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On the Minimax Disparity Obtaining OWA Operator Weights

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Abstract

The determination of the associated weights in the theory of ordered weighted averaging (OWA) operators is one of the important issue. Recently, Wang and Parkan [Information Sciences 175 (2005) 20-29] proposed a minimax disparity approach for obtaining OWA operator weights and the approach is based on the solution of a linear program (LP) model for a given degree of orness. Recently, Liu [International Journal of Approximate Reasoning, accepted] showed that the minimum variance OWA problem of Fullér and Majlender [Fuzzy Sets and Systems 136 (2003) 203-215] and the minimax disparity OWA problem of Wang and Parkan always produce the same weight vector using the dual theory of linear programming. In this paper, we give an improved proof of the minimax disparity problem of Wang and Parkan while Liu's method is rather complicated. Our method gives the exact optimum solution of OWA operator weights for all levels of orness, $0 \leq \alpha \leq 1$, whose values are piecewise linear and continuous functions of α.

Key words : OWA operator; Operator weights; Degree of orness; Minimax disparity

1. Introduction

Yager [10] introduced a new aggregation technique based on the ordered weighted averaging(OWA) operators. An OWA operator of dimension n is a mapping $F : R^n \to R$ that has an associated weighting vector $W = (w_1, \cdots, w_n)^T$ of having the properties $w_1 + \cdots + w_n = 1,\ 0 \leq w_i \leq 1,\ i = 1, \cdots, n$, and such that

$$F(a_1, \cdots, a_n) = \sum_{i=1}^{n} w_i a_i,$$

where b_j is the jth largest element of the collection of the aggregated objects $\{a_1, \cdots, a_n\}$. In [12], Yager introduced a measure of "orness" associated with the weighting vector W of an OWA operator, defined as

$$orness(W) = \sum_{i=1}^{n} \frac{n-i}{n-1} w_i,$$

and it characterizes the degree to which the aggregation is like an *or* operation. One important issue in the theory of ordered weighted averaging operators is the determination of the associated weights. A number of approaches have been suggested for obtaining the associated weights, i.e., quantifier guided aggregation [11, 12], exponential smoothing [1] and learning [12]. Another approaches, suggested by O' Hagan [5], determines a special class of OWA operators having maximal entropy of the OWA weights for a given level of orness; algorithmically it is based on the solution of a constrained optimization problem. Fullér and Majlender [3, 4] showed that the maximum entropy model could be transformed into a polynomial equation that can be solved analytically and suggested a minimum variance approach to obtain the minimal variability OWA weights. Recently, Hong [6] gave a new proof of the minimum variance problem. Liu and Chen [7] suggested a parametric geometric approach that could be used to obtain maximum entropy weights. Recently, Wang and Parkan [13] proposed a minimax disparity approach for obtaining OWA operator weights. They transferred the minimax disparity model to a linear programming model, obtained weights for some special values of orness and proved the dual property of OWA. Recently, Liu [8] showed that the minimum variance problem of Fullér and Majlender [4] and the minimax disparity OWA problem of Wang and Parkan [13] always produce the same weight vector using the dual theory of linear programming [9]. In Section 2, we reconsider the proof of the minimum variance OWA problem of Fullér and Majlender [4]. In Section 3, we give an improved proof of the minimax disparity OWA problem of Wang and Parkan [13].

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2. Preliminaries

Recently, to obtain minimal variability OWA weights under given level of orness, Fullér and Majlender [4] considered the following constrained mathematical programming problem

$$\text{minimize} \quad D^2(W) = \sum_{i=1}^{n} \frac{1}{n}(w_i - E(W))^2 = \frac{1}{n}\sum_{i=1}^{n} w_i^2 - \frac{1}{n^2}.$$

subject to

$$orness(W) = \sum_{i=1}^{n} \frac{n-i}{n-1} w_i = \alpha, \quad 0 \le \alpha \le 1, \quad (1)$$

$$w_1 + \cdots + w_n = 1, 0 \le w_i, i = 1, \cdots, n,$$

where $E(W) = (w_1 + \cdots + w_n)/n$ stands for the arithmetic mean of weights.

We first consider the following disjunctive partition of unit interval (0, 1) presented in Fullér and Majlender [4]:

$$(0,1) = \bigcup_{r=2}^{n-1} J_{r,n} \cup J_{1,n} \cup \bigcup_{s=2}^{n-1} J_{1,s}, \quad (2)$$

where

$$\begin{aligned} J_{r,n} &= (1 - \frac{1}{3}\frac{2n+r-2}{n-1}, 1 - \frac{1}{3}\frac{2n+r-3}{n-1}], \\ &\quad r = 2, \cdots, n-1 \\ J_{1,n} &= (1 - \frac{1}{3}\frac{2n-1}{n-1}, 1 - \frac{1}{3}\frac{n-2}{n-1}), \\ J_{1,s} &= [1 - \frac{1}{3}\frac{s-1}{n-1}, 1 - \frac{1}{3}\frac{s-2}{n-1}), \\ &\quad s = 2, \cdots, n-1 \end{aligned}$$

Suppose that $\alpha \in J_{r,s}$ for some r and s from partition (2). Such r and s always exist for any $\alpha \in (0, 1)$, furthermore, $r = 1$ or $s = n$ should hold. Fullér and Majlender [4] found that the associated minimal variability weighting vector is

$$W^* = (0, \cdots, 0, w_r^*, \cdots, w_s^*, 0, \cdots, 0)$$

where

$$\begin{aligned} w_j^* &= 0 \ \ if \ \ j \notin I_{\{r,s\}} = \{r, \cdots, s\}, & (3) \\ w_r^* &= \frac{2(2s+r-2) - 6(n-1)(1-\alpha)}{(s-r+1)(s-r+2)} & (4) \\ w_s^* &= \frac{6(n-1)(1-\alpha) - 2(s+2r-4)}{(s-r+1)(s-r+2)} & (5) \\ w_j^* &= \frac{s-j}{s-r} w_r^* + \frac{j-r}{s-r} w_s^* \ \ if \ \ j \in I_{\{r+1,s-1\}}. & (6) \end{aligned}$$

According to (6), we have that $w_i^* = a^*i + b^*$ for $i \in I_{\{r,s\}}$ for some a^*, b^*. Using formulas (4) and (5) we find

$$\begin{aligned} & w_r^*, w_s^* \in [0,1] \Leftrightarrow \\ & \alpha \in [1 - \frac{1}{3}\frac{2s+r-2}{n-1}, 1 - \frac{1}{3}\frac{s+2r-4}{n-1}]. \end{aligned} \quad (7)$$

Then, it is also easy to check that $a^*i + b^* \le 0$ for $i \notin I_{\{r,s\}}$. Hence, it is considered that Fullér and Majlender [4] proved the following result. Hong [6] also gave a new simple proof.

Theorem 2.1. ([4, 6]) The optimal weight for the constrained optimization problem (1) should satisfy the equations

$$w_i^* = \begin{cases} a^*i + b^*, & \text{if } i \in I_{\{r,s\}} = \{r, \cdots, s\}, \\ 0 & \text{elsewhere.} \end{cases}$$

for some a^*, b^* satisfying $a^*i + b^* \le 0$ for $i \notin I_{\{r,s\}}$ and $r = 1$ or $s = n$.

Here, a^* and b^* can be obtained as (see [6])

$$a^* = \frac{6(r+s-2n+2n\alpha-2\alpha)}{(s-r+1)(r-s)(s-r+2)}, \quad (8)$$

$$b^* = \frac{2[3(r+s)(n-n\alpha+\alpha)-(2s^2+2r^2+2rs+s-r)]}{(s-r+1)(r-s)(s-r+2)}. \quad (9)$$

Then, from the equality $w_i^* = a^*i + b^*, i = r, \cdots, s$, we can get

$$\begin{aligned} w_j^* &= 0 \ \text{ if } \ j \notin I_{\{r,s\}}, \\ w_r^* &= a^*r + b^* = \frac{2(2s+r-2)-6(n-1)(1-\alpha)}{(s-r+1)(s-r+2)} \\ w_s^* &= a^*s + b^* = \frac{6(n-1)(1-\alpha)-2(s+2r-4)}{(s-r+1)(s-r+2)} \\ w_j^* &= a^*j + b^* = \frac{s-j}{s-r} w_r^* + \frac{j-r}{s-r} w_s^* \ \text{ if } \ j \in I_{\{r+1,s-1\}}. \end{aligned}$$

3. Obtaining minimax disparity OWA operator weights

The approach suggested by Wang and Parkan [13] is based on the solution of the following constrained mathematical programming problem:

$$\text{Minimize} \quad Max_{i\in\{1,\cdots,n-1\}} |w_i - w_{i+1}|$$

subject to

$$orness(W) = \sum_{i=1}^{n} \frac{n-i}{n-1} w_i = \alpha, \quad (10)$$

$$0 \le \alpha \le 1,$$

$$w_1 + \cdots + w_n = 1, 0 \le w_i, i = 1, \cdots, n.$$

They transferred the minimax disparity model (7) to a LP model shown below:

$$\begin{aligned}
&\text{Minimize} \quad \delta,\\
&\text{subject to}\\
&orness(W) = \sum_{i=1}^{n} \frac{n-i}{n-1} w_i = \alpha, 0 \leq \alpha \leq 1, \quad (11)\\
&w_1 + \cdots + w_n = 1,\\
&w_i - w_{i+1} - \delta \leq 0,\ i = 1, \cdots, n-1\\
&w_i - w_{i+1} + \delta \geq 0,\ i = 1, \cdots, n-1\\
&w_i \geq 0, i = 1, \cdots, n.
\end{aligned}$$

And Wang and Parkan [13] have proven the following theorem on the LP model (11).

Theorem 3.1. ([13]) For an OWA operator weight vector $W = (w_1, \cdots, w_n)^T$ determined by the LP (11)

$$\begin{aligned}
&\text{if } orness(W) = 1, \text{ then } W = (1, 0, \cdots, 0)^T;\\
&\text{if } orness(W) = 0, \text{ then } W = (0, \cdots, 0, 1)^T;\\
&\text{if } orness(W) = 0.5, \text{ then } W = (1/n, \cdots, 1/n)^T.
\end{aligned}$$

Liu [8] used the following dual problem of (11):

$$\begin{aligned}
&\text{Maximize} \quad \omega = \alpha\lambda_1 + \lambda_2,\\
&\text{subject to} \quad \lambda_1 + \lambda_2 + \mu_1 + \mu_2 \leq 0, \quad (12)\\
&\frac{n-i}{n-1}\lambda_1 + \lambda_2 - \mu_{2i-3} + \mu_{2i-2} + \mu_{2i-1} - \mu_{2i} \leq 0,\\
&\qquad i = 1,\ 2, \cdots, n-1,\\
&\lambda_2 - \lambda_{2n-3} + \lambda_{2n-2} \leq 0,\\
&-\sum_{i=1}^{2(n-1)} \mu_i = 1,\\
&\qquad \mu_i \leq 0,\ \ i = 1,\ 2, \cdots, 2(n-1).
\end{aligned}$$

With the dual theory of linear programming [9], Liu [8] proved the optimal solution of (10) is also the optimal solution of (12) in the five cases for different orness levels respectively.

In this paper, we shall give a simple direct proof of the constrained optimization problem (10) analytically for all levels of orness, $0 \leq \alpha \leq 1$.

Let us consider the constrained optimization problem (10).

Theorem 3.2. The optimal weight for the constrained optimization problem (1) for a given level of $\alpha = orness(W)$ should satisfy the equations

$$w_i^* = \begin{cases} a^*i + b^*, & \text{if } i \in \{r, \cdots, s\},\\ 0 & \text{elsewhere}\end{cases}$$

where either $r = 1$ (for $0.5 \leq \alpha \leq 1$) or $s = n$ (for $0 \leq \alpha \leq 0.5$), equivalently, $w_i^* = max\{a^*i + b^*, 0\}$, $i = 1, \cdots, n$, and hence we have that

$$Minimize\{Max_{i\in\{1,\cdots,n-1\}}|w_i - w_{i+1}|\} = |a^*|.$$

Proof. If $\alpha = 1$, it is clear that $w_1^* = a^* + b^* = 1$ and 0, otherwise. Now, by the dual property [13] of OWA operator to generate OWA operator weights, we may assume that $0.5 \leq \alpha < 1$. Then there exist a^*, b^* and such that $a^* \leq 0$ and

$$w_i^* = max\{a^*i + b^*, 0\},\ i = 1, \cdots, n,$$

satisfying

$$\begin{aligned}
\sum_{i=1}^{n} i w_i^* &= n - (n-1)\alpha,\\
\sum_{i=1}^{n} w_i^* &= 1,\ 0 \leq w_i^*,\ i = 1, \cdots, n.
\end{aligned}$$

Then we clearly have that

$$Max_{i\in\{1,\cdots,n-1\}}|w_i^* - w_{i+1}^*| = |a^*|$$

Actually, we show that $W^* = (w_1^*, \cdots, w_s^*, 0, , \cdots, 0)$ is the unique optimum solution of the constrained optimization problem (10) that satisfies the orness constraint.

Let $w_i, i = 1, \cdots, n$ satisfy

$$\sum_{i=1}^{n} i w_i = n - (n-1)\alpha, \quad (13)$$

$$\sum_{i=1}^{n} w_i = 1,\ 0 \leq w_i,\ i = 1, \cdots, n. \quad (14)$$

and suppose that

$$Max_{i\in\{1,\cdots,n-1\}}|w_i - w_{i+1}| < |a^*|. \quad (15)$$

Then we should have that $w_1 < w_1^* = a^* + b^*$. Assume that $w_1 \geq w_1^* = a^* + b^*$. Then by (15), we have

$$w_2 > w_1 - |a^*| \geq w_1^* - |a^*| = w_1^* + a^* = w_2^*$$

Similarly, by induction, we have that $w_i > w_i^*$ for all $i = 3, \cdots, n$. Then we have that

$$\sum_{i=1}^{n} w_i > \sum_{i=1}^{n} w_i^* = 1,$$

which is contradictory to (14). It follows that $w_1 < w_1^* = a^* + b^*$.

Now, since $w_1 < w_1^*$ and $\sum_{i=1}^{n} w_i = \sum_{i=1}^{n} w_i^* = 1$, there exists i_0 such that $w_{i_0} > w_{i_0}^*$ and $w_i \leq w_i^*$ for $i = 1, \cdots, i_0 - 1$. Then by (15), we have

$$w_{i_0+1} > w_{i_0} - |a^*| \geq w_{i_0}^* - |a^*| = w_{i_0}^* + a^* = w_{i_0+1}^* \quad (16)$$

Similarly, by induction, we have $w_i > w_i^*$ for all $i = i_0+2,\cdots,n$. Then we have that

$$\begin{aligned}\sum_{i=1}^{n} iw_i - \sum_{i=1}^{n} iw_i^* &= \sum_{i=1}^{n} i(w_i - w_i^*)\\ &= \sum_{i<i_0} i(w_i - w_i^*) + \sum_{i\geq i_0} i(w_i - w_i^*)\\ &\geq \sum_{i<i_0} i(w_i - w_i^*) + i_0\sum_{i\geq i_0}(w_i - w_i^*) \qquad (17)\\ &= \sum_{i<i_0} i(w_i - w_i^*) - i_0\sum_{i<i_0}(w_i - w_i^*)\\ &= \sum_{i<i_0}(i-i_0)(w_i - w_i^*) > 0,\end{aligned}$$

where the second equality comes from the fact that $\sum_{i<i_0}(w_i - w_i^*) = -\sum_{i\geq i_0}(w_i - w_i^*)$. This means that $\sum_{i=1}^{n} iw_i > \sum_{i=1}^{n} iw_i^* = n-(n-1)\alpha$, which is contradictory to (13). It follows that

$$Minimize\{Max_{i\in\{1,\cdots,n-1\}}|w_i - w_{i+1}|\} = |a^*|.$$

Now, we show that W^* is the unique feasible solution satisfying

$$Minimize\{Max_{i\in\{1,\cdots,n-1\}}|w_i - w_{i+1}|\} = |a^*|.$$

Let $w_i, i=1,\ \cdots,n$ satisfy

$$\begin{aligned}\sum_{i=1}^{n} iw_i &= n-(n-1)\alpha,\\ \sum_{i=1}^{n} w_i &= 1,\ 0\leq w_i,\ i=1,\cdots,n. \qquad (18)\end{aligned}$$

and suppose that

$$Max_{i\in\{1,\cdots,n-1\}}|w_i - w_{i+1}| = |a^*|. \qquad (19)$$

If we assume that $w_1 < w_1^*$, then we have a contradiction by a similar way as in (16) and (17). So we may assume that $w_1 \geq w_1^*$. Then by (19), we have

$$w_2 \geq w_1 - |a^*| \geq w_1^* - |a^*| = w_1^* + a^* = w_2^* \qquad (20)$$

Similarly, by induction, we have that $w_i \geq w_i^*$ for all $i=3,\cdots,n$. Then by (18)

$$0 = \sum_{i=1}^{n}(w_i - w_i^*).$$

Since $w_i - w_i^* \geq 0,\ i=1,2,\cdots,n$, we have that $w_i = w_i^*$ for all $i=1,\cdots,n$, which completes the proof. □

4. Numerical example

We consider the same numerical example that illustrates the application of the minimax disparity approach as Wang and Parkan [13] presented. Suppose n = 5. Using (7)(or see [4]), we find that

$$\begin{aligned}\alpha &\in [\frac{11}{12},\frac{12}{12}) &\Leftrightarrow\quad r=1,s=2,\\ \alpha &\in [\frac{10}{12},\frac{11}{12}) &\Leftrightarrow\quad r=1,s=3,\\ \alpha &\in [\frac{9}{12},\frac{10}{12}) &\Leftrightarrow\quad r=1,s=4,\\ \alpha &\in (\frac{3}{12},\frac{9}{12}) &\Leftrightarrow\quad r=1,s=5,\\ \alpha &\in [\frac{2}{12},\frac{3}{12}) &\Leftrightarrow\quad r=2,s=5,\\ \alpha &\in (\frac{1}{12},\frac{2}{12}] &\Leftrightarrow\quad r=3,s=5,\\ \alpha &\in (\frac{0}{12},\frac{1}{12}] &\Leftrightarrow\quad r=4,s=5.\end{aligned}$$

Let $\alpha \in [11/12,\ 12/12)$ then $r=1, s=2$. Using formulas (4)-(6), (8) and (9), we have the optimum solution of OWA operator weights;

$$a^* = 7-8\alpha,\ \ b^* = 12\alpha - 10,$$
$$w_1^* = 4\alpha-3,\ \ w_2^* = 4-4\alpha,\ \ w_3^* = w_4^* = w_5^* = 0.$$

The OWA operator weights for all levels of orness are shown in Table 1, and plotted in Fig.1. Using LP, Wang and Parkan [13] obtained a series of OWA operator weights that satisfy different levels of orness : $\alpha = 0,\ 0.1,\ \cdots,\ 0.9,\ 1$. It was shown in Fig.1 [13] that they are linear functions whose graphs are the straight line connecting $(k/10, w_i(k/10))$ to $((k+1)/10, w_i((k+1)/10))$ between $k/10$ and $(k+1)/10$ where are $i=1,\cdots,5$ and $k=0,\cdots,9$. In fact, when using LP, we do not know that w_i's are linear between $k/10$ and $(k+1)/10$. But we can guess they are linear if we try to solve for a large number of orness levels of orness. However, our method completely overcomes this problem and gives the exact optimum solution of OWA operator weights for all levels of orness, $0\leq\alpha\leq 1$, whose values are piecewise linear and continuous functions of α. Actually, the graph of OWA operator weights in Fig.1 of Wang and Parkan [13] is not clear. According to our result, the break points of levels of orness are $\alpha = 1/12,\ 2/12,\ 3/12,\ 9/12,\ 10/12,\ 11/12$.

W	orness(W)=α $[\frac{11}{12}, \frac{12}{12})$	$[\frac{10}{12}, \frac{11}{12})$	$[\frac{9}{12}, \frac{10}{12})$	$(\frac{3}{12}, \frac{9}{12})$	$[\frac{2}{12}, \frac{3}{12})$	$(\frac{1}{12}, \frac{2}{12}]$	$(\frac{0}{12}, \frac{1}{12}]$
r, s	$1, 2$	$1, 3$	$1, 4$	$1, 5$	$2, 5$	$3, 5$	$4, 5$
a^*	$7 - \frac{8}{\alpha}$	$\frac{3}{2} - \frac{2}{\alpha}$	$\frac{1}{2} - \frac{4}{5}\alpha$	$\frac{1}{5} - \frac{2}{5}\alpha$	$\frac{3}{10} - \frac{4}{5}\alpha$	$\frac{1}{2} - 2\alpha$	$1 - 8\alpha$
b^*	$-10 + 12\alpha$	$-\frac{8}{3} + 4\alpha$	$-1 + 2\alpha$	$-\frac{2}{5} + \frac{6}{5}\alpha$	$-\frac{4}{5} + \frac{14}{5}\alpha$	$-\frac{5}{3} + 8\alpha$	$-4 + 36\alpha$
w_1^*	$-3 + 4\alpha$	$-\frac{7}{6} + 2\alpha$	$-\frac{2}{1} + \frac{6}{5}\alpha$	$-\frac{1}{5} + \frac{4}{5}\alpha$	0	0	0
w_2^*	$4 - 4\alpha$	$-\frac{1}{3}$	$\frac{2}{5}\alpha$	$\frac{2}{5}\alpha$	$-\frac{1}{5} + \frac{6}{5}\alpha$	0	0
w_3^*	0	$\frac{11}{6} - 2\alpha$	$\frac{1}{2} - \frac{2}{5}\alpha$	$\frac{1}{5}$	$\frac{1}{10} + \frac{2}{5}\alpha$	$-\frac{1}{6} + 2\alpha$	0
w_4^*	0	0	$1 + \frac{6}{5}\alpha$	$\frac{2}{5} + \frac{2}{5}\alpha$	$\frac{2}{5} + \frac{2}{5}\alpha$	$\frac{1}{3}$	4α
w_5^*	0	0	0	$\frac{3}{5} - \frac{4}{5}\alpha$	$\frac{7}{10} - \frac{6}{5}\alpha$	$\frac{5}{6} - 2\alpha$	$1 - 4\alpha$

Table I. The OWA operator weights generated by the minmax disparity approach

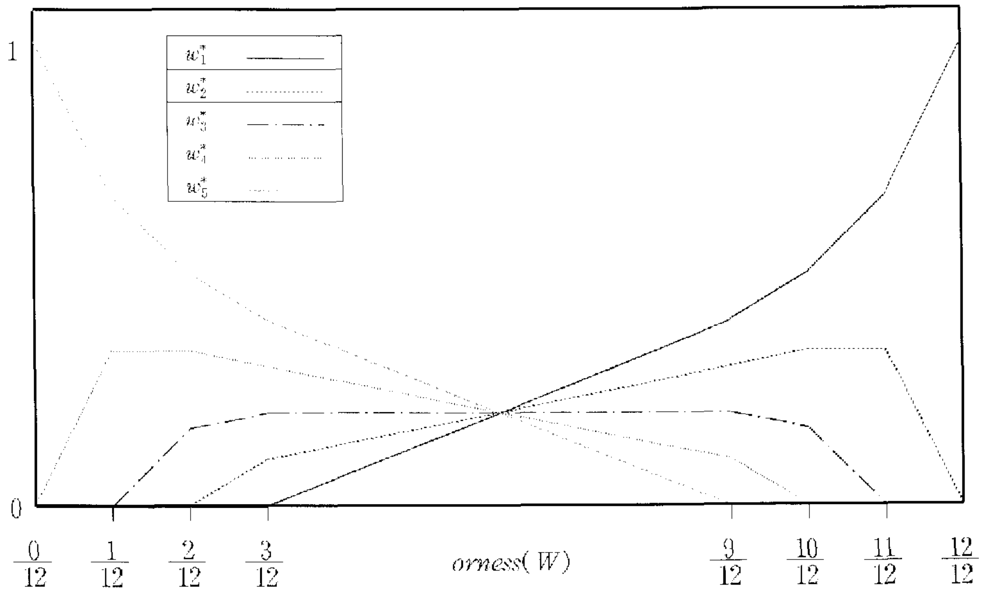

Figure 1. The variation of the minimax disparity OWA operator weights.

5. Conclusion

This paper proposed an improvement proof for the solution equivalence of the minimum variance OWA problem and the minimax disparity OWA problem. Our method give the exact optimum solution of OWA operator weights for all levels of orness, $0 \leq \alpha \leq 1$, whose values are piecewise linear and continuous functions of α. We have illustrated the application of our approach by the same numerical example and verified our approach gives more precise information in generating valid OWA operator weights.

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References

[1] D. Filev & R. R. Yager, *On the issue of obtaining OWA operator weights*, Fuzzy Sets and Systems **94**(1988), 157-169.

[2] D. Filev & R. R. Yager, *Analytic approach of maximal entropy OWA operators*, Information Sciences **85**(1995), 11-27.

[3] R. Fullér & P. Majlender, *An analytic approach for obtaining maximal entropy OWA operators weights*, Fuzzy Sets and Systems **124**(2001), 53-57.

[4] R. Fullér & P. Majlender, *On obtaining minimal variability OWA operator weights*, Fuzzy Sets and Systems **136**(2003), 203-215.

[5] M. O' Hagan, *Aggregating template or rule antecedents in real-time expert systems with fuzzy set logic*, in:Proc. 22nd annual IEEE Asilomar Conf. on Signals, Systems , Computers, Pacific Grove, CA(1988), 81-689.

[6] D. H. Hong, *A note on the minimal variability OWA operator weights*, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems **14**(2006), 747-752.

[7] X. Liu & L. Chen, *On the properties of parametric geometric OWA operator*, International Journal of Approximate Reasoning **35**(2004), 163-178.

[8] X. Liu, *The solution equivalence of minimax disparity and minimum variance problems for OWA operators*, International Journal of Approximate Reasoning(2006), doi:10.1016/j.ijar.2006.06.004.

[9] M. Payne, *Operations Research*: Applications and Algorithms, second ed., PWS-Kent publishing company.

[10] R. R. Yager, *Ordered weighted averaging aggregation operators in multi-criteria decision making*, IEEE Trans. Systems Man Cybernet. **18**(1988), 183-190.

[11] R. R. Yager, *Families of OWA operators*, Fuzzy Sets and Systems **59**(1993), 125-148.

[12] R. R. Yager & D. Filev, *Induced ordered weighted averaging operators*, IEEE Trans. Systems, Man Cybernet. -Part B: Cybernetics **29**(1999), 141-150.

[13] Y. M. Wang & C.A. Parkan, *Minimax disparity approach obtaining OWA operator weights*, Information Sciences **175**(2005), 20-29.

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