with the market is achieved by using a volatility function instead of a constant. The local volatility model, proposed by Dupire (1994) and Derman and Kani (1994) is one of the most widely used smile-consistent models with which a large number of the positions in portfolios of exotic options are delta-hedged.

When market-provided implied volatility data are given, one of the most popular way to estimate local volatility models is first to employ available smoothing methods to estimate the implied volatility surface and then to apply the Dupire formula to estimate the resulting local volatility model as in Fengler (2005). However, the constructed implied volatility surface (correspondingly, the local volatility surface) are highly variable to the employed smoothing techniques.

In this paper, we suggest a way to use a nonparametric smoothing method to estimate an implied volatility

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surface and then to construct a less-variable local volatility via model averaging strategy with a pre-set model prior. The resulting surface is provided with the confidence interval information that might be useful in analyzing a model risk.

The organization of this paper is as follows. In Section 2, we review the local volatility model. Then we describe the whole methodology employed in this paper. In Section 4, we describe the ELW data and show the experimental results in Section 5. Section 6 concludes the paper.

2. REVIEW OF LOCAL VOLATILITY

As special cases of restricted-stochastic-volatility models, local volatility models proposed by Dupire (1994) and Derman and Kani (1994) are very widely-used smile-consistent models. They are developed to price and hedge more complex derivative products by designing an asset price dynamics that are consistent with the volatility information observed from the plain vanilla option prices (correspondingly the IVS).

There are two widely used methods for constructing the local volatility models: the Derman-Kani (DK) tree case for discrete time modeling and the Dupire formula case for continuous-time modeling. The DK tree is formulated in a binomial model with discrete time and stock-price steps. It provides tree-based algorithms to extract the local volatility function from today' s quoted prices of a series of plain-vanilla options of different strikes and maturities. The DK construction is very simple to implement and automatically provides a pricing engine. However, it requires a very 'delicate' numerical implementation due to the difficulty of separating numerical noise from financially meaningful signals as noted in Rebonato (2004). The obtained local volatility data can be used to construct a smooth local volatility surface by applying some interpolation techniques as in Kim *et al*. (2006). However, its direct use requires a numerical scheme such as finite difference methods to compute option values corresponding to a local volatility input or rather inexact option values utilizing an ad-hoc practical Black-Sholes formula with local volatility input instead of implied volatility input. Moreover, it is often highly sensitive to the variation of local volatility input data and generates rather implausible shapes for the local volatility surface (Coleman *et al.*, 1999).

In contrast, a method based on the Dupire's formula uses as input a smooth implied volatility surface via a nonparametric method. Then it calculates the local volatility using the Dupire's formula. Although this approach does not automatically provide a pricing engine, the information about the local volatility enables us to build a local volatility model where a numerical scheme can be used to develop an option pricing algorithm.

We next review a Dupire's formula to construct a local volatility surface from an implied volatility surface. Under the equivalent martingale measure asset prices follow the SDE:

$$\frac{dS_t}{S_t} = (r-\delta)dt + \sigma(S_t, t, \cdot)d\bar{W}_t^{(0)}, \tag{1}$$

where $\bar{W}_t^{(0)}$ denotes the Brownian motion, which drives the asset price, under the risk neutral measure Q, and S_t does the asset price at time t. The interest rate and the continuously compounded dividend yield are denoted by r and δ, respectively. Then the local volatility function can be completely represented in terms of observed call prices (C_t^{BS}) and their derivatives.

$$\sigma_{K,T}^2(S_t, t) = 2\frac{\dfrac{\partial C_t^{BS}}{\partial T} + \delta C_t^{BS} + K(r-\delta)\dfrac{\partial C_t^{BS}}{\partial K}}{K^2\dfrac{\partial^2 C_t^{BS}}{\partial K^2}} \tag{2}$$

where $C_t^{BS} = C_t^{BS}(K, T)$ and K and T mean the strike price and the maturity date of C_t^{BS}, respectively. This is the so-called *Dupire formula*.

We can recover the LVS, which in principle is unobservable, from the easily observable IVS by inserting the BS formula and its derivatives into the Dupire formula as follows:

Theorem 1. (Dupire formula from IV) *Assume that* $C(S, t, K, T, \sigma, r, \delta) = C^{BS}(S_t, t, K, T, \Sigma(K, T), r, \delta)$ *and that local volatility is a deterministic function. Then the LVS is completely represented in terms of IVS:*

$$\sigma_{K,T}^2(S_t, t) = \frac{\dfrac{\Sigma}{\tau} + 2\dfrac{\partial\Sigma}{\partial T} + 2K(r-\delta)\dfrac{\partial\Sigma}{\partial K}}{K^2\{\dfrac{1}{K^2\Sigma_\tau} + 2\dfrac{d_1}{K\Sigma\sqrt{\tau}}\dfrac{\partial\Sigma}{\partial K} + \dfrac{d_1 d_2}{\Sigma}(\dfrac{\partial\Sigma}{\partial K})^2 + \dfrac{\partial^2\Sigma}{\partial K^2}\}}. \tag{3}$$

Proof. There are many sources for the proof of this theorem. Here for the completeness of the presentation, we will provide a rather simplified version of the proof (up to our knowledge). Applying the chain rule of differentiation and inserting the analytical expressions for the BS (Black-Scholes) formula and its K-and T-derivatives, we obtain for the numerator of the Dupire formula:

$$\begin{aligned} &2\{\frac{\partial C_t^{BS}}{\partial T} + \frac{\partial C_t^{BS}}{\partial \Sigma}\frac{\partial \Sigma}{\partial T} + \delta C_t^{BS} + (r-\delta)K(\frac{\partial C_t^{BS}}{\partial K} + \frac{\partial C_t^{BS}}{\partial \Sigma}\frac{\partial \Sigma}{\partial K})\} \\ &= 2\frac{\partial C_t^{BS}}{\partial \Sigma}\{\frac{\partial \Sigma}{2\tau} + \frac{\partial \Sigma}{\partial T} + (r-\delta)K\frac{\partial \Sigma}{\partial K}\}, \end{aligned} \tag{4}$$

where $\tau = T - t$. Note that we can simplify the computations by observing that the constant volatility of the Black-Sholes model is equal to its local volatility, so the Dupire formula becomes

$$\frac{\partial C_t^{BS}}{\partial T} + \delta C_t^{BS} + (r-\delta)K\frac{\partial C_t^{BS}}{\partial K} = \frac{1}{2}K^2\Sigma^2\frac{\partial^2 C_t^{BS}}{\partial K^2}. \tag{5}$$

Using $\frac{\partial^2 C_t^{BS}}{\partial K^2} = \frac{\partial C_t^{BS}}{\partial \Sigma} / (K^2 \Sigma \tau)$ and differentiating the denominator yield:

$$\begin{aligned} & K^2 \{ \frac{\partial^2 C_t^{BS}}{\partial K^2} + 2 \frac{\partial^2 C_t^{BS}}{\partial K \partial \Sigma} \frac{\partial \Sigma}{\partial K} + \frac{\partial^2 C_t^{BS}}{\partial \Sigma^2} (\frac{\partial \Sigma}{\partial K})^2 + \frac{\partial C_t^{BS}}{\partial \Sigma} \frac{\partial^2 \Sigma}{\partial K^2} \} \\ & = K^2 \frac{\partial C_t^{BS}}{\partial \Sigma} \{ \frac{1}{K^2 \Sigma \tau} + 2 \frac{d_1}{K \Sigma \sqrt{\tau}} \frac{\partial \Sigma}{\partial K} + \frac{d_1 d_2}{\Sigma} (\frac{\partial \Sigma}{\partial K})^2 + \frac{\partial^2 \Sigma}{\partial K^2} \} \end{aligned} \quad (6)$$

Finally inserting the analytical BS derivatives in terms of the BS vega into the denominator yields the result. □

3. PROPOSED METHODOLOGY

In this section, we explain our method for estimating local volatility of options with estimators' confidence interval. The method consists of four steps, which are explained below. The overall flow of the process is provided in Figure 1. After Step 1, Step 2, 3 and 4 are repeated N times. N is the number of different bandwidth values.

3.1 Data Preprocessing

We preprocess the option data before employing the raw data directly. The options with zero trading volume are removed in the experiment. We assume no dividend effects. The no-arbitrage price of the underlying index in a frictionless market without dividends is given by

$$S_t = \exp(-r_{T_F,t}(T_F - t))F_t, \quad (7)$$

where S_t and F_t denote the spot and the future price respectively, T_F the maturity date of the futures contract, and $r_{T,t}$ the interest rate with maturity T-t. We use KORIBOR rates as risk-free rates and they are linearly interpolated. With this notation, we can use simple moneyness measure

$$\kappa = \frac{K}{S_t}, \quad (8)$$

where K is the strike index of an option.

3.2 Estimating Implied Volatility Surface

To estimate the IVS, we apply nonparametric smoothing methods in particular bivariate local quadratic method. In the following section, we describe the nonparametric smoothing methods briefly.

3.2.1 Nonparametric Smoothing Methods

In nonparametric estimation, the objective is to find the regression relationship given a data set $\{(x_i, y_i)\}_{i=1}^n$

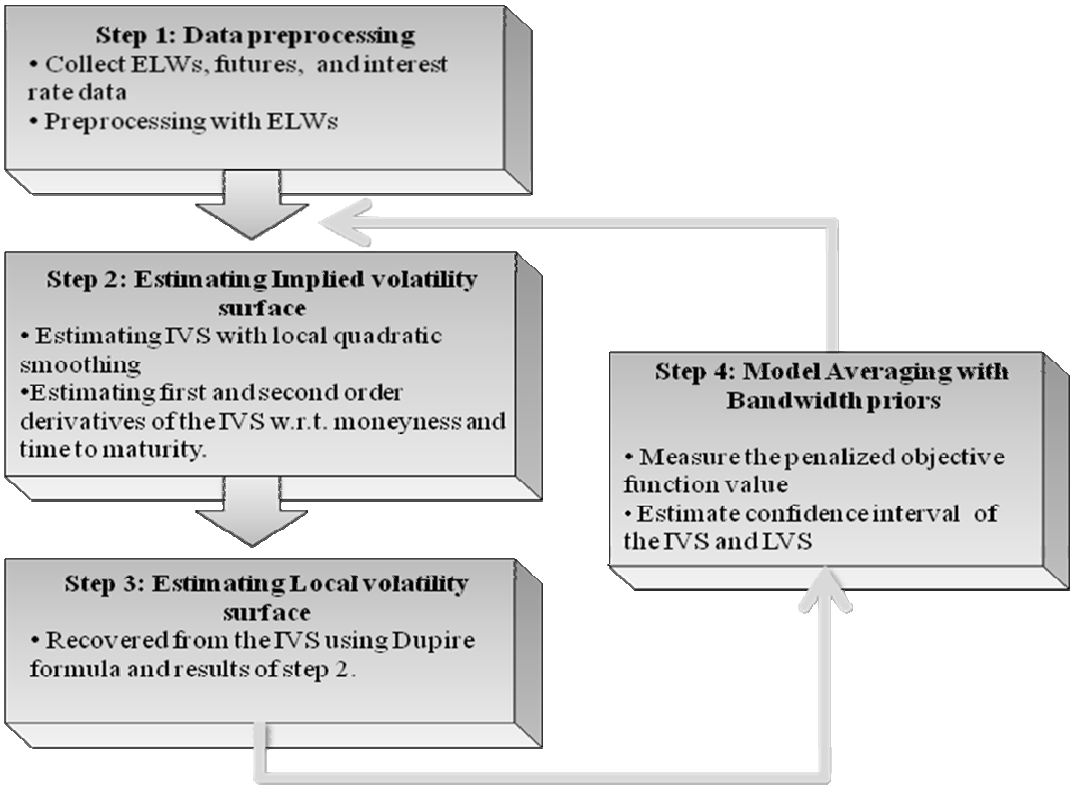

Figure 1. The overall flow of the process.

$$y_i = f(x_i) + \varepsilon_i, \qquad i = 1, \cdots, n. \tag{9}$$

In this paper, the predictor variable $x_i \in \mathbb{R}^2$ is moneyness measure and time to maturity and the response variable y_i is implied volatility (IV). Nonparametric estimation obtains an estimate $\hat{f}(x)$ by locally averaging the data. The weighting is achieved by *kernel functions* $K(\cdot)$. Common kernel functions employed in nonparametric smoothing are *quartic* kernel, the *Epanechnikov* kernel and the *Gaussian* kernel. We employed the Gaussian kernel with infinite support, which is given by:

$$K(u) = \frac{1}{\sqrt{2\pi}} e^{-u^2/2}. \tag{10}$$

In this paper, we use bivariate model, so we need two dimensional kernels. It is obtained by products of univariate kernels:

$$K(u_1, u_2) = K(u_1)K(u_2). \tag{11}$$

In nonparametric estimation, the bandwidth h is involved in the kernel functions via

$$\frac{1}{h_n} K(\frac{x - x_i}{h_n}), \qquad i = 1, \cdots, n. \tag{12}$$

Thus the degree of localization or smoothing is controlled by the bandwidth h. For simplicity, we will use the abbreviation

$$K_h(u) = \frac{1}{h} K(\frac{u}{h}). \tag{13}$$

3.2.2 Local Quadratic Smoothing

In this paper, we employed bivariate local quadratic smoothing method. The predictor variables are moneyness measure and time to maturity. Assume that the regression function is continuous up to order two. By expanding equation (9) in a bivariate Taylor series, we obtain

$$\begin{aligned} f(\xi_1, \xi_2) &\approx f(x_1, x_2) + (\xi_1 - x_1) f_x(x_1, x_2) + (\xi_2 - x_2) f_y(x_1, x_2) \\ &+ \frac{1}{2!}[(\xi_1 - x_1)^2 f_{xx}(x_1, x_2) + 2(\xi_1 - x_1)(\xi_2 - x_2) f_{xy}(x_1, x_2) \\ &+ (\xi_2 - x_2)^2 f_{yy}(x_1, x_2)], \end{aligned} \tag{14}$$

for (ξ_1, ξ_2) in the neighborhood of (x_1, x_2) where the subscripts denote the respective partial derivatives. Thus, an estimator of f(x) can be formulated in terms of the quadratic minimization problem employing kernel weights

$$\begin{aligned} &\min_{\beta \in \mathbb{R}^6} \sum_{i=1}^{n} \{ y_i - \beta_0 - \beta_1(x_1^i - x_1) - \beta_2(x_2^i - x_2) - \beta_3(x_1^i - x_1)^2 \\ &- \beta_4(x_1^i - x_1)(x_2^i - x_2) - \beta_5(x_2^i - x_2)^2 \}^2 K_h(x_i - x), \end{aligned} \tag{15}$$

where $\beta = (\beta_0, \cdots, \beta_5)^T$ denotes the vector of coefficients.

We introduce the following matrix notation:

$$X = \begin{pmatrix} 1 & x_1^1 - x_1 & x_2^1 - x_2 & (x_1^1 - x_1)^2 & (x_1^1 - x_1)(x_2^1 - x_2) & (x_2^1 - x_2)^2 \\ 1 & x_1^2 - x_1 & x_2^2 - x_2 & (x_1^2 - x_1)^2 & (x_1^1 - x_1)(x_2^1 - x_2) & (x_2^2 - x_2)^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_1^n - x_1 & x_2^n - x_2 & (x_1^n - x_1)^2 & (x_1^1 - x_1)(x_2^1 - x_2) & (x_2^n - x_2)^2 \end{pmatrix} \tag{16}$$

and $y = (y_1, \cdots, y_n)^T$, and finally

$$W = \begin{pmatrix} K_h(x - x_1) & 0 & \cdots & 0 \\ 0 & K_h(x - x_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & K_h(x - x_n) \end{pmatrix}. \tag{17}$$

Then the solution of (15) is obtained as

$$\hat{\beta}(x) = (X^T W X)^{-1} X^T W y. \tag{18}$$

From (14) and (15) the local quadratic estimator for the regression function is given by

$$\hat{f}(x) = \hat{\beta}_0(x). \tag{19}$$

An important advantage of local quadratic estimators is that the derivatives of regression function are obtained as byproduct of estimators.

$$\hat{f}_x(x) = \hat{\beta}_1(x), \cdots, \hat{f}_{xy}(x) = \hat{\beta}_4(x), \hat{f}_{yy}(x) = 2!\hat{\beta}_5(x). \tag{20}$$

3.2.3 Estimating the IVS

To estimate the LVS using Dupire formula, we need the derivatives of the IVS. Using bivariate local quadratic smoothing methods, we can achieve the derivatives of implied volatility function with respect to moneyness measure and time to maturity. Also, the bias problem visible in the sparse region is less present in local quadratic smoothing (Fengler, 2005). Thus we use bivariate local quadratic smoothing to estimate the IVS. In experiment, we multiply the kernel weights by log trading volume of each data. Thus we can rewrite the notation in section 3.2.2.

$$X = \begin{pmatrix} 1 & \kappa_1 - \kappa & \tau_1 - \tau & (\kappa_1 - \kappa)^2 & (\kappa_1 - \kappa)(\tau_1 - \tau) & (\tau_1 - \tau)^2 \\ 1 & \kappa_2 - \kappa & \tau_2 - \tau & (\kappa_2 - \kappa)^2 & (\kappa_2 - \kappa)(\tau_2 - \tau) & (\tau_2 - \tau)^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \kappa_n - \kappa & \tau_n - \tau & (\kappa_n - \kappa)^2 & (\kappa_n - \kappa)(\tau_n - \tau) & (\tau_n - \tau)^2 \end{pmatrix} \tag{21}$$

and

$$W = \begin{pmatrix} \log(V_1) K_h(x - x_1) & 0 & \cdots & 0 \\ 0 & \log(V_2) K_h(x - x_2) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \log(V_n) K_h(x - x_n) \end{pmatrix}, \tag{22}$$

where V is the trading volume of each data. Then $\hat{\beta}$ can be obtained via (18).

3.3 Estimating Local Volatility Surface

Local volatility surface is recovered from implied volatility surface by Dupire formula in *Theorem 1*. The derivatives of the IVS are estimated as derivatives of local quadratic estimators which are used to smooth the IVS.

3.4 Model Averaging with Bandwidth Priors

We applied model averaging method to estimate the IVS and LVS from option data. The estimated regression function values are varied with bandwidth h. The art in nonparametric estimation is the bandwidth choice. This involves trading off the bias and the variance of the estimate. To find the optimal bandwidth, we can employ the penalizing methods. In penalizing approaches a weighted version of the resubstitution estimate is employed

$$G(h) = \frac{1}{n}\sum_{i=1}^{n}\{y_i - \hat{f}(x_i)\}^2 \tilde{w}(x_i)\Xi(\frac{1}{n}w_{i,n}(x_i)) \tag{23}$$

where $\tilde{w}(\cdot)$ is some weight function and the correction function $\Xi(\cdot)$. We use the *Akaike information criterion* as the correction function $\Xi(\cdot)$

$$\Xi_{AIC}(u) = \exp(2u), \tag{24}$$

and

$$w_{i,n}(x) = \frac{K_h(x - x_i)}{n^{-1}\sum_{j=1}^{n} K_h(x - x_j)}. \tag{25}$$

For other asymptotically equivalent choices of $\Xi(\cdot)$, see Härdle *et al*. (2004).

The above penalized objective function values can be the measure of suitability of the model. The minimizer of (23) can be interpreted as optimal bandwidth. However, the function values increase as the bandwidth h increases. Furthermore, the estimated LVS becomes complex number with too narrow bandwidth. Thus we use the inverse of penalized objective function value as bandwidth priors. Then we can calculate the weighted mean of IVS and LVS and the confidence band of each data points. Using N different bandwidth values, we can form a 90% pointwise confidence band from the percentiles at each point: we find the N×0.05th largest and smallest values at each point. In the experiment, we used 100 different bandwidth values.

4. DATA DESCRIPTION

In this section we describe the data we've employed in the paper.

4.1 Equity-Linked Warrant (ELW)

An ELW is a kind of option that gives the holder the right but not the obligation to buy or sell an underlying asset at a set price, on or before an expiry date. The Korea Exchange (KRX) opened the ELW market on December 1, 2005. Since then, the Korean ELW market has surged to rank fourth in the world by turnover value, trailing Hong Kong, Germany and Italy. There are 2,518 ELWs Listed with an average daily turnover value more than 500 billion won in first quarter of the year 2009. The aggregate value is about 5170 billion won which increases by 10% in the last quarter.

4.2 Data Summary

For the application, we have used daily market data of KOSPI200 ELWs obtained from the Korea Investors Service (KIS). The KOSPI200 ELW market is liquid market. The number of item underlying KOSPI200 ELW occupies 20.5% of total and the volume of daily transaction is about 68% of total volume. The KOSPI200 futures contract and interest rate data in daily frequency, i.e. one, three, six and twelve month KORIBOR rates, are obtained from the Band of Korea. We use ELW data from the date 20081106.

In Table 2, we give an overview of the data. The statistics are in form of the IV data not in form of the price data. The distribution of the ELW data across moneyness appears in Figure 2. Solid line is for all observations, dashed line is for puts, and the dotted line is for calls only. The densities are obtained via a nonparametric density estimator, and bandwidths are chosen by Silverman's rule of thumb (Hardle *et al*., 2004). All densities are shifted to

Table 1. Growth of the Korean ELW market.

Year	2005	2006	2007	2008 3/4	2008 4/4	2009 1/4
Trading volume(hundreds/day)	13,665	225,177	391,423	1,033,395	810,197	1,002,121
Volume of daily transaction(billion won)	21	185	276	410	449	541
Aggregate value (billion won)	662	3,826	4,843	4,363	4,701	5,167
The number of listed stock(million)	391	5,229	6,495	12,473	10,416	10,665
The number of item	72	1,387	1,646	3,107	2,613	2,518

Table 2. Statistics of KOSPI200 index ELW data in form of the IV data.

Observation Date	Time to Expiry (days)	Min	Max	Mean	Standard Deviation	Total number of Observations	Calls
20081106	35	0.5062	1.4367	0.6683	0.1673	62	45
	63	0.4370	0.8317	0.5962	0.0920	44	30
	98	0.3507	1.3706	0.6967	0.1895	48	20
	126	0.4490	1.0079	0.6907	0.1655	28	16

the right instead of nicely distributed with moneyness 1-centered. This may be due to the depressed market. Put and call densities are also shifted. This is due to the higher liquidity of ATM and OTM.

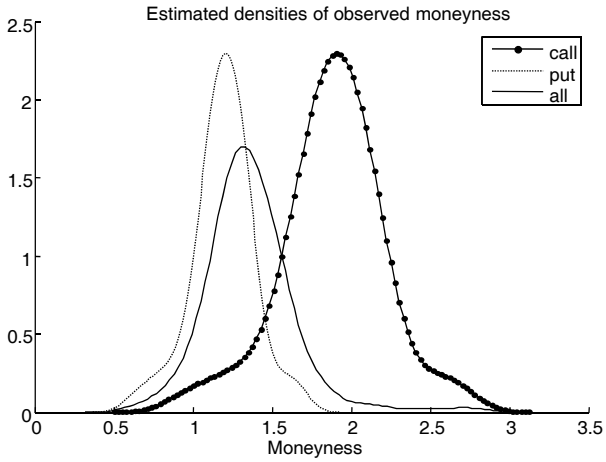

Figure 2. Nonparametrically estimated densities of observed moneyness for 20081106.

5. EXPERIMENTAL RESULTS

In this section, we present the results of applying the above presented four step method to estimate the LVS of ELW with confidence interval. In Figure 3, we present two surfaces, the implied volatility surface estimate and local volatility surface estimate. These are the weighted mean values of the fitted estimator. The single circles present IV data and the size of the circle is proportional to the log trading volume. Here different values of the ELW data for different liquidity providers (LPs) are all integrated in the same data set not to give fair market values but to simplify our experiments. Figure 4 shows some slices from the surface in the direction of the index (or strike) level. The left panels and the right panels in Figure 4 present the implied volatility smile and local volatility smile respectively. The upper panels are for time 0.104 and lower panels for 0.176. The thick lines are weighted mean values and the thin lines are 90% confidence intervals. The x-axis represents the index level for the local volatility function and the strike for the implied volatility and as for the 'time', it is calendar time for the local volatility, but maturity for the implied volatility. The confi-

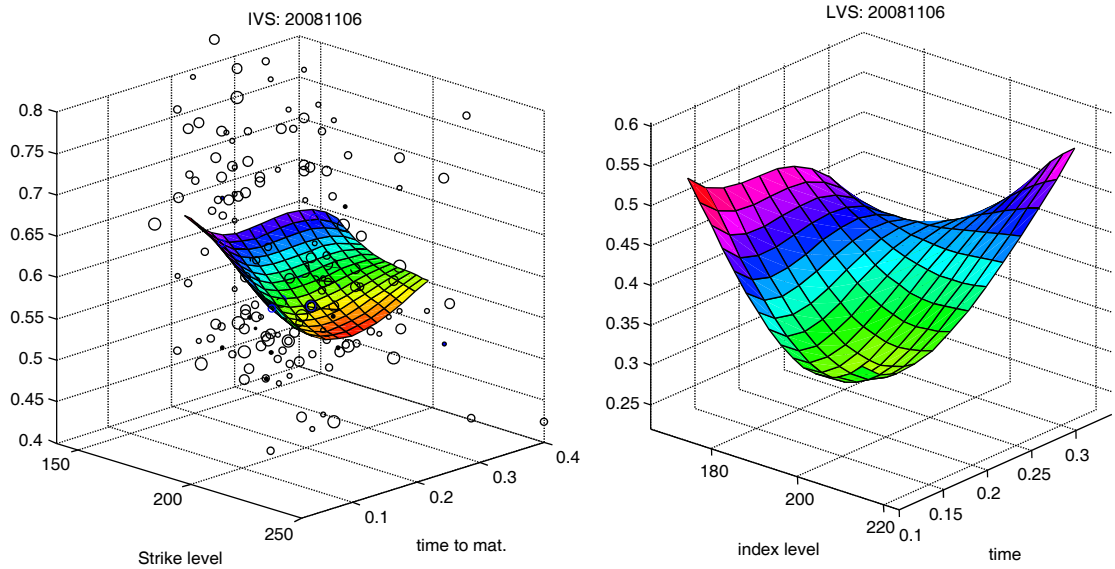

Figure 3. Left panel: IVS fit for 20081106; right panel: LVS fit for 20081106. The single circles denote IV data obatained by inverting the BS formula separately for each observation.

dence interval around end points is not much wide in the IVS. This is due to that the bias problem is not visible in local quadratic estimator. However, the number of observations becomes smaller and smaller the interval is wider and the interval is narrow around ATM which is very liquidity. The pattern is similar in LVS. For comparison to the method without model averaging, we present the smile function for IV and LV in Figure 5. Like Figure 4, the left panels and the right panels present the IV smile and LV smile respectively and the upper panels

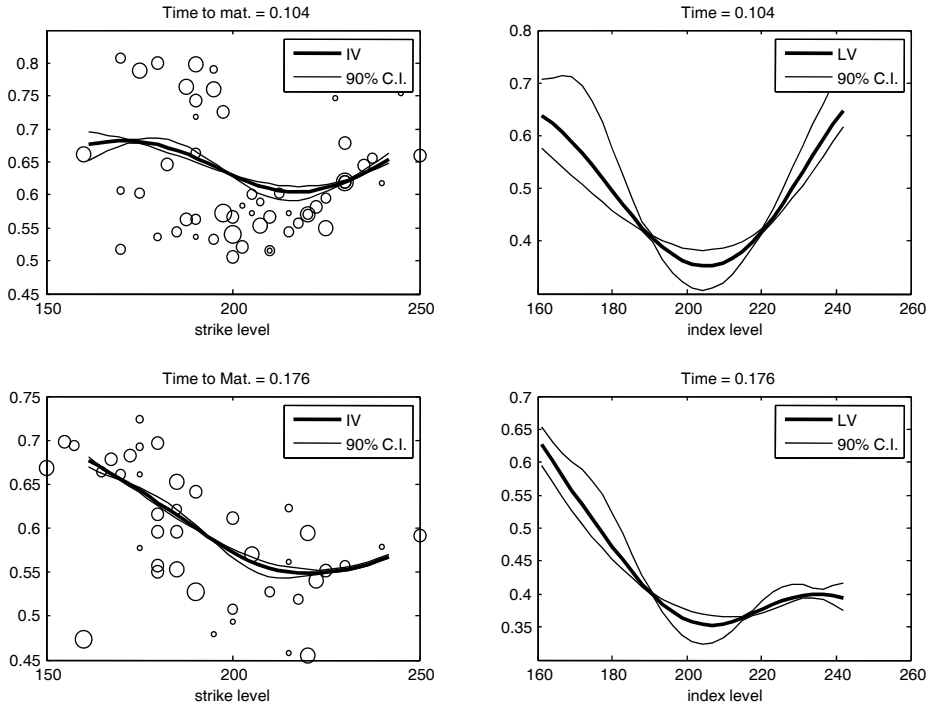

Figure 4. The implied volatility smile and local volatility smile with 90% confidence interval.

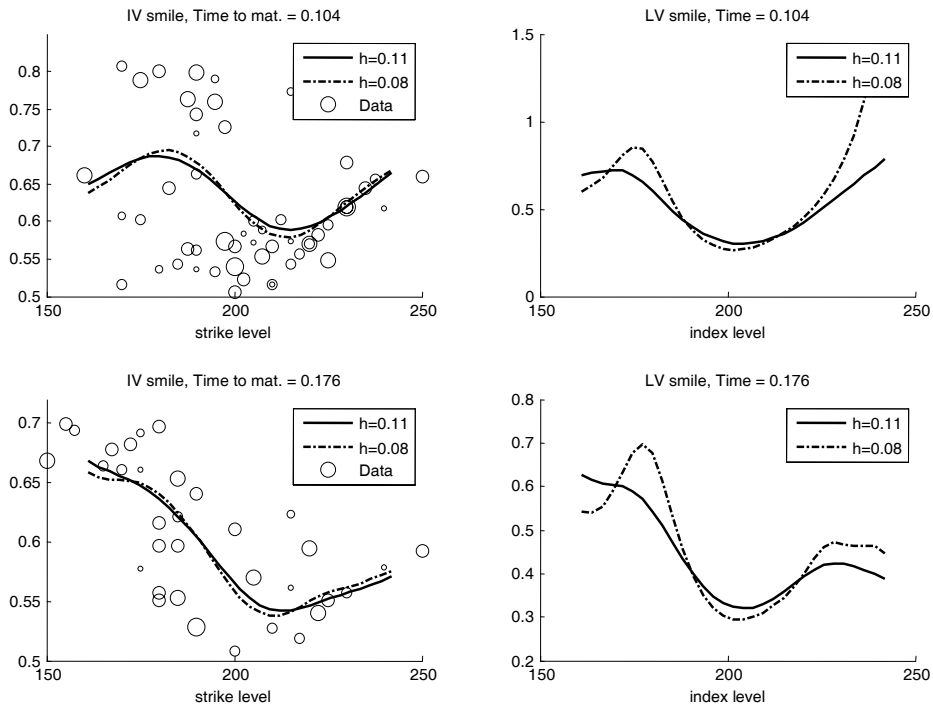

Figure 5. The volatility smile using a fixed bandwidth.

are for time 0.104 and lower panels for 0.176. The thick solid lines are estimators when the bandwidth is 0.11 in index level direction and the dashed lines are estimators when the bandwidth is 0.08 in index level direction. The bandwidth is 0.1 in time direction in both cases. As the bandwidth changes little, the IV smile is not much affected because the local quadratic estimates are relatively robust against the bandwidth change. However we can find that the LV smile is more affected by the change in Figure 5. Also, the LV function is not smile any more. Thus, it can be important to find the reasonable bandwidth again although the estimator for the IV is robust. Using model averaging method, we can overcome this difficulty. Figure 6 displays slices from both IVS and LVS at the time of 0.1 and 0.2. In Figure 5, the steepness of the LVS looks similar to that of the IVS due to the different scales. However, as shown in Figure 6, the LVS is steeper than the IVS. It is known as the two-times-IV-slope-rule for local volatility. It was reported as an empirical regularity in equity markets (Derman *et al*., 1996).

6. CONCLUSION

In the paper, we suggested a non-parametric method for the IVS and LVS. There are two features in our proposed method. One is that the suggested method provides the information about the confidence interval of estimators as well as a rather less variable weighted mean value for the IVS and LVS. It also utilizes the trading volume to give the weight to each data in non-parametric smoothing. Our experiment shows the level of bandwidth influences the estimated LVS. That is, simply applying the local quadratic smoothing produces no robust LVS unlike IVS. The other advantage is that our proposed method resolves this problem by means of model averaging method using bandwidth priors. Furthermore, we find that the good side effect, 'two-times-IV-slope-rule', appear as seen in Figure 6.

For the future works, more reliable and enhanced estimation techniques, e.g. kernel-based techniques as in Lee *et al*. (2006) or Lee *et al*. (2007) needs to be further investigated with suitable modifications or extensions. Also more realistic priors with better prediction should be sought with theoretical analysis. Applying the proposed strategy to real dynamic-hedging problems and assessing their performance with respect to various financial derivatives markets will be another important issue for the further study.

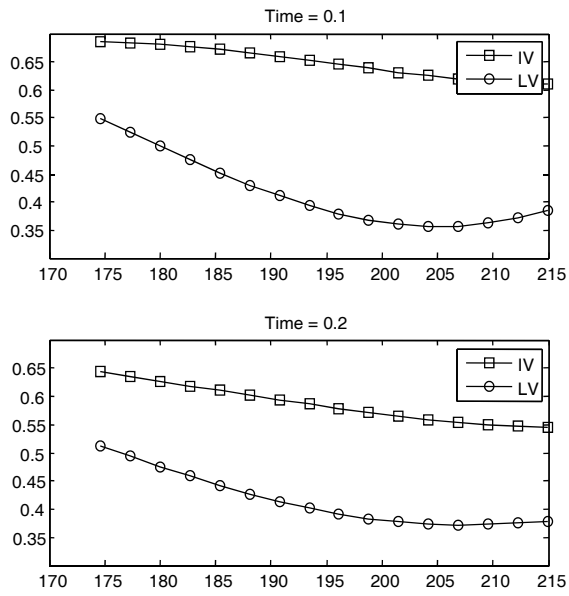

Figure 6. The implied and local volatility functions estimated using our proposed method.

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