Journal of the Korean Institute of Industrial Engineers
Vol. 35, No. 1, pp. 73-86, March 2009.

Designing Container Blocks with Automated Rail-Mounted Gantry Cranes in Container Terminals

Byung Kwon Lee · Kap Hwan Kim†

Department of Industrial Engineering, Pusan National University, Busan 609-735, Korea

컨테이너 터미널에서 자동화 야드 크레인이 설치된 블록의 설계

이병권 · 김갑환

부산대학교 산업공학과

This paper discusses a method of determining the optimal design of a block. A horizontal layout of blocks is assumed in which transfer points are located at a side of the block. Each block has several transfer points (TPs) each of which is assigned to a group of adjacent bays and located at the center of the assigned group. The goal is to find the optimal size of a block and the optimal number of TPs while minimizing the total cost consisting of the fixed and operational cost of yard cranes (YCs), the operational cost of internal trucks, and the installation cost of TPs. Constraints on the maximum expected system time of trucks are imposed for the optimization. Formulas for estimating handling operation cycle times of a YC are derived analytically. Numerical experiments are conducted to illustrate optimal block designs for a given set of data.

Keywords: Design of a Block, Transfer Points, Yard Cranes, Cycle Times

1. Introduction

There are many factors to be considered when designing a transport facility, such as size, layout and position. In container terminals, the operational productivity of yard cranes is determined by size, layout, positions of TPs, and the number of TPs on the block.

There are many studies dealing with the design of a facility system. Bozer and White (1990) suggested a design algorithm in end-of-aisle order-picking systems to determine the optimal number of aisles based on throughput and storage capacity. Le-Duc (2005) proposed a travel time model for the storage and retrieval (S/R) machine and the conveyor to design a storage system which has minimum travel time. Kim and Kim (2002) developed a cost model to determine the optimal storage space and number of YCs for import containers. Kim *et al.* (2008) proposed an optimal design of the entire yard mathematically. They tried to minimize rehandling by YCs and travel time by internal trucks. Petering and Murty (2008) use simulation tests to compare the performance of YCs based on various sizes of blocks.

Many studies consider equipment travel time to evaluate the performance of the design of a system. Bozer and White (1984) proposed formulas for estimating travel times during single-and dual-cycle operations by a storage and retrieval (S/R) machine. Kim (1997) proposed a formula to estimate the total num-

This paper was supported by the Ministry of Land, Transport and Maritime Affairs in Korea through the project 'Development of Intelligent Port Logistics Systems'.
† Corresponding author : Kap Hwan Kim, Department of IndustrialEngineering, Pusan National University, Busan 609-735, Korea,
Fax : +82-51-512-7603, E-mail : kapkim@pusan.ac.kr
Received October 17, 2008; Revision Received January 5, 2009; Accepted January 8, 2009.

ber of handlings, including rehandles, required to retrieve all of the inbound containers from a yard-bay. Kim (2006) estimated the expected cycle times of YCs for storage and retrieval operations. He considered two kinds of blocks: blocks with a transfer point on the side of each bay, and blocks with transfer points at the ends of the block. Lee and Kim (2007) also estimated cycle times of a YC analytically. Unlike Kim (2006), who proposed formulas for estimating only the expected cycle times of storage and retrieval operations, Lee and Kim (2007) classify operations into receiving, loading, discharging and delivery operations and provide formulas for estimating the expected cycle time and the variance in the cycle time for each operation type.

This paper deals with a method for simultaneously determining the size of a block and the optimal number of TPs under a given layout. Several cost functions including construction, installation, and operation cost are suggested, and an optimal design which minimizes total cost is proposed.

The remainder of this paper is organized as follows: Section 2 describes the process of finding an optimal design. Section 3 proposes formulas to estimate expected cycle times and the expected variances of a YC. Numerical experiments are conducted in Section 4. Conclusions are drawn in Section 5.

2. Formulations for the design of a block

The performance of a block is determined by the size and layout of the block and the mechanical specification of YCs installed at the block. The size of a block is a combination of the number of bays in the block and the number of tiers and rows in a bay. The mechanical specification of YCs is represented by the speed of the hoisting/lowering of the spreader, the speed of trolley movement, and the speed of gantry travel. The block layout is determined by the location of TPs. The conventional layout is a block with a TP at each bay. Most automated container terminals have blocks with TPs at the ends. However, in this study, we consider a specific type of block layout, which is a block with intermittent TPs along a side of the block.

<Figure 1> shows the layout of a sample block. The number of TPs is less than the number of bays at one side of a block. Each TP is located at the middle of several bays. This means that road trucks always park to deliver or receive containers within those bays. On the other side, an internal truck parks at the side of a bay where a container will be picked up from (put down onto) the truck. With this block layout, when a road truck with a container arrives at the TP nearest to

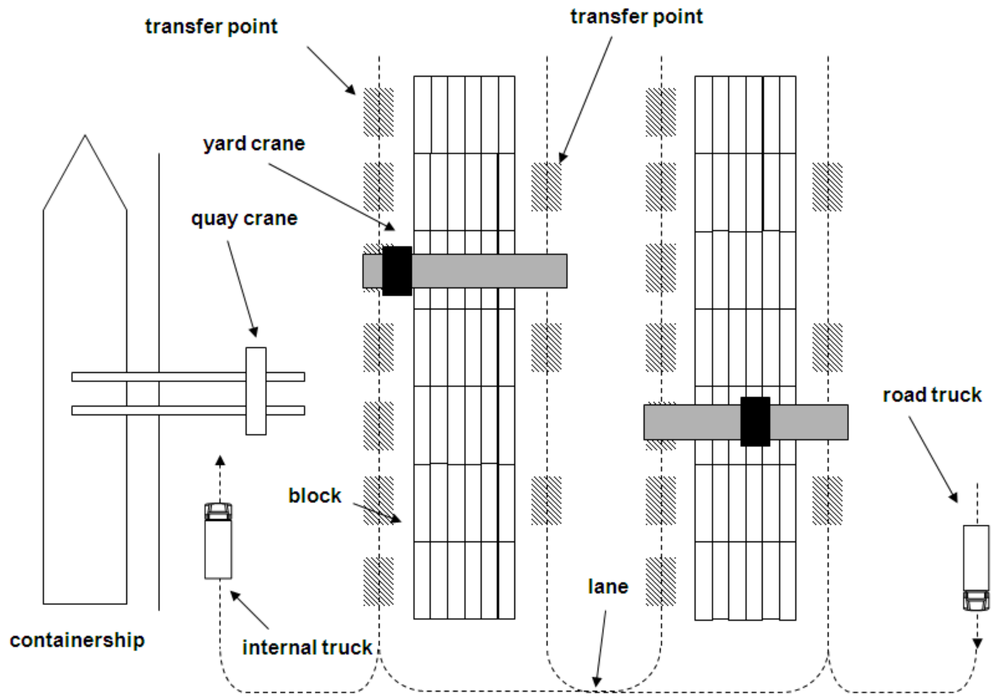

Figure 1. Blocks with intermittent TPs along side of several designated bays

the bay where the container is to be stored, a YC travels to that TP and picks up the container, then travels to the storage location for the container. Thus, there are only receiving and delivery operations of a YC at designated TPs in one side of a block. Internal trucks can use every bay for loading and discharging operations on the other side of a block.

This study proposed an optimal layout of a block with a given size and YC specification. This means that we determine the optimal number of TPs to minimize total cost. To derive cost functions, basic notations are as follows. In this study, a block is defined to be a group of stacks which are assigned to a YC. Thus, when two YCs are deployed to a block in the conventional meaning, a block in this study corresponds to a half of a block in the conventional meaning.

Input parameters

n_r = Number of containers moving from the hinterland to vessels during a year

n_t = Number of containers discharged from a vessel and then loaded onto another vessel during a year

n_d = Number of containers discharged from a vessel and then moved to the hinterland during a year

s_O = Total storage space requirement (TEU) for outbound containers including transshipment containers

s_I = Total storage space requirement (TEU) for inbound containers (containers from vessels to the hinterland)

a_r = Arrival rate of road trucks for receiving containers per minute per block

a_d = Arrival rate of road trucks for delivery containers per minute per block

a_v = Arrival rate of internal trucks per minute per block during the vessel operation

t_r = Maximum allowed system time of road trucks (*min*)

t_v = Maximum allowed system time of internal trucks (*min*)

f_{TP} = Installation (fixed) cost of a TP per year

f_G = Construction cost of the ground space equivalent to a square meter per year

c_{YC} = Operation cost per unit time (*min*) of a YC

c_{TR} = Operation cost per unit time (*min*) of an internal truck

l_t = Length of a truck including space allowance between adjacent trucks (*m*)

l_b = Length of a bay (*m*)

w_r = Width of a stack (*m*)

w_d = Width of a lane for truck traveling (*m*)

w_e = Width of a lane between adjacent blocks (m)

Decision variables

B = Number of bays, which are assigned to a YC, in a block

T = Number of tiers at stacks

R = Number of rows in a bay

X = Number of TPs (decision variable)

$S(Y)$ = Set of Y under consideration, where Y can be B, T, R, or X

Functions of decision variables

$T_r(B, T, R, X)$ = Cycle time of a YC for receiving a container under a given combination of B, T, R, and X (*min*)

$T_O(B, T, R)$ = Cycle time of a YC for transferring an outbound container to an internal truck or receiving a transshipment container from an internal truck under a given combination of B, T, and R (*min*)

$T_I(B, T, R)$ = Cycle time of a YC for receiving an inbound container from an internal truck under a given combination of B, T, and R (*min*)

$T_d(B, T, R, X)$ = Cycle time for delivery a container by a YC under a given combination of B, T, R, and X (*min*)

$f_{YC}(T, R)$ = Installation (fixed) cost of a YC per year under a given combination of T and R

$W_r(B, T, R, X)$ = Wait-time of a road truck for transferring an outbound container to a YC under a given combination of B, T, R, and X (*min*)

$W_d(B, T, R, X)$ = Wait-time of a road truck for receiving an inbound container from a YC under a given combination of B, T, R, and X (*min*)

$W_O(B, T, R)$ = Wait-time of an internal truck for receiving an outbound container from a YC or transferring an transshipment container to a YC under a given combination of B, T, and R (*min*)

$W_I(B, T, R)$ = Wait-time of an internal truck for transferring an inbound container to a YC under a given combination of B, T, and R (min)

The goal is to minimize the total cost under some constraints related to handling performance. The handling performance may be expressed by the maximum expected waiting time of internal and road trucks. The cost terms include the construction cost of the ground space, installation (fixed) cost of a YC, installation (fixed) cost of a TP, the operational cost of a YC, and the operational cost of internal trucks.

The following assumptions are introduced for the formulation :

1. A truck delivers one container at a time.
2. Blocks for inbound containers are separated from those for outbound containers.
3. Transshipment containers are stored at blocks for outbound containers.

In the following, we introduce a formulation for designing a block for outbound containers. Suppose that blocks of (B, T, R, X) are provided for outbound containers. The objective function includes the following terms.

The construction cost of the ground space for outbound containers

$$\frac{s_O\left(l_b B+w_d\right)\left(w_r R+w_e\right) f_G}{BTR} \tag{1}$$

The fixed overhead cost of YCs for outbound containers

$$\frac{s_O f_{YC}\left(T,\ R\right)}{BTR} \tag{2}$$

The fixed overhead cost of TPs for outbound containers

$$\frac{s_O f_{TP} X}{BTR} \tag{3}$$

The operational cost of YCs for outbound containers

It is assumed that c_{YC} consists of the labor fee, the fuel cost, and the maintenance cost.

$$c_{YC}\left\{n_r E\left[T_r\left(B,\ T,\ R,\ X\right)\right]+\left(n_r+2n_t\right) E\left[T_O\left(B,\ T,\ R\right)\right]\right\} \tag{4}$$

The operational cost of internal trucks for outbound containers

The c_{TR} also includes the labor fee, the fuel cost, and the maintenance cost. In addition, the fixed overhead cost is included.

$$c_{TR}\left(n_r+2n_t\right)\left\{E\left[W_O\left(B,\ T,\ R\right)\right]+E\left[T_O\left(B,\ T,\ R\right)\right]\right\} \tag{5}$$

The objective function is restricted by the following constraints.

A constraint on the maximum system time of internal trucks

$$E\left[W_O\left(B,\ T,\ R\right)\right]+E\left[T_O\left(B,\ T,\ R\right)\right] \le t_v \tag{6}$$

A constraint on the maximum system time of road trucks

$$E\left[W_r\left(B,\ T,\ R,\ X\right)\right]+E\left[T_r\left(B,\ T,\ R,\ X\right)\right] \le t_r \tag{7}$$

A constraint on the maximum number of TPs

$$l w_b B / X \ge l_t \tag{8}$$

Therefore, we can formulate the problem for a block for outbound containers (PO) as follows :

$$\begin{aligned}
&(PO)\ \underset{B,T,R,X}{Minimize}\ \frac{s_O\left(l_b B+w_d\right)\left(w_r R+w_e\right) f_G}{BTR} \\
&+\frac{s_O f_{YC}(T,\ R)}{BTR}+\frac{s_O f_{TP} X}{BTR} \\
&+c_{YC}\left\{n_r E\left[T_r\left(B,\ T,\ R,\ X\right)\right]+\left(n_r+2n_t\right) E\left[T_O\left(B,\ T,\ R\right)\right]\right\} \\
&+c_{TR}\left(n_r+2n_t\right)\left\{E\left[W_O\left(B,\ T,\ R\right)\right]+E\left[T_O\left(B,\ T,\ R\right)\right]\right\}
\end{aligned}$$

$$\begin{aligned}
&\textit{subject to :} \\
&\quad E\left[W_O\left(B,\ T,\ R\right)\right]+E\left[T_O\left(B,\ T,\ R\right)\right] \le t_v \\
&\quad E\left[W_r\left(B,\ T,\ R,\ X\right)\right]+E\left[T_r\left(B,\ T,\ R,\ X\right)\right] \le t_r \\
&\quad l_b B / X \ge l_t \\
&\textit{where } B \in S(B),\ T \in S(T),\ R \in S(R),\ \textit{and } X \in S(X)
\end{aligned} \tag{9}$$

The purpose is to find B, T, R and X which guarantee the optimal design of a block, minimizing total cost. The problem for a block for inbound containers (PI) can be formulated similarly. Formulas for estimating the expected cycle times and variances of cycle times are necessary for solving problems PO and PI. Section 3 derives formulas for the expected values and the variances in the cycle times for receiving, loading, discharging, and delivery operations. These cycle times are also used to estimate expected waiting times of trucks.

3. Estimating cycle times of a YC and waiting times of trucks

3.1 Basic notations

The following introduces various parameters required for deriving the expressions for the expectation and the variance of the cycle times of YCs. Also, the basic parameters are illustrated in <Figure 2> and <Figure 3>, and the notations are as follows :

c_w = Width of a container (m)
c_h = Height of a container (m)
c_l = Length of a container (m)
d_c = Distance between the end of bay and the center of the chassis (m)
g_b = Empty gap between two consecutive bays (m)
g_r = Empty gap between two consecutive rows (m)
h_c = Height of a chassis (m)
v_g^e = Speed of empty gantry travel of a YC (m/min)
v_g^l = Speed of loaded gantry travel of a YC (m/min)
v_t^e = Speed of empty trolley move of a YC (m/min)
v_t^l = Speed of loaded trolley move of a YC (m/min)
v_h^e = Speed of empty hoisting of a YC (m/min)
v_h^l = Speed of loaded hoisting of a YC (m/min)
s_g = Time required for a spreader to grasp a container (min)
s_r = Time required for a spreader to release a container (min)
h_{max} = Height of the spreader at the top position (m)
$h_{max} = c_h(t+1)+1.5$. Note that ($t+1$) is the height including the distance for rehandling, and 1.5 is the allowance distance.
d_{max}^h = Distance between the top position of the spreader and the position of the spreader when the YC picks up a container from a chassis (m)
$d_{max}^h = h_{max} - (h_c + c_h)$
b_l = The gantry travel distance between two ends of a block (m)
$b_l = (c_l + g_b)(b-1)$
b_w = The trolley moving distance between two ends of a bay (m)
$b_w = (c_w + g_r)(r-1)$
D_t^h = Distance between the top position of the spreader and the pickup (releasing) position of a container from (to) a bay (m). This is a random variable.
$E(D_t^h)$ = The expected hoisting (lowering) distance (m)
$$E(D_t^h) = h_{max} - c_h\left(\frac{t+1}{2}\right)$$
$Var(D_t^h)$ = The variance of hoisting (lowering) distance (m)
$$Var(D_t^h) = c_h^{\,2}\left(\frac{t^2-1}{2}\right)$$

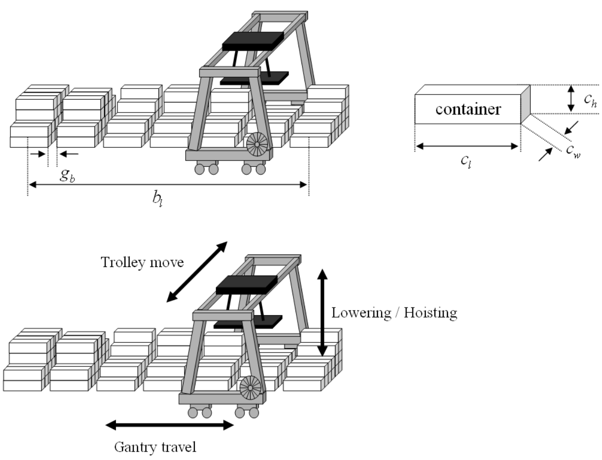

Figure 2. Illustration of notations on blocks

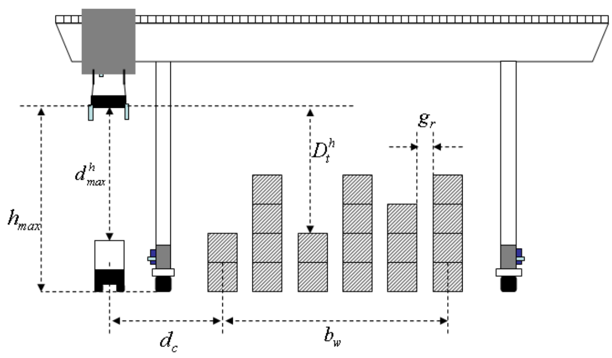

Figure 3. Illustration of notations of a bay

R_{rt} = Number of rehandles required to pick up a random container from a bay with t tiers and r rows, which is a random variable.

$E(R_{rt})$ = The expected number of rehandles in a bay (Kim, 1997)

$$E(R_{rt}) = \frac{t-1}{4} + \frac{t+2}{16r}$$

$Var(R_{rt})$ = The variance in the number of rehandles in a bay (Lee and Kim, 2007)

$$Var(R_{rt}) = -0.0186r + 0.0585t^2 + 0.2169$$

3.2 Notations for handling elements

Each handling element of a YC is represented by "T" with subscripts or superscripts. Subscripts indicate the handling part and its state (loaded or empty). Superscripts indicate the starting and ending positions for the movement. When "t" is used to represent handling time instead of "T," it indicates that the time has a constant value. The time element represented by "T" is a random variable.

T_{ge}^{rr} = Time required for a YC to travel between two random positions while empty

T_{gl}^{nw} = Time required for a YC to travel between a TP and a random position within several bays which are covered by the TP with a container

T_{te}^{er} = Time required for a trolley to move from the end of a bay to a random position while empty

T_{hl}^{tc} = Time required for hoisting a spreader from the top position on the chassis with a container

T_{hl}^{tr} = Time required for hoisting a spreader from the top position to a random position with a container

$T^{(re)}$ = Time required for a rehandle

t_{sg} = Time required for a spreader to grasp a container

t_{sr} = Time required for a spreader to release a container

$E(\cdot)$ = Expected handling time for a handling element

$Var(\cdot)$ = Variance of handling time for a handling element

The handling elements for the movement of a YC are listed in <Table 1>.

This study assumes that the trolley of a YC moves while the YC travels in the gantry direction. This type of movement is known as Tchebychev travel. For safety reasons, however, these movements cannot be done simultaneously with the hoisting/lowering movement of the spreader.

Table 1. Handling elements in blocks with intermittent TPs

Order	Receiving	Delivery	Loading	Discharging	Rehandle
1	$Max(T_{te}^{re},\ T_{ge}^{rn})$	$Max(T_{te}^{er},\ T_{ge}^{rr})$	$Max(T_{te}^{er},\ T_{ge}^{rr})\left(f = \frac{1}{l_0}\right)$	$Max(T_{te}^{re},\ T_{ge}^{rr})\left(f = \frac{1}{s_{rt}}\right)$	T_{he}^{tr}
2	t_{he}^{tc}	$T^{(re)}(f = R_{rt})$	T_{he}^{tr}	t_{he}^{tc}	t_{sg}
3	t_{sg}	T_{he}^{tr}	t_{sg}	t_{sg}	T_{hl}^{rt}
4	t_{hl}^{ct}	t_{sg}	T_{hl}^{rt}	t_{hl}^{ct}	T_{tl}^{rr}
5	$Max(T_{tl}^{er},\ T_{gl}^{nw})$	T_{hl}^{rt}	T_{tl}^{re}	T_{tl}^{er}	T_{hl}^{tr}
6	T_{hl}^{tr}	$Max(T_{tl}^{re},\ T_{gl}^{wn})$	t_{hl}^{tc}	T_{hl}^{tr}	t_{sr}
7	t_{sr}	t_{hl}^{tc}	t_{sr}	t_{sr}	T_{he}^{rt}
8	T_{he}^{rt}	t_{sr}	t_{he}^{ct}	T_{he}^{rt}	T_{te}^{rr}
9		t_{he}^{ct}	$T_{te}^{er}\left(f = \frac{l_0 - 1}{l_0}\right)$	$T_{te}^{re}\left(f = \frac{s_{rt} - 1}{s_{rt}}\right)$	

3.3 The expected cycle time and the variance for a receiving operation

$Max\left(T_{te}^{re},\ T_{ge}^{rn}\right)$ corresponds to the empty trolley movement during the empty gantry travel of a YC. The first travel is from the position where the transfer of the previous container was completed to the position of the TP for the next receiving operation. For the expected value and the variance, refer to case 3 in the Appendix.

t_{he}^{tc} is the handling time it takes for the YC to lower its spreader without a container from its maximum height to a container on a truck chassis, which we consider to be constant as follows :

$$t_{he}^{tc} = d_{max}^{h} \frac{1}{v_h^e} \tag{10}$$

t_{sg} is the time it takes the YC spreader to grasp a target container. We assume this time value is a constant, too. Thus

$$t_{sg} = s_g \tag{11}$$

t_{hl}^{ct} is the time it takes the YC to hoist its loaded spreader from the position of a container on a truck chassis to its maximum height.

$$t_{hl}^{ct} = d_{max}^{h} \frac{1}{v_h^l} \tag{12}$$

$Max\left(T_{tl}^{er},\ T_{gl}^{nw}\right)$ is the time it takes the YC to travel from the current TP to a random bay for storage with its loaded spreader. Formulas for estimating cycle times can be derived by referring to case 1 in the Appendix.

T_{hl}^{tr} is the time it takes for the YC to lower its spreader with a container from the top position to a random tier of a stack designated for container storage.

$$E\left(T_{tl}^{er}\right) = \left(\frac{b_w}{2} + d_c\right)\frac{1}{v_t^l} \tag{13}$$

$$Var\left(T_{tl}^{er}\right) = \frac{b_w^{\ 2}}{12}\left(\frac{1}{v_t^l}\right)^2 \tag{14}$$

t_{sr} is the time it takes the YC spreader to release a container onto a stack, and is assumed to be constant.

$$t_{sr} = s_r \tag{15}$$

T_{hl}^{tr} is the time it takes for the YC to lower its spreader with a container from the top position to a random tier of a stack designated for container storage.

$$E\left(T_{hl}^{tr}\right) = E\left(D_t^h\right)\frac{1}{v_h^l}$$

$$\text{where } E\left(D_t^h\right) = h_{max} - c_h\left(\frac{t+1}{2}\right) \tag{16}$$

$$Var\left(T_{hl}^{tr}\right) = Var\left(D_t^h\right)\left(\frac{1}{v_h^l}\right)^2$$

$$\text{where } Var\left(D_t^h\right) = c_h^{\ 2}\left(\frac{t^2-1}{12}\right) \tag{17}$$

In terms of the handling elements of the receiving operation in <Table 1>, the expected cycle time for the receiving operation with a given size of a block, $E[T_r(B, T, R, X)]$, is

$$\begin{aligned} E\left[T_r\left(B,\ T,\ R,\ X\right)\right] &= E\left[Max\left(T_{te}^{re},\ T_{ge}^{rn}\right)\right] \\ &+ t_{he}^{tc} + t_{sg} + t_{hl}^{ct} + E\left[Max\left(T_{tl}^{er},\ T_{gl}^{nw}\right)\right] \\ &+ E\left(T_{hl}^{tr}\right) + t_{sr} + E\left(T_{he}^{rt}\right) \end{aligned} \tag{18}$$

The variance of the cycle time, $Var[T_r(B,T,R,X)]$ can be expressed as :

$$\begin{aligned} Var\left[T_r\left(B,\ T,\ R,\ X\right)\right] &= Var\left[Max\left(T_{te}^{re},\ T_{ge}^{rn}\right)\right] \\ &+ Var\left[Max\left(T_{tl}^{er},\ T_{gl}^{nw}\right)\right] + Var\left(T_{hl}^{tr}\right) + Var\left(T_{he}^{rt}\right) \end{aligned} \tag{19}$$

3.4 The expected cycle time and the variance for a delivery operation

The cycle time for the delivery operation also includes the rehandling time. The rehandling work is repeated until all the blocking containers are removed. The expected value of the cycle time for a delivery operation with a given size of block, $E[T_d(B, T, R, X)]$, can be expressed as follows.

$$\begin{aligned} E\left[T_d\left(B,\ T,\ R,\ X\right)\right] &= E\left[Max\left(T_{te}^{er},\ T_{ge}^{rr}\right)\right] \\ &+ E\left(R_{rt}\right)E\left(T^{(re)}\right) + E\left(T_{he}^{tr}\right) + t_{sg} + E\left(T_{hl}^{rt}\right) \\ &+ E\left[Max\left(T_{tl}^{re},\ T_{gl}^{wn}\right)\right] + t_{hl}^{tc} + t_{sr} + t_{he}^{ct} \end{aligned} \tag{20}$$

By referring to Ross (1996), the variance in the rehandling work is

$$Var\left(\sum_{j=1}^{R_{rt}}\left(T^{(re)}\right)_j\right) = E\left(R_{rt}\right)Var\left(T^{(re)}\right)$$

$$+E^2\left(T^{(re)}\right)Var\left(R_{rt}\right) \tag{21}$$

Detailed elements of the rehandling work are enumerated in the “Rehandle” column in <Table 1>. Thus, the variance of the cycle time for a delivery operation, $Var[T_d(B, T, R, X)]$, can be expressed as follows.

$$\begin{aligned} Var\left[T_d\left(B,\ T,\ R,\ X\right)\right] &= Var\left[Max\left(T_{te}^{er},\ T_{ge}^{rr}\right)\right] \\ &+ E\left(R_{rt}\right)Var\left(T^{(re)}\right) + E^2\left(T^{(re)}\right)Var\left(R_{rt}\right) \\ &+ Var\left(T_{he}^{tr}\right) + Var\left(T_{hl}^{rt}\right) + Var\left[Max\left(T_{tl}^{re},\ T_{gl}^{wn}\right)\right] \end{aligned} \tag{22}$$

The expected cycle times for a loading operation, $E[T_O(B, T, R)]$, and a discharging operation, $E[T_I(B, T, R)]$, and their variances, $Var[T_O(B, T, R)]$ and $Var[T_I(B, T, R)]$, are referred to Lee and Kim (2007). Cycle times in this section are used following numerical experiments to compare various scenarios for designing a block.

3.5 Estimating the expected waiting time of trucks

Using the expected values and variances of cycle times derived in this study, we can estimate the expected waiting time of trucks in the yard for various truck arrival rates, which is an important performance measure of container terminals. The average truck waiting time can be estimated by using the following formula (Gross and Harris, 1998) :

$$W_q = \frac{\rho E(S)}{2(1-\rho)}\left(1+\frac{Var(S)}{E^2(S)}\right) \tag{23}$$

Here, W_q is the expected truck waiting time, ρ is the traffic intensity (the average arrival rate of trucks multiplied by the expected cycle time of a YC). Note that $E(S)$ is the expected cycle time of a type of operation, and $Var(S)$ is the variance.

4. Numerical experiments

A numerical experiment was conducted to compare the cycle times according to the number of TPs in a block. It was assumed that b = 34, t = 6, r = 9, $g_b = g_r$ = 0.4 m, d_c = 6 m, h_c = 1.5 m, $v_g^e = v_g^l$ = 180 m/min, v_t^e = 140 m/min, v_t^l = 100 m/min, v_h^e = 120 m/min, v_h^l = 80 m/min, and $s_g = s_r$ = 2 sec. The size of a 20ft container is c_l = 6.058 m, c_w = 2.438 m, and c_h = 2.591 m.

Expected variances of cycle times for two operations were compared for different numbers of TPs. As shown in <Figure 4>～<Figure 7>, there is no significant difference among those. If the number of TPs increases above six, expected cycle times of a receiving or a delivery operation become almost the same.

Another experiment was conducted to find the optimal design of a block. It was assumed that n_r = 360.788 $moves/block$, n_t = 757,813 $moves/block$, n_d = 412,906 $moves/block$, s_O = 13,800 TEU, s_I = 13,800 TEU, f_{TP} = 1,000,000 ₩/$year$, f_G = 28,890 ₩/$year$, c_{YC} = 569 ₩/$min/year$, c_{TR} = 481 ₩/$min/year$, and l_t = 25 m. The inter-arrival time for loading/discharging operations is set to 2.4 min based on the cycle time of a QC. The expected inter-arrival time for receiving/delivery operations is set to 10 min, based on the average data collected from a container terminal. It was also assumed

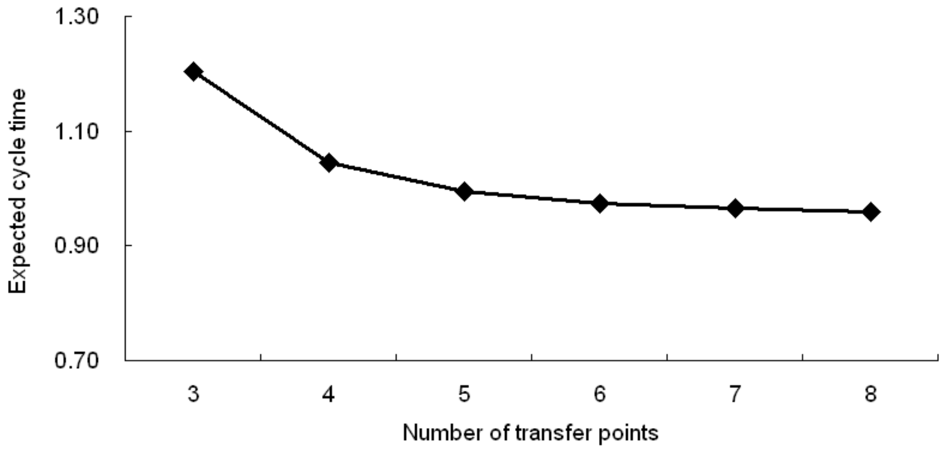

Figure 4. Expected cycle times for a receiving operation for different number of TPs

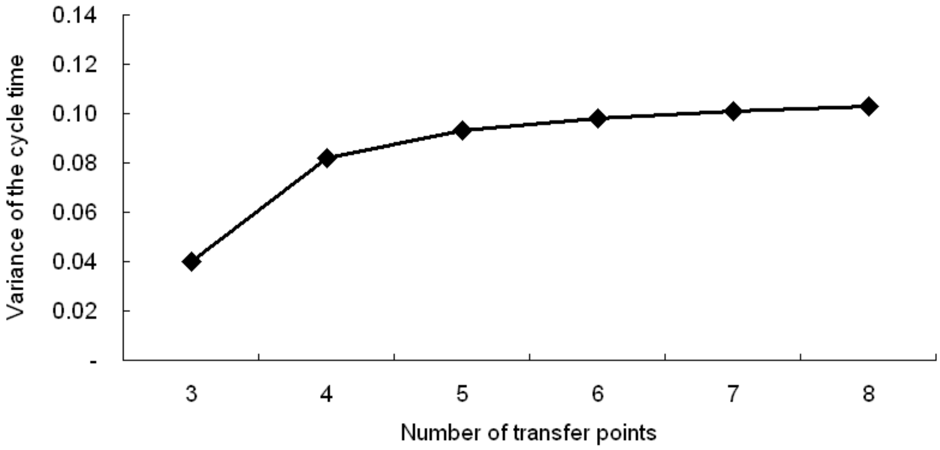

Figure 5. Variances of cycle times for a receiving operation for different number of TPs

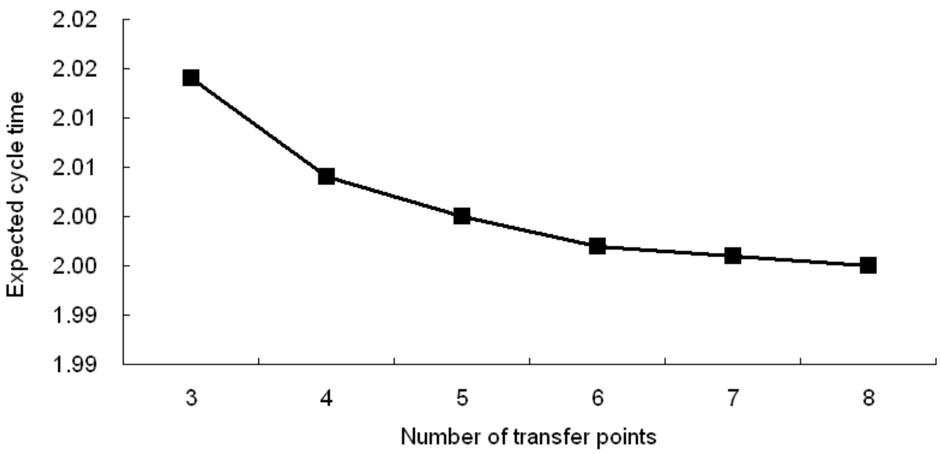

Figure 6. Expected cycle times for a delivery operation for different number of TPs

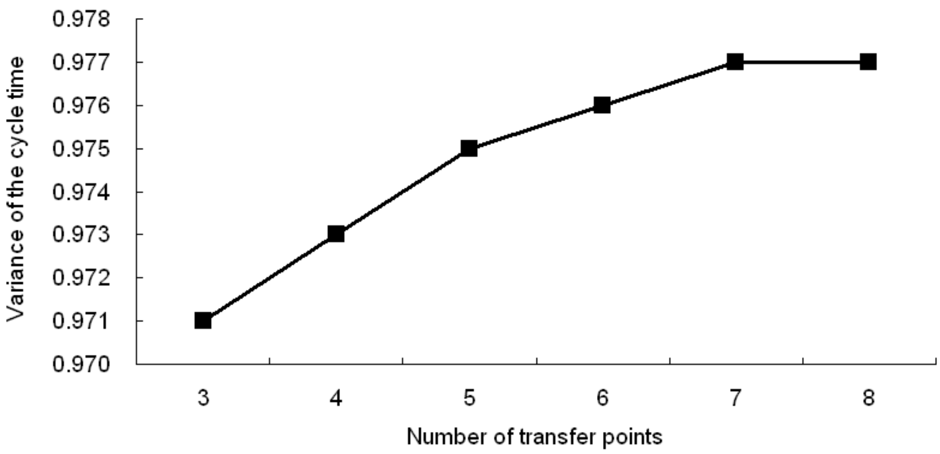

Figure 7. Variances of cycle times for a delivery operation for different number of TPs

that $t_r = t_v = 1$ min at the block for outbound containers, and $t_r = 1.8$ m and $t_v = 1$ min at the block for inbound containers.

Normally, the unit length of a berth is measured by 300 *m*. Thus, it is suitable to give $S(B)$ integer values from 25 to 50. In addition, both $S(T)$ and $S(X)$ are set from 3 to 8, and $S(R)$ is set from 6 to 15. Under these ranges of decision variables, the optimal configuration was 50 bays, 4 tiers, 9 rows, and 3 TPs at the block for outbound containers, and 49 bays, 4 tiers, 8 rows, and 3

TPs at the block for inbound containers. Because the number of bays and the number of TPs have extreme values, it is necessary to extend the search boundary for the further analysis. <Table 2> and <Table 3> show the results of this experiment.

When the upper boundary for the number of bays ex-

Table 2. Effect of the extension of S(B) and S(X) at the block for outbound containers

S(B)	*S(X)*	*(B, T, R, X)*	$E[T_r(B, T, R, X)]$	$E[T_O(B, T, R)]$	$E[Wr(B, T, R, X)]$	$E[W_O(B, T, R)]$
25~50	1~10	(50, 4, 9, 3)	0.92 *min*	0.79 *min*	0.05 *min*	0.20 *min*
25~60	1~10	(60, 3, 11, 3)	0.94	0.77	0.05	0.19
25~70	1~10	(60, 3, 11, 3)	0.94	0.77	0.05	0.19
25~80	1~10	(60, 3, 11, 3)	0.94	0.77	0.05	0.19
25~90	1~10	(60, 3, 11, 3)	0.94	0.77	0.05	0.19
25~100	1~10	(60, 3, 11, 3)	0.94	0.77	0.05	0.19

Table 3. Effect of the extension of S(B) and S(X) at the block for inbound containers

S(B)	*S(X)*	*(B, T, R, X)*	$E[T_d(B, T, R, X)]$	$E[T_I(B, T, R)]$	$E[W_d(B, T, R, X)]$	$E[W_I(B, T, R)]$
25~50	1~10	(50, 3, 13, 2)	1.53 *min*	0.77 *min*	0.15 *min*	0.19 *min*
25~60	1~10	(56, 3, 13, 2)	1.62	0.78	0.17	0.19
25~70	1~10	(56, 3, 13, 2)	1.62	0.78	0.17	0.19
25~80	1~10	(56, 3, 13, 2)	1.62	0.78	0.17	0.19
25~90	1~10	(56, 3, 13, 2)	1.62	0.78	0.17	0.19
25~100	1~10	(56, 3, 13, 2)	1.62	0.78	0.17	0.19

Table 4. Effect of the limitation of system times of trucks at the block for outbound containers

t_v	t_r	*(B, T, R, X)*	$E[T_r(B, T, R, X)]$	$E[T_O(B, T, R)]$	$E[W_r(B, T, R, X)]$	$E[W_O(B, T, R)]$
1.5 *min*	1.5 *min*	(100, 3, 10, 5)	1.17 *min*	0.80 *min*	0.08 *min*	0.21 *min*
	1.2	(93, 3, 10, 5)	1.12	0.79	0.07	0.20
	1	(60, 3, 11, 3)	0.94	0.77	0.05	0.19
	0.75	(32, 3, 8, 2)	0.72	0.67	0.03	0.13
1.2	1.5	(100, 3, 10, 5)	1.17	0.80	0.08	0.21
	1.2	(93, 3, 10, 5)	1.12	0.79	0.07	0.20
	1	(60, 3, 11, 3)	0.94	0.77	0.05	0.19
	0.75	(32, 3, 8, 2)	0.72	0.67	0.03	0.13
1	1.5	(98, 3, 10, 5)	1.16	0.79	0.08	0.21
	1.2	(93, 3, 10, 5)	1.12	0.79	0.07	0.20
	1	(60, 3, 11, 3)	0.94	0.77	0.05	0.19
	0.75	(32, 3, 8, 2)	0.72	0.67	0.03	0.13
0.75	1.5	(42, 3, 6, 2)	0.81	0.63	0.04	0.12
	1.2	(42, 3, 6, 2)	0.81	0.63	0.04	0.12
	1	(42, 3, 6, 2)	0.81	0.63	0.04	0.12
	0.75	(37, 3, 6, 3)	0.71	0.63	0.03	0.11

Table 5. Effect of the limitation of system times of trucks at the block for inbound containers

t_v	t_r	(B, T, R, X)	$E[T_d(B, T, R, X)]$	$E[T_l(B, T, R)]$	$E[W_d(B, T, R, X)]$	$E[W_l(B, T, R)]$
2.2 *min*	2.2 *min*	(80, 3, 15, 3)	1.93 *min*	0.83 *min*	0.25 *min*	0.23 *min*
	1.8	(53, 3, 15, 2)	1.61	0.82	0.17	0.22
	1.5	(41, 3, 11, 2)	1.36	0.73	0.12	0.16
	1.2	(29, 3, 7, 2)	1.11	0.63	0.08	0.11
1.8	2.2	(80, 3, 15, 3)	1.93	0.83	0.25	0.23
	1.8	(53, 3, 15, 2)	1.61	0.82	0.17	0.22
	1.5	(41, 3, 11, 2)	1.36	0.73	0.12	0.16
	1.2	(29, 3, 7, 2)	1.11	0.63	0.08	0.11
1.5	2.2	(80, 3, 15, 3)	1.93	0.83	0.25	0.23
	1.8	(53, 3, 15, 2)	1.61	0.82	0.17	0.223
	1.5	(41, 3, 11, 2)	1.36	0.73	0.12	0.163
	1.2	(29, 3, 7, 2)	1.11	0.63	0.08	0.11
1.2	2.2	(80, 3, 15, 3)	1.93	0.83	0.25	0.23
	1.8	(53, 3, 15, 2)	1.61	0.82	0.17	0.22
	1.5	(41, 3, 11, 2)	1.36	0.73	0.12	0.16
	1.2	(29, 3, 7, 2)	1.11	0.63	0.08	0.11

ceeds a certain value, the optimal number of bays was bounded by the constraint on the maximum system time of trucks. Unlike our expectation, the optimal number of TPs was observed to be two or three. This means that the performance of a YC is not sensitive to the number of TPs.

The sensitivity analysis is conducted to observe the effect of constraints for system times of trucks on the optimal solution. <Table 4> and <Table 5> show the results of the experiment. It was found that the length of a block is sensitive to the constraint for the maximum system time of trucks.

5. Conclusions

This study proposed an optimization method to determine the design of a block with transfer points (TPs) along a side only at several designated bays. Each TP is assigned to a group of adjacent bays and is located at the middle position of the group of the bays. A cost model was proposed considering construction of the yard, installation and operation costs of various equipment. The maximum expected system time, including waiting time and service time, of trucks was considered as constraints in the optimization model. Decision variables were the number of bays in a block, the number of tiers and rows of a bay, and the number TPs. Detailed formulas for estimating the expectation and variance of cycle times of a YC are derived.

The expected cycle time of a YC was not reduced when we increase the number of TPs beyond six. As a result, the optimal configuration of a block was not sensitive to the number of TPs. However, the optimal configuration of a block was sensitive to the constraint on the maximum expected system time of trucks.

The results of this study may be extended to the problem of determining the optimal layout of the entire container yards or determining the optimal number of YCs to be deployed to the yard. Also, other types of layouts different from that in this study may be analyzed in future studies.

Appendix : Handling time models of a YC with Tchebychev metric

Bozer and White (1984) derived mathematical models to estimate the cycle time of an S/R machine in the AS/RS system using the Tchebychev metric. Based on their study, we estimate the expected cycle time and the variance of the cycle time. The notations are as follows :

t_h = The time required for the trolley to move from one end to the other end of a bay (along the x-axis)

t_v = The time required for the YC to travel from one end to the other end (along the y-axis)

T = $Max(t_h, t_v)$. (Because we assume that $t_h < t_v$, $T = t_v$.)

b = Shape factor ($0 \le b \le 1$). (This inequality holds because we assume that $t_h < t_v$, $b = t_h / t_v$.)

t_{xy} = $Max(t_x, t_y)$, where t_x and t_y are the travel time in the x- and y-direction, respectively.

$H(\cdot)$ = Cumulative distribution function for the random variable t_{xy}, $P(t_{xy} \le z)$

$h(\cdot)$ = Probability density function of the random variable t_{xy}

Assuming t_x and t_y are mutually independent, $H(z)$ can be represented as follows :

$$H(z) = P(t_{xy} \le z) = P(t_x \le z)P(t_y \le z) \quad (24)$$

Case 1 : *Both the trolley and gantry movement time follows U(0, b) and U(0, 1), respectively.*

$$P(t_x \le z) = \begin{cases} \dfrac{z}{b}, & 0 \le z \le b \\ 1, & b < z \le 1 \end{cases} \quad (25)$$

$$P(t_y \le z) = z \quad (26)$$

Hence $H(z)$ and $h(z)$ can be derived as follows :

$$H(z) = \begin{cases} \dfrac{z^2}{b}, & 0 \le z \le b \\ z, & b < z \le 1 \end{cases} \quad (27)$$

$$h(z) = \begin{cases} \dfrac{2z}{b}, & 0 \le z \le b \\ 1, & b < z \le 1 \end{cases} \quad (28)$$

The expected handling time and the variance of the cycle time are obtained as follows :

$$E(z) = \int_0^1 zh(z)\,dz = \frac{1}{6}b^2 + \frac{1}{2} \quad (29)$$

$$Var(z) = \int_0^1 z^2 h(z)\,dz - E^2(z) = \frac{1}{6}\left[-\frac{1}{6}b^2\left(b^2 - 6b + 6\right) + \frac{1}{2}\right] \quad (30)$$

Case 2 : *When the trolley movement time follows U(0, b) and the gantry travel time is the difference between the two random variables that follow U(0, 1), respectively.*

$$P(t_x \le z) = \begin{cases} \dfrac{z}{b}, & 0 \le z \le b \\ 1, & b < z \le 1 \end{cases} \quad (31)$$

$$P(t_y \le z) = 2z - z^2 \quad (32)$$

Thus, the expected handling time and its variance are as follows :

$$E(z) = -\frac{1}{12}b^3 + \frac{1}{3}b^2 + \frac{1}{3} \quad (33)$$

$$Var(z) = -\frac{1}{10}b^4 + \frac{1}{3}b^3 + \frac{1}{6} - \left(-\frac{1}{12}b^3 + \frac{1}{3}b^2 + \frac{1}{3}\right)^2 \quad (34)$$

Case 3 : *When the trolley movement time follows U(0, b), the gantry travel time is also the difference between a random variable that follows U(0, 1) and one of several designated variables, respectively.*

$$P(t_x \le z) = \begin{cases} \dfrac{z}{b}, & 0 \le z \le b \\ 1, & b < z \le 1 \end{cases} \quad (35)$$

$$\begin{aligned} P(t_y \le z) &= \frac{1}{x}\sum_{j=1}^{x}\int_{Max\left(\frac{2j-1}{2x}-z,\,0\right)}^{Min\left(\frac{2j-1}{2x}+z,\,1\right)}\left|w - \frac{2j-1}{2x}\right| dw \\ &= \frac{1}{x}\sum_{j=1}^{x}\left[\int_{Max\left(\frac{2j-1}{2x}-z,\,0\right)}^{\frac{2j-1}{2x}}\left(\frac{2j-1}{2x} - w\right)dw + \int_{\frac{2j-1}{2x}}^{Min\left(\frac{2j-1}{2x}+z,\,1\right)}\left(w - \frac{2j-1}{2x}\right)dw\right] \\ &= \frac{1}{x}\sum_{j=1}^{x}\left(\frac{2j-1}{2x}\right)^2 + \frac{1}{2x}\sum_{j=1}^{x}Min^2\left(\frac{2j-1}{2x}+z,\,1\right) \\ &\quad + \frac{1}{2x}\sum_{j=1}^{x}Max^2\left(\frac{2j-1}{2x}-z,\,0\right) \\ &\quad - \frac{1}{x}\sum_{j=1}^{x}\left(\frac{2j-1}{2x}\right)Min\left(\frac{2j-1}{2x}+z,\,1\right) \\ &\quad - \frac{1}{x}\sum_{j=1}^{x}\left(\frac{2j-1}{2x}\right)Max\left(\frac{2j-1}{2x}-z,\,0\right) \end{aligned} \quad (36)$$

w indicates the current position of a YC and x is the number of TPs. As described before, a TP is located in the middle of several bays. For simplification, formula (36) can be fractionated in detail by following the segment range of z.

Ⅰ. $0 \le z \le \frac{1}{2x}$

$$P\left(t_y \le z\right) = \frac{1}{x}\sum_{j=1}^{x}\left(\frac{2j-1}{2x}\right)^2 + \frac{1}{2x}\sum_{j=1}^{x}\left(\frac{2j-1}{2x}+z\right)^2 + \frac{1}{2x}\sum_{j=1}^{0}1^2 + \frac{1}{2x}\sum_{j=1}^{x}\left(\frac{2j-1}{2x}-z\right)^2 + \frac{1}{2x}\sum_{j=1}^{0}0^2 - \frac{1}{x}\left(\sum_{j=1}^{x}\left(\frac{2j-1}{2x}\right)\left(\frac{2j-1}{2x}+z\right)+\sum_{j=1}^{0}\left(\frac{2j-1}{2x}\right)\cdot 1\right) - \frac{1}{x}\left(\sum_{j=1}^{x}\left(\frac{2j-1}{2x}\right)\left(\frac{2j-1}{2x}-z\right)+\sum_{j=1}^{0}\left(\frac{2j-1}{2x}\right)\cdot 0\right) = z^2 \quad (37)$$

Ⅱ. $\frac{2k-1}{2x} < z \le \frac{2\left(k+1\right)-1}{2x}$, k is an integer value satisfying $1 \le k \le x-1$.

$$P\left(t_y \le z\right) = \frac{1}{x}\sum_{j=1}^{x}\left(\frac{2j-1}{2x}\right)^2 + \frac{1}{2x}\sum_{j=1}^{x-k}\left(\frac{2j-1}{2x}+z\right)^2 + \frac{1}{2x}\sum_{j=1}^{k}1^2 + \frac{1}{2x}\sum_{j=k+1}^{x}\left(\frac{2j-1}{2x}-z\right)^2 + \frac{1}{2x}\sum_{j=1}^{k}0^2 - \frac{1}{x}\left(\sum_{j=1}^{x-k}\left(\frac{2j-1}{2x}\right)\left(\frac{2j-1}{2x}+z\right)+\sum_{j=1}^{k}\left(\frac{2j-1}{2x}\right)\cdot 1\right) = \frac{2}{x}\left(x-k\right)z \quad (38)$$

Ⅲ. $\frac{2x-1}{2x} < z \le 1$

$$P\left(t_y \le z\right) = \frac{1}{x}\sum_{j=1}^{x}\left(\frac{2j-1}{2x}\right)^2 + \frac{1}{2x}\sum_{j=1}^{0}\left(\frac{2j-1}{2x}+z\right)^2 + \frac{1}{2x}\sum_{j=1}^{x}1^2 + \frac{1}{2x}\sum_{j=x+1}^{x}\left(\frac{2j-1}{2x}-z\right)^2 + \frac{1}{2x}\sum_{j=1}^{x}0^2 - \frac{1}{x}\left(\sum_{j=1}^{0}\left(\frac{2j-1}{2x}\right)\left(\frac{2j-1}{2x}+z\right)+\sum_{j=1}^{x}\left(\frac{2j-1}{2x}\right)\cdot 1\right) = \frac{1}{x}\left(\sum_{j=1}^{x}\left(\frac{2j-1}{2x}\right)^2 - \sum_{j=1}^{x}\left(\frac{2j-1}{2x}\right)\right) + \frac{1}{2} \quad (39)$$

By using formulas (35) through (39), we can obtain the cumulated density function, $H(z)$, and the probability distribution function, $h(z)$. Therefore, the expected handling time and the variance in handling time, respectively, can be derived as follows.

Ⅰ. $0 \le b \le \frac{1}{2x}$

$$E\left(z\right) = \frac{3}{64}\frac{1}{f}\frac{1}{x^4} + \sum_{k=1}^{x-1}\frac{1}{6}\frac{1}{x^4}\left(x-k\right)\left(12k^2+1\right) \quad (40)$$

$$Var\left(z\right) = \frac{3}{160}\frac{1}{f}\frac{1}{x^5} + \sum_{k=1}^{x-1}\frac{1}{2}\frac{k}{x^5}\left(x-k\right)\left(4k^2+1\right) - E^2\left(z\right) \quad (41)$$

Ⅱ. $\frac{2k-1}{2x} < b \le \frac{2\left(k+1\right)-1}{2x}$, h is also an integer value satisfying $1 \le h \le x-1$.

$$E\left(z\right) = \frac{3}{64}\frac{1}{f}\frac{1}{x^4} + \sum_{k=1}^{h}\frac{1}{f}\frac{k}{x^3}\left(\frac{k}{2}\left(1-\frac{k}{x}\right) + \frac{3}{4}\frac{1}{x^2}\left(x-k\right)\left(4k^2+1\right) + \frac{1}{12}\frac{k}{x^2}\left(4k^2-1\right)\right) + \sum_{k=h+1}^{x-1}\frac{1}{6}\frac{1}{x^4}\left(x-k\right)\left(12k^2+1\right) \quad (42)$$

$$Var\left(z\right) = \frac{3}{160}\frac{1}{f}\frac{1}{x^5} + \sum_{k=1}^{h}\frac{1}{8}\frac{1}{f}\frac{1}{x^4}\left(\frac{k}{3}\left(1-\frac{k}{x}\right)\left(12k^2+1\right) + \frac{3}{10}\frac{1}{x^2}\left(x-k\right)\left(80k^4+40k^2+1\right) + \frac{1}{18}\frac{k}{x^2}\left(4k^2-1\right)\left(12k^2+1\right)\right) + \sum_{k=h+1}^{x-1}\frac{1}{2}\frac{k}{x^5}\left(x-k\right)\left(4k^2+1\right) - E^2\left(z\right) \quad (43)$$

Ⅲ. $\frac{2x-1}{2x} < b \le 1$

$$E\left(z\right) = \frac{3}{64}\frac{1}{f}\frac{1}{x^4} + \sum_{k=1}^{x-1}\frac{1}{f}\frac{k}{x^3}\left(\frac{k}{2}\left(1-\frac{k}{x}\right) + \frac{3}{4}\frac{1}{x^2}\left(x-k\right)\left(4k^2+1\right) + \frac{1}{12}\frac{k}{x^2}\left(4k^2-1\right)\right) + \frac{1}{96}\frac{1}{f}\frac{1}{x^4}\left(4x^2-1\right)\left(4x-1\right) \quad (44)$$

$$Var\left(z\right) = \frac{3}{160}\frac{1}{f}\frac{1}{x^5} + \sum_{k=1}^{x-1}\frac{1}{8}\frac{1}{f}\frac{1}{x^4}\left(\frac{k}{3}\left(1-\frac{k}{x}\right)\left(12k^2+1\right) + \frac{3}{10}\frac{1}{x^2}\left(x-k\right)\left(80k^4+40k^2+1\right) + \frac{1}{18}\frac{k}{x^2}\left(4k^2-1\right)\left(12k^2+1\right)\right) + \frac{1}{288}\frac{1}{f}\frac{1}{x^5}\left(4x^2-1\right)\left(12x^2-6x+1\right) - E^2\left(z\right) \quad (45)$$

References

Bozer, Y. A. and White J. A. (1984), Travel time models for automated storage/retrieval systems, *IIE Transactions*, **16** (4), 329-338.

Bozer, Y. A. and White J. A. (1990), Design and performance models for end-of-aisle order picking systems, *Management Science*, **36**(7), 852-866.

Gross, D. and Harris, C. M. (1998), *Fundamentals of Queueing Theory*, 3rd ed., New York : Wiley.

Kim, K. H. (1997), Evaluation of the number of rehandles in container yards, *Computers & Industrial Engineering*, **32**(4), 701-711.

Kim, K. H. and Kim, H. B. (2002), The optimal sizing of the storage space and handling facilities for import containers, *Transportation Research*, **36B**(9), 821-835.

Kim, K. H., Park, Y. M., and Jin, M. J. (2008), An optimal layout of container yards, *OR Spectrum*, **30**(4), 675-695.

Kim, K. Y. (2006), Evaluation models for the container handling times of the automated transfer crane in container terminal, *IE Interfaces*, **19**(3), 214-224.

Le-Duc, T. (2005), *Design and control of efficient order picking processes*, Ph. D. Dissertation, Erasmus University Rotterdam.

Lee, B. K. and Kim, K. H. (2007), Cycle time models for yard cranes considering block layouts in container terminals, *Journal of Korean Institute of Industrial Engineers*, **33**(1), 110-125.

Ross, S. M. (1996), *Stochastic Processes*, 2nd ed., New York : Wiley.

Petering, M. E. H. and Murty, K. G. (2008), Effect of block length and yard crane deployment systems on overall performance at a seaport container transshipment terminal, *Computers and Operations Research*, DOI : 10.1016/j.cor.2008.04.007.