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Minimization of Inspection Cost in an Inspection System Using a Time-based Flow Analysis

Moonhee Yang† · Sun-uk Kim

Dept. of Industrial Eng., Dankook University, Cheon-an, 330-714, Korea

In this paper, we address an optimization problem and a case study for minimizing the cost of inspections incurred throughout an inspection system, which includes a K-stage inspection system, a source inspection shop, and a re-inspection shop. In order to formulate the inspection cost function, we make a time-based flow analysis between nodes (or shops), and derive the limiting sizes of flows between nodes and limiting defective rates by solving a set of nonlinear balance equations. It turns out that the number of items reworked throughout the inspection system is invariant irrespective of the defective rate of items moved through the K-stage inspection system. Hence we define the inspection cost as the total number of items inspected, and we provide an enumeration method for determining an optimal value of K which minimizes the number of items inspected.

Keywords: Inspection cost, Rework cost, K-stage Inspection System

1. Introduction

In order to reduce the outgoing quality rate, small business suppliers must reduce basically the defective rates of their production lines. However, they can hardly do the activities because of insufficient investment cost and the absence of experts. The only way to survive has been just the strategy to select good items and to rework bad items if they are detected. If we assign many workers at the end of production lines and let them select good items as many as possible and rework bad items when detected, then we may reduce more items inspected and reworked in the remaining processes after the production lines. If not, then we may inspect and/or rework much more items in the remaining processes. Thus, small business suppliers would like to know how to design the selecting/reworking processes to reduce the outgoing quality rate as well as how to forecast in advance the amount of work inspected and reworked throughout their factory.

Since most of papers related to an inspection system assume different designs and operations in addition to limited constraints, it is not easy to search and utilize the published results from previous papers. However, some papers slightly related with an inspection system such as Raz and Thomas (1982), Jaraiedi *et al*. (1987), Avinadav and Raz (2003) may be recommended. In order to select good items efficiently and to attain the specified outgoing quality rate, Yang (2007) suggested a K-stage inspection system which was composed of K stages, each of which included an inspection process and a rework process. He determined the smallest integer value of K which could achieve a given target defective rate. However, he did not consider the effect of the defective rate of items moved through the K-stage inspection system on the remaining processes after the system.

In this paper, partly utilizing the results of Yang's K-stage inspection system, we deal with an optimization problem and a case study of an inspection system for minimizing the inspection cost. Our inspection system includes a K-stage inspection system, a source inspection shop and a re-inspection shop, each of which is interconnected with other shops forming a network.

In Section 2, we describe briefly our problem for the inspection system. In Section 3, we formulate our cost objective function by making some rational assumptions. First, we make a time-based flow analysis between nodes, and we derive the limiting sizes of flows between nodes and limiting defective rates by solving a set of nonlinear limiting balance equations. Second, we show that the number of items reworked throughout the inspection system is invariant irrespective of the defective rate of items moved through the K-stage

† Corresponding author : Moonhee Yang, Dept. of Industrial Eng., Dankook University, Fax : +82-41-550-3570,
E-mail : myfriend@dankook.ac.kr
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inspection system. Finally, we redefine the inspection cost as the number of items inspected, and we provide an enumeration method for determining an optimal value of K which minimizes the number of items inspected. In Section 4, a case study will be given and analyzed.

2. Problem Statement

Yang (2007) suggested a K-stage inspection system consisting of K stages, each of which includes an inspection process and a rework process as shown in <Figure 1>. For convenience, operation or storage areas will be represented as a node. In the first stage, if an item coming off from an production line is classified as good by the first inspector, then it is sent to Node 2, a storage area represented by a reversed triangle. Otherwise, it is sent to the first rework process. After reworked by the first reworker, it is sent to the second inspector. If the reworked item is classified as good by the second inspector, then it is sent to Node 2. Otherwise, it is sent to the second rework process and so on. At the last K-th stage, an item classified as good is sent to Node 2 and an item classified as bad is reworked and sent immediately to Node 2 without inspection. Assuming that inspectors are perfect in the sense that both type I error and type II error are zeros and using his results, we can reduce the average defective rate of items stored at Node 2 as

$$q_K = q_0 q_R^K \tag{1}$$

where q_0 = the average defective rate of items produced from production lines, q_R = the average defective rate of the items reworked. Throughout this paper, we assume that $0 < q_0,\, q_R < 1$. It follows that $0 < q_K < 1$.

As shown in <Figure 2>, our factory consists of the production lines, the K-stage inspection system, the source inspection shop, and finally the re-inspection shop, each of which includes several operations and/or storages. Items stored in Node 2 in the K-stage inspection system are packed into lots and transferred to Node 3 in the source inspection shop, where they are stored. If demands arrive, an inspector (called as the source inspector, who is sent by a buyer and works in the source inspection shop) starts to inspect samples

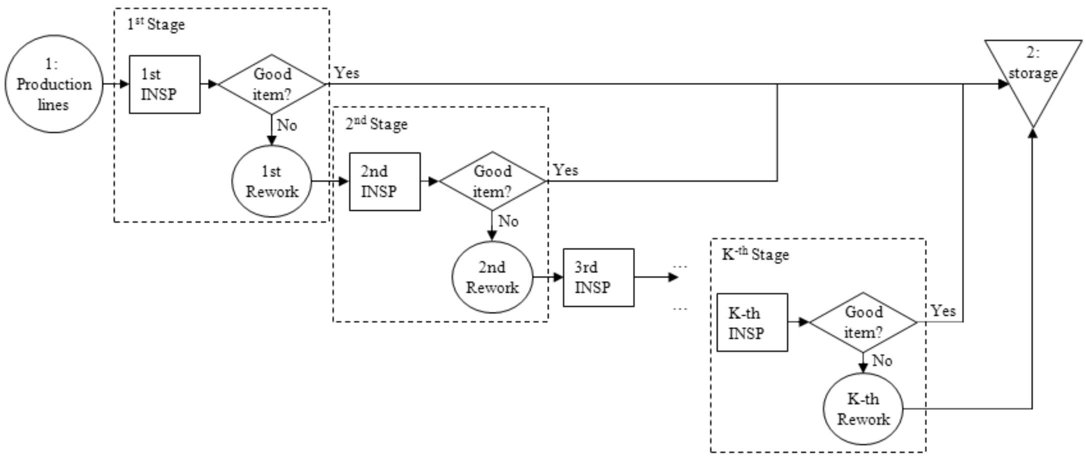


Figure 1. A conceptual process diagram of the K-stage inspection system

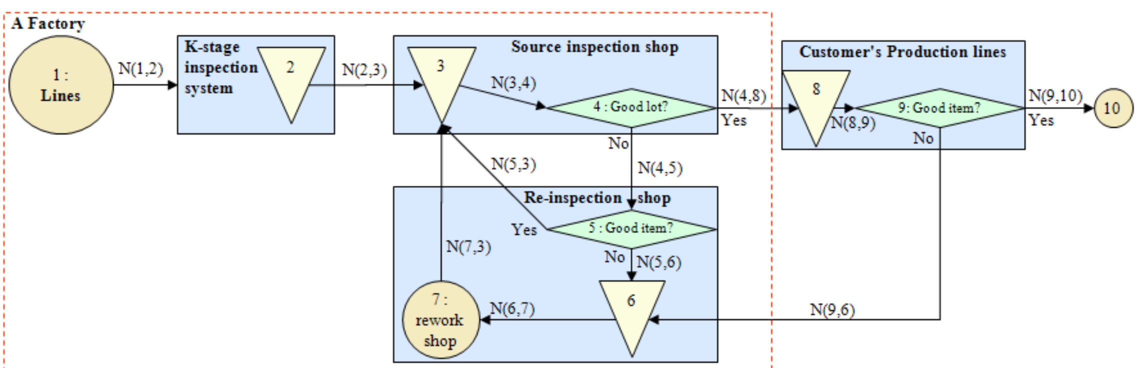


Figure 2. A conceptual process diagram of an inspection system and a customer

drawn from each lot. If all the samples drawn from a lot are judged as good by the source inspector, the lot is accumulated and transported to the consumer's production lines just in time. Otherwise, lots are transferred to Node 5 in the re-inspection shop where all the items are re-inspected again. If those items in Node 5 are classified as good by several inspectors, they are transferred to Node 3, the storage area in the source inspection shop. If not, they are sent to Node 6 and reworked in Node 7, located in the re-inspection shop. The bad items returned from customers (or Node 9) are also reworked together with the items sent from Node 5. Note that Yang (2007) estimated the defective rate of items stored in Node 2. However, we are interested partly in estimating the defective rate of items stored in Node 3.

Define $TC(K)$ to be the total inspection plus rework cost incurred at four nodes; the K-stage inspection system, Node 4 in the source inspection shop, and both Node 5 and Node 7 in the re-inspection shop. It may be conjectured that if K increases, the cost incurred at the K-stage inspection system increases while the cost incurred at Node 4, Node 5, and Node 7 decreases. Otherwise, reverse phenomenon will happen. Hence it can be expected that there exists an optimal value of K minimizing $TC(K)$, and our problem can be stated as follows; Find the optimal value of K, denoted by K^*, so that we minimize $TC(K)$.

3. Flow and Cost Analysis

In this section, we will make a time-based flow analysis in order to estimate the number of items inspected or reworked in Node 4, Node 5, and Node 7 in the long run. We will make several assumptions to derive a set of time-based flow balance equations at nodes for obtaining the limiting size of flow and the limiting defective rate in Node 3 as time goes to infinity. After the flow analysis, we will formulate and analyze our cost function and provide a procedure for determining K^*.

3.1 Flow Analysis

In order to facilitate our flow analysis throughout the inspection system and a customer, we define t to be a positive integer, i.e., t = 1, 2, ⋯, throughout this paper if we do not mention it especially. Also we define,

$NG_t(i, j)$ = the number of good items sent from Node i to Node j in period t,
$NB_t(i, j)$ = the number of bad items sent from Node i to Node j in period t,
$N_t(i, j) = NG_t(i, j) + NB_t(i, j)$,
$NG_t(i) = NG_t(i, i)$ = the number of good items in Node i, produced or stored or inspected in period t,
$NB_t(i) = NB_t(i, i)$ = the number of bad items in Node i, produced or stored or inspected in period t
$N_t(i) = N_t(i, i) = NG_t(i) + NB_t(i)$

Consider Node 2, the storage area in the K-stage inspection system as shown in <Figure 2>. Suppose that R_0 units are produced in period (t-1) and they are inspected/reworked in the K-stage inspection system and stored in Node 2 in the same period (t-1), that is, $N_{t-1}(2) = R_0$ for a positive integer R_0. Assume that all the items available in Node 2 are sent to Node 3 in period (t-1), i.e., $N_{t-1}(2, 3) = R_0$. Then using Eq. (1), we have

$NG_{t-1}(2, 3) = (1 - q_K)R_0$, and
$NB_{t-1}(2, 3) = q_K R_0$

Consider Node 3, the storage area in the source inspection shop. Node 3 has three inflows from Node 5 and Node 7 as well as Node 2. Assume that all the items re-inspected in Node 5 in period (t-1) and all the items reworked in Node 7 in period (t-1) are available in Node 3 in period t. Further assume that the number of bad items sent from Node 5 in period (t-1) is zero, that is, $NB_{t-1}(5, 3) = 0$ for all t. Then we have,

$$NG_t(3) = NG_{t-1}(2, 3) + NG_{t-1}(5, 3) + NG_{t-1}(7, 3) \quad (2)$$
$$NB_t(3) = NB_{t-1}(2, 3) + NB_{t-1}(7, 3) \quad (3)$$
$$N_t(3) = NG_t(3) + NB_t(3) \quad (4)$$

For initial conditions, we assume that $NG_0(2, 3) = (1 - q_K)R_0$, $NB_0(2, 3) = q_K R_0$, and $NG_0(5, 3) = NG_0(7, 3) = NB_0(7, 3) = NB_0(7, 3) = 0$. It follows that $N_1(3) = R_0$. The defective rate of items stored in Node 3 in period t can be expressed as

$$q_t(3) = \frac{NB_t(3)}{N_t(3)} \quad (5)$$

Consider Node 4. Assume that all the lots are immediately sent from Node 3 to Node 4 for inspection in period t, i.e., $N_t(3, 4) = N_t(3)$. Suppose that a lot is accepted only if all the sampled n_S items per lot are judged as good by the source inspector. Note that even if a rejected lot may have some good items, they are not partially accepted. Assuming that the defective rate of a lot is a constant $q_t(3)$ in period t, we can approximate the probability that a lot is accepted in period t by the source inspector as $\{1-q_t(3)\}^{n_s}$. Assume that all the items accepted or rejected in Node 4 are sent to Node 5 or Node 8 in period t respectively, and assume that $N_t(3,4)$ is very large enough. Define N_{LOT} to be the number of items in a lot and let $\lfloor x \rfloor$ be the greatest integer less than or equal to x. Since the average number of lots accepted will be $\lfloor N_t(3,4)/N_{LOT} \rfloor \{1-q_t(3)\}^{n_s}$, we have,

$$N_t(4,8) = N_{LOT} \lfloor \frac{N_t(3,4)}{N_{LOT}} \rfloor \{1-q_t(3)\}^{n_s} \approx \{1-q_t(3)\}^{n_s} N_t(3)$$

$$N_t(4,5) \approx \left[1-\{1-q_t(3)\}^{n_s}\right] N_t(3) \quad (6)$$

Since the difference between two values computed respectively by the approximation and equality equations is usually within an allowed tolerance, we will use the equality sign instead of the approximation sign from now on.

Consider Node 8. We assume that as soon as the items corresponding to $N_t(4,8)$ are moved to Node 8, they are temporarily stored in Node 8 and are inspected and loaded into a customer's production line. In other words, we assume that there are actually no stored stocks in Node 8 due to a JIT production strategy, and we represent the series of these activities as $N_t(4,8) = N_t(8) = N_t(8,9)$. Since $q_t(8) = q_t(3)$, we have

$$NG_t(8) = \{1-q_t(3)\}^{n_s+1} N_t(3)$$

$$NB_t(8) = q_t(3)\{1-q_t(3)\}^{n_s} N_t(3) \quad (7)$$

$$N_t(8) = \{1-q_t(3)\}^{n_s} N_t(3) \quad (8)$$

Consider Node 9. Assume that if items are good, then they are sent to Node 10 in period t and otherwise they are returned to the inspection system in period t. Then using Eq. (8), we have,

$$N_t(9,10) = \{1-q_t(3)\}^{n_s+1} N_t(3) \quad (9)$$

$$N_t(9,6) = q_t(3)\{1-q_t(3)\}^{n_s} N_t(3) \quad (10)$$

Consider Node 5. Assume that all the good items coming into Node 5 are sent to Node 3 and all the bad items coming into Node 5 are sent to Node 6 by inspectors. Assume that the defective rate of item coming into Node 5 is $q_t(3)$. Then, using Eq. (6), we have,

$$NG_t(5,3) = \{1-q_t(3)\}\left[1-\{1-q_t(3)\}^{n_s}\right] N_t(3)$$

$$N_t(5,3) = \{1-q_t(3)\}\left[1-\{1-q_t(3)\}^{n_s}\right] N_t(3)$$

$$NB_t(5,6) = q_t(3)\left[1-\{1-q_t(3)\}^{n_s}\right] N_t(3)$$

$$N_t(5,6) = q_t(3)\left[1-\{1-q_t(3)\}^{n_s}\right] N_t(3) \quad (11)$$

Consider Node 7. Assume that $N_t(6,7) = N_t(7) = N_t(7,3)$. Assuming that all the items coming into Node 7 are reworked in period t, from Eq. (10) and Eq. (11), we have

$$N_t(7,3) = q_t(3) N_t(3)$$

$NG_t(7,3) = (1-q_R) q_t(3) N_t(3)$ and

$$NB_t(7,3) = q_R q_t(3) N_t(3)$$

Now we return to the Node 3. We can reduce Eq. (2), Eq. (3) and Eq. (4) respectively to; for t = 2, 3, 4, ⋯

$$NG_t(3) = (1-q_K)R_0 + \left[1 - q_R q_{t-1}(3) - \{1-q_{t-1}(3)\}^{n_s+1}\right] N_{t-1}(3)$$

$$NB_t(3) = q_K R_0 + q_R q_{t-1}(3) N_{t-1}(3) \quad (12)$$

$$N_t(3) = R_0 + \left[1-\{1-q_{t-1}(3)\}^{n_s+1}\right] N_{t-1}(3) \quad (13)$$

Using Eq. (12) and Eq. (13), we have

$$q_t(3) = \frac{q_K R_0 + q_R q_{t-1}(3) N_{t-1}(3)}{R_0 + \left[1-\{1-q_{t-1}(3)\}^{n_s+1}\right] N_{t-1}(3)}$$

for t = 2, 3, 4, ⋯ (14)

For consistency, we assume that $q_1(3) = q_K$. It can be observed from Eq. (13) and Eq. (14) that it is not easy to represent $N_t(3)$ and $q_t(3)$ as functions of t since two equations are nonlinearly related each other. However, the limiting values of $N_t(3)$ and $q_t(3)$, which will be used for our limiting cost, can be obtained. We define $N_E(3)$ and $q_E(3)$ to be $\lim_{t\to\infty} N_t(3)$ and $\lim_{t\to\infty} q_t(3)$ respectively. Then the following lemma and theorem hold.

Lemma 1: If $0 < q_R, q_K < 1$ and $0 < x < 1$, then for positive integer n_S, the following equation has a single unique solution; $f(x) = q_K(1-x)^{n_s+1} - (1-q_R)x = 0$.

Proof : Since $1-x > 0$, $q_K > 0$, and $1-q_R > 0$, the first order derivative of $f(x)$, $f'(x) = -\left\{q_K(n_s+1)(1-x)^{n_s} + (1-q_R)\right\} < 0$. It follows that $f(x)$ is a strictly decreasing function. Since $f(0) = q_K > 0$ and $f(1) = -(1-q_R) < 0$, there exists a unique solution, x^* in (0, 1) such that $f(x^*) = 0$. □

Theorem 2 : If $0 < q_R, q_K < 1$, then for a positive integer n_S, there exists a unique limiting solution $(N_E(3),\ q_E(3))$ such that

(i) $q_E(3)$ satisfies the equation ;

$f(q_E(3)) = q_K(1-q_E(3))^{n_s+1} - (1-q_R)q_E(3) = 0$

(ii) $N_E(3) = \dfrac{R_0}{\{1-q_E(3)\}^{n_s+1}} = \dfrac{q_K R_0}{(1-q_R)q_E(3)}$.

Proof : Suppose that there exist both $N_E(3)$ and $q_E(3)$. Then $\lim_{t\to\infty} N_{t-1}(3) = \lim_{t\to\infty} N_t(3) = N_E(3)$, and $\lim_{t\to\infty} q_{t-1}(3) = \lim_{t\to\infty} q_t(3) = q_E(3)$. Eq. (13) and Eq. (14) can be reduced to Eq. (15) and Eq. (16) respectively.

$$N_E(3)\{1-q_E(3)\}^{n_s+1} = R_0 \quad (15)$$

$$q_E(3) = \frac{q_K R_0 + q_R q_E(3) N_E(3)}{N_E(3)} \text{ or}$$

$$f(q_E(3)) = q_K(1-q_E(3))^{n_s+1} - (1-q_R)q_E(3) = 0 \quad (16)$$

From Eq. (15), Eq. (16) and Lemma 1, part (i) holds. From Eq. (15), part (ii) holds. From Lemma 1, Eq. (16) has a single unique solution, $q_E(3)$, and so does Eq. (15). Now suppose that the sequence of $q_t(3)$ diverges. Then since the sequence of $q_t(3)$ is bounded as $0 < q_t(3) < 1$, the limit of $q_t(3)$ must have at least two values and this contradicts Lemma 1. Hence the sequence of $q_t(3)$ must be convergent. Suppose that the sequence of $N_t(3)$ diverges to infinity. Then using Eq. (14), we have,

$$\lim_{t\to 0} q_t(3) = \lim_{t\to\infty} \frac{q_K R_0 + q_R q_{t-1}(3) N_{t-1}(3)}{N_t(3)} = q_R \lim_{t\to\infty} q_t(3)$$

It follows that $q_R = 1$ and this contradicts to our assumption that $0 < q_R < 1$. If the sequence of $N_t(3)$ diverges from one value n_3 to another value n'_3, then since $q_t(3)$ converges to $q_E(3)$, using Eq. (13), we have $n_3 = n'_3$ and this is a contradiction to that $n_3 \neq n'_3$. Therefore the sequence of $N_t(3)$ must converge to one and only one value. □

Corollary 3 : $\lim_{t\to\infty} N_t(9,10) = R_0$.

Proof : Taking limits on both sides of Eq. (9) and using Theorem 2, we have, $\lim_{t\to\infty} N_t(9,10) = R_0$. □

This corollary indicates that the number of items consumed is equal to the number of items produced in the long run. Using the above theorem and corollary, we can compute easily the limiting value, $N_E(i,j)$, of $N_t(i,j)$ given K.

From Theorem 2, the sequences of $N_t(3)$ and $q_t(3)$ are convergent but it is not known whether $N_t(3)$ and $q_t(3)$ are decreasing functions of t or not. It is clear from Theorem 2-(ii) that $N_E(3) > N_1(3) = R_0$ since $0 < q_E(3) < 1$, but it is not clear whether $q_E(3) < q_1(3) = q_K$ or not. If our inspection system gives $q_E(3)$ greater than q_K, then there is no reason for the rework shop corresponding to Node 7 to exist. In fact, the following proposition gives the necessary and sufficient condition for its existence.

Proposition 4 : $q_E(3) < q_K$ if and only if $q_R < 1-(1-q_K)^{n_s+1}$ and $0 < q_R, q_K < 1$.

Proof : If $q_R < 1-(1-q_K)^{n_s+1}$, then $q_K\{(1-q_K)^{n_s+1} - (1-q_R)\} = f(q_K) < 0$. Since $f(x)$ is a decreasing function and $f(q_E(3)) = 0$, $f(q_K) < 0$ means that $q_E(3) < q_K$. Now if $q_E(3) < q_K$, then since $f(x)$ is a decreasing function and $f(q_E(3)) = 0$, it follows that $f(q_K) < 0$. In other words, $f(q_K) = q_K\{(1-q_K)^{n_s+1} - (1-q_R)\} < 0$. Since $q_K > 0$, we have, $q_R < 1-(1-q_K)^{n_s+1}$. □

Since both $N_E(3)$ and $q_E(3)$ depend upon K, we change these symbols to $N_E(3:K)$ and $q_E(3:K)$ respectively. Define K to be nonnegative integer throughout this paper. We have the following basic properties for the shapes of $N_E(3:K)$ and $q_E(3:K)$.

Corollary 5 : If $0 < q_0, q_R < 1$, then

(i) both $q_E(3:K)$ and $N_E(3:K)$ are strictly decreasing functions of K,

(ii) $\lim_{K\to\infty} q_E(3:K) = 0$, and

(iii) $\lim_{K\to\infty} N_E(3:K) = R_0$.

Proof : Using Eq. (1) and Eq. (16), and taking the first order derivative of $q_E(3:K)$, we have

$$\frac{\partial}{\partial K} q_E(3:K) = \frac{q_0 q_R^K (\ln q_R)\{1-q_E(3:K)\}^{n_s+1}}{(1-q_R) + q_0 q_R^K (n_S+1)\{1-q_E(3:K)\}^{n_s}} < 0 \quad (17)$$

Note that q_K is a function of K. From Eq. (17), $q_E(3:K)$ is a strictly decreasing function of K. Using Theorem 2-(ii), and taking the derivative of $N_E(3:K)$, we have,

$$\frac{\partial}{\partial K} N_E(3:K) = \frac{(n_S+1)R_0}{\{1-q_E(3:K)\}^{n_s+2}} \frac{\partial}{\partial K} q_E(3:K) < 0 \quad (18)$$

Hence, $N_E(3:K)$ is also a strictly decreasing function of K. Since q_K converges to zero as K increases, we have $(1-q_R)q_E(3:K)=0$ from Eq. (16). Hence, Corollary 5-(ii) holds. It follows that Corollary 5-(iii) holds from Theorem 2-(ii). □

From Corollary 5, the shapes of $q_E(3:K)$ and $N_E(3:K)$ may be drawn as in <Figure 3>. Note that the first left parts of the shapes can be slightly different from the figures since those values of $\frac{\partial^2}{\partial K^2}q_E(3:K)$ and $\frac{\partial^2}{\partial K^2}N_E(3:K)$ can be either positive or negative depending upon K and the input values of (q_R, q_0, n_S).

3.2 Cost Analysis

Consider the total relevant cost of items reworked, $NRW(K)$. Define $NRW(1:K)$ and $NRW(2:K)$ to be the number of items reworked at the K-stage inspection system and at the re-inspection shop respectively. Assuming that the unit rework cost occurred at two different places are same, we define $NRW(K)$ to be the sum of $NRW(1:K)$ and $NRW(2:K)$. Then the following proposition indicates that $NRW(K)$ is invariant irrespective of the value of K and that we may exclude $NRW(K)$ when making a cost analysis. In addition, it indicates that the only way to reduce $NRW(K)$ is to reduce q_0 and/or q_R.

Proposition 6 : $NRW(K)=\frac{q_0R_0}{1-q_R}$.

Proof : For K = 0, 1, 2, ⋯, we can express $NRW(1:K)$ and $NRW(2:K)$ respectively as,

$$NRW(1:K)=\frac{(1-q_R^K)q_0R_0}{1-q_R} \text{ (from Yang(2007))}$$

$$NRW(2:K)=N_E(6,7)=q_E(3)N_E(3)=\frac{q_KR_0}{1-q_R}$$

(from Theorem 2-(ii))

Using Eq. (1), we have, $NRW(K)=\frac{q_0R_0}{1-q_R}$. □

Our total inspection plus rework cost, $TC(K)$, can be now redefined as the inspection costs incurred at three shops; the K-stage inspection system, the source inspection shop, and the re-inspection shop. Utilizing the results of Yang(2007), the number of items inspected at the K-stage inspection system, denoted by $NIN(1:K)$, can be expressed as

$$NIN(1:K)=0 \quad \text{if K = 0}$$

$$=\left\{1+\frac{(1-q_R^{K-1})q_0}{1-q_R}\right\}R_0 \quad \text{if K} \geq 1 \quad (19)$$

Assume that the source inspector must examine all of the n_S samples for each lot even though he may happen to find a defective and reject the lot without inspecting the remaining samples. Then, the number of items inspected in Node 4, denoted by $NIN(2:K)$, can be expressed as,

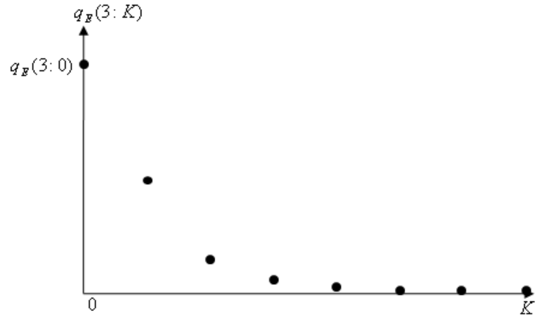
$$NIN(2:K)=n_S\left\lfloor\frac{N_E(3:K)}{N_{LOT}}\right\rfloor=\frac{n_SN_E(3:K)}{N_{LOT}}$$

From Eq. (6), the number of items inspected in Node 5, denoted by $NIN(3:K)$, can be expressed as,

$$NIN(3:K)=N_E(4,5)$$
$$=\left[1-\{1-q_E(3:K)\}^{n_S}\right]N_E(3:K)$$

Hence, assuming that the unit inspection costs occurred at three different places are same, we can express the total relevant inspection cost as $TC(K)=\sum_{i=1}^{3}NIN(i:K)$. It is not easy to make a graph of $TC(K)$. However, the following property might be useful to sketch and explain an approximated shape of $TC(K)$.

Proposition 7 : If $0<q_0, q_R<1$, then



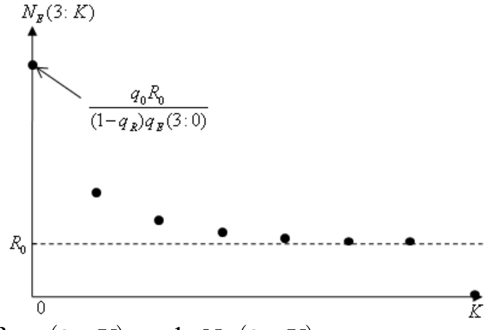


Figure 3. The shapes of $q_E(3:K)$ and $N_E(3:K)$

(i) $NIN(1:K)$ is a strictly increasing concave function of K and

(ii) $\lim_{K\to\infty} NIN(1:K) = \left(1+\frac{q_0}{1-q_R}\right)R_0$,

(iii) $NIN(2:K)$ is a strictly decreasing function of K and

(iv) $\lim_{K\to\infty} NIN(2:K) = \frac{n_S R_0}{N_{LOT}}$,

(v) $NIN(3:K)$ is a strictly decreasing function of K, and

(vi) $\lim_{K\to\infty} NIN(3:K) = 0$.

Proof : For K = 0 and 1, Proposition 7-(i) is clear since $NIN(1:0)$ = 0 and $NIN(1:1) = R_0$ from Eq. (19). For K $\geq$ 1, using Eq. (19), we have,

$$\frac{\partial}{\partial K}NIN(1:K) = -\frac{q_0(\ln q_R)R_0 q_R^{K-1}}{1-q_R} > 0 \text{ and}$$
$$\frac{\partial^2}{\partial K^2}NIN(1:K) = -\frac{q_0(\ln q_R)^2 R_0 q_R^{K-1}}{1-q_R} < 0$$

Hence, $NIN(1:K)$ is a strictly increasing concave function of K. Since $\lim_{K\to\infty} q_R^{K-1} = 0$, Proposition 7-(ii) holds. Taking the derivatives of $NIN(2:K)$, we have

$$\frac{\partial}{\partial K}NIN(2:K) = \frac{n_S}{N_{LOT}}\frac{\partial N_E(3:K)}{\partial K} < 0$$

Hence, $NIN(2:K)$ is a strictly decreasing function of K. From Corollary 5-(iii), Proposition 7-(iv) holds. Taking the first order derivative of $NIN(3:K)$ and using Eq. (17) and Eq. (18), we have

$$\begin{aligned}&\frac{\partial}{\partial K}NIN(3:K)\\ &= n_S\{1-q_E(3:K)\}^{n_S-1}N_E(3:K)\frac{\partial q_E(3:K)}{\partial K}\\ &\quad + \left[1-\{1-q_E(3:K)\}^{n_S}\right]\frac{\partial N_E(3:K)}{\partial K} < 0\end{aligned}$$

It follows that $NIN(3:K)$ is a strictly decreasing function of K. From Corollary 5-(ii) and (iii), Proposition 7-(vi) holds. □

From Proposition 7, The shape of $NIN(i:K)$ may be drawn as in <Figure 4>. Note that the first left parts of the shapes of $NIN(2:K)$ and $NIN(3:K)$ can be slightly different since those values of $\frac{\partial^2}{\partial K^2}NIN(2:K)$ and $\frac{\partial^2}{\partial K^2}NIN(3:K)$ can be either positive or negative depending upon K and the input values of (q_R, q_0, n_S).

Corollary 8 : If $0 < q_0, q_R < 1$, then $\lim_{K\to\infty} TC(K) = R_0\left(1+\frac{q_0}{1-q_R}+\frac{n_S}{N_{LOT}}\right)$.

Proof : It is clear from Proposition 7.

3.3 A Procedure for determining K

Since the candidate value of K is very limited, an enumeration method for determining K may work well. Hence we suggest the following procedure; For an appropriate value of K_{MAX} which is the biggest integer satisfying $q_R < 1-(1-q_{K_{MAX}})^{n_S+1}$,

Step 1. For K = 1 to K_{MAX}
 Begin
 Find a solution of the equation in Lemma 1 and let $q_E(3:K)$ be the solution.
 Compute $N_E(3:K) = \frac{R_0}{\{1-q_E(3:K)\}^{n_S+1}}$.
 Compute $TC(K)$ using $q_E(3:K)$ and $N_E(3:K)$.
 End

Step 2. Find K^* which minimizes $TC(K)$.

4. Case Study of a BLU inspection system

After collecting the data accumulated for six months from a

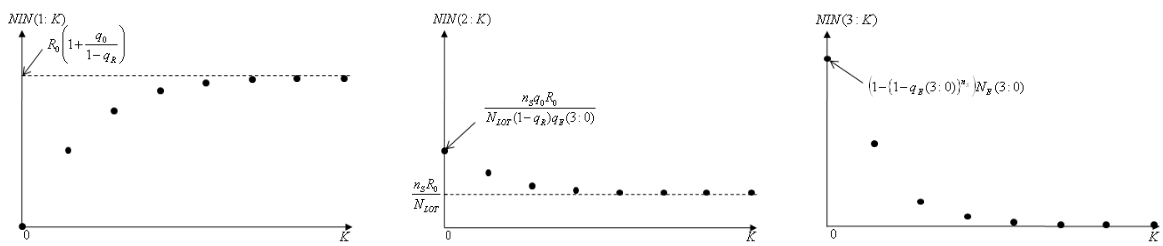


Figure 4. The shapes of $NIN(1:K)$, $NIN(2:K)$, and $NIN(3:K)$

selected BLU (Back-light unit) supplier, we estimate $(R_0, q_0, q_R, N_{LOT}, n_S)$ as (9,600 units/year, 16.1%, 5.0%, 240 units, 16 units). As shown in <Table 1>, as K increases, q_K decreases and converges to zero. Similarly, as K increases, $q_E(3:K)$ and $N_E(3:K)$ also decrease and converge to zero and R_0 respectively. As proved in Proposition 7, NIN $NIN(1:K)$ increases and converges to 11,227 units, and $NIN(2:K)$ and $NIN(3:K)$ decrease and converge to 640 units and zero respectively. The values of $TC(K)$ for 0 ≦ K ≦ 6 are computed sequentially as 18,904, 11,558, 11,855, 11,866 units and so on, and consequently K^* = 1 with TC (K^*) = 11,558 units in this case study. Note that K^* must be either zero or one by Proposition 4 since $q_E(3:K)$ is greater than q_K for K ≧ 2.

It can be observed that there is the greatest reduction of $TC(K)$ when K changes from zero to one as shown in <Figure 5>. In detail, the number of items inspected in the K-stage inspection system has increased from zero to only 9,600 units while the number of items inspected in the source inspection shop and the re-inspection shop has decreased from 18,904 units to 1,958 units. Hence the number of inspected items reduced throughout the inspection system becomes as many as 16,945 units in total. After K≧2, $TC(K)$ increases a little and converges very rapidly to the value of 11,867 units, which can be verified using Corollary 8. Note that $NRW(K)$ is computed as 1,627 units and that it is invariant irrespective of K.

Since the amount of output data are so massive, we summarize $N_t(i, j)\,(t \leq 7)$ in <Table 2> only when K = 1. As proved in Theorem 2 and Corollary 5, $q_t(3)$ and $N_t(3)$ decrease and converge to 7,461 PPM(Part(s) per million) and 10,993 units respectively. Using $q_t(3)$ and $N_t(3)$, we can compute successively all the size of flows between nodes as shown in <Table 2>.

Define $T_E(i, j)$ to be the smallest integer value t which satisfies $|N_t(i, j) - N_{t-1}(i, j)| < 0.5$. For example, $T_E(3, 3) = 4$, $T_E(9, 6) = 2$, and $T_E(4, 5) = 3$ as shown in <Table 2>. When t ≧ 4, there are almost no changes between $N_t(i, j)$ and $N_{t-1}(i, j)$ for all i and j, and all the sequences of flows converge very rapidly. Define the time of convergence, $T_{MAX}(K)$, as the maximum time of $T_E(i, j)$ for all i and j. The values of $T_E(i, j)$ are computed as one to four as shown in <Table 2>, and it follows that $T_{MAX}(1) = 4$. In our experiment, it turns out that $T_{MAX}(K)$ decreases

Table 1. Computational results of $TC(K)$

K	0	1	2	3	4	5	6
q_K(PPM)	16,100	8,050	403	20	1	0	0
$N_E(3:K)$	27,293	10,903	9,669	9,603	9,600	9,600	9,600
$q_E(3:K)$	5.9611%	0.7461%	0.0421%	0.0021%	0.0001%	0.0001%	0.0000%
$NIN(1:K)$	0	9,600	11,146	11,223	11,227	11,227	11,227
$NIN(2:K)$	1,820	727	645	640	640	640	640
$NIN(3:K)$	17,084	1,231	65	3	0	0	0
$TC(K)$	18,904	11,558	11,855	11,866	11,867	11,867	11,867

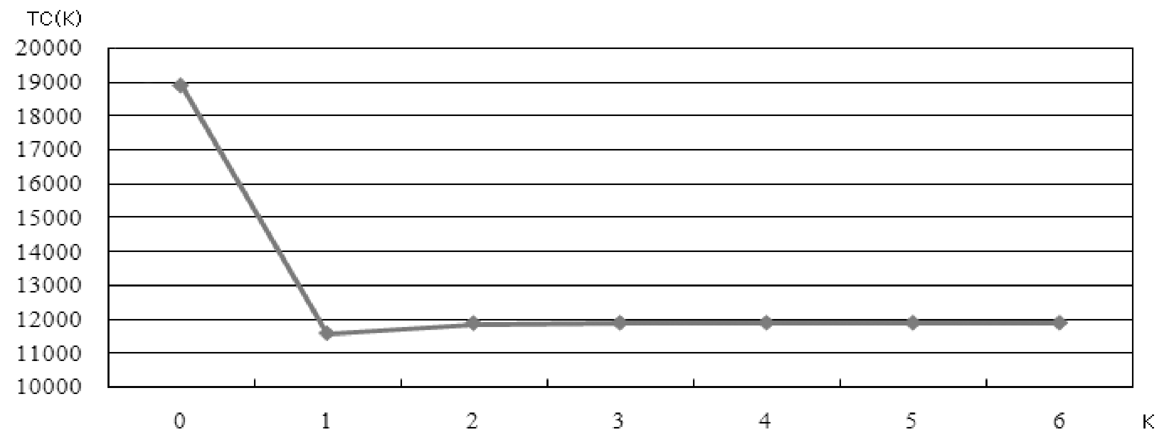


Figure 5. $TC(K)$ given $(R_0, q_0, q_R, N_{LOT}, n_S)$ = (9,600 units/day, 16.1%, 5.0%, 240 units, 16 units)

Table 2. The sizes of flows in period t and their limiting values when K = 1

Iteration	1	2	3	4	5	6	7	$T_E(i, j)$
$N_t(3)$	9,600	10,832	10,900	10,903	10,903	10,903	10,903	4
$NG_t(3)$	9,523	10,751	10,818	10,822	10,822	10,822	10,822	4
$NB_t(3)$	77	81	81	81	81	81	81	2
$q_t(3)$	0.8050%	0.7491%	0.7462%	0.7461%	0.7461%	0.7461%	0.7461%	-
$N_t(3, 4)$	9,600	10,832	10,900	10,903	10,903	10,903	10,903	4
$N_t(4, 8)$	8,435	9,605	9,669	9,672	9,672	9,672	9,672	4
$NG_t(4, 8)$	8,368	9,533	9,597	9,600	9,600	9,600	9,600	4
$NB_t(4, 8)$	68	72	72	72	72	72	72	2
$N_t(9, 10)$	8,368	9,533	9,597	9,600	9,600	9,600	9,600	4
$N_t(9, 6)$	68	72	72	72	72	72	72	2
$N_t(4, 5)$	1,165	1,228	1,231	1,231	1,231	1,231	1,231	3
$N_t(5, 3)$	1,155	1,219	1,222	1,222	1,222	1,222	1,222	3
$N_t(5, 6)$	9	9	9	9	9	9	9	1
$N_t(6, 7)$	77	81	81	81	81	81	81	2
$NG_t(7, 3)$	73	77	77	77	77	77	77	2
$NB_t(7, 3)$	4	4	4	4	4	4	4	1

as K increases as shown in <Table 3>.

Table 3. The values of $T_{MAX}(K)$

K	0	1	2	3	4	5	6
$T_{MAX}(K)$	11	4	3	2	1	1	1

5. Concluding Remarks

In this paper, we derived the limiting sizes of flows in nodes and the limiting defective rate in Node 3. Based on the limiting sizes of flows, we proved that the number of items which must be reworked throughout the inspection system was constant irrespective of the value of K. Note that this result holds true only if our assumptions hold. Hence excluding the rework cost, we formulated and minimized our objective function in terms of the total number of items inspected throughout the inspection system.

However, when the defective rate coming off from the K-stage inspection system decreases in an inspection system, we may measure other several benefits such as the decreased amounts of moving cost, storage cost, recall cost, and so on. Further research may be concentrated on the problems maximizing the combination of different benefits mentioned. Moreover, since one of our assumptions is that inspectors are perfect in the sense that both type I error and type II error are zeros, this assumption may be relaxed and very complicated results could be derived in the future. Our methodology used in this paper to derive the limiting sizes of flows between nodes could be applied and extended to similar situations with slight modification.

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