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Empty Containers Distribution Problem considering the Container Ship Route

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컨테이너선의 경로를 고려한 공 컨테이너의 분배 문제에 대한 연구

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Today international trade through maritime transportation is significantly increasing. Due to this increase, shipping companies are faced with problems concerning the repositioning of empty containers from import dominant ports. The liner shipping service network has been provided to transport containers which load customers' freights. Container ships are moved on the liner shipping service network by observing the pre-determined route and transportation time. This research deals with the empty containers distribution problem considering the container ship route. A mathematical model based on the mixed integer program has been introduced in this study. The objective is to minimize the total relevant costs of empty containers such as handling, leasing, and inventory holding, etc. Due to the complexity of the problem, a genetic algorithm has been suggested to solve large sized problems within a reasonable time. Numerical experiments have been conducted to show the efficiency of the genetic algorithm.

Keywords: Empty Containers Distribution, Container Ship Route, Genetic Algorithm, Mixed Integer Program

1. Introduction

Shipping companies try to satisfy their customers' demands at all times. The container demands increase as international trade grows. In 2007, worldwide container cargo quantities increased by 11.7% compared to 2006 and 492million TEUs (Twenty foot Equivalent Units) of cargo quantities were carried out in 2007. It is expected that in the North East Asian Region (EA), 199 million TEUs will be carried out in 2008, an increase of 13.2% compared to 2007, while in the North American Region (US), 53 million TEUs will be carried out, an increase of 7.0% compared to 2007 and in the Western European Region (EU) 100 million TEUs will be carried out, an increase of 9.7% compared to 2007. Numerous shipping companies have been established to meet the demands for containers and this has eventually led to a fierce competition among them. More competition results in complex problems such as a difficulty in forecasting demands, and an imbalance of international trade, etc. Therefore, shipping companies always tend to have extra containers to cope with the shortage of empty containers, which is obviously a needless extravagance in terms of the need for an efficient use of containers.

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A successful plan to distribute empty containers will be one of the most important ways for shipping companies to maintain a dominant position within the transportation industry. Shipping companies usually use several methods of operating their container ships. These methods need to be considered in order to propose a successful plan for distributing empty containers. In reality, however, one of the most frequently used methods is to employ the liner shipping service network, whereby a schedule is made for container ships to arrive at a certain port, helping to reserve containers for customers. The shipping companies provide a liner shipping service network that provides many lines by considering the most profitable way to use the predetermined routes and schedules. <Figure 1> shows the container ships that sail through the assigned routes according to schedules in a liner shipping service network.

There are two types of ports, those of export- dominant or import-dominant ports. If full containers arrive at an import-dominant port, they will soon be unloaded to become empty containers within a certain period of time. The time it takes for full containers to become empty containers is called the devanning time. On the other hand, a shortage of empty containers may occur in the export-dominant ports. When the rate of exports is more than that of imports, an imbalance occurs in using containers. The containers are a limited resource of transporting freight from the shipping companies' point of view. Empty container distribution planning involves repositioning empty containers from import-dominant ports to export-dominant ports at a minimum cost. The shipping companies try to enhance this type of activity because they can make more profit by using containers efficiently. A number of empty container problems have recently been studied. Li *et al.* (2004) dealt with the empty container allocation problem at a port. They applied a discrete Markov chain process to a problem while proving the optimal quality of the (U, D) policy and considering a number of conditions as a type of inventory problem. The (U, D) policy concerns the importing of empty containers up to the U level when the number of empty containers in the port is less than the U level, or exporting empty containers to the D level when the number of empty containers is more than that of the D level. In addition, Li *et al.* (2007) extended Li *et al.'s* (2004) study to the multi port problem concerned with the allocation of empty containers from supply ports to demand ports. Shen and Khoong (1995) developed a decision support system for an empty container distribution planning problem. They proposed the multi period distribution of empty containers using network optimization models. Luca *et al.* (2006) focused on a fleet management problem in an inland situation. They modeled an integer program and the proposed solution approach was based on the decomposition in three simpler sub problems associated with resource assignment costs, routing costs, and the container repositioning costs considered. Hossein *et al.* (2006) studied inland empty containers movement in a case at Los Angeles and Long Beach port. They suggested a mathematical model to reduce congestion by optimizing empty container reuse. Koich *et al.* (2007) presented

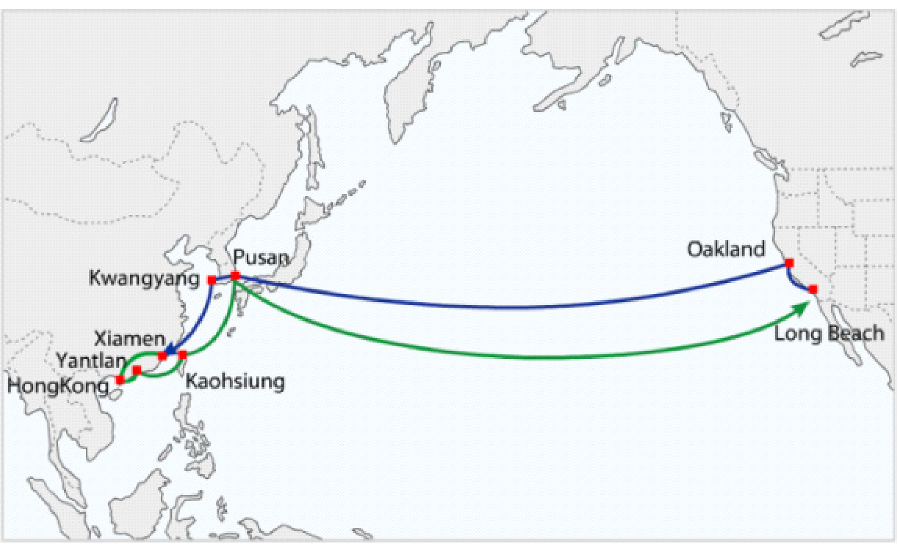

Figure 1. Example of liner shipping service network

the design of container liner shipping service networks by taking into account empty container repositioning. They formulated a mathematical model as a two-stage problem, and a genetic algorithm based heuristic was developed. Wan and Levary (1995) studied shipper contracting in perspective of shipping companies. A linear programming based model was proposed for a minimum amount of time on price negotiation. Crainic *et al.* (1989) introduced the multi commodity location problem with balancing requirements. The study considered balancing movements of empty containers among depots with empty containers supply and demand. Crainic and Delorme (1993) extended Crainic *et al.'s* (1989) study with a dual ascent procedure heuristic. Crainic *et al.* (1993) also proposed an empty container allocation model for inland transportation. They considered several container movements such as customer and depot movement, depot and depot movement, depot and port movement and so on. Gendron and Crainic (1997) introduced a parallel branch and bound algorithm for a multi commodity location problem considering balancing requirements. They formulated a mathematical model and developed a branch and bound based heuristic with a flip flop procedure.

In most previous studies on empty container distribution planning and designing of liner shipping, the route configuration has not been considered. In this present study, the route configuration is considered in developing the empty containers distribution model. The format of the remainder of the paper is as follows. Section 2 presents a mathematical model for an empty containers distribution problem considering the container ship route. In section 3, a genetic algorithm is suggested. Computational experiments are conducted for comparison of the mathematical model and the proposed genetic algorithm in section 4. Section 5 concludes this paper.

2. Mathematical model

2.1 Problem description

<Figure 2> shows empty containers positioning without consideration of routes. Nodes 1, 2 and 3 are shortage ports and the other nodes are surplus ports. In this case, we can transport empty containers from any surplus port to any shortage port. However, <Figure 3> shows restrictions on the repositioning of empty containers in the liner shipping service network. For example, we cannot directly transport empty containers from node 6 to node 1. As a result, nodes 2 and 3 are candidates for transshipment to transport empty containers to node 1. In our study, we will develop an empty containers distribution model by considering a route configuration in a liner shipping service network.

2.2 Assumptions

We use the following assumptions :

i) A single commodity model is considered.
ii) The demands for each period and port are known during the planning horizon and these must be satisfied. If the inventory level of empty containers is lower than the demand in a certain period, leasing is an option to satisfy the demand.
iii) There is no limitation for leasing. The leased containers are used until the end of planning horizon.
iv) The route used for transporting is fixed.
v) The transportation time for each route is determined by the liner shipping service network.
vi) On a liner shipping service network, there are schedules for the arrival ports for each container

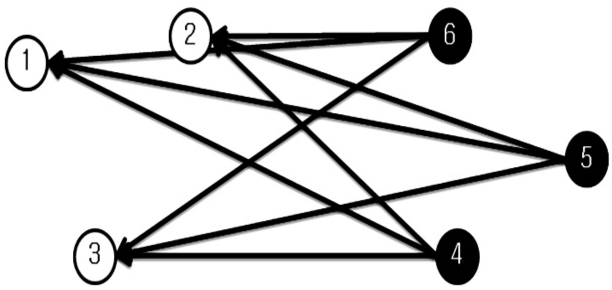

Figure 2. Example of empty containers positioning without routes

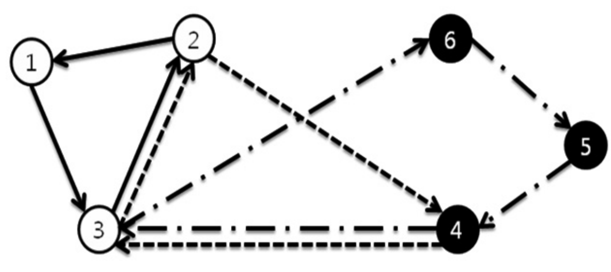

Figure 3. Example of routes configuration in a liner shipping service network

ship.

vii) We consider one container ship on each route.

viii) The handling cost consists of loading and unloading costs.

ix) The unloaded full containers are changed into empty containers within the devanning time.

x) All costs are independent of time periods.

2.3 Notation

The notation to be used is as follows;

Indices :

$i,\ j$: indices for ports, $i,\ j \in \{1,\ 2,\ 3,\ \cdots,\ N\}$

r : index for routes, $r \in \{1,\ 2,\ 3,\ \cdots,\ R\}$

t : index for time, $t \in \{1,\ 2,\ 3,\ \cdots,\ T\}$

Parameters :

dev : time for changing from a full container to an empty container (devanning time)

c_{ijr} : empty containers transportation cost from port i to port j using route r

lc_i : handling cost for loading container at port i

uc_i : handling cost for unloading container at port i

h_i : inventory holding cost at port i

o_i : leasing cost at port i

tt_{ijr} : transportation time from port i to port j using route r

CAP_r : transportation capacity for container ship using route r

d_{ijrt} : demands to be transported from port i to port j using route r at time t

v_{irt} $\begin{cases} 1 : & \text{if a container ship on route } r \text{ arrives at port } i \text{ at time } t, \\ 0 : & \text{otherwise} \end{cases}$

y^F_{ijrt} : number of full containers loading on container ship from port i to port j using route r at time t; This is a predetermined parameter which is calculated by

$$y^F_{ijrt} = d_{ijrt} \cdot v_{irt} \quad \forall i, \forall j, \forall r, \forall t$$

u^F_{jrt} : number of unloaded full containers at port j using route r at time t; This is a predetermined parameter which is calculated by

$$u^F_{jrt} = \sum_{i=1, i\neq j}^{N} y^F_{ijr(t-tt_{ijr})} \quad \forall j, \forall r, \forall t$$

M : sufficiently large number

Decision Variables :

I^E_{it} : inventory level of empty containers at port i at time t

y^E_{ijrt} : number of empty containers loading on a container ship from port i to port j using route r at time t

u^E_{irt} : number of unloaded empty containers at port i using route r at time t

w_{rt} : number of containers on the container ship using route r at time t

z_{it} : number of leasing containers at port i at time t

x_{ijrt} $\begin{cases} 1 : & \text{if route } r \text{ is selected for transporting containers from port } i \text{ to port } j \text{ at time } t, \\ 0 : & \text{otherwise} \end{cases}$

2.4 Model development

The mathematical model can be formulated as follows :

$$Min \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\sum_{r=1}^{R}\sum_{t=1}^{T} c_{ijr}\cdot x_{ijrt} + \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\sum_{r=1}^{R}\sum_{t=1}^{T} lc_i \cdot y^E_{ijrt} + \sum_{i=1}^{N}\sum_{r=1}^{R}\sum_{t=1}^{T} uc_i \cdot u^E_{irt} + \sum_{i=1}^{N}\sum_{t=1}^{T} o_i \cdot z_{it} + \sum_{i=1}^{N}\sum_{t=1}^{T} h_i \cdot I^E_{it} \tag{1}$$

Subject to

$$I^E_{it} = I^E_{i(t-1)} + z_{it} + \sum_{r=1}^{R} u^F_{ir(t-dev)} + \sum_{r=1}^{R} u^E_{irt} - \sum_{j=1,j\neq i}^{N}\sum_{r=1}^{R} y^E_{ijrt} - \sum_{j=1,j\neq i}^{N}\sum_{r=1}^{R} y^F_{ijrt} \quad \forall i, \forall t \tag{2}$$

$$w_{rt} = w_{r(t-1)} + \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} (y^E_{ijrt} + y^F_{ijrt}) - \sum_{j=1}^{N} (u^E_{jrt} + u^F_{jrt}) \quad \forall r, \forall t \tag{3}$$

$$x_{ijrt} \leq v_{irt} \cdot v_{jr(t+tt_{ijr})} \quad \forall i, \forall j, \forall r, \forall t \tag{4}$$

$$y^E_{ijrt} \leq M \cdot x_{ijrt} \quad \forall i, \forall j, \forall r, \forall t \tag{5}$$

$$u^E_{jrt} = \sum_{i=1,i\neq j}^{N} y^E_{ijr(t-tt_{ijr})} \quad \forall j, \forall r, \forall t \tag{6}$$

$$w_{rt} \leq CAP_r \quad \forall r, \forall t \tag{7}$$

$$\forall y^E_{ijrt},\ u^E_{irt},\ z_{it},\ I^E_{it} \geq 0,\ \forall x_{ijrt} \in \{0,\ 1\} \tag{8}$$

The objective function (1) is to minimize the relevant costs including the transportation cost, leasing cost, handling cost and inventory holding cost. Constraint (2) represents the empty container inventory level at port i at time t. The inventory level consists of the inventory level of the previous period, the number of leasing containers, the number of devanning containers, and the unloaded empty containers from the container ship, then subtracting the number of empty and full containers transported from port i. Constraint (3) represents the inventory level of the container ship. Loading and unloading activities happen in each port when the container ships arrive. Constraints (4) and (5) guarantee selecting empty containers transporting from port i to port j using route r at time t. The container ship must arrive at port j after departing from port i considering transportation time using route r. Constraint (6) guarantees that the transported empty containers must be unloaded at destination ports. The container ships have limited capacity for transporting containers that consist of full and empty containers of which the destination is determined. Constraint (7) guarantees that the container ship cannot transport in excess of the level of capacity. Constraint (8) shows non-negative decision variables and binaries.

3. Genetic algorithm

It takes much time to solve the mixed integer program in Section 2 as the number of ports, the number of periods, and the number of routes increase. Therefore, we need to develop a heuristic algorithm or a meta-heuristic algorithm to find a near-optimal solution

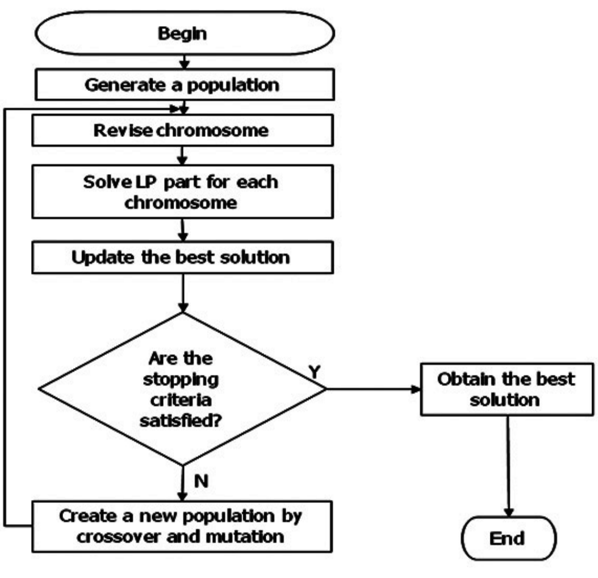

Figure 4. Structure of the genetic algorithm

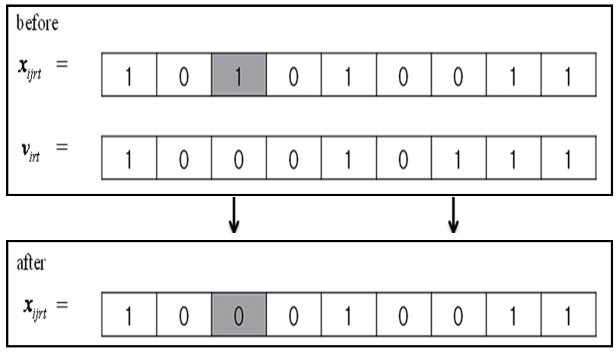

Figure 5. Example of revision procedure

quickly. In this section, we propose a genetic algorithm approach for solving the empty container distribution problem efficiently. The genetic algorithm is based on the idea of genetic evolution in biology. Over the past three decades, genetic algorithms have been widely used in various areas with the modification or updating of some parts. The overall procedure of the genetic algorithm for the empty container distribution problem is illustrated in <Figure 4>.

Firstly, the genetic algorithm generates chromosomes randomly as an initial population. In the next step, chromosomes need to be revised according to the container ship's schedule as shown in <Figure 5>. For a binary variable, the genetic algorithm is used for selecting whether or not empty containers are being loaded at a certain time. The container ships must arrive at a port to load not only empty containers but also full containers. Since the binary variables are determined, the MIP model is converted to the LP model. Therefore, we solve the LP part by the optimization software (Lingo 10) to find the number of empty containers being loaded and unloaded. Secondly, we evaluate the fitness values of the chromosomes, and update the best solution. Finally, the algorithm checks the terminating condition. If the terminating condition is satisfied, the genetic algorithm is terminated. Otherwise, a new population is generated by crossover and mutation.

3.1 Representation chromosome

The correct representation of a solution plays a key role in the development of a genetic algorithm, and the choice of solution representation affects the method of transformation and evaluation. A string that consists of binary values is a solution (a chromosome) similar to a traditional string. This string is a complex structure

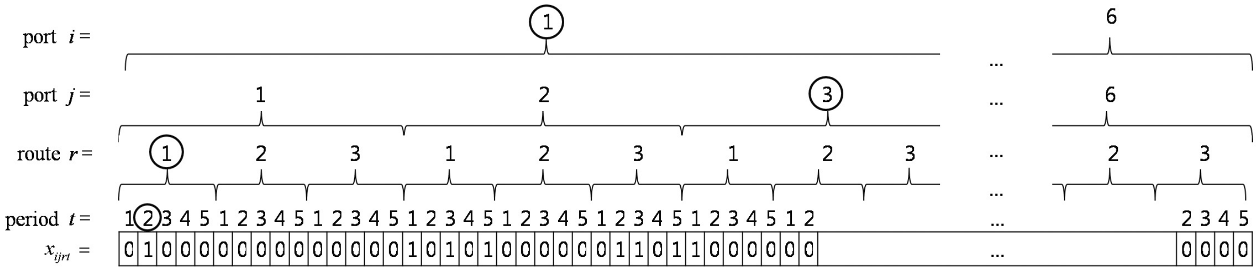

Figure 6. Example of the represented chromosome

following sequence. Firstly, origin candidate ports and destination candidate ports are both represented on the right hand side. Because a set of all destination candidate ports are employed for one origin candidate port, it is iterative as the number of origin candidate ports. The routes are also represented in the same way as the destination candidate ports. Each decision is made for the planning horizon in each route. For example, $x_{1312}=1$ in <Figure 6> means that route 1 is selected for transporting empty containers from port 1 to port 3 at period 2.

3.2 Fitness function

The role of the fitness function is to evaluate each chromosome. Each fitness value in the individual chromosome in the population is observed by a fitness function. A fitness function is computed for each string in the population and the objective is to find the string with the minimum fitness function value. The relevant cost with empty container distribution can be calculated for a given string. The objective function used in the mathematical model is shown below :

$$\text{eval (A)} = \text{Minimize Z}$$

3.3 Reproduction, crossover and mutation

There are several methods for selection such as a roulette wheel, tournament, ranked selection, etc. In this research, we use the ranked selection and the roulette wheel selection. The roulette wheel selection is a genetic operator for selecting potentially useful solutions for recombination. In fitness proportionate selection, as in all selection methods, the fitness function assigns a fitness value to possible solutions or chromosomes. This fitness level is used to associate a probability of selection with each individual chromosome (Goldberg, 1989). The ranked selection is used for the elite parent, and the roulette selection is used for overcoming local optimality. After two chromosomes are selected, they will produce two new chromosomes. We use one point crossover for the selected two chromosomes to generate the new chromosomes. Mutation is performed after the crossover operation. The mutation operator also uses several methods. In this research, the uniform method (randomly performing mutation for each chromosome) is used to maintain genetic diversity.

3.4 Parameters of genetic algorithm

In order to choose the appropriate parameter values in our genetic algorithm for the distribution of empty containers considering container ship route with the objective of minimizing the relevant cost, the above mentioned crossover operator, mutation operator, and cloning were employed with the following parameters.

- Population size : 1000
- Crossover probability : 0.5
- Mutation probability : 0.2
- Cloning Rate : 0.1

The terminating condition is used to stop the algorithm when the number of generations reaches 10000 or when improvement of the best individual is less than 0.1% over 200 generations.

4. Computational experiments

In this section, we compare the results of the mathematical model and the proposed genetic algorithm for an empty containers distribution problem. A small sized problem is to be solved to compare the efficiency of the mathematical model and the proposed

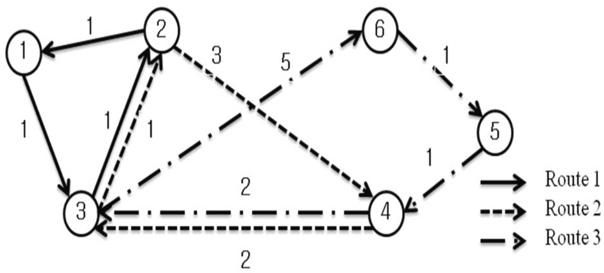

Figure 7. Example problem

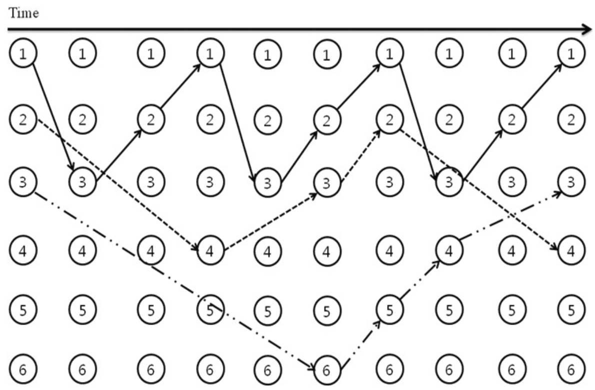

Figure 8. Container ships movements

genetic algorithm. The mathematical model was implemented and solved using LINGO version 10. The proposed genetic algorithm has been developed using C# based on .Net Framework version 3.5. The computational experiments are conducted using Intel Pentium T2400, 2GB RAM with Windows XP.

<Figure 7> represents a liner shipping service network. There are 6 nodes and each node represents a port. The various shapes of arrows represent different routes. The numbers on the arrows refer to the transportation time between linked nodes. The container ships are sailing on the predetermined routes horizontally as shown in <Figure 8.>

Ports 1, 2 and 3 are shortage ports within a planning horizon, while ports 4, 5 and 6 are surplus ports. The result of leasing is shown in <Table 1>. The empty containers repositioning is shown in <Figure 8>. At period 4, 100 empty containers are transported from port 4 to port 2 by using route 2. There are 900 empty containers being transported from port 6 to port 3 and 100 empty containers are moving from port 5 to port 3 using route 3. The container ship has a limitation of capacity, and consequently fewer containers are moving from port 5.

We obtain an optimal solution using the proposed genetic algorithm as shown in <Table 2>. However, the efficiency of the proposed genetic algorithm is less than that of the mixed integer program for small sized problems because the LP part is solved for each chromosome.

<Table 3> shows a summary of the comparison results of the mathematical model and the proposed genetic algorithm with randomly generated problems.

Table 1. Result of leasing containers

port \ period	1	2	3	4	5	6	7	8	9	10
1				14			8			
2		6			13					
3							8			21
4										
5										
6										

Table 2. Comparison results of mathematical model and GA

	Number of ports	Number of routes	Planning horizon	Total relevant cost	Computational time*	Remark (penalty)
MIP	6	3	10	$ 6,154,670	2 sec.	Optimal
GA	6	3	10	$ 6,154,670	49 sec.	Optimal

Note) * : average of 5 evaluations.

Table 3. Comparison of computational experiments

No.	Port	Route	Planning Horizon	Objective value		Computational time (sec.)		Penalty
				Mathematical model	Genetic algorithm	Mathematical model	Genetic algorithm	
1	6	3	10	6,154,670	6,154,670	2	49	0%
2	6	3	20	9,043,100	9,043,100	4	63	0%
3	6	3	30	10,608,990	10,608,990	5	68	0%
4	6	3	40	16,088,020	16,731,824	6	70	3.84%
5	6	3	50	19,004,650	19,532,054	8	71	2.70%
6	9	3	20	6,181,240	6,181,240	2	72	0%
7	9	3	40	14,904,200	15,676,716	6	89	4.92%
8	9	3	60	23,333,760	24,362,418	8	103	4.22%
9	9	3	80	33,485,790	34,735,302	15	112	3.59%
10	9	3	100	47,056,700	49,206,226	23	148	4.36%
11	12	4	20	4,878,620	5,045,096	34	82	3.29%
12	12	4	40	5,967,890	6,186,610	141	118	3.53%
13	12	4	60	9,668,180	10,120,524	192	149	4.47%
14	12	4	80	16,455,770	17,004,106	288	213	3.22%
15	12	4	100	21,794,500	22,608,562	412	294	3.60%
16	16	4	40	16,262,040	17,117,698	1,542	524	4.99%
17	16	4	60	23,186,350	24,006,804	4,097	632	3.42%
18	16	4	80	-	32,508,496	-	819	-
19	16	4	100	-	39,459,646	-	1,028	-
20	16	4	120	-	50,163,128	-	1,335	-

However, from problem number 16 to number 20, we use PSW (Pacific Southwest), PCX (Pacific China Express), SAX (Southeast Asia Express) and PS2 (Pacific Southwest 2) routes data, which are used in shipping companies (HMM, 2008). Gaps between results in small sizes problems are either quite small or nonexistent. For the mathematical model, however, too much time is taken to calculate large sized problems. Moreover, optimal solutions could not be found for large sized problems in which the number of ports is 16, the number of routes is 4, and the number of planning horizon is more than 80. This computational result shows that the genetic algorithm can be useful for a real situation in whereby the number of ports is large.

5. Conclusions

This research has dealt with an empty containers distribution problem considering the container ship route. A mathematical model and a genetic algorithm have been developed and a number of numerical examples have been solved to evaluate their performances. The difference in the objective values between the mixed integer programming model and the proposed genetic algorithm is either very small or nonexistent for small sized problems. Finding an optimal solution, however, to the mixed integer program is either too time consuming or an optimal solution cannot be found in the case of large sized problems. The proposed genetic algorithm can find a near optimal solution within a 5% penalty. While a single commodity model was considered in this research, it can be extended to a multi commodity case. Also, leasing can be distinguished between short term and long term. The proposed model can be extended to a more complex model considering the different types of leasing. In this research, demands are assumed to be deterministic. Hence, the proposed genetic can also be extended to stochastic demand cases.

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