ADAPTIVE MOMENT-OF-FLUID METHOD : A NEW VOLUME-TRACKING METHOD FOR MULTIPHASE FLOW COMPUTATION

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A novel adaptive mesh refinement(AMR) strategy based on the Moment-of-Fluid(MOF) method for volume-tracking dynamic interface computation is presented. The Moment-of-Fluid method is a new interface reconstruction and volume advection method using volume fraction as well as material centroid. The adaptive mesh refinement is performed based on the error indicator, the deviation of the actual centroid obtained by interface reconstruction from the reference centroids given by moment advection process. Using the AMR-MOF method, the accuracy of volume-tracking computation with evolving interfaces is improved significantly compared to other published results.

Key Words : Adaptive Mesh Refinement(AMR), Volume-of-Fluid(VOF), Moment-of-Fluid(MOF), Volume Tracking, Multi-phase Flow, Multi-material Flow

1. INTRODUCTION

A popular strategy of improving accuracy in computational physics is to use adaptive mesh refinement(AMR). Although the flows with evolving interface is considered a very appropriate class of problem with potential adaptivity, the application of AMR on such problem is relatively rare compared to the flow problems without interfacial phenomena. Here, we present a novel adaptive mesh refinement technique based on the moment-of-fluid method(AMR-MOF) for multi-phase / multi - material interfacial flow simulation.

The MOF method[4,6-7] can be thought of as a generalization of VOF method[1]. In VOF method, volume(the zeroth moment) is advected with local velocity and the interface is reconstructed based on the updated(reference) volume fraction data. In MOF method, volume(zeroth moment) as well as centroid(ratio of the first moment with respect to the zeroth moment) are advected and the interface is reconstructed based on the updated moment data(reference volume and reference centroid).

In the MOF method, the computed interface is chosen to exactly match the reference volume and to provide the best possible approximation to the reference centroid of the material.

By using the centroid information, volume tracking with dynamic interfaces can be computed much more accurately. Furthermore with this conceptual extension of using the moment data, the interface in a particular cell can be reconstructed independently from its neighboring cells. With the advantages of MOF method over the VOF method, our opinion is that the MOF method is a next generation volume-tracking interfacial flow computation method evolved from VOF method.

In this paper, we present a very accurate and efficient adaptive mesh refinement strategy for volume-tracking interfacial flow computations based on the moment-of-fluid method.

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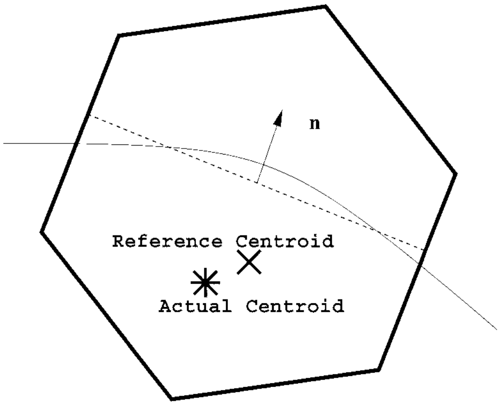


Fig. 1 Stencil for MOF in two dimensions. The stencil for MOF interface reconstruction is composed of only the cell under consideration

2. ADATIVE MOMENT-OF-FLUID METHOD

In volume-tracking multiphase flow simulations, the essential part of the algorithm is the representation of sharp interface using given information(volume fraction and/or centroid). This process is called interface reconstruction. In this section we present the basic ideas of moment-of-fluid interface reconstruction method, and next the design principle of the adaptive moment-of-fluid method.

2.1 PIECEWISE LINEAR INTERFACE CALCULATION(PLIC)

Almost all volume-of-fluid types methods utilize piecewise linear interface calculation(PLIC) for sharp interface representation. In PLIC methods, each mixed cell interface between two materials is represented by plane(line in 2D). It is convenient to specify this plane in Hessian normal form

$$\mathrm{n} \cdot \mathrm{r} + d = 0 \tag{1}$$

where $\mathrm{r} = (x,y)$ is a point on the interface, $\mathrm{n} = (n_x, n_y)$ is the unit normal to the interface, and d is the signed distance from the origin to the interface.

The principal reconstruction constraint is local volume conservation, i.e. the reconstructed interface must truncate the cell, c, with a volume equal to the reference volume V_c^{ref} of the material.

PLIC methods differ in how the interface normal n is computed. In VOF method, the interface normal for cell-c is computed from the volume fraction data on the stencil, composed of cell-c as well as its neighbors. In MOF method, the interface normal is computed from moment data, i.e. volume fraction and material centroids, in cell-c only.

Once the interface normal is computed, the interface is uniquely defined by computing d satisfying the reference volume V_c^{ref} exactly.

2.2 MOMENT-OF-FLUID INTERFACE RECONSTRUCTION

The moment-of-fluid(MOF) interface reconstruction method was first introduced in [6], for interface reconstruction in 2D. The 3D extension for the arbitrary polyhedral mesh and multi-material case is described in [3-4]. To describe main idea of MOF method we need to introduce some definitions. For a given material region, Ω, the zeroth moment (volume) and first moment are defined as follow

$$M_0(\Omega) = \int_\Omega dV, \tag{2}$$

$$M_1(\Omega) = \int_\Omega \mathrm{x}\, dV. \tag{3}$$

Centroid of the material region Ω is the ratio of the first and the zeroth moments

$$x_\Omega = \frac{M_1(\Omega)}{M_0(\Omega)}. \tag{4}$$

Let us assume that for each mixed cell we know not only the reference volume fraction f_c^{ref} but also the reference centroid x_c^{ref}. We need to emphasize that for interface reconstruction reference volume fraction and reference centroid are input data, which is supplied by some other algorithm(advection, for example). Therefore these quantities have errors and moreover there may be no real material configuration which matches exactly both reference volume fraction and reference centroid.

In the MOF method, the computed interface is chosen to match the reference volume exactly and to provide the best possible approximation to the reference centroid of the material. That is, in MOF, the interface normal, n, is computed by minimizing(under the constraint that the corresponding pure subcell has exactly the reference volume fraction in the cell) the following functional:

$$E_c^{MOF}(\mathrm{n}) = \|\mathrm{x}_c^{ref} - \mathrm{x}_c(\mathrm{n})\|^2 \tag{5}$$

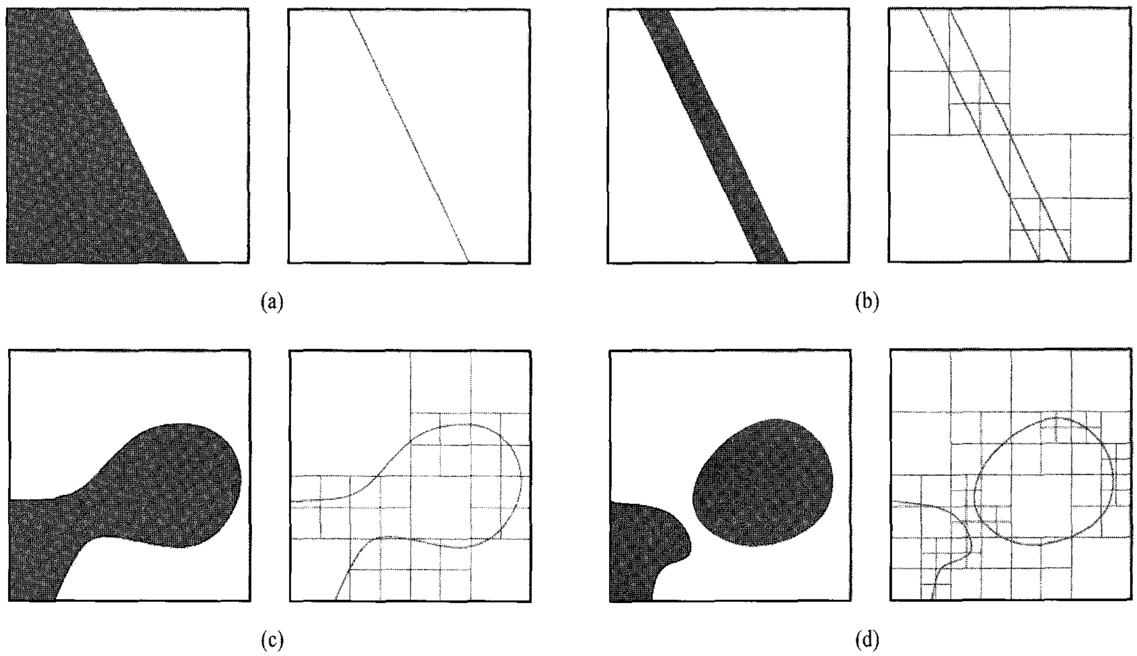


Fig. 2 Subcell scale interface featureswith different curvature and topology. Left column -- material configuration, right column -- possible AMR-MOF refinement pattern. Four representative interface features within a square cell are illustrated: (a) one piece of the material inside the cell --- interface is the segment of the straight line (curvature is zero); (b) two disjoint pieces of the white material --- subcell thickness filament of dark material, curvature has meaning only for each segment of the straight lines and equal to zero, but one curvature per cell does not make sense; (c) one piece of dark material with complicated shape, only average averaged curvature makes sense; (d) disjoint pieces of dark material (subcell size droplet), each of pieces has high average curvature

where x_c^{ref} is the reference material centroid and $\mathrm{x}_c(\mathrm{n})$ is the actual (reconstructed) material centroid with given interface normal n.

The implementation of MOF method requires the minimization of the non-linear function(of one variable in 2D and of two variables in 3D). The computation of $E_c^{MOF}(\mathrm{n})$ requires the following steps. First step, for a given n, is to find the parameter d of the plane such that the volume fraction in cell-c exactly matches f_c^{ref}. Second, we compute the centroid of the resulting subcell containing reference material. This is a simple calculation, described, for example, in [3-4]. Finally, one computes the distance between actual and reference centroids. The MOF method is linearity-preserving, that is, it reconstructs linear interfaces exactly.

The MOF method is a local scheme. The MOF method uses information about the volume fraction, f_c^{ref} as well as centroid, x_c^{ref} of the material, but only from the cell c under consideration. As illustrated in Fig. 1, no information from neighboring cells is used. In context of AMR meshes, this means that MOF method does not care about mesh data structures on interface reconstruction stage. The MOF method can be used for arbitrary polygonal cells(polyhedral cells in 3D). The solid curved line represents the true interface, and the dashed straight line represents the piece-wise linear, volume fraction matching interface at the cell.

2.3 Adaptive Mesh Refinement for Moment of Fluid Method

In general, required level of mesh adaptation has to depend on the complexity of the interface, two immediate examples being curvature and topology of the interface. Fig. 2 illustrates representative interface features.

We note that all features illustrated in Fig. 2 are in subcell scale(their length scale is less then those of unrefined mesh) and also independent from the features of their neighboring cells(neighboring cell may not have similar features).

The design principal of adaptive MOF method is for the development of an intelligent adaptation algorithm for detecting the sub-cell scale features, namely curvature and topology changes.

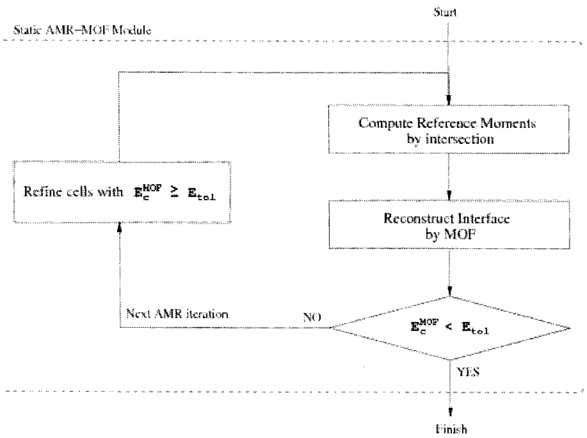


Fig. 3 Flow-chart for static AMR-MOF interface reconstruction for initial representation of material configuration on AMR mesh

3. RESULTS

3.1 STATIC INTERFACE RECONSTRUCTION

The statement of the problem for AMR-MOF static interface reconstruction is as follows: for a given original material configuration, represent the reconstructed material region by PLIC on adaptively refined mesh.

The flowchart for the static AMR-MOF interface reconstruction of a given geometry is presented in Fig. 3.

We note that the static AMR-MOF interface reconstruction, described in Fig. 3, is only for the initial representation of given material configuration on AMR mesh.

In this Section we present static interface reconstruction for multi-element airfoil configuration. The AMR-MOF reconstruction starts with a single cell $[0,1]^2$ level-0 mesh. Adaptive refinement is performed up to level-8 from the level-0 mesh. First eight levels of AMR-MOF interface reconstruction is displayed in Fig. 4.

3.2 DYNAMIC INTERFACE RECONSTRUCTION

In order to apply the MOF interface reconstruction to volume-tracking evolving interfaces, a moment advection scheme is required. Similar to VOF, various types of advection schemes can be devised for MOF. For example, a flux-based approach or a Lagrangian-backtracking + remap approach. The advection scheme presented here is based on Lagrangian-backtracking + remap approach because this approach is often observed to be less diffusive compared with other methods, and the algorithm can be extended to handle multi-material($n_{mat} \ge 3$) cases

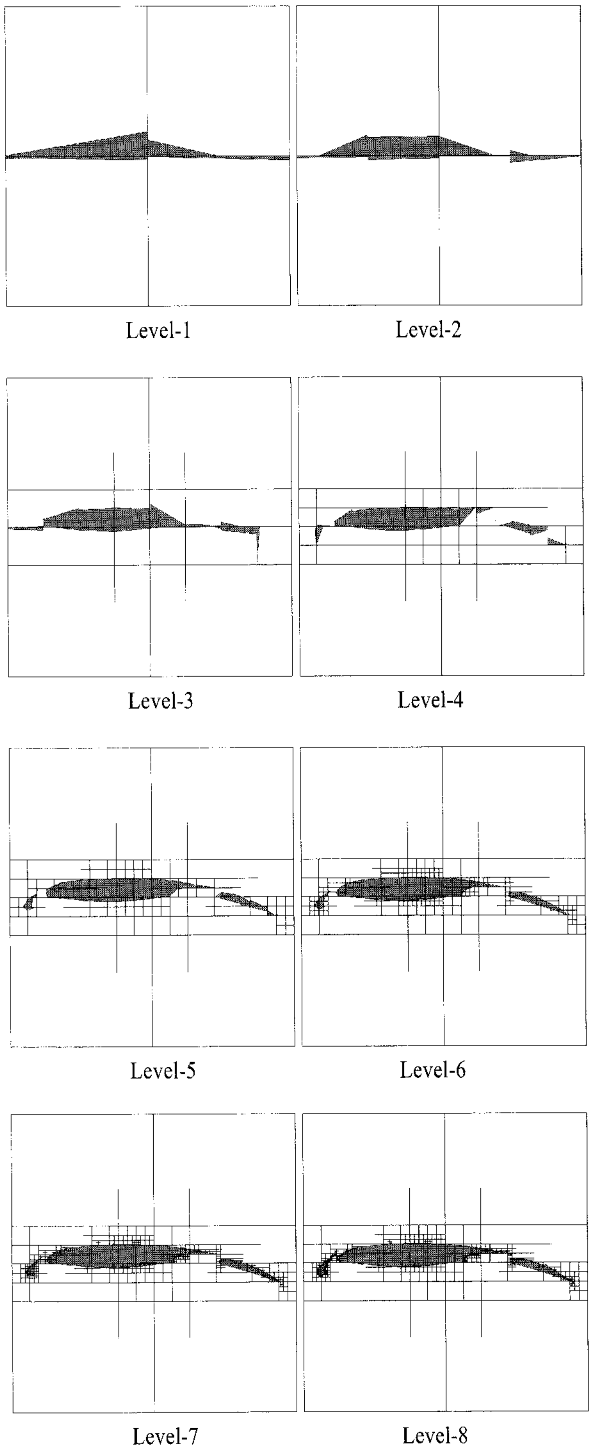
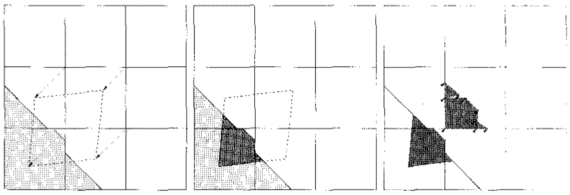


Fig. 4 AMR-MOF interface reconstruction of multi-element airfoil configuration starting with one cell, i.e. the level-0 mesh is 1×1 covering the domain of $[0,1]^2$. Different levels of AMR-MOF reconstruction process are displayed. E is used as the refinement criterion



(a) (b) (c)

Fig. 5 Moment advection by Lagrangian-backtracking + remap strategy. The moment advection process for the central cell-c on 3×3 local stencil is illustrated. (a) Lagrangian-backtracking defines the departure volume indicated by dashed polygon, (b) polygon intersection results in the pure material regions to be remapped on to the cell-c after the advection, and (c) illustrates the centroid advection process

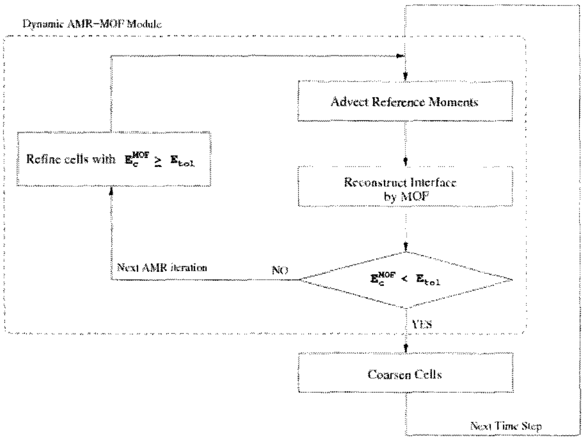
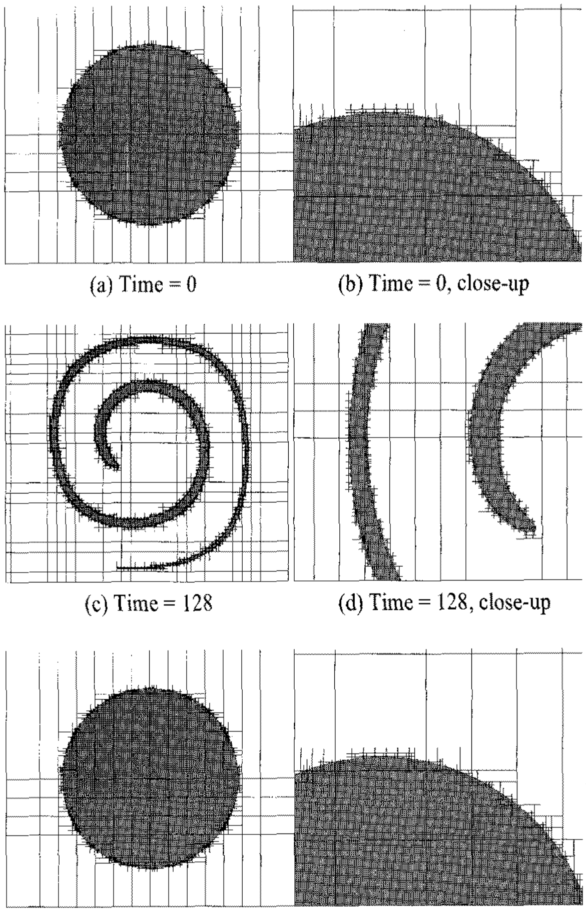


Fig. 6 Flow-chart for dynamic AMR-MOF interface reconstruction and moment advection



(a) Time = 0 (b) Time = 0, close-up

(c) Time = 128 (d) Time = 128, close-up

(e) Time = 256 (f) Time = 256, close-up

Fig. 7 Single vortex flow, $T = 8$. Left column – perspective view, right column – close-up view. Level-0 mesh is 32×32 and maximum 4 level of AMR is allowed (maximum effective mesh resolution is 512×512). $E_{tol} = 1.e-20$ is used as the refinement criterion

in a straightforward manner.

The moment advection is performed only for the potentially mixed cells, i.e. the cells that may contain a material interface. The potentially mixed cells are determined by checking the following two conditions:

1. if cell-c or any of its neighbors is mixed, i.e. contains more than one material,
2. if cell-c is a pure cell, and any of its neighbors is a pure cell but with a different material.

These two conditions are devised to detect when material interface exists inside of the local neighbor stencil of cell-c. Hence, the local stencil is composed of only the immediate neighbors and the number of potentially mixed cells that reside within a narrow band around the interface.

The present advection method is composed of the following steps:

1. Lagrangian-backtracking.
2. Polygon intersection (reference volume computation).
3. Centroid advection (reference centroid computation).

The three conceptual stages of the advection scheme are illustrated in Fig. 5.

The algorithm of the AMR-MOF for dynamically evolving interface is illustrated in Fig. 6. The difference of the dynamic AMR-MOF module from the static AMR-MOF module, as shown in Fig. 3, is reference moment computation step. For dynamic case, the reference moment is computed by advection step, as indicated with gray box.

In order to test the accuracy and applicability of the AMR-MOF method for dynamically evolving interfaces, we consider reversible vortex problem, which is a

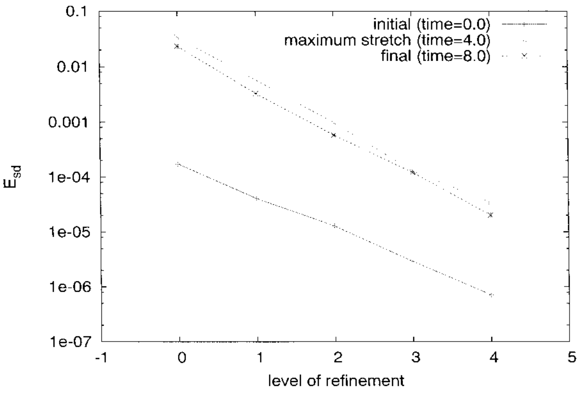


Fig. 8 Reduction of error with respect to the maximum level of adaptive mesh refinement allowed. The level-0 initial mesh is 32×32 and refinement is allowed up to level-4, i.e. the maximum effective mesh resolution 512×512

standard test case described by Rider and Kothe[2]. The initial configuration of the material is defined by a circle with radius $r=0.15$ centered at $(0.5, 0.75)$ within the square domain of $[0,1]^2$. The circular region deforms under the nonlinear unsteady velocity field defined by the following stream function,

$$\Psi(x,y,t) = \frac{1}{\pi}\sin^2(\pi x)\sin^2(\pi y)\cos\left(\frac{\pi t}{T}\right) \tag{6}$$

which results in the nonlinear divergence-free vertical velocity field.

The reversible vortex problem is presented with period, $T=8$. Time steps of $\Delta t = 1/32$ is used for all AMR-MOF computation. 32×32 initial level-0 mesh is used to cover the square domain of $[0,1]^2$. The result of AMR-MOF computation, with maximum refinement up to level-4, is displayed in Fig. 7 at various time steps.

The reduction of error for the successive level of adaptive mesh refinement is displayed in Fig. 8. The error is measured by the area of symmetric difference between the true material configuration(T) and the reconstructed material configuration(R) defined as follows

$$E_{sd} = \sum_{c\in M}|T_c \triangle R_c| = \sum_{c\in M}|(T_c\cup R_c)-(T_c\cap R_c)|, \tag{7}$$

where M is the set of cells, T_c is the true material region in cell-c, and R_c is the reconstructed material region in cell-c. It is clear that for each level of adaptive mesh refinement, the error is reduced by the factor of four which indicates the second-order accuracy.

4. CONCLUSIONS

A new adaptive mesh refinement strategy based on the moment-of-fluid method was presented. Numerical examples demonstrate that error in the centroid position can correctly detect not only regions with high curvature of the interface but also regions with subcell structures like filaments. In [5] we have coupled standard MOF without AMR with incompressible Navier-Stokes solver for two materials. In the future we are planning to couple AMR-MOF with incompressible Navier-Stokes AMR solver.

ACKNOWLEDGMENTS

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