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Interval-Valued Fuzzy M-Continuity and Interval-Valued Fuzzy M^*-open mappings

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Abstract

We introduce the concepts of interval-valued fuzzy M-continuity and interval-valued fuzzy M^* (M)-open mappings. And we study some characterizations and basic properties of such functions.

Key words : interval-valued fuzzy minimal spaces, IVF M-continuous functions, IVF M^*-open mapping, IVF M-open mapping

1. Introduction and preliminaries

Zadeh [5] introduced the concept of fuzzy set and several researchers were concerned about the generalizations of the concept of fuzzy sets, intuitionistic fuzzy sets [1] and interval-valued fuzzy sets [3].

In [2], Alimohammady and Roohi introduced fuzzy minimal structures and fuzzy minimal spaces and some results are given. In this paper, we introduce the concept of interval valued fuzzy minimal structure (simply, IVF minimal structure) as a generalization of interval-valued fuzzy topology introduced by Mondal and Samanta [6]. In [5], we introduced the concepts of IVF m-continuity and IVF m-compactness and we studied some results about them. In this paper, we introduce the concepts of interval-valued fuzzy M-continuity and interval-valued fuzzy M^*-open mappings defined on interval-valued fuzzy minimal spaces. And we study some characterizations and basic properties of such functions.

Let $D[0,1]$ be the set of all closed subintervals of the interval $[0,1]$. The elements of $D[0,1]$ are generally denoted by capital letters $M, N, \cdots$ and note that $M = [M^L, M^U]$, where M^L and M^U are the lower and the upper end points respectively. Especially, we denote $\mathbf{0} = [0,0], \mathbf{1} = [1,1]$, and $\mathbf{a} = [a,a]$ for $a \in (0,1)$. We also note that

1. $(\forall M, N \in D[0,1])(M = N \Leftrightarrow M^L = N^L, M^U = N^U)$.
2. $(\forall M, N \in D[0,1])(M \leq N \Leftrightarrow M^L \leq N^L, M^U \leq N^U)$.

For every $M \in D[0,1]$, the complement of M, denoted by M^c, is defined by $M^c = 1 - M = [1 - M^U, 1 - M^L]$.

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Let X be a nonempty set. A mapping $A : X \to D[0,1]$ is called an interval-valued fuzzy set (simply, IVF set) in X. For each $x \in X$, $A(x)$ is a closed interval whose lower and upper end points are denoted by $A(x)^L$ and $A(x)^U$, respectively. For any $[a,b] \in D[0,1]$, the IVF set whose value is the interval $[a,b]$ for all $x \in X$ is denoted by $\widetilde{[a,b]}$. In particular, for any $a \in [a,b]$, the IVF set whose value is $\mathbf{a} = [a,a]$ for all x X is denoted by simply $\widetilde{a}$. For a point $p \in X$ and for $[a,b] \in D[0,1]$ with $b > 0$, the IVF set which takes the value $[a,b]$ at p and $\mathbf{0}$ elsewhere in X is called an interval-valued fuzzy point (simply, IVF point) and is denoted by $[a,b]_p$. In particular, if $b = a$, then it is also denoted by a_p. Denoted by IVF(X) the set of all IVF sets in X.

For every $A, B \in IVF(X)$, we define

$$A = B \Leftrightarrow (\forall x \in X)([A(x)]^L = [B(x)]^L$$

and

$$[A(x)]^U = [B(x)]^U),$$

$$A \subseteq B \Leftrightarrow (\forall x \in X)([A(x)]^L \subseteq [B(x)]^L$$

and

$$[A(x)]^U \subseteq [B(x)]^U).$$

The complement A^c of A is defined by

$$[A^c(x)]^L = 1 - [A(x)]^U \text{and } [A^c(x)]^U = 1 - [A(x)]^L$$

for all $x \in X$.

For a family of IVF sets $\{A_i : i \in J\}$ where J is an index set, the union $G = \cup_{i \in J} A_i$ and $F = \cap_{i \in J} A_i$ are defined by

$$(\forall x \in X)\ ([G(x)]^L = \sup_{i \in J}[A_i(x)]^L,\quad [G(x)]^U = \sup_{i \in J}[A_i(x)]^U),$$

$$(\forall x \in X)\ ([F(x)]^L = \inf_{i \in J}[A_i(x)]^L,\quad [F(x)]^U = \inf_{i \in J}[A_i(x)]^U),$$

respectively.

Let $f : X \to Y$ be a mapping and let A be an IVF set in X. Then the image of A under f, denoted by $f(A)$, is defined as follows

$$[f(A)(y)]^L = \begin{cases} \sup_{f(x)=y}[A(x)]^L, & \text{if } f^{-1}(y) \neq \emptyset,\ y \in Y \\ 0, & \text{otherwise}. \end{cases}$$

$$[f(A)(y)]^U = \begin{cases} \sup_{f(x)=y}[A(x)]^U, & \text{if } f^{-1}(y) \neq \emptyset,\ y \in Y \\ 0, & \text{otherwise}, \end{cases}$$

for all $y \in Y$.

Let B be an IVF set in Y. Then the inverse image of B under f, denoted by $f^{-1}(B)$, is defined as follows

$$(\forall x \in X)([f^{-1}(B)(x)]^L = [B(f(x))]^L,\quad [f^{-1}(B)(x)]^U = [B(f(x))]^U).$$

Definition 1.1 ([6]). A family τ of IVF sets in X is called an *interval-valued fuzzy topology* on X if it satisfies:
(1) $\mathbf{0}, \mathbf{1} \in \tau$.
(2) $A, B \in \tau \Rightarrow A \cap B \in \tau$.
(3) For $i \in J$, $A_i \in \tau \Rightarrow \cup_{i \in J} A_i \in \tau$.

Every member of τ is called an IVF open set. An IVF set A is called an IVF closed set if the complement of A is an IVF open set. And (X, τ) is called an *interval-valued fuzzy topological space.*

In an IVF topological space (X, τ), for an IVF set A in X, the IVF closure and the IVF interior of A, denoted by $Cl(A)$ and $Int(A)$, respectively, are defined as

$$Cl(A) = \cap\{B \in IVF(X) : B^c \in \tau \text{ and } A \subseteq B\},$$

$$Int(A) = \cup\{B \in IVF(X) : B \in \tau \text{ and } B \subseteq A\}.$$

Let (X, τ) be an IVF topological space. An IVF set A in X is said to be *IVF compact* if every IVF open cover $\mathcal{A} = \{A_i : i \in J\}$ of B has a finite IVF subcover. And an IVF set A in X is said to be *almost IVF compact* (resp., *nearly IVF compact*) if for every IVF open cover $\mathcal{A} = \{A_i : i \in J\}$ of B, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} Cl(A_i)$ (resp., $A \subseteq \cup_{i \in J_0} Int(Cl(A_i))$).

Definition 1.2 ([4]). An IVF set A in an IVF topological space (X, τ) is called
(1) an IVF semiopen set in X if there exists $B \in \tau$ such that $B \subseteq A \subseteq Cl(B)$);
(2) an IVF preopen set in X if $A \subseteq Int(Cl(A))$;
(3) an IVF α-open set in X if $A \subseteq Int(Cl(Int(A)))$.

And an IVF set A is called an IVF semiclosed (resp., IVF preclosed, IVF α-closed) set if the complement of A is an IVF semiopen (resp., IVF preopen, IVF α-open) set. Denoted by IVFSO(X) (resp., IVFPO(X), IVFα(X)) the set of all IVF semiopen (resp., IVF preopen, IVF α-open) sets. Denoted by IVFSC(X) (resp., IVFPC(X), IVFαC(X)) the set of all IVF semiclosed (resp., IVF preclosed, IVF α-closed) sets.

Definition 1.3 ([5]). A family $\mathcal{M}$ of interval-valued fuzzy sets in X is called an *interval-valued fuzzy minimal structure* on X if

$$\mathbf{0}, \mathbf{1} \in \mathcal{M}.$$

In this case, $(X, \mathcal{M})$ is called an *interval-valued fuzzy minimal space* (simply, *IVF minimal space*). Every member of $\mathcal{M}$ is called an IVF m-open set. An IVF set A is called an IVF m-closed set if the complement of A (simply, A^c) is an IVF m-open set.

Let (X, τ) be an IVF topological space. Then τ, IVFSO(X), IVFSC(X), IVFPO(X) and IVFPC(X) are all interval-valued fuzzy minimal spaces.

Let $(X, \mathcal{M})$ be an IVF minimal space and A in IVF(X). The IVF minimal-closure and the IVF minimal-interior of A [5], denoted by $mCl(A)$ and $mInt(A)$, respectively, are defined as

$$mCl(A) = \cap\{B \in IVF(X) : B^c \in \mathcal{M} \text{ and } A \subseteq B\},$$

$$mInt(A) = \cup\{B \in IVF(X) : B \in \mathcal{M} \text{ and } B \subseteq A\}.$$

Theorem 1.4 ([5]). Let $(X, \mathcal{M})$ be an IVF minimal space and A, B in IVF(X).
(1) $mInt(A) \subseteq A$ and if A is an IVF m-open set, then $mInt(A) = A$.
(2) $A \subseteq mCl(A)$ and if A is an IVF m-closed set, then $mCl(A) = A$.
(3) If $A \subseteq B$, then $mInt(A) \subseteq mInt(B)$ and $mCl(A) \subseteq mCl(B)$.
(4) $mInt(A) \cap mInt(B) \supseteq mInt(A \cap B)$ and

$mCl(A) \cup mCl(B) \subseteq mCl(A \cup B)$.

(5) $mInt(mInt(A)) = mInt(A)$ and $mCl(mCl(A)) = mCl(A)$.

(6) $\mathbf{1} - mCl(A) = mInt(\mathbf{1} - A)$ and $\mathbf{1} - mInt(A) = mCl(\mathbf{1} - A)$.

2. IVF M-continuous functions and IVF M^*-open mappings

Definition 2.1. Let $(X, \mathcal{M}_X)$ be an IVF minimal space and let (Y, τ) be an IVF topological space. Then $f : X \to Y$ is said to be *interval-valued fuzzy M-continuous* (simply, IVF M-continuous) if for every $A \in \tau$, $f^{-1}(A)$ is in $\mathcal{M}_X$.

Theorem 2.2. Let $f : X \to Y$ be a function on an IVF minimal space $(X, \mathcal{M}_X)$ and an IVF topological space (Y, τ). Then we have the following:

(1) f is IVF M-continuous.

(2) $f^{-1}(B)$ is an IVF m-closed set, for each IVF closed set B in Y.

(3) $f(mCl(A)) \subseteq Cl(f(A))$ for $A \in IVF(X)$.

(4) $mCl(f^{-1}(B)) \subseteq f^{-1}(Cl(B))$ for $B \in IVF(Y)$.

(5) $f^{-1}(Int(B)) \subseteq mInt(f^{-1}(B))$ for $B \in IVF(Y)$.

Then (1) $\Leftrightarrow$ (2) $\Rightarrow$ (3) $\Leftrightarrow$ (4) $\Leftrightarrow$ (5).

Proof. (1) $\Leftrightarrow$ (2) Obvious.

(2) $\Rightarrow$ (3) For $A \in IVF(X)$,

$f^{-1}(Cl(f(A))) = f^{-1}(\cap\{F \in IVF(X) : f(A) \subseteq F$ and F is an IVF closed set$\}) = \cap\{f^{-1}(F) \in IVF(X) : A \subseteq f^{-1}(F)$ and F is an IVF closed set $\} \supseteq \cap\{K \in IVF(X) : A \subseteq K$ and K is an IVF m-closed set$\}$ $= mCl(A)$.

Hence $f(mCl(A)) \subseteq Cl(f(A))$.

(3) $\Rightarrow$ (4) Let $B \in IVF(Y)$. From (3) it follows that

$$f(mCl(f^{-1}(B))) \subseteq Cl(f(f^{-1}(B))) \subseteq Cl(B).$$

Hence we get (4). Similarly, we get (4) $\Rightarrow$ (3).

(4) $\Leftrightarrow$ (5) From Theorem 1.4, it is obvious. □

Example 2.3. Let $X = \{a, b\}$. Let A, B and C be IVF sets defined as follows

$$A(a) = [0.1, 0.6], A(b) = [0.2, 0.5],$$

$$B(a) = [0.2, 0.5], B(b) = [0.3, 0.4],$$

$$C(a) = [0.2, 0.6], C(b) = [0.3, 0.5].$$

Note $C = A \cup B$. Consider an IVF m-structure $\mathcal{M}_X = \{\emptyset, A, B, X\}$ and an IVF topological space $\tau = \{\emptyset, A, B, C, X\}$. Let $f : (X, \mathcal{M}_X) \to (X, \tau)$ be a function defined as follows $f(x) = x$ for each $x \in X$. Then f satisfies the condition (5) in Theorem 2.2, but it is not IVF M-continuous because C is not in $\mathcal{M}_X$.

Corollary 2.4. Let $f : X \to Y$ be a function on an IVF minimal space $(X, \mathcal{M}_X)$ and an IVF topological space (Y, τ). Then the following equivalent:

(1) $f(A) \subseteq Cl(f(A))$ for each IVF m-closed set A in X.

(2) $f^{-1}(F) = mCl(f^{-1}(F))$ for each IVF closed set F in Y.

(3) $f^{-1}(B) = mInt(f^{-1}(B))$ for each IVF open set B in Y.

Proof. Obvious. □

Definition 2.5. Let $\mathcal{M}_X$ be an IVF minimal structure on X. Then $\mathcal{M}_X$ said to have property $(\mathcal{B})$ if the union of any family of IVF sets belong to $\mathcal{M}_X$ belongs to $\mathcal{M}_X$.

Remark 2.6. IVFSO(X), IVFPO(X) and IVFα(X) are all IVF minimal structures on X with property $(\mathcal{B})$ in an IVF topological space (X, τ).

Lemma 2.7. Let $\mathcal{M}_X$ be an IVF minimal structure on X. Then the following are equivalent.

(1) $\mathcal{M}_X$ has property $(\mathcal{B})$.

(2) If $mInt(B) = B$, then $B \in \mathcal{M}_X$.

(3) If $mCl(F) = F$, then $X - F \in \mathcal{M}_X$.

Proof. Obvious. □

Corollary 2.8. Let $f : X \to Y$ be a function on an IVF minimal space $(X, \mathcal{M}_X)$ and an IVF topological space (Y, τ). If $\mathcal{M}_X$ has property $(\mathcal{B})$, then the following are equivalent:

(1) f is IVF M-continuous.

(2) $f^{-1}(B)$ is an IVF m-closed set, for each IVF closed set B in Y.

(3) $f(mCl(A)) \subseteq Cl(f(A))$ for $A \in IVF(X)$.

(4) $mCl(f^{-1}(B)) \subseteq f^{-1}(Cl(B))$ for $B \in IVF(Y)$.

(5) $f^{-1}(Int(B)) \subseteq mInt(f^{-1}(B))$ for $B \in IVF(Y)$.

Proof. By Lemma 2.7, it is obvious. □

Definition 2.9. Let $(X, \mathcal{M}_X)$ be an IVF minimal space and (Y, τ) be an IVF topological space. Then $f : X \to Y$ is called an *interval-valued fuzzy M^*-open* (simply, IVF M^*-open) mapping if for every IVF m-open set A in X, $f(A)$ is IVF open in Y.

Theorem 2.10. Let $f : X \to Y$ be a function on an IVF minimal space $(X, \mathcal{M}_X)$ and an IVF topological space (Y, τ). Then the following are equivalent:
(1) f is IVF M^*-open.
(2) $f(mInt(A)) \subseteq Int(f(A))$ for $A \in IVF(X)$.
(3) $mInt(f^{-1}(B)) \subseteq f^{-1}(Int(B))$ for $B \in IVF(Y)$.

Proof. (1) $\Rightarrow$ (2) For $A \in IVF(X)$,

$f(mInt(A)) = f(\cup\{B \in IVF(X) : B \subseteq A, B$ is an IVF m-open set $\}) = \cup\{f(B) \in IVF(X) : f(B) \subseteq f(A), f(B)$ is an IVF open set $\} \subseteq \cup\{U \in IVF(X) : U \subseteq f(A), U$ is an IVF open set $\} = Int(f(A))$.

Hence $f(mInt(A)) \subseteq Int(f(A))$.

(2) $\Rightarrow$ (1) By Theorem 1.4 (1), f is IVF M^*-open.

(2) $\Rightarrow$ (3) For $B \in IVF(Y)$, from (3) it follows that

$$f(mInt(f^{-1}(B))) \subseteq Int(f(f^{-1}(B))) \subseteq Int(B).$$

Hence we get (3). Similarly, we get (3) $\Rightarrow$ (2). □

Corollary 2.11. Let $f : X \to Y$ be a function on an IVF minimal space $(X, \mathcal{M}_X)$ and an IVF topological space (Y, τ). If f is IVF M^*-open, then $f(A) = Int(f(A))$ for every IVF m-open set A in X.

Proof. From Theorem 1.4 and Theorem 2.10, it is obvious. □

Definition 2.12. Let (X, τ) be an IVF topological space and $(Y, \mathcal{M}_Y)$ be an IVF minimal space. Then $f : X \to Y$ is called an *interval-valued fuzzy M-open* (simply, IVF M-open) mapping if for every IVF open set A in X, $f(A)$ is IVF m-open in Y.

Theorem 2.13. Let $f : X \to Y$ be a function on an IVF topological space (X, τ) and an IVF minimal space $(Y, \mathcal{M}_Y)$.
(1) f is IVF M-open.
(2) $f(Int(A)) \subseteq mInt(f(A))$ for $A \in IVF(X)$.
(3) $Int(f^{-1}(B)) \subseteq f^{-1}(mInt(B))$ for $B \in IVF(Y)$.
Then (1) $\Rightarrow$ (2) $\Leftrightarrow$ (3).

Proof. (1) $\Rightarrow$ (2) For $A \in IVF(X)$,

$f(Int(A)) = f(\cup\{B \in IVF(X) : B \subseteq A, B$ is an IVF open set $\}) = \cup\{f(B) \in IVF(X) : f(B) \subseteq f(A), f(B)$ is an IVF m-open set $\} \subseteq \cup\{U \in IVF(X) : U \subseteq f(A), U$ is an IVF m-open set $\} = mInt(f(A))$. Hence $f(Int(A)) \subseteq mInt(f(A))$.

(2) $\Rightarrow$ (3) For $B \in IVF(Y)$, from (3) it follows that

$$f(Int(f^{-1}(B))) \subseteq mInt(f(f^{-1}(B))) \subseteq mInt(B).$$

Hence we get (3). Similarly, we get (3) $\Rightarrow$ (2). □

In Theorem 2.13, the implication (2) $\Rightarrow$ (1) is not true in general, as seen in the following example.

Example 2.14. In Example 2.3, consider function $f : (X, \tau) \to (X, \mathcal{M}_X)$ defined as follows $f(x) = x$ for each $x \in X$. Then f satisfies the condition (2) in Theorem 2.13, but it is not IVF M-open.

Corollary 2.15. Let $f : X \to Y$ be a function on an IVF topological space (X, τ) and an IVF minimal space $(Y, \mathcal{M}_Y)$. If f is IVF M-open, then $f(A) = mInt(f(A))$ for every IVF open set A in X.

Proof. It follows from Theorem 1.4 and Theorem 2.13. □

Corollary 2.16. Let $f : X \to Y$ be a function on an IVF topological space (X, τ) and an IVF minimal space $(Y, \mathcal{M}_Y)$. If $\mathcal{M}_Y$ has property $(\mathcal{B})$, then the following are equivalent:
(1) f is IVF M-open.
(2) $f(Int(A)) \subseteq mInt(f(A))$ for $A \in IVF(X)$.
(3) $Int(f^{-1}(B)) \subseteq f^{-1}(mInt(B))$ for $B \in IVF(Y)$.

Proof. Obvious. □

Definition 2.17. Let $(X, \mathcal{M}_X)$ be an IVF minimal space and (Y, τ) be an IVF topological space. Then $f : X \to Y$ is called an *interval-valued fuzzy M^*-closed* (simply, IVF M^*-closed) mapping if for every IVF m-closed set A in X, $f(A)$ is an IVF closed set in Y.

Theorem 2.18. Let $f : X \to Y$ be a function on an IVF minimal space $(X, \mathcal{M}_X)$ and an IVF topological space (Y, τ). Then the following are equivalent:
(1) f is IVF M^*-closed.
(2) $Cl(f(A)) \subseteq f(mCl(A))$ for $A \in IVF(X)$.
(3) $f^{-1}(Cl(B)) \subseteq mCl(f^{-1}(B))$ for $B \in IVF(Y)$.

Proof. It follows from Theorem 1.4 and Theorem 2.10. □

Corollary 2.19. Let $f : X \to Y$ be a function on an IVF minimal space $(X, \mathcal{M}_X)$ and an IVF topological space (Y, τ). If f is IVF M^*-closed, then $f(A) = Cl(f(A))$ for every IVF m-closed set A in X.

Proof. It is obvious. □

Definition 2.20. Let (X, τ) be an IVF topological space and $(Y, \mathcal{M}_Y)$ be an IVF minimal space. Then $f : X \to Y$ is called an *interval-valued fuzzy M-closed* (simply, IVF M-closed) mapping if for every IVF closed set A in X, $f(A)$ is IVF m-closed in Y.

Theorem 2.21. Let $f : X \to Y$ be a function on an IVF topological space (X, τ) and an IVF minimal space $(Y, \mathcal{M}_Y)$.
(1) f is IVF M-closed.
(2) $mCl(f(A)) \subseteq f(Cl(A))$ for $A \in IVF(X)$.
(3) $f^{-1}(mCl(B)) \subseteq Cl(f^{-1}(B))$ for $B \in IVF(Y)$.
Then $(1) \Rightarrow (2) \Leftrightarrow (3)$.

Proof. It follows from Theorem 1.4 and Theorem 2.13. □

In Theorem 2.21, the implication $(2) \Rightarrow (1)$ is not true in general, as seen in Example 2.14.

Corollary 2.22. Let $f : X \to Y$ be a function on an IVF topological space (X, τ) and an IVF minimal space $(Y, \mathcal{M}_Y)$. If f is IVF M-closed, then $f(A) = mCl(f(A))$ for every IVF closed set A in X.

Proof. It is obvious. □

Corollary 2.23. Let $f : X \to Y$ be a function on an IVF topological space (X, τ) and an IVF minimal space $(Y, \mathcal{M}_Y)$. If $\mathcal{M}_Y$ has property $(\mathcal{B})$, then the following are equivalent:
(1) f is IVF M-closed.
(2) $mCl(f(A)) \subseteq f(Cl(A))$ for $A \in IVF(X)$.
(3) $f^{-1}(mCl(B)) \subseteq Cl(f^{-1}(B))$ for $B \in IVF(Y)$.

Proof. Obvious. □

Definition 2.24 ([5]). Let $(X, \mathcal{M}_X)$ be an IVF minimal space. An IVF set A in X is said to be *IVF m-compact* if every IVF m-open cover $\mathcal{A} = \{A_i : i \in J\}$ of B has a finite IVF subcover. And an IVF set A in X is said to be *almost IVF m-compact* (resp., *nearly IVF m-compact*) if for every IVF m-open cover $\mathcal{A} = \{A_i : i \in J\}$ of B, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} mCl(A_i)$ (resp., $A \subseteq \cup_{i \in J_0} mInt(mCl(A_i))$).

Theorem 2.25. Let $f : X \to Y$ be an IVF M-continuous function on an IVF minimal space $(X, \mathcal{M}_X)$ and an IVF topological space (Y, τ). If A is an IVF m-compact set, then $f(A)$ is an IVF compact set.

Proof. Let $\{B_i : i \in J\}$ be an IVF open cover of $f(A)$ in Y. Then since f is an IVF M-continuous function, $\{f^{-1}(B_i) : i \in J\}$ is an IVF m-open cover of A in X. By IVF m-compactness, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} f^{-1}(B_i)$. Hence $f(A) \subseteq \cup_{i \in J_0} B_i$. □

Theorem 2.26. Let $f : X \to Y$ be an IVF M-continuous function on an IVF minimal space $(X, \mathcal{M}_X)$ and an IVF topological space (Y, τ). If A is an almost IVF m-compact set, then $f(A)$ is an almost IVF compact set.

Proof. Let $\{B_i : i \in J\}$ be an IVF open cover of $f(A)$ in Y. Then $\{f^{-1}(B_i) : i \in J\}$ is an IVF m-open cover of A in X. By almost IVF m-compactness, there exists $J_0 = \{1, 2, \cdots, n\} \subseteq J$ such that $A \subseteq \cup_{i \in J_0} mCl(f^{-1}(B_i))$. From Theorem 2.2, it follows

$$\cup_{i \in J_0} mCl(f^{-1}(B_i)) \subseteq \cup_{i \in J_0} f^{-1}(Cl(B_i)) = f^{-1}(\cup_{i \in J_0} Cl(B_i)).$$

Hence $f(A) \subseteq \cup_{i \in J_0} Cl(B_i)$. □

Theorem 2.27. Let $f : X \to Y$ be an IVF M-continuous and IVF M^*-open function on an IVF minimal space $(X, \mathcal{M}_X)$ and an IVF topological space (Y, τ). If A is a nearly IVF m-compact set, then $f(A)$ is a nearly IVF compact set.

Proof. Let $\{B_i : i \in J\}$ be an IVF open cover of $f(A)$ in Y. Then $\{f^{-1}(B_i) : i \in J\}$ is an IVF m-open cover of A in X. By nearly IVF m-compactness, there exists $J_0 = \{J_1, J_2, \cdots J_n\} \subseteq J$ such that $A \subseteq \cup_{i\in J_0} mInt(mCl(f^{-1}(B_i)))$. From Theorem 2.2 and Theorem 2.10, it follows

$$\begin{aligned} f(A) &\subseteq \cup_{i\in J_0} f(mInt(mCl(f^{-1}(B_i)))) \\ &\subseteq \cup_{i\in J_0} Int(f(mCl(f^{-1}(B_i)))) \\ &\subseteq \cup_{i\in J_0} mInt(f(f^{-1}(Cl(B_i)))) \\ &\subseteq \cup_{i\in J_0} Int(Cl(B_i)). \end{aligned}$$

Hence the proof is completed. □

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