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퍼지이론과 버블기법을 이용한 3차원 구조물의 유한요소해석을 위한 요소생성기법

Mesh Generation Methodology for FE Analysis of 3D Structures Using Fuzzy Knowledge and Bubble Method

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요 약

본 논문은 3차원구조물의 유한요소해석을 위한 자동 유한요소 생성에 관한 것으로 퍼지이론과 버블요소 생성기법, 상용 솔리드모델러로 구성되어진다. 새로운 요소생성과정은 (a) 해석모델인 형상모델링 정의, (b) 버블생성, 그리고 (c) 요소생성으로 이루어진다. 형상모델링에는 상용 솔리드모델러를 이용하였으며 버블은 각 지점에서의 버블간격함수에 의해 생성되어진다. 버블간격 함수는 지식처리수법에 의해 조절되어 진다. 요소생성을 위해서는 기본적으로 데로우니방법을 도입하였다. 이러한 3차원 구조물에 대한 유한요소의 자동생성은 해석을 위해 큰 잇점이 있다. 실제적인 현 시스템의 효용성을 검증하기 위해 3차원 형상에 대한 예를 제시하였다.

키워드 : 자동요소생성, 버블기법, 퍼지지식처리, 데로우니삼각화법, 유한요소해석

Abstract

This paper describes an automatic finite element mesh generation for finite element analysis of three-dimensional structures. It is consisting of fuzzy knowledge processing, bubble meshing and solid geometry modeler. This novel mesh generation process consists of three subprocesses: (a) definition of geometric model, i.e. analysis model, (b) generation of bubbles, and (c) generation of elements. One of commercial solid modelers is employed for three-dimensional solid structures. Bubble is generated if its distance from existing bubble points is similar to the bubble spacing function at the point. The bubble spacing function is well controlled by the fuzzy knowledge processing. The Delaunay method is introduced as a basic tool for element generation. Automatic generation of finite element for three-dimensional solid structures holds great benefits for analyses. Practical performances of the present system are demonstrated through several mesh generations for 3D geometry.

Key Words : Automatic Mesh Generation, Bubble Method, Fuzzy Knowledge Processing, Delaunay Triangulation Method, Finite Element Analysis

1. Introduction

Loads for pre-processing and post-processing are increasing rapidly in accordance with an increase of scale and complexity of analysis models to be solved. Particularly, the mesh generation process, which influences computational accuracy as well as efficiency and whose fully automation is very difficult in three-dimensional (3D) cases, has become the most critical issue in a whole process of the finite element (FE) analyses. In this respect, various researches[1-7] have been performed on the development of automatic mesh generation techniques. Also, many researchers have endeavored to improve the performance of the finite element method.

Among mesh generation methods, the tree model method[6] can generate graded meshes and it uses a reasonably small amount of computer time and storage. However, it is, by nature, not possible to arbitrarily control the changing rate of mesh size with respect to location, so that some smaller projection and notch etc. are sometimes omitted. Also, domain decomposition method[7] does not always succeed, and a designation of such sub-domains is very tedious for uses in

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three-dimensional cases.

The present authors have developed an automatic FE mesh generation technique, which is based on the fuzzy theory[8,9] and computational geometry technique, is incorporated into the system, together with one of commercial solid modelers. In the present study, to support the FE analysis system which require such special mesh, an automatic bubble mesh generation combined with the automatic mesh generation system.

2. Bubble Method

Bubble meshing is summarized as a sequence of two steps: (1) packing circles or spheres, called bubbles, closely in a domain, and (2) connecting their centers by Delaunay triangulation, which selects the best topological connection for a set of nodes by avoiding small angles.

2.1 Bubble Packing

The key element of bubble meshing lies in the first step, that is, the optimization of mesh node locations by close packing bubbles. In this method, bubbles move in a domain until forces between bubbles are stabilized, and the Delaunay triangulation is then applied to generate a mesh connecting the nodes defined by the bubble packing. A repulsive or attractive force much like an intermolecular van der Waals force is assumed to exist between two adjacent bubbles. A globally stable configuration of tightly packed bubbles is determined by solving the equation of motion.

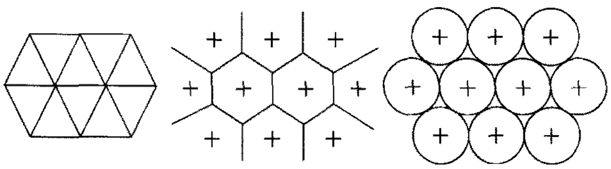


Fig. 1. Schematics of the Delaunay triangulation, the Voronoi Polygons, and packed bubbles.

Fig. 1 shows the Delaunay triangulation and the bubble packing method, and Fig. 2 shows the procedure of the bubble packing method. Bubble meshing generates a two-dimensional triangular mesh by the following two steps: (a) Solving the equation of motion on vertices, edges, and faces(or loops) in that order, (b) Generation of triangular mesh by connecting the center points of bubbles by Delaunay Triangulation. Similar steps are also applied to the generation of three-dimensional tetrahedral meshes. In this procedure, the mesh density is needed to determine the radius of bubble. To handle general bubble spacing, we adopted a function of bubble density distribution. In the present system each bubble data are stored as a tree structure of domain such as vertices, edges, surfaces. In general, it is not so easy to well control element size for a complex geometry. A bubble density distribution over a whole geometry model can be constructed. A user selects some of local bubble patterns, depending on their analysis purposes, and designates where to locate them.

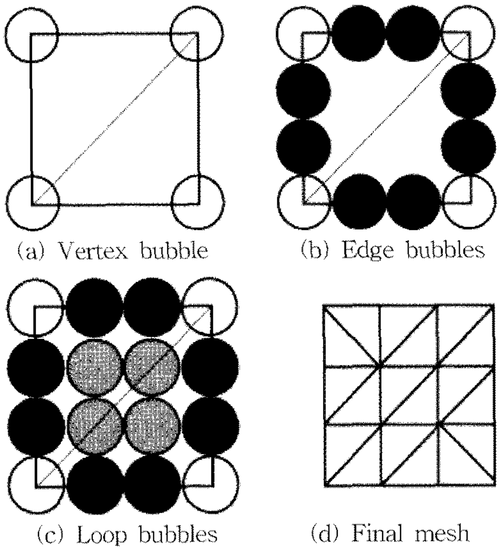


Fig. 2. Procedure of the bubble mesh

2.2 Dynamic bubbles

A force function f(r) between two adjacent bubbles is shown in Fig. 3. If f(r)=r0, where r0 is defined to be

$$r_0 = 0.5(d_i + d_j) \qquad (1)$$

where d_i, d_j denote diameters of two adjacent bubbles.

The two bubbles are defined to be in a stable distance. If f(r) is larger than zero, a repulsive force is assumed to exist between two bubbles, and if f(r) is smaller than zero, a attractive force is assumed to exist between them. The kinetic equation is written as follows:

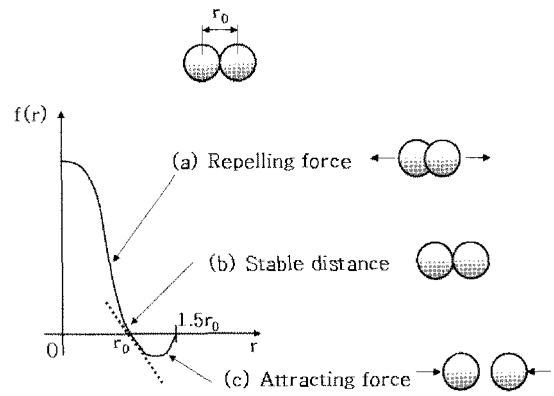


Fig. 3. Interbubble proximity-based forces

$$m_i \frac{d^2 s_i}{dt^2} + c\frac{ds_i}{dt} = f_i(t), \qquad i = 1, 2, ..., n \qquad (2)$$

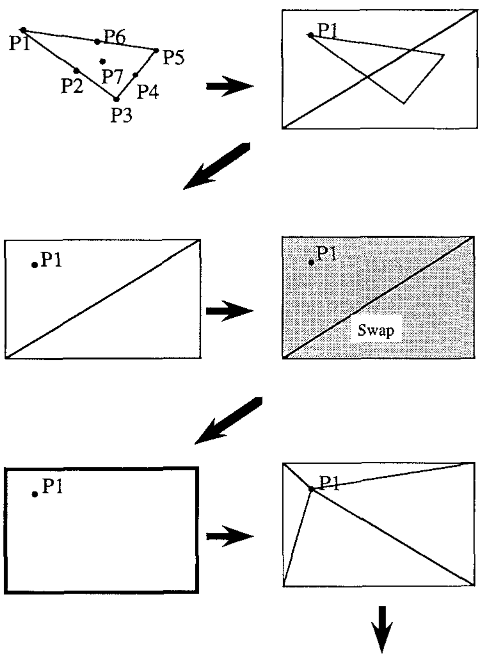
where m_i denotes the mass of bubble, c the coefficient of viscosity, s_i the position of the ith bubble and $f_i(t)$ the force between two adjacent bubbles. The system of equation (2) describes the process of physical relaxation, which eventually moves the bubbles to proper equilibrium positions. The force $f_i(t)$, which depends on the position s_i and the distances from its center to the centers of the neighboring bubbles, is modeled by the van der Waals force.

By solving above equaiton by Runge-Kutta method, the optimized bubble configuration is obtained. In the process of the optimization, population of bubbles is adaptively controlled using fuzzy knowledge process. That is, excess bubbles which significantly overlap their neighbors are removed, and new bubbles are added around open bubbles which lack the appropriate number of neighboring bubbles. After the optimization of the bubble configuration, a triangular mesh is generated by using Delaunay triangulation.

3. Element Generation

The Delaunay triangulation method [1,3] is utilized to generate tetrahedral elements from numerous bubbles given in a geometry. In this section, briefly describes this method.

Let N be a set of nodes, it has the property that the circumcircle of any triangle in the triangulation contains no point of N in its interior. The remaining points in N will be iteratively added to the triangulation. After each point is added, it will be connected to the vertices of its enclosing triangle. (See Fig. 4) All internal edges of a triangulation of a finite set N are locally optimal if no point of N is interior to any circumcircle of a triangle.



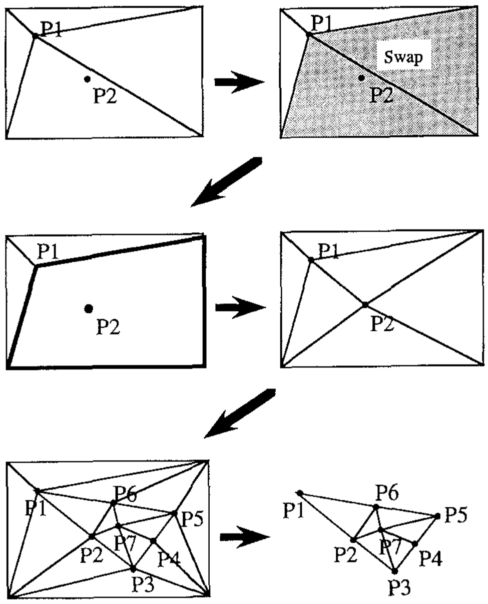


Fig. 4. Example showing a Delaunay triangulation

The speed of element generation by the Delaunay triangulation method is proportional to the number of nodes. If this method is utilized to generate elements in a geometry with indented shape, elements are inevitably generated even outside the geometry. However, such mis-match elements can be removed by performing the IN / OUT check for gravity center points of such elements. In addition, it is necessary to avoid the generation of those mis-match elements crossing domain boundary by setting node densities on edges to be slightly higher than those inside the domain near the boundaries.

4. Fuzzy Theory

4.1 Control of bubble patterns by fuzzy knowledge

In this section, the connecting process of locally optimum bubble images is dealt with using the fuzzy knowledge processing technique. Performances of automatic mesh generation methods based on node generation algorithms depend on how to control node spacing functions or node density distributions and how to generate nodes. The basic concept of the present mesh generation algorithm is originated from the imitation of mesh generation processes by human experts of finite element analyses. One of the aims of this algorithm is to transfer such experts' techniques to beginners.

In general, it is not so easy to well control element size for a complex geometry. A node density distribution over a whole geometry model is constructed as follows. The present system stores several local bubble patterns such as the pattern suitable to well capture stress concentration, the pattern to subdivide a finite domain uniformly, and the pattern to subdivide a whole domain uniformly. A user selects some of those local bubble patterns, depending on their analysis purposes, and designates where to locate them.

When these stress concentration fields exist closely to each other in the same analysis domain, a simple superposition of both local bubble patterns.

In the present method, the field A close to the hole and the field B close to the crack-tip are defined in terms of the membership functions used in the fuzzy set theory. For the purpose of simplicity, each membership function is given a function in the figure. In practice the membership function can be expressed as $\mu(x, y)$ in this particular example, and in 3D cases it is a function of 3D coordinates, i.e. $\mu(x, y, z)$. This procedure of node generation, i.e. the connection procedure of both node patterns, is summarized as follows :

- If $\mu A(xp, yp) \geq \mu B(xp, yp)$ for a node p(xp, yp) belonging to the pattern A, then the node p is generated, and otherwise p is not generated.
- If $\mu A(xq, yq) \geq \mu B(xq, yq)$ for a node q(xq, yq) belonging to the pattern B, then the node q is generated, and otherwise q is not generated.

It is apparent that the above algorithm can be easily extended to 3D problems and any number of node patterns. In addition, since finer node patterns are generally required to place near stress concentration sources, it is convenient to let the membership function correspond to node density as well.

2.2 Fuzzy control of bubble position

The fuzzy rules employed here can be generalized as :

$$RULE^i : IF\ p\ is\ A^i\ ,\ THEN\ q\ is\ B^i$$

where $RULE^i$ is the i-th fuzzy rule, A^i and B^i the fuzzy variables, p the value of node, and $\triangle p$ the difference of the current and the next values of p, i.e. $|p(n+1)-p(n)|$(n: the iteration number of node), respectively. The labels of the fuzzy variables are defined as follows.

As for A^i,

LARGE → p is much larger than 1.0.
MEDIUM → p is larger than 1.0.
SMALL → p is little larger than 1.0.

As for B^i,

LARGE → q is positive and large.
MEDIUM → q is positive and medium.
SMALL → q is positive and small.

As shown in Fig. 5, trapezoid type membership functions are utilized as those of labes of A^i and B^i from the viewpoint of simplicity.

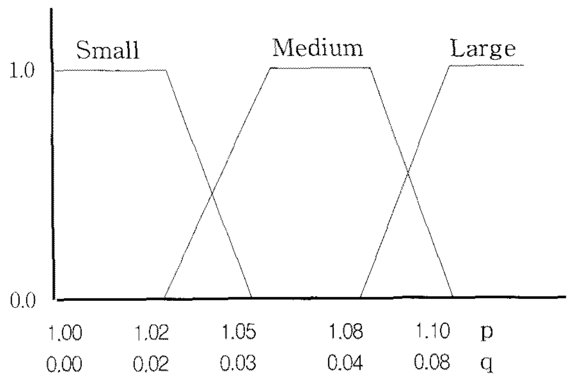


Fig. 5. Membership functions of labels of $A^i(p)$ and $B^i(q)$

5. Examples and Discussions

The performance of the system is demonstrated through the mesh generation of several geometries. Fig. 6 shows the screen of the system, and Figs. 7 and 8 show the uniform bubble and mesh.

In case of a complex geometry as shown in Fig. 9, a uniform mesh and a nonuniform mesh were connected very smoothly. Bubble and elements are generated in about 2 minutes and in about 3 minutes, respectively. The mesh consists of 6,896 tetrahedral elements.

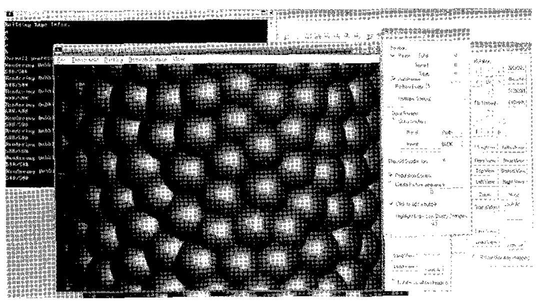


Fig. 6. Screen of the system

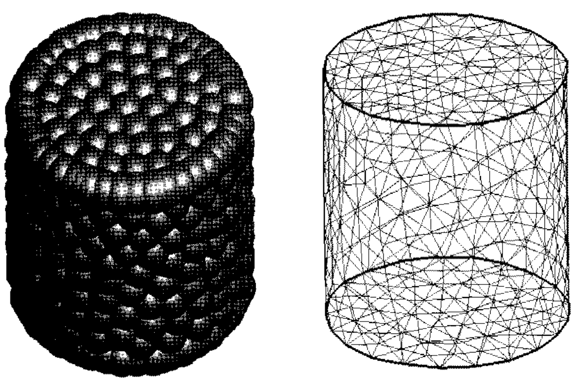


Fig. 7. Bubble image and mesh of cylinder

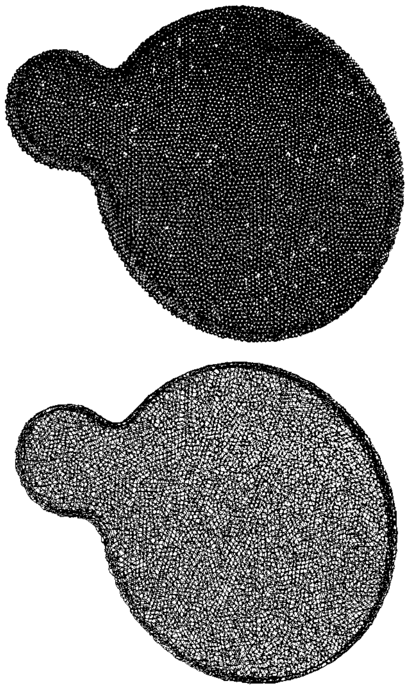


Fig. 8. Bubble image and mesh

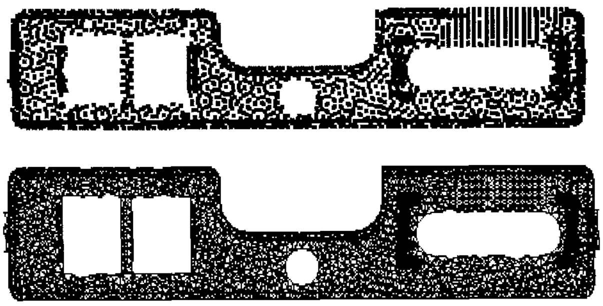


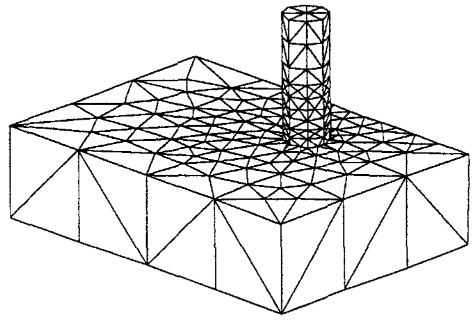
Fig. 9. Bubble image and mesh of cylinder

Also, Fig. 10, finer elements are also observed around the edge of the plate on this side. When the cylinder is connected to the plate near its edge, such a mesh pattern is desirable to calculate accurately the influence of the plate's edge to the stress concentration field around the junction.

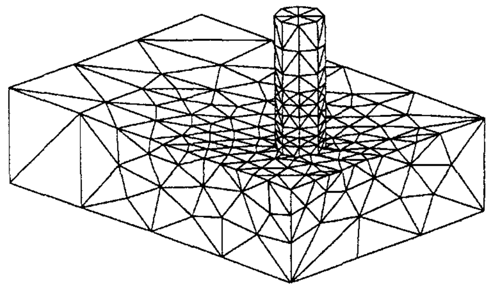
To complete this mesh, the following two bubble patterns are utilized ; (a) the base bubble pattern in which nodes are generated with uniform spacing over a whole analysis domain, (b) a special bubble pattern for stress concentration of edge corners.

6. Conclusions

A novel automatic finite element mesh generation using bubble packing is developed. Bubbles on domain can be controlled in order to obtain an improve quality of mesh in three-dimensional problems. The key features of the present algorithm are an easy control of complex 3D bubble density distribution with a fewer input data by means of the fuzzy knowledge processing technique. The effective of the present system is demonstrated through several mesh generations for 3D structures.



(a) Cylinder is connected around the center of the plate



(b) Cylinder is connected near the edge of the plate

Fig. 10. Example of mesh subdivisions of a junction of cylinder and plate

In the next version, the present system will be complemented to hexahedral mesh generation.

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