Fuzzy S-weakly (r,s)-Continuous Mappings 관한 연구

On Fuzzy S-weakly (r,s)-Continuous Mappings

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요 약

Sostak개념의 intuitionistic fuzzy topological space에서 fuzzy S-weakly (r,s)-continuous mapping의 개념과 연산을 소개하며 기본적인 성질을 조사한다.

Abstract

In this paper, we introduce the concept of fuzzy S-weakly (r,s)-continuous mapping on an intuitionistic fuzzy topological space in Sostak's sense and investigate some properties of such mappings.

Key Words: fuzzy S-weakly (r,s)-continuous, fuzzy weakly (r,s)-continuous, fuzzy (r,s)-semiopen.

1. Intorduction

The concept of a fuzzy set was introduced by Zadeh [13]. As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Chang [2] defined fuzzy topological spaces using fuzzy sets. In [3], Chattopadhyay, Hazra and Samanta introduced the concept of smooth fuzzy topological spaces which are a generalization of fuzzy topological spaces. Coker and his colleagues [4, 5, 6, 7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. In [5], Coker and Demirci introduced intuitionistic fuzzy topological spaces in Sostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces. The author introduced the concept of fuzzy weakly (r,s)-continuous mappings and studied characterizations for them in [11].

In this paper, we introduce fuzzy S-weakly (r,s)-continuous mappings on the intuitionistic fuzzy topological space in Sostak's sense and investigate some properties. The concept of fuzzy S-weakly (r,s)-continuous mappings is an extended concept of fuzzy weakly (r,s)-continuous mappings.

2. Preliminaries

Let I be the unit interval [0,1] of the real line. A member μ of I^X is called a fuzzy set of X. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and 1,

접수일자: 2008년 8월 15일 완료일자: 2008년 11월 25일 respectively. For any $\mu \in I^X$, μ^c denotes the complement $\tilde{1} - \mu$. All other notations are standard notations of fuzzy set theory.

Let X be a nonempty set. An intuitionistic fuzzy set A is an ordered pair $A = \{ \langle x, \mu_A(x), \gamma_A(x) \rangle : x \in X \}$

(simply,
$$A=(\mu_A, \gamma_A)$$
)

where the functions $\mu_A: X \to I$ and $\gamma_A: X \to I$ denote the degree of membership and the degree of non-membership, respectively, and $0 \le \mu_A(x) + \gamma_A(x) \le 1$ for $x \in X$.

An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X is an intuitionistic fuzzy set

$$x_{(\alpha,\beta)} = (\mu_A, \gamma_A)$$

where the functions the functions $\mu_A: X \to I$ and $\gamma_A: X \to I$ are defined as follows:

$$(\mu_A(y), \gamma_A(y)) = \begin{cases} (\alpha, \beta), & \text{if } y = x, \\ (0, 1), & \text{if } y \neq x; \end{cases}$$
$$0 \le \alpha + \beta \le 1.$$

An intuitionistic fuzzy point $x_{(\alpha,\beta)}$ is said to belong to an intuitionistic fuzzy set $A=(\mu_A,\gamma_A)$ in X, denoted by $x_{(\alpha,\beta)} \in A$, if $\mu_A(x) \geq \alpha$ and $\gamma_A(x) \leq \beta$ for $x \in X$.

An intuitionistic fuzzy set A in X is the union of all intuitionistic fuzzy points which belong to A.

Definition 2.1 ([1]) Let $A=(\mu_A,\gamma_A)$ and $B=(\mu_B,\gamma_B)$ be intuitionistic fuzzy sets on X. Then

- (1) $A \subseteq B$ iff $\mu_A \le \mu_B$ and $\gamma_A \ge \gamma_B$.
- (2) A = B iff $A \subseteq B$ and $B \subseteq A$.

- (3) $A^c = (\gamma_A, \mu_A)$.
- (4) $A \cap B = (\mu_A \wedge \mu_B, \ \gamma_A \vee \gamma_B).$
- (5) $A \cup B = (\mu_A \vee \mu_B, \ \gamma_A \wedge \gamma_B).$
- (6) $0 = (\tilde{0}, \tilde{1})$ and $1 = (\tilde{1}, \tilde{0})$.

Let f be a map from a set X to a set Y. Let $A = (\mu_A, \gamma_A)$ be an intuitionistic fuzzy set of X and $B = (\mu_B, \gamma_B)$ an intuitionistic fuzzy set of Y. Then:

(1) The image of A under f, denoted by f(A) is an intuitionistic fuzzy set in Y defined by

$$f(A)=(f(\mu_A), \tilde{1}-f(\tilde{1}-\gamma_A)).$$

(2) The inverse image of B under f, denoted by $f^{-1}(B)$ is an intuitionistic fuzzy set in X defined by

$$f^{-1}(B) = (f^{-1}(\mu_B), f^{-1}(\gamma_B)).$$

A smooth fuzzy topology [12] on X is a map $T:I^X \to I$ which satisfies the following properties:

- (1) $T(\tilde{0}) = T(\tilde{1}) = 1$.
- (2) $T(\mu_1 \wedge \mu_2) \ge T(\mu_1) \wedge T(\mu_2)$ for $\mu_1, \mu_2 \in I^X$.
- (3) $T(\vee \mu_i) \ge \wedge T(\mu_i)$ for $\mu_i \in I^X$.

The pair (X, T) is alled a smooth fuzzy topological.

An intuitionistic fuzzy topology on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties:

- (1) 0_{\sim} , $1_{\sim} \in T$.
- (2) If A_1 , $A_2 \in T$, then $A_1 \cap A_2 \in T$.
- (3) If $A_i \in T$ for all $i \in I$, then $\bigcup A_i \in T$.

The pair (X,T) is called an intuitionistic fuzzy topological space.

Let I(X) be a family of all intuitionistic fuzzy sets of X and let $I \otimes I$ be the set of the pair (r,s) such that $r,s \in I$ and $0 \le r+s \le 1$.

Definition 2.2 ([6]) Let X be a nonempty set. An intuitionistic fuzzy topology in Sostak's sense (SoIFT for short) $T=(T_1, T_2)$ on X is a map $T: I(X) \to I \otimes I$ which satisfies the following properties:

- (1) $T_1(0_{\sim}) = T_1(1_{\sim}) = 1$ and $T_2(0_{\sim}) = T_2(1_{\sim}) = 0$.
- (2) $T_1(A \cap B) \ge T_1(A) \wedge T_1(B)$ and $T_2(A \cap B) \le T_2(A) \vee T_2(B)$.
- (3) $T_1(\cup A_i) \ge \wedge T_1(A_i)$ and $T_2(\cup A_i) \le \vee T_2(A_i)$.

Then the $(X, T)=(X, T_1, T_2)$ is said to be an intuitionistic fuzzy topological space in Sostak's sense (SoIFTS for short). Also, we call $T_1(A)$ a gradation of openness of A and $T_2(A)$ a gradation of nonopenness

of A.

The fuzzy (r,s)-closure and the fuzzy (r,s)-interior of A, denoted by $\operatorname{cl}(A,r,s)$ and $\operatorname{int}(A,r,s)$, respectively, are defined as

$$\operatorname{cl}(A, r, s) = \bigcap \{B \in \operatorname{IF}(X) \colon A \subseteq B \text{ and } B \text{ is fuzzy } (r,s) \text{-closed}\},$$
 $\operatorname{int}(A, r, s) = \bigcup \{B \in \operatorname{IF}(X) \colon B \subseteq A \text{ and } B \text{ is fuzzy } (r,s) \text{-open}\}.$

Definition 2.3 Let A be an intuitionistic fuzzy set in an SoIFTS (X,T_1,T_2) and $(r,s)\in I\otimes I$. Then A is said to be

- (1) fuzzy (r,s)-semiopen [8] if there is a fuzzy (r,s)-open set B in X such that $B \subseteq A \subseteq \operatorname{cl}(B, r, s)$,
- (2) fuzzy (r,s)-preopen [9] if $A \subseteq \operatorname{int}(\operatorname{cl}(A,r,s), r,s)$,
- (3) fuzzy (r,s)-regular open [10] if $A = \operatorname{int}(\operatorname{cl}(A, r, s), r, s),$
- (4) fuzzy $(r,s)-\beta-open$ [11] if $A \subseteq \operatorname{cl}(\operatorname{int}(\operatorname{cl}(A,r,s),r,s),r,s).$

Let A be an intuitionistic fuzzy set in an SoIFTS (X, T_1, T_2) and $(r, s) \in I \otimes I$.

The fuzzy (r,s)-semi-closure and the fuzzy (r,s)-semi-interior of A, denoted by scl(A, r, s) and sint (A, r, s), respectively, are defined as

$$\operatorname{scl}(A, r, s) = \bigcap \{B \in \operatorname{I\!F}(X) \colon A \subseteq B \text{ and } B \text{ is } (r, s) \text{-semiclosed}\},$$

 $\operatorname{sint}(A, r, s) = \bigcup \{B \in \operatorname{I\!F}(X) \colon B \subseteq A \text{ and } B \text{ is } (r, s) \text{-semiopen}\}.$

The following relationships are obtained:

$$\operatorname{int}(A, r, s) \subseteq \operatorname{sint}(A, r, s) \subseteq A \subseteq \operatorname{scl}(A, r, s)$$

 $\subseteq \operatorname{cl}(A, r, s)$

Definition 2.4. ([10]) Let $f:(X, T_1, T_2) \to (Y, U_1, U_2)$ be a mapping from an SoIFTS X to another SoIFTS Y and $(r,s) \in I \otimes I$. Then f is said to be fuzzy weakly (r,s)-continuous if for each fuzzy (r,s)-open set B of Y, $f^{-1}(B) \subseteq \operatorname{int}(f^{-1}(\operatorname{cl}(B, r,s)), r,s)$.

3. Main Results

Definition 3.1. Let $f:(X, T_1, T_2) \to (Y, U_1, U_2)$ be a mapping from SoIFTS's X, Y and $(r,s) \in I \otimes I$. Then f is said to be fuzzy S-weakly (r,s)-continuous if for each fuzzy (r,s)-open set B of Y, $f^{-1}(B) \subseteq \text{sint } (f^{-1}(\text{cl}(B, r,s)), r,s)$.

Remark 3.2. Every fuzzy weakly (r,s)-continuous mapping is fuzzy S-weakly (r,s)-continuous but the converse is not always true.

fuzzy (r,s)-continuous \Rightarrow fuzzy weakly (r,s)-continuous \Rightarrow fuzzy S-weakly (r,s)-continuous

Example 3.3. Let $X=\{x,y\}$ and A_1 , A_2 , σ and μ be intuitionistic fuzzy sets of X defined as

$$\begin{split} &A_1(x) \!\!=\!\! (0.2,\!0.8), \ A_1(y) \!\!=\!\! (0.5,\,0.5); \\ &A_2(x) \!\!=\!\! (0.5,\!0.2), \ A_2(y) \!\!=\!\! (0.5,\!0.5); \\ &\mu(x) \!\!=\!\! (0.6,\!0.2), \quad \mu(y) \!\!=\!\! (0.5,\!0.5); \\ &\sigma(x) \!\!=\!\! (0.2,\!0.7), \quad \sigma(y) \!\!=\!\! (0.5,\!0.5). \end{split}$$

Define an SoIFT $T_1: I(X) \to I \otimes I$ by

$$\begin{split} T_1(A) &= (\ V_1(A), \quad V_2(A)) \\ &= \begin{cases} (1,0), & \text{if } A = 0_-, 1_-, \\ (\frac{1}{2},\frac{1}{3}), & \text{if } A = A_1, A_2, A_1 \cup A_2, \\ (0,1), & \text{otherwise} : \end{cases} \end{split}$$

and an $T_2: I(X) \to I \otimes I$ by

$$\begin{split} T_2(A) &= (U_1(A), \ U_2(A)) \\ &= \begin{cases} (1,0), & \text{if } A = 0_{\sim}, 1_{\sim}, \\ (\frac{1}{2}, \frac{1}{3}), & \text{if } A = \mu, \ \sigma, \\ (0,1), & otherwise. \end{cases} \end{split}$$

Consider a mapping $f:(X,V_1,V_2)\to (X,U_1,U_2)$ defined as follows f(x)=x for all $x\in X$.

Since $\operatorname{cl}(\mu,\frac{1}{2},\frac{1}{3})=\sigma^c$ is a fuzzy $(\frac{1}{2},\frac{1}{3})$ -semiopen set in an SoIFTS (X,T_1,T_2) , clearly f is a fuzzy S-weakly $(\frac{1}{2},\frac{1}{3})$ -continuous mapping but it is not fuzzy weakly $(\frac{1}{2},\frac{1}{3})$ -continuous.

Theorem 3.4. Let $f:(X,T_1,T_2) \to (Y,U_1,U_2)$ be a mapping on two SoIFTS's X, Y and (α,β) , $(r,s) \in I \otimes I$. Then f is fuzzy S-weakly (r,s)-continuous if and only if for every intuitionistic fuzzy point $x_{(\alpha,\beta)}$ and each fuzzy (r,s)-open set V containing $f(x_{(\alpha,\beta)})$, there exists a fuzzy (r,s)-semiopen set U containing $x_{(\alpha,\beta)}$ such that $f(U) \subseteq \operatorname{cl}(V,r,s)$.

Proof. Suppose that f is a fuzzy S-weakly (r,s)- continuous mapping. Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and V a fuzzy (r,s)-semiopen set containing $f(x_{(\alpha,\beta)})$; then there exists a fuzzy (r,s)-open set B such that $f(x_{(\alpha,\beta)}) \in B \subseteq V$. Since f is fuzzy S-weakly (r,s)-continuous, it follows

$$f^{-1}(B) \subseteq \operatorname{sint}(f^{-1}(\operatorname{cl}(B,r,s)),r,s)$$
$$\subseteq \operatorname{sint}(f^{-1}(\operatorname{cl}(V,r,s)),r,s).$$

Set $U=\operatorname{sint}(f^{-1}(\operatorname{cl}(V,r,s)),r,s);$ then U is a fuzzy

(r,s)-semiopen set such that $x_{(\alpha,\beta)} \in f^{-1}(B) \subseteq U \subseteq f^{-1}(cl(V,r,s))$. So $f(U) \subseteq cl(V,r,s)$.

For the converse, let V be a fuzzy (r,s)-open set in Y. For each $x_{(\alpha,\beta)} \in f^{-1}(V)$, by hypothesis, there exists a fuzzy (r,s)-semiopen set $U_{x_{(\alpha,\beta)}}$ containing $x_{(\alpha,\beta)}$ such that $f(U_{x_{(\alpha,\beta)}}) \subseteq \operatorname{cl}(V,r,s)$. This implies $x_{(\alpha,\beta)} \in U_{x_{(\alpha,\beta)}} \subseteq f^{-1}(\operatorname{cl}(V,r,s))$. Thus we have

$$\begin{split} f^{-1}(V) &\subseteq \cup \{ U_{x_{(\alpha,\beta)}} &: x_{(\alpha,\beta)} \in f^{-1}(V) \} \\ &\subseteq f^{-1}(\operatorname{cl}(V,r,s)). \end{split}$$

Since \cup { $U_{x_{(\alpha,\beta)}}$: $x_{(\alpha,\beta)}$ \in $f^{-1}(V)$ } is a fuzzy (r,s) -semiopen set,

$$f^{-1}(V) \subseteq \operatorname{sint}(f^{-1}(\operatorname{cl}(V,r,s)),r,s).$$

Hence f is fuzzy S-weakly (r, s)-continuous.

Theorem 3.5. Let $f:(X,T_1,T_2)\to (Y,U_1,U_2)$ be a mapping on two SoIFTS's X, Y and (α,β) , $(r,s)\in I\otimes I$. Then the following statements are equivalent:

- (1) f is fuzzy S-weakly (r,s)-continuous.
- (2) $\operatorname{scl}(f^{-1}(\operatorname{int}(P,r,s)),r,s) \subseteq f^{-1}(P)$ for each fuzzy (r,s) -closed set P in Y.
- (3) $\operatorname{cl}(f^{-1}(\operatorname{int}(\operatorname{cl}(B,r,s),r,s)),r,s) \subseteq f^{-1}(\operatorname{cl}(B,r,s))$ for each fuzzy intuitionistic fuzzy set B in Y.
- (4) $f^{-1}(\operatorname{int}(B,r,s)) \subseteq \operatorname{sint}(f^{-1}(\operatorname{cl}(\operatorname{int}(B,r,s),r,s)),r,s)$ for each fuzzy intuitionistic fuzzy set B in Y.
- (5) $\operatorname{scl}(f^{-1}(V),r,s) \subseteq f^{-1}(\operatorname{cl}(V,r,s))$ for a fuzzy (r,s) -open set V in Y.

Proof. (1) \Rightarrow (2) Let P be any fuzzy (r,s)-closed set of Y. Then $1_{\sim} P$ is a fuzzy (r,s)-open set in Y and by (1),

$$\begin{split} f^{-1}(1_{\sim}-P) &\subseteq & \operatorname{sint}(f^{-1}(\operatorname{cl}(1_{\sim}-P,\ r,s)),r,s) \\ &= & \operatorname{sint}(f^{-1}(1_{\sim}-\operatorname{int}(P,r,s)),\ r,s) \\ &= & \operatorname{sint}(1_{\sim}-f^{-1}(\operatorname{int}(P,r,s)),r,s) \\ &= & 1_{\sim}-\operatorname{scl}(f^{-1}(\operatorname{int}(P,r,s)),r,s). \end{split}$$

Hence we have $\operatorname{scl}(f^{-1}(\operatorname{int}(P,r,s)),r,s)\subseteq f^{-1}(P).$

(2) \Rightarrow (3) Let B be any intuitionistic fuzzy set in Y. Since $\mathrm{cl}(B,r,s)$ is a fuzzy (r,s)-closed set in Y, by (2),

$$scl(f^{-1}(int(cl(B,r,s),r,s))) \subseteq f^{-1}(cl(B,r,s)).$$

(3) \Rightarrow (4) Let B be any intuitionistic fuzzy set of Y. Then, from (3),

$$f^{-1}(\text{int}(B,\mathbf{r},\mathbf{s})) = 1_{\sim} - (f^{-1}(\text{cl}(1_{\sim} - B, r, s)))$$

$$\subseteq 1_{\sim} - \text{scl}(f^{-1}(\text{int}(\text{cl}(1_{\sim} - B, r, s), r, s)), r, s)$$

$$= \text{sint}(f^{-1}(\text{cl}(\text{int}(B, r, s), r, s)), r, s).$$

Hence, (4) is obtained.

(4) \Rightarrow (5) Let V be any fuzzy (r,s)-open set of Y. Then from (4) and $(V,r,s)\subseteq \operatorname{int}(\operatorname{cl}(V,r,s),r,s)$, it follows:

$$\begin{split} 1_{\sim} - f^{-1}(\text{cl}(\ V, r, s)) &= f^{-1}(\text{int}(1_{\sim} - V, r, s)) \\ &\subseteq \text{sint}(f^{-1}(\text{cl}(\text{int}(1_{\sim} - V, r, s), r, s)), r, s)) \\ &= \text{sint}(1_{\sim} - (f^{-1}(\text{int}(\text{cl}(\ V, r, s), r, s)), r, s)) \\ &= 1_{\sim} - \text{scl}(f^{-1}(\text{int}(\text{cl}(\ V, r, s), r, s)), r, s)) \\ &\subseteq 1_{\sim} - \text{scl}(f^{-1}(\ V, r, s). \end{split}$$

Hence we have $\operatorname{scl}(f^{-1}(V),r,s) \subseteq f^{-1}(\operatorname{cl}(V,r,s))$.

(5) \Rightarrow (1) Let V be a fuzzy (r,s)-open set in Y. By (V, r,s) \subseteq int(cl(V,r,s),r,s)) and (5),

$$\begin{split} f^{-1}(\ V) &\subseteq f^{-1}(\mathrm{int}(\mathrm{cl}(\ V,r,s),r,s)) \\ &= 1_{\sim} - f^{-1}(\mathrm{cl}(1_{\sim} - \mathrm{cl}(\ V,r,s),r,s)) \\ &\subseteq 1_{\sim} - \mathrm{scl}(f^{-1}(1_{\sim} - \mathrm{cl}(\ V,r,s)),r,s) \\ &= \mathrm{sint}(f^{-1}(\mathrm{cl}(\ V,r,s)),r,s). \end{split}$$

Hence f is a fuzzy S-weakly (r,s)-continuous.

Theorem 3.6 Let $f:(X,T_1,T_2) \to (Y,U_1,U_2)$ be a mapping on two SoIFTS's X, Y and (α,β) , $(r,s) \in I \otimes I$. Then the following statements are equivalent:

- (1) f is fuzzy S-weakly (r,s)-continuous.
- (2) $\operatorname{scl}(f^{-1}(\operatorname{int}(\operatorname{cl}(G,r,s),r,s)),r,s)\subseteq f^{-1}(\operatorname{cl}(G,r,s))$ for each fuzzy (r,s)-open set G in Y.
- (3) $\operatorname{scl}(f^{-1}(\operatorname{int}(\operatorname{cl}(V,r,s),r,s)),r,s) \subseteq f^{-1}(\operatorname{cl}(V,r,s))$ for each fuzzy (r,s)-preopen set V in Y.
- (4) $\operatorname{scl}(f^{-1}(\operatorname{int}(K,r,s)),r,s) \subseteq f^{-1}(K)$ for each fuzzy (r,s)-regular closed set K in Y.
- (5) $\operatorname{scl}(f^{-1}(\operatorname{int}(\operatorname{cl}(G,r,s),r,s)),r,s) \subseteq f^{-1}(\operatorname{cl}(G,r,s))$ for each fuzzy $(r,s)-\beta$ -open set G in Y.
- (6) $\operatorname{scl}(f^{-1}(\operatorname{int}(\operatorname{cl}(G,r,s),r,s)),r,s) \subseteq f^{-1}(\operatorname{cl}(G,r,s))$ for each fuzzy (r,s)-semiopen set G in Y.

Proof. (1) \Rightarrow (2) Let G be a fuzzy (r,s)-open set of Y. Then by Theorem 3.5 (3), we have

$$\operatorname{scl}(f^{-1}(\operatorname{int}(\operatorname{cl}(G,r,s),r,s)),r,s) \subseteq f^{-1}(\operatorname{cl}(G,r,s)).$$

(2) \Rightarrow (3) Let V be a fuzzy (r,s)-preopen set of Y. Set $A=\operatorname{int}(\operatorname{cl}(V,r,s),r,s)$. Then since A is a fuzzy (r,s)-open set, from (2), it follows

$$\operatorname{scl}(f^{-1}(\operatorname{int}(\operatorname{cl}(A,r,s),r,s)),r,s) \subseteq f^{-1}(\operatorname{cl}(A,r,s)).$$

Since cl(A,r,s)=cl(V, r,s), we have

$$\operatorname{scl}(f^{-1}(\operatorname{int}(\operatorname{cl}(V,r,s),r,s)),r,s) \subseteq f^{-1}(\operatorname{cl}(V,r,s)).$$

(3) \Rightarrow (4) Let K be a fuzzy (r,s)-regular closed set of Y. Since $\operatorname{int}(K,r,s)$ is fuzzy (r,s)-preopen, by (3),

$$\operatorname{scl}(f^{-1}(\operatorname{int}(\operatorname{cl}(\operatorname{int}(K,r,s),r,s),r,s)),r,s) \subseteq f^{-1}(\operatorname{cl}(\operatorname{int}(K,r,s),r,s)).$$

Then from int(K,r,s)=int(cl(int(K,r,s),r,s),r,s), we

have $scl(f^{-1}(int(K,r,s)),r,s) \subseteq f^{-1}(K)$.

- (4) \Rightarrow (5) Let G be a fuzzy $(r,s)-\beta$ -open set. Then $G \subseteq \operatorname{cl}(\operatorname{int}(\operatorname{cl}(G,r,s),r,s),r,s)$ and $\operatorname{cl}(G,r,s)$ is a fuzzy (r,s)-regular closed set. Hence by (4), we have $\operatorname{scl}(f^{-1}(\operatorname{int}(\operatorname{cl}(G,r,s),r,s)),r,s)\subseteq f^{-1}(\operatorname{cl}(G,r,s)).$
- $(5) \Rightarrow (6)$ It is obvious.
- (6) \Rightarrow (1) Let V be a fuzzy (r,s)-open set; then since V is a fuzzy (r,s)-semiopen set, by (6) and $V\subseteq$ int(cl(V,r,s),r,s), we have

$$\operatorname{scl}(f^{-1}(V), r, s) \subseteq \operatorname{scl}(f^{-1}(\operatorname{int}(\operatorname{cl}(V, r, s), r, s)), r, s)$$

$$\subseteq f^{-1}(\operatorname{cl}(V, r, s)).$$

Hence, f is fuzzy S-weakly (r,s)-continuous.

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