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Multiple Attribute Group Decision Making Problems Based on Fuzzy Number Intuitionistic Fuzzy Information

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Abstract

Fuzzy number intuitionistic fuzzy sets (FNIFSs), each of which is characterized by a membership function and a non-membership function whose values are trigonometric fuzzy number rather than exact numbers, are a very useful means to describe the decision information in the process of decision making. Wang [10] developed some arithmetic aggregation operators, such as the fuzzy number intuitionistic fuzzy weighted averaging (FIFWA) operator, the fuzzy number intuitionistic fuzzy ordered weighted averaging (FIFOWA) operator and the fuzzy number intuitionistic fuzzy hybrid aggregation (FIFHA) operator. In this paper, based on the FIFHA operator and the FIFWA operator, we investigate the group decision making problems in which all the information provided by the decision-makers is presented as fuzzy number intuitionistic fuzzy decision matrices where each of the elements is characterized by fuzzy number intuitionistic fuzzy numbers, and the information about attribute weights is partially known. An example is used to illustrate the applicability of the proposed approach.

Key words : Multiple attribute group decision making, fuzzy number intuitionistic fuzzy decision matrix, fuzzy number intuitionistic fuzzy number, FIFHA operator, FIFWA operator.

1. Introduction

Interval-valued intuitionistic fuzzy sets, introduced by Atanassov and Gargov [1], each of which is characterized by a membership function and a non-membership function whose values are intervals rather than exact numbers, are a very useful means to describe the decision information in the process of decision making. Some researcher have applied the interval-valued intuitionistic fuzzy set theory to the field of decision making. Xu and Chen [2] defined some operational laws of interval-valued intuitionistic fuzzy values and, based on these operational laws, developed some arithmetic aggregation operators, such as the interval-valued intuitionistic fuzzy weighted averaging (IIFWA) operator, the interval-valued intuitionistic fuzzy ordered weighted averaging (IIFOWA) operator and the interval-valued intuitionistic fuzzy hybrid aggregation (IIFHA) operator for aggregating interval-valued intuitionistic fuzzy information, and gave an application of the IIFHA operator to multiple attribute group decision making with interval-valued intuitionistic fuzzy information. Xu [3] developed some geometric aggregation operator, such as the interval-valued intuitionistic fuzzy geometric (IIFG) operator and the interval-valued intuitionistic fuzzy weighted geometric (IIFWG) operator and applied them to multi-attribute group decision making with interval-valued intuitionistic fuzzy information. Xu and Chen [4] and Wei and Wang [5], respectively, developed some geometric aggregation operator, such as the interval-valued intuitionistic fuzzy ordered weighted geometric (IIFOWG) operator and the interval-valued intuitionistic fuzzy hybrid geometric (IIFHG) operator and applied them to multi-attribute group decision making with interval-valued intuitionistic fuzzy information. Park et al. [6] investigated the group decision making problems in which all the information provided by the decision-makers is presented as interval-valued intuitionistic fuzzy decision matrices where each of the elements is characterized by interval-valued intuitionistic fuzzy number, and the information about attribute weights is partially known.

Recently, Liu and Yuan [7] also extended the IFS and introduced the fuzzy number intuitionistic fuzzy set

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(FNIFS) whose fundamental characteristic is that the values of its membership function and non-membership function are trigonometric fuzzy numbers rather than exact numbers. There is a little investigation on aggregation operators for aggregating fuzzy number intuitionistic fuzzy information. Wang [8] developed some geometric aggregation operator, such as the fuzzy number intuitionistic fuzzy weighted geometric (FIFWG) operator, the fuzzy number intuitionistic fuzzy ordered weighted geometric (FIFOWG) and the fuzzy number intuitionistic fuzzy hybrid geometric (FIFHG) operator, and applied them to multiple attribute decision making with fuzzy number intuitionistic fuzzy information. Park et al. [9] investigated the group decision making problems in which the information about attribute weights is partially known based on the FIFHG and FIFWG operators.

In [10], Wang also developed some arithmetic aggregation operators, such as the fuzzy number intuitionistic fuzzy weighted averaging (FIFWA) operator, the fuzzy number intuitionistic fuzzy ordered weighted averaging (FIFOWA) operator and the fuzzy number intuitionistic fuzzy hybrid aggregation (FIFHA) operator, and gave an application of the FIFHA operator to multiple attribute decision making with fuzzy number intuitionistic fuzzy information. However, he used the FIFHA operator in the situation where the information about attribute weights is completely known. In this paper, we investigate the group decision making problems in which all the information provided by the decision-makers is presented as fuzzy number intuitionistic fuzzy decision matrices where each of the elements is characterized by fuzzy number intuitionistic fuzzy number (FNIFN), and the information about attribute weights is partially known. First, we use the FIFHA operator to aggregate all individual fuzzy number intuitionistic fuzzy decision matrices provided by the decision-makers into the collective fuzzy number intuitionistic fuzzy decision matrix, and then we use the score function to calculate the score of each attribute value and construct the score matrix of the collective fuzzy number intuitionistic fuzzy decision matrix. From the score matrix and the given attribute weight information, we establish some optimization models to determine the weights of attributes, and then we use the obtained attribute weights and the FIFWA operator to fuse the fuzzy number intuitionistic fuzzy information in the collective fuzzy number intuitionistic fuzzy decision matrix to get the overall fuzzy number intuitionistic fuzzy values of alternatives by which the raking of all the given alternatives can be found. Finally, a numerical example is used to illustrate the applicability of the proposed method.

2. Basic concepts and relations

Let a set X be fixed and $D[0,1]$ be the set of all closed subintervals of the interval $[0,1]$. An interval-valued intuitionistic fuzzy set (IVIFS) [1, 11] A in X is an object having the form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}, \tag{1}$$

where $\mu_A : X \to D[0,1]$, $\nu_A : X \to D[0,1]$ with the condition $\sup \mu_A(x) + \sup \nu_A(x) \leq 1$ for any $x \in X$.

The intervals $\mu_A(x)$ and $\nu_A(x)$ denote, respectively, the degree of membership and the degree of non-membership of the element x to A. Then for each $x \in X$, $\mu_A(x)$ and $\nu_A(x)$ are closed intervals and their lower and upper end points are denoted by $\mu_{AL}(x)$, $\mu_{AU}(x)$, $\nu_{AL}(x)$ and $\nu_{AU}(x)$, respectively, and thus we can replace (1) with

$$A = \{\langle x, [\mu_{AL}(x), \mu_{AU}(x)], [\nu_{AL}(x), \nu_{AU}(x)]\rangle : x \in X\},$$

where $0 \leq \mu_{AU}(x) + \nu_{AU}(x) \leq 1$ for any $x \in X$.

However, interval lacks gravity center, cannot emphasis center point which is most impossible to be given value, and so it is more suitable that the degree of membership and the degree of non-membership are expressed by trigonometric fuzzy number [12]. In such cases, Liu and Yuan [7] defined the notion of FNIFS as follows.

The trigonometric fuzzy number [12] α on $I = [0,1]$, denoted by $\alpha = (l, p, q)$, is fuzzy set with its membership function defined by

$$\mu_\alpha(x) = \begin{cases} \frac{x-l}{p-l}, & l \leq x \leq p, \\ \frac{x-q}{p-q}, & p \leq x \leq q, \\ 0, & \text{otherwise}, \end{cases} \tag{2}$$

where $x \in I$, $0 \leq l \leq p \leq q \leq 1$, l is lower limit of α, q is upper limit of α, and p is gravity center of α. The set of all trigonometric fuzzy numbers on I will be denoted $F(I)$.

On the basis of document [12], the mean of α is defined as follows:

$$E(\alpha)^{(\theta)} = \frac{(1-\theta)l + p + \theta q}{2}, \quad \theta \in [0,1], \tag{3}$$

where θ is an index that reflects the decision-maker's risk-bearing attitude. If $\theta > 0.5$, then the decision-maker is a risk lover. If $\theta < 0.5$, then the decision maker is a risk averter. In general, let $\theta = 0.5$, then the attitude of the decision-maker is neutral to the risk, and

$$E(\alpha)^{(\theta)} = \frac{l + 2p + q}{4}. \tag{4}$$

A fuzzy number intuitionistic fuzzy set (FNIFS) [7] A in X is an object having the form:

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}, \tag{5}$$

where $\mu_A(x) = (\mu_A^1(x), \mu_A^2(x), \mu_A^3(x))$ and $\nu_A(x) = (\nu_A^1(x), \nu_A^2(x), \nu_A^3(x))$ are two trigonometric fuzzy numbers satisfying the condition $0 \leq \mu_A^3(x) + \nu_A^3(x) \leq 1$ for every $x \in X$. Especially, if each of the trigonometric fuzzy numbers $\mu_A(x)$ and $\nu_A(x)$ contains exactly one element, then the given FNIFS is transformed to intuitionistic fuzzy set [13, 14].

For each FNIFS A in X, we will call

$$\pi_A(x) = 1 - \frac{\mu_A^1(x) + 2\mu_A^2(x) + \mu_A^3(x)}{4} - \frac{\nu_A^1(x) + 2\nu_A^2(x) + \nu_A^3(x)}{4} \quad (6)$$

the fuzzy number intuitionistic index of x in A. It is hesitancy degree of x to A and $0 \le \pi_A(x) \le 1$ for every $x \in X$.

For convenience, we call $\tilde{\alpha} = \langle(a,b,c),(l,p,q)\rangle$ a FNIFN [8], where

$$(a,b,c) \in F(I),\ (l,p,q) \in F(I),\ 0 \le c+q \le 1 \quad (7)$$

and let Ω be the set of all FNIFNs.

Xu [3] and Wei and Wang [5] defined a score function and an accuracy function to measure an interval-valued intuitionistic fuzzy number, respectively. Based on these, Wang [10] defined a score function s to measure a FNIFN $\tilde{\alpha}$ as follows:

$$s(\tilde{\alpha}) = \frac{a+2b+c}{4} - \frac{l+2p+q}{4}, \quad (8)$$

where $s(\tilde{\alpha}) \in [-1,1]$. The larger the value of $s(\tilde{\alpha})$, the higher the FNIFN $\tilde{\alpha}$. Especially, if $s(\tilde{\alpha}) = 1$, then $\tilde{\alpha} = \langle(1,1,1),(0,0,0)\rangle$, which is the largest FNIFN; if $s(\tilde{\alpha}) = -1$, then $\tilde{\alpha} = \langle(0,0,0),(1,1,1)\rangle$, which is the smallest FNIFN.

Now, we define an accuracy function h to evaluate the accuracy degree of a FNIFN $\tilde{\alpha}$ as:

$$h(\tilde{\alpha}) = \frac{a+2b+c}{4} + \frac{l+2p+q}{4}, \quad (9)$$

where $h(\tilde{\alpha}) \in [0,1]$. The larger the value of $h(\tilde{\alpha})$, the higher the degree of accuracy of the IVIFN $\tilde{\alpha}$.

From (6), we define the hesitancy degree of the FNIFN $\tilde{\alpha} = \langle(a,b,c),(l,p,q)\rangle$ as:

$$\pi(\tilde{\alpha}) = 1 - \frac{a+2b+c}{4} - \frac{l+2p+q}{4}.$$

Then we get the relation between the hesitancy degree and the accuracy degree of the FNIFN $\tilde{\alpha}$

$$\pi(\tilde{\alpha}) = 1 - \frac{a+2b+c}{4} - \frac{l+2p+q}{4} = 1 - h(\tilde{\alpha}),$$

i.e.,

$$\pi(\tilde{\alpha}) + h(\tilde{\alpha}) = 1. \quad (10)$$

From (10), we know that the higher the accuracy degree $h(\tilde{\alpha})$, the lower the hesitancy degree $\pi(\tilde{\alpha})$.

Let $\tilde{\alpha} = \langle(a,b,c),(l,p,q)\rangle$, $\tilde{\alpha}_1 = \langle(a_1,b_1,c_1),(l_1,p_1,q_1)\rangle$ and $\tilde{\alpha}_2 = \langle(a_2,b_2,c_2),(l_2,p_2,q_2)\rangle$ be three FNIFNs, Wang [10] defined two operational laws of FNIFNs defined as:

1. $\tilde{\alpha}_1 \oplus \tilde{\alpha}_2 = \langle(a_1+a_2-a_1a_2, b_1+b_2-b_1b_2, c_1+c_2-c_1c_2),(l_1l_2, p_1p_2, q_1q_2)\rangle$;

2. $\lambda\tilde{\alpha} = \langle(1-(1-a)^\lambda, 1-(1-b)^\lambda, 1-(1-c)^\lambda),(l^\lambda, p^\lambda, q^\lambda)\rangle$, $\lambda > 0$,

which can ensure that the operational results are also FNIFNs. Moreover, we also define a method to compare two FNIFNs, which is based on the score function and the accuracy function:

Let $s(\tilde{\alpha}_1) = \frac{a_1+2b_1+c_1}{4} - \frac{l_1+2p_1+q_1}{4}$ and $s(\tilde{\alpha}_2) = \frac{a_1+2b_2+c_2}{4} - \frac{l_2+2p_2+q_2}{4}$ be the score of $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$, respectively, and let $h(\tilde{\alpha}_1) = \frac{a_1+2b_1+c_1}{4} + \frac{l_1+2p_1+q_1}{4}$ and $h(\tilde{\alpha}_2) = \frac{a_1+2b_2+c_2}{4} + \frac{l_2+2p_2+q_2}{4}$ be the accuracy degrees of $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$, respectively, then

- if $s(\tilde{\alpha}_1) < s(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1$ is smaller than $\tilde{\alpha}_2$, denoted by $\tilde{\alpha}_1 < \tilde{\alpha}_2$;
- if $s(\tilde{\alpha}_1) = s(\tilde{\alpha}_2)$, then
  1) if $h(\tilde{\alpha}_1) = h(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1$ and $\tilde{\alpha}_2$ represent the same information, i.e., $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$, $l_1 = l_2$, $p_1 = p_2$ and $q_1 = q_2$, denoted by $\tilde{\alpha}_1 = \tilde{\alpha}_2$;
  2) if $h(\tilde{\alpha}_1) < h(\tilde{\alpha}_2)$, then $\tilde{\alpha}_1$ is smaller than $\tilde{\alpha}_2$, denoted by $\tilde{\alpha}_1 < \tilde{\alpha}_2$.

3. Multiple attribute group decision making with incomplete attribute weights

For multiple attribute group decision making problem, let $O = \{O_1, O_2, \ldots, O_n\}$ be the set of n alternatives, $D = \{d_1, d_2, \ldots, d_l\}$ be the set of l decision-makers, and $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_l)^T$ be the weight vector of decision-makers, where $\lambda_k \ge 0$, $k = 1,2,\ldots,l$, and $\sum_{k=1}^{l} \lambda_k = 1$. Let $U = \{u_1, u_2, \ldots, u_m\}$ be the set of m attributes. In general, the decision-makers need to determine the importance degrees of a set U of m attributes. Thus we suppose that the decision-makers provide the attribute weight information may be presented in the following forms [15, 16], for $i \ne j$:

1. A weak ranking: $\{w_i \ge w_j\}$;
2. A strict ranking: $\{w_i - w_j \ge \delta_i (> 0)\}$;
3. A ranking with multiples: $\{w_i \ge \delta_i w_j\}$, $0 \le \delta_i \le 1$;
4. An interval form: $\{\delta_i \le w_i \le \delta_i + \epsilon_i\}$, $0 \le \delta_i \le \delta_i + \epsilon_i \le 1$;
5. A ranking of differences: $\{w_i - w_j \ge w_k - w_l\}$, for $j \ne k \ne l$.

For convenience, we denote by H the set of the known information about attribute weights provided by the decision-makers. Let $R^{(k)} = (\tilde{r}_{ij}^{(k)})_{m\times n}$ be an fuzzy number intuitionistic fuzzy decision matrix, where $\tilde{r}_{ij}^{(k)} =$

$\langle(a_{ij}^{(k)}, b_{ij}^{(k)}, c_{ij}^{(k)}), (l_{ij}^{(k)}, p_{ij}^{(k)}, q_{ij}^{(k)})\rangle$ is a FNIFN, provided by the decision-maker $d_k \in D$ for the alternative O_j with respect to the attribute $u_i \in U$, $(a_{ij}^{(k)}, b_{ij}^{(k)}, c_{ij}^{(k)})$ indicates the degree that the alternative $O_j \in O$ satisfy the attribute u_i, expressed by the decision-maker d_k, while $(l_{ij}^{(k)}, p_{ij}^{(k)}, q_{ij}^{(k)})$ indicates the degree that the alternative $O_j \in O$ does not satisfy the attribute u_i, expressed by the decision-maker d_k, and

$$(a_{ij}^{(k)}, b_{ij}^{(k)}, c_{ij}^{(k)}) \in F(I), (l_{ij}^{(k)}, p_{ij}^{(k)}, q_{ij}^{(k)}) \in F(I), \quad (11)$$
$$c_{ij}^{(k)} + q_{ij}^{(k)} \leq 1,\ i = 1, 2, \ldots, m, j = 1, 2, \ldots, n.$$

To make a final decision in the process of group decision making, we need to fuse all individual decision opinion into group opinion. To do this, we use the FIFHA operator [10] to aggregate all individual fuzzy number intuitionistic fuzzy decision matrices $R^{(k)} = (\tilde{r}_{ij}^{(k)})_{m\times n}$ $(k = 1, 2, \ldots, l)$ into the collective fuzzy number intuitionistic fuzzy decision matrix $R = (\tilde{r}_{ij})_{m\times n}$, where

$$\begin{aligned}\tilde{r}_{ij} &= \mathrm{FIFHA}_{\omega,\lambda}(\tilde{r}_{ij}^{(1)}, \tilde{r}_{ij}^{(2)}, \ldots, \tilde{r}_{ij}^{(l)}) \quad (12)\\ &= \omega_1 \dot{\tilde{r}}_{ij}^{(\sigma(1))} \oplus \omega_2 \dot{\tilde{r}}_{ij}^{(\sigma(2))} \oplus \cdots \oplus \omega_l \dot{\tilde{r}}_{ij}^{(\sigma(l))}\\ &= \Bigg\langle \Big(1 - \prod_{k=1}^{l}(1 - \dot{a}_{ij}^{(\sigma(k))})^{\omega_k},\\ &\quad 1 - \prod_{k=1}^{l}(1 - \dot{b}_{ij}^{(\sigma(k))})^{\omega_k}, 1 - \prod_{k=1}^{l}(1 - \dot{c}_{ij}^{(\sigma(k))})^{\omega_k}\Big),\\ &\quad \Big(\prod_{k=1}^{l}(\dot{l}_{ij}^{(\sigma(k))})^{\omega_k}, \prod_{k=1}^{l}(\dot{p}_{ij}^{(\sigma(k))})^{\omega_k}, \prod_{k=1}^{l}(\dot{q}_{ij}^{(\sigma(k))})^{\omega_k}\Big)\Bigg\rangle,\\ &\quad i = 1, 2, \ldots, m; j = 1, 2, \ldots, n,\end{aligned}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_l)^T$ is weight vector of FIFHA operator with $\omega_k > 0$ $(k = 1, 2, \ldots, l)$ and $\sum_{k=1}^{l}\omega_k = 1$, and $\dot{\tilde{r}}_{ij}^{(\sigma(k))} = \langle(\dot{a}_{ij}^{(k)}, \dot{b}_{ij}^{(k)}, \dot{c}_{ij}^{(k)}), (\dot{l}_{ij}^{(k)}, \dot{p}_{ij}^{(k)}, \dot{q}_{ij}^{(k)})\rangle$ is the kth largest of the weighted FNIFNs $\dot{\tilde{r}}_{ij}^{(k)}$ $(\dot{\tilde{r}}_{ij}^{(k)} = l\lambda_k \tilde{r}_{ij}^{(k)}$, $k = 1, 2, \ldots, l$, l is the balancing coefficient). Here, we denote $\tilde{r}_{ij} = \langle(a_{ij}, b_{ij}, c_{ij}), (l_{ij}, p_{ij}, q_{ij})\rangle$, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$.

In the cases that the information about attribute weights is completely known, that is, the weight vector $w = (w_1, w_2, \ldots, w_m)^T$ of the attributes u_k $(k = 1, 2 \ldots, m)$ can be completely determine in advance, then based on the collective fuzzy number intuitionistic fuzzy decision matrix $R = (\tilde{r}_{ij})_{m\times n}$, we can use the FIFWA operator [10]:

$$\begin{aligned}\tilde{r}_j &= \mathrm{FIFWA}_w(\tilde{r}_{1j}, \tilde{r}_{2j}, \ldots, \tilde{r}_{mj}) \quad (13)\\ &= w_1\tilde{r}_{ij} \oplus w_2\tilde{r}_{2j} \oplus \cdots \oplus w_m\tilde{r}_{mj}\\ &= \Bigg\langle \Big(1 - \prod_{i=1}^{m}(1 - a_{ij})^{w_i},\\ &\quad 1 - \prod_{i=1}^{m}(1 - b_{ij})^{w_i}, 1 - \prod_{i=1}^{m}(1 - c_{ij})^{w_i}\Big),\\ &\quad \Big(\prod_{i=1}^{m}(l_{ij})^{w_i}, \prod_{i=1}^{m}(p_{ij})^{w_i}, \prod_{i=1}^{m}(q_{ij})^{w_i}\Big)\Bigg\rangle,\\ &\quad j = 1, 2, \ldots, n\end{aligned}$$

to obtain the overall the alternative O_j. The greater the value of r_j, the better the alternative O_j will be.

3.1 Model for determining attribute weights

However, the information about attribute weights provided by the decision-makers is usually incomplete (see, [15, 16]). So an interesting and important issue is how to utilize the collective fuzzy number intuitionistic fuzzy decision matrix and the known weight information to find the most desirable alternative(s).

In the following, we present an approach to determining the weight of attributes.

Definition. Let $R = (\tilde{r}_{ij})_{m\times n}$ be the collective fuzzy number intuitionistic fuzzy decision matrix. Then we call $S = (s_{ij})_{m\times n}$ the score matrix of $R = (\tilde{r}_{ij})_{m\times n}$, where

$$\begin{aligned}s_{ij} &= s(\tilde{r}_{ij}) \quad (14)\\ &= \frac{a_{ij} + 2b_{ij} + c_{ij}}{4} - \frac{l_{ij} + 2p_{ij} + q_{ij}}{4},\\ &\quad i = 1, 2, \ldots, m; j = 1, 2, \ldots, n\end{aligned}$$

, and $s(\tilde{r}_{ij})$ is called the score of $\tilde{r}_{ij}$.

Based on the score matrix, we present the overall score values of each alternatives O_j $(j = 1, 2, \ldots, m)$:

$$s_j(w) = \sum_{i=1}^{m} w_i s_{ij},\ \ j = 1, 2, \ldots, n. \quad (15)$$

Obviously, the greater the value $s_j(w)$, the better the alternative O_j. When we only consider the alternative O_j, then a reasonable vector of attribute weights $w = (w_1, w_2, \ldots, w_m)^T$ should be determined. Thus, we establish the following optimization model to maximize $s_j(w)$:

$$\textbf{(M)} \quad \text{Maximize } s_j(w) = \sum_{i=1}^{m} w_i s_{ij}$$
$$\text{Subject to}: w = (w_1, \ldots, w_m)^T \in H, w_i \geq 0,$$
$$i = 1, \ldots, m, \sum_{i=1}^{m} w_i = 1.$$

By solving the model (M), we obtain the optimal solution $w^{(j)} = (w_1^{(j)}, w_2^{(j)}, \ldots, w_m^{(j)})^T$ corresponding to the alternative O_j. However, in the process of determining the weight vector $w = (w_1, w_2, \ldots, w_m)^T$, we need to consider all the alternatives O_j $(j = 1, 2, \ldots, n)$ as a whole. Thus, we construct weight matrix $W = (w_i^{(j)})_{m\times n}$ of the optimal solutions $w^{(j)} = (w_1^{(j)}, w_2^{(j)}, \ldots, w_m^{(j)})^T$ $(j = 1, 2, \ldots, n)$ as:

$$W = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & w_1^{(n)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & w_2^{(n)} \\ \vdots & \vdots & \vdots & \vdots \\ w_m^{(1)} & w_m^{(2)} & \cdots & w_m^{(n)} \end{pmatrix}$$

and we calculate the normalized eigenvector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ of the matrix $(S^TW)^T(S^TW)$, and then we construct a combined weight vector as follows:

$$w = W\omega = \begin{pmatrix} w_1^{(1)} & w_1^{(2)} & \cdots & w_1^{(n)} \\ w_2^{(1)} & w_2^{(2)} & \cdots & w_2^{(n)} \\ \vdots & \vdots & \vdots & \vdots \\ w_m^{(1)} & w_m^{(2)} & \cdots & w_m^{(n)} \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{pmatrix} \quad (16)$$
$$= \omega_1 w^{(1)} + \omega_2 w^{(2)} + \cdots + \omega_n w^{(n)},$$

and thus we derive the weight vector $w = (w_1, w_2, \ldots, w_m)^T$ of the attributes u_k $(k = 1, 2 \ldots, m)$.

3.2 An approach to multiple attribute group decision making with partially known attribute weights

Based on the analysis above, in the following we present an approach to multiple attribute fuzzy number intuitionistic fuzzy group decision making with incomplete attribute weight information:

Step 1. Utilize the FIFHA operator (12) to aggregate all individual fuzzy number intuitionistic fuzzy decision matrices $R^{(k)} = (\tilde{r}_{ij}^{(k)})_{m\times n}$ $(k = 1, 2, \ldots, l)$ into a collective fuzzy number intuitionistic fuzzy decision matrix $R = (\tilde{r}_{ij})_{m\times n}$.

Step 2. Calculate the score matrix $S = (s_{ij})_{m\times n}$ of the collective fuzzy number intuitionistic fuzzy decision matrix R.

Step 3. Utilize the model (M) to obtain the optimal weight vectors $w^{(j)} = (w_1^{(j)}, w_2^{(j)}, \ldots, w_m^{(j)})^T$ $(j = 1, 2, \ldots, n)$ corresponding to the alternatives O_j $(j = 1, 2, \ldots, n)$, and then construct the weight matrix W.

Step 4. Calculate the normalized eigenvector $\omega = \omega_1, \omega_2, \ldots, \omega_n)^T$ of the matrix $(S^TW)^T(S^TW)$.

Step 5. Utilize (16) to derive the weight vector $w = (w_1, w_2, \ldots, w_m)^T$.

Step 6. Use the FIFWA operator (13) to get the overall values $\tilde{r}_j$ of the alternatives O_j $(j = 1, 2, \ldots, n)$.

Step 7. Use the score function (8) to calculate the scores $s(\tilde{r}_j)$ of the overall values $\tilde{r}_j$ of the alternatives O_j $(j = 1, 2, \ldots, n)$.

Step 8. Utilize the scores $s(\tilde{r}_j)$ to rank the alternatives O_j $(j = 1, 2, \ldots, n)$, and then select the most desirable one(s) (if two scores $s(\tilde{r}_i)$ and $s(\tilde{r}_j)$ are identical, then we calculate the accuracy degrees $h(\tilde{r}_i)$ and $h(\tilde{r}_j)$ of the overall values $\tilde{r}_i$ and $\tilde{r}_j$, respectively, and then rank the alternatives O_i and O_j according to the accuracy degrees $h(\tilde{r}_i)$ and $h(\tilde{r}_j)$).

4. Illustrative example

In this section, we discuss a problem concerning with a manufacturing company, searching the best global supplier for one of its most critical parts used in assembling process [17]. The attributes which are considered here in selection of three potential global suppliers O_j $(j = 1, 2, 3)$ are: (1) u_1: Overall cost of the product; (2) u_2: Quality of the product; (3) u_3: Service performance of supplier; (4) u_4: Supplier's profile; and (5) u_5: Risk factor.

An expert group is formed which consists of four experts d_k $(k = 1, 2, 3, 4)$ (whose weight vector is $\lambda = (0.3, 0.2, 0.3, 0.2)^T$) from each strategic decision area. The experts d_k $(k = 1, 2, 3, 4)$ represent, respectively, the characteristics of the potential global suppliers O_j $(j = 1, 2, 3)$ by the FNIFNs $\tilde{r}_{ij}^{(k)}$ $(i = 1, 2, 3, 4, 5; j = 1, 2, 3)$ with respect to the attributes u_i $(i = 1, 2, 3, 4, 5)$, listed in Tables 1-4 (i.e., fuzzy number intuitionistic fuzzy decision matrices $R^{(k)} = (\tilde{r}_{ij}^{(k)})_{5\times 3}$ $(k = 1, 2, 3, 4)$).

	O_1	O_2	O_3
u_1	$\langle(0.4, 0.5, 0.6), (0.1, 0.2, 0.3)\rangle$	$\langle(0.2, 0.3, 0.4), (0.4, 0.5, 0.6)\rangle$	$\langle(0.3, 0.4, 0.5), (0.3, 0.4, 0.5)\rangle$
u_2	$\langle(0.2, 0.4, 0.5), (0.3, 0.4, 0.5)\rangle$	$\langle(0.1, 0.2, 0.3), (0.2, 0.3, 0.4)\rangle$	$\langle(0.4, 0.6, 0.7), (0.1, 0.2, 0.3)\rangle$
u_3	$\langle(0.5, 0.6, 0.7), (0.2, 0.2, 0.3)\rangle$	$\langle(0.2, 0.3, 0.4) (0.3, 0.4, 0.5)\rangle$	$\langle(0.5, 0.6, 0.8), (0.1, 0.2, 0.2)\rangle$
u_4	$\langle(0.5, 0.6, 0.7), (0.1, 0.2, 0.2)\rangle$	$\langle(0.1, 0.2, 0.3), (0.5, 0.6, 0.7)\rangle$	$\langle(0.4, 0.5, 0.6), (0.2, 0.2, 0.3)\rangle$
u_5	$\langle(0.1, 0.2, 0.4), (0.3, 0.4, 0.5)\rangle$	$\langle(0.6, 0.7, 0.8), (0.1, 0.1, 0.2)\rangle$	$\langle(0.4, 0.5, 0.6), (0.2, 0.3, 0.4)\rangle$

Table 1. Fuzzy number intuitionistic fuzzy decision matrix $R^{(1)}$

	O_1	O_2	O_3
u_1	$\langle(0.4,0.5,0.6),$ $(0.2,0.3,0.4)\rangle$	$\langle(0.3,0.4,0.5),$ $(0.4,0.4,0.5)\rangle$	$\langle(0.4,0.5,0.6),$ $(0.2,0.3,0.4)\rangle$
u_2	$\langle(0.2,0.3,0.4),$ $(0.4,0.5,0.6)\rangle$	$\langle(0.1,0.2,0.3),$ $(0.3,0.5,0.7)\rangle$	$\langle(0.6,0.7,0.8)$ $(0.1,0.2,0.2)\rangle$
u_3	$\langle(0.5,0.6,0.7),$ $(0.1,0.2,0.3)\rangle$	$\langle(0.2,0.3,0.4)$ $(0.4,0.5,0.6)\rangle$	$\langle(0.6,0.7,0.8),$ $(0.1,0.2,0.2)\rangle$
u_4	$\langle(0.4,0.6,0.7),$ $(0.1,0.2,0.3)\rangle$	$\langle(0.2,0.3,0.3),$ $(0.4,0.6,0.7)\rangle$	$\langle(0.4,0.5,0.6),$ $(0.1,0.2,0.4)\rangle$
u_5	$\langle(0.1,0.2,0.3),$ $(0.2,0.4,0.5)\rangle$	$\langle(0.5,0.7,0.8),$ $(0.1,0.2,0.2)\rangle$	$\langle(0.3,0.4,0.6),$ $(0.2,0.3,0.4)\rangle$

Table 2. Fuzzy number intuitionistic fuzzy decision matrix $R^{(2)}$

	O_1	O_2	O_3
u_1	$\langle(0.4,0.5,0.7),$ $(0.1,0.2,0.3)\rangle$	$\langle(0.3,0.4,0.5),$ $(0.2,0.3,0.4)\rangle$	$\langle(0.2,0.3,0.4),$ $(0.3,0.4,0.5)\rangle$
u_2	$\langle(0.3,0.4,0.5),$ $(0.2,0.3,0.4)\rangle$	$\langle(0.2,0.3,0.4),$ $(0.3,0.4,0.5)\rangle$	$\langle(0.6,0.7,0.8)$ $(0.1,0.1,0.2)\rangle$
u_3	$\langle(0.4,0.5,0.7),$ $(0.1,0.2,0.3)\rangle$	$\langle(0.3,0.4,0.5)$ $(0.2,0.3,0.5)\rangle$	$\langle(0.5,0.6,0.7),$ $(0.1,0.2,0.3)\rangle$
u_4	$\langle(0.4,0.6,0.7),$ $(0.1,0.2,0.3)\rangle$	$\langle(0.1,0.2,0.3),$ $(0.4,0.5,0.7)\rangle$	$\langle(0.4,0.5,0.7),$ $(0.1,0.2,0.3)\rangle$
u_5	$\langle(0.3,0.4,0.5),$ $(0.2,0.3,0.5)\rangle$	$\langle(0.5,0.6,0.7),$ $(0.1,0.2,0.3)\rangle$	$\langle(0.6,0.7,0.8),$ $(0.1,0.1,0.2)\rangle$

Table 3. Fuzzy number intuitionistic fuzzy decision matrix $R^{(3)}$

	O_1	O_2	O_3
u_1	$\langle(0.4,0.6,0.7),$ $(0.1,0.2,0.3)\rangle$	$\langle(0.3,0.4,0.5),$ $(0.2,0.4,0.5)\rangle$	$\langle(0.2,0.4,0.5),$ $(0.2,0.3,0.5)\rangle$
u_2	$\langle(0.3,0.4,0.5),$ $(0.2,0.3,0.5)\rangle$	$\langle(0.1,0.2,0.3),$ $(0.2,0.3,0.5)\rangle$	$\langle(0.5,0.6,0.7)$ $(0.1,0.2,0.3)\rangle$
u_3	$\langle(0.6,0.7,0.8),$ $(0.1,0.2,0.2)\rangle$	$\langle(0.3,0.4,0.5)$ $(0.3,0.4,0.5)\rangle$	$\langle(0.4,0.5,0.7),$ $(0.1,0.2,0.3)\rangle$
u_4	$\langle(0.4,0.5,0.7),$ $(0.1,0.2,0.3)\rangle$	$\langle(0.1,0.2,0.3),$ $(0.4,0.5,0.6)\rangle$	$\langle(0.4,0.5,0.6),$ $(0.1,0.3,0.4)\rangle$
u_5	$\langle(0.1,0.2,0.3),$ $(0.5,0.6,0.7)\rangle$	$\langle(0.4,0.5,0.7),$ $(0.1,0.2,0.3)\rangle$	$\langle(0.5,0.6,0.7),$ $(0.1,0.2,0.3)\rangle$

Table 4. Fuzzy number intuitionistic fuzzy decision matrix $R^{(4)}$

Assume that the information about attribute weights, given by decision-makers, is shown as follows, respectively:

$$d_1 : w_1 \le 0.3,\ 0.2 \le w_3 \le 0.5;$$
$$d_2 : 0.1 \le w_2 \le 0.2,\ w_5 \le 0.4;$$
$$d_3 : w_3 - w_2 \ge w_5 - w_4,\ w_4 \ge w_1;$$
$$d_4 : w_3 - w_1 \le 0.1,\ 0.1 \le w_4 \le 0.3.$$

Then the set H of the known information about attribute weights provided by the decision-makers is

$$H = \{w_1 \le 0.3,\ 0.2 \le w_3 \le 0.5,\ 0.1 \le w_2 \le 0.2,\ w_5 \le 0.4, w_3 - w_2 \ge w_5 - w_4,\ w_4 \ge w_1,\ w_3 - w_1 \le 0.1,\ 0.1 \le w_4 \le 0.3\}.$$

Step 1. Utilize the FIFHA operator (12) (let $\omega = (0.155,\ 0.345, 0.345, 0.155)^T$ be its weight vector of the FIFHA operator derived by the normal distribution based method [18]) to aggregate the individual fuzzy number intuitionistic fuzzy decision matrices $R^{(k)} = (\tilde{r}_{ij}^{(k)})_{5\times 3}$ $(k = 1,2,3,4)$ into the collective fuzzy number intuitionistic fuzzy decision matrix $R = (\tilde{r}_{ij})_{5\times 3}$ (Table 5).

	O_1	O_2	O_3
u_1	$\langle(0.400,0.530,0.650),$ $(0.109,0.210,0.311)\rangle$	$\langle(0.260,0.360,0.461),$ $(0.290,0.416,0.517)\rangle$	$\langle(0.279,0.392,0.493)$ $(0.255,0.357,0.470)$
u_2	$\langle(0.248,0.388,0.489),$ $(0.258,0.369,0.491)\rangle$	$\langle(0.120,0.220,0.320),$ $(0.227,0.337,0.475)\rangle$	$\langle(0.514,0.650,0.751)$ $(0.100,0.176,0.249)$
u_3	$\langle(0.493,0.595,0.732),$ $(0.114,0.200,0.268)\rangle$	$\langle(0.248,0.348,0.448),$ $(0.288,0.390,0.491)\rangle$	$\langle(0.519,0.620,0.751)$ $(0.100,0.200,0.249)$
u_4	$\langle(0.420,0.589,0.700),$ $(0.100,0.200,0.278)\rangle$	$\langle(0.113,0.213,0.300),$ $(0.439,0.552,0.671)\rangle$	$\langle(0.400,0.500,0.621)$ $(0.133,0.210,0.337)$
u_5	$\langle(0.141,0.242,0.383),$ $(0.265,0.399,0.521)\rangle$	$\langle(0.509,0.640,0.751),$ $(0.100,0.176,0.249)\rangle$	$\langle(0.461,0.563,0.675)$ $(0.145,0.219,0.325)$

Table 5. Collective fuzzy number intuitionistic fuzzy decision matrix R

Step 2. Calculate the score matrix $S = (s_{ij})_{5\times 3}$ of the collective fuzzy number intuitionistic fuzzy decision matrix R (Table 6):

	O_1	O_2	O_3
u_1	0.317	-0.049	0.029
u_2	0.011	-0.124	0.466
u_3	0.408	-0.041	0.440
u_4	0.380	-0.343	0.283
u_5	-0.144	0.460	0.339

Table 6. Collective score matrix S

Step 3. Use the method (M) to obtain the optimal weight vectors $w^{(j)} = (w_1^{(j)}, w_2^{(j)}, w_3^{(j)}, w_4^{(j)}, w_5^{(j)})^T$ $(j = 1,2,3)$ corresponding to the alternatives O_j $(j = 1,2,3)$

$$w^{(1)} = (0.25, 0.10, 0.35, 0.30, 0.00)^T,$$
$$w^{(2)} = (0.16, 0.10, 0.26, 0.16, 0.32)^T,$$
$$w^{(3)} = (0.10, 0.20, 0.20, 0.25, 0.25)^T$$

and construct the weight matrix

$$W = \begin{pmatrix} 0.25 & 0.16 & 0.10 \\ 0.10 & 0.10 & 0.20 \\ 0.35 & 0.26 & 0.20 \\ 0.30 & 0.16 & 0.25 \\ 0.00 & 0.32 & 0.25 \end{pmatrix}$$

then

$$(S^TW)^T(S^TW) = \begin{pmatrix} 0.2195 & 0.1430 & 0.1595 \\ 0.1430 & 0.1356 & 0.1381 \\ 0.1595 & 0.1381 & 0.1459 \end{pmatrix}.$$

Step 4. Calculate the normalized eigenvectors ω of the matrix $(S^TW)^T(S^TW)$:

$$\omega = (0.3813, 0.2991, 0.3196)^T.$$

Step 5. Use (16) to derive the weight vector w:

$$w = W\omega = \begin{pmatrix} 0.25 & 0.16 & 0.10 \\ 0.10 & 0.10 & 0.20 \\ 0.35 & 0.26 & 0.20 \\ 0.30 & 0.16 & 0.25 \\ 0.00 & 0.32 & 0.25 \end{pmatrix} \begin{pmatrix} 0.3813 \\ 0.2991 \\ 0.3196 \end{pmatrix}$$
$$= (0.1751, 0.1320, 0.2751, 0.2421, 0.1756)^T.$$

Step 6. Use the FIFWA operator (13) to obtain the overall values $\tilde{r}_j$ $(j = 1, 2, 3)$ of the alternatives O_j $(j = 1, 2, 3)$:

$$\tilde{r}_1 = \langle (0.3766, 0.5080, 0.6360), (0.1415, 0.2462, 0.3380) \rangle,$$
$$\tilde{r}_2 = \langle (0.2607, 0.3726, 0.4797), (0.2571, 0.3658, 0.4722) \rangle,$$
$$\tilde{r}_3 = \langle (0.4434, 0.5529, 0.6729), (0.1349, 0.2237, 0.3136) \rangle.$$

Step 7. Use the score function (8) to calculate the score $s(\tilde{r}_j)$ $(j = 1, 2, 3)$ of the overall values $\tilde{r}_j$ $(j = 1, 2, 3)$ of the alternatives O_j $(j = 1, 2, 3)$:

$$s(\tilde{r}_1) = 0.2642, s(\tilde{r}_2) = 0.0062, s(\tilde{r}_3) = 0.3315$$

and thus,

$$s(\tilde{r}_3) > s(\tilde{r}_1) > s(\tilde{r}_2).$$

Step 8. Use the scores $s(\tilde{r}_j)$ $(j = 1, 2, 3)$ to rank the alternatives O_j $(j = 1, 2, 3)$:

$$O_3 > O_1 > O_2$$

and then the most desirable alternative is O_3.

5. Conclusions

We have investigated the multiple attribute group decision making problems under fuzzy number intuitionistic fuzzy environment, and developed an approach to handling the situations where the attribute values are characterized by FNIFNs, and the information about attribute weights is partially known. The approach first fuses all individual fuzzy number intuitionistic fuzzy decision matrices into collective fuzzy number intuitionistic fuzzy decision matrix by using the FIFHA operator. In the situations where the information about attribute weights is incomplete, we have constructed the score matrix of the collective fuzzy number intuitionistic fuzzy decision matrix, and established a optimization model to determine the attribute weights. Then we have utilized the obtained attribute weights and the FIFWA operator to find the ranking of the alternatives and then select the most desirable one. All these procedures have been illustrated by a numerical example concerning with a manufacturing company, searching the best global supplier for one of its most critical parts used in assembling process. In future, we shall continue working in the application of the FIFHG and FIFWG operators to other domains.

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