

# 가중 ITAE 지수를 사용한 2관성 모터시스템의 속도제어기 설계

## Speed Controller Design of a Two-Inertia Motor System Using Weighted ITAE Index

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**Abstract:** In a two-inertia motor system with flexible shaft, a torsional vibration is often generated as a quick speed response is required. This vibration makes it difficult to achieve a quick response of speed and disturbance rejection. The objective of this paper is to provide a systematic analysis and design of the three kinds of speed controllers such as I-P, I-PD, and state feedback control by using the weighted ITAE performance index. Some simulation and experiment results verify the effectiveness of the proposed design.

**Keywords:** two-inertia motor system, pole placement controller, ITAE (Integral of Time multiplied by the Absolute Error)

### I. INTRODUCTION

A two-inertia motor system, such as an industrial rolling machine with a flexible shaft, has very low natural resonant frequency because of the long shaft and low stiffness between the motor and load. This makes it difficult to achieve the quick speed response due to torsional vibration. If we increase the controller gain to reduce the settling time of the system, torsional vibration may occur and the resultant stress on the shaft may cause damage to the shaft. Hence, many researchers have focused on reducing oscillation and settling time in a two-inertia motor system[1-8].

The pole placement technique was developed in order to design an I-P and an I-PD controller for a two-inertia system[1]. The Kalman filter and LQ-based speed controller for torsional vibration suppression were also developed[2]. The vibration suppression by using feedback of the imperfect derivative of the estimated torsion torque was also studied[3]. For better performance than the result studied in [3], the slow resonance ratio control of the torsional system was discussed[4]. In [5], after identifying system parameters based on an ARX model, a controller for vibration suppression using these parameters was developed. In [6], the development of the technique of how to tune parameters of controller and observer in a 2-degree of freedom control system using genetic algorithms was described.

In this paper, the systematic analysis and design technique of speed controller for a two-inertia motor system is presented. In particular, a description of how to assign closed-loop poles of three controllers such as I-P, I-PD and state feedback controller with a weighted ITAE(Integral of Time multiplied by the Absolute Error) performance index is presented.

### II. TWO-INERTIA MOTOR SYSTEM

In this section, we describe a model of a two-inertia motor system and the derivation of optimal controller gains by utilizing pole placement for three kinds of controllers.

#### 1. Model of two-inertia motor system

A motor and load coupled by a shaft with a stiffness is shown in Fig. 1, in which

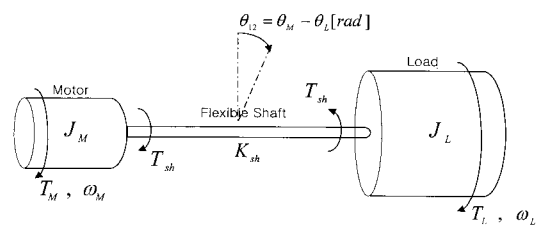


그림 1. 유연한 샤프트로 결합된 2관성 시스템의 모델.  
Fig. 1. Two-inertia motor system coupled by flexible shaft.

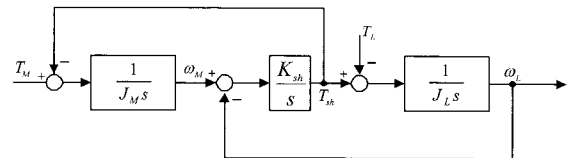


그림 2. 2관성 시스템의 블록도.  
Fig. 2. Block diagram of a two-inertia system.

$J_M$  motor inertia;  $\omega_M$  motor speed;  $T_M$  motor torque;  
 $K_{sh}$  torsion stiffness of the drive shaft;  $T_L$  load torque;  
 $J_L$  load inertia;  $\omega_L$  load speed;  $T_{sh}$  shaft torque;  
 $\theta_M$  motor angle;  $\theta_L$  load angle;

Fig. 2 is a simple block diagram representation of a two-inertia motor system. The friction is neglected. The state equation of a mechanical plant for a two-inertia system is given by

$$\begin{aligned} \dot{X}(t) &= AX(t) + BT_M(t) + ET_L(t) \\ Y(t) &= CX(t) \end{aligned} \quad (1)$$

where output vector  $Y = \omega_M$ , state vector  $X = [\omega_M \ \omega_L \ \theta_{12}]^T$  and  $\theta_{12} = \theta_M - \theta_L$ .

$$A = \begin{bmatrix} 0 & 0 & \frac{K_{sh}}{J_M} \\ 0 & 0 & \frac{K_{sh}}{J_L} \\ 1 & -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{J_M} \\ 0 \\ 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ -\frac{1}{J_L} \\ 0 \end{bmatrix}, \quad C = [1 \ 0 \ 0].$$

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The transfer function from  $T_M$  to  $\omega_M$  in Fig. 2 can be calculated as follows:

$$T(s) = \frac{s^2 + \omega_a^2}{J_M s(s^2 + \omega_0^2)} \quad (2)$$

where  $\omega_a$  and  $\omega_0$  represent the anti-resonant frequency and the resonant frequency, respectively. The resonance ratio,  $R$ , and the inertia ratio of load to motor,  $K$ , are defined as follows:

$$\omega_0 = \omega_a \sqrt{1+K}, K = \frac{J_L}{J_M}, \omega_a = \sqrt{\frac{K_{sh}}{J_L}}, R = \frac{\omega_0}{\omega_a} = \sqrt{1+K} \quad (3)$$

2. Analysis and design of controllers

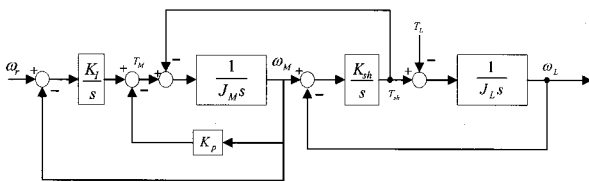
In this section, we describe the pole placement technique for the three kinds of controllers and a comparison of them. In particular, the design criterion of each controller is described to suppress the overshoot and oscillation of a closed-loop system by using a weighted ITAE performance index. Fig. 3 represents the structure of each speed control system using I-P, I-PD, and state feedback controllers, respectively. The closed-loop transfer functions are in order given by

$$\frac{\omega_L}{\omega_r} = \frac{K_I \omega_a^2}{J_M s^2(s^2 + \frac{K_P}{J_M} s + \omega_0^2) + K_P \omega_a^2 s + K_I(s^2 + \omega_a^2)} \quad (4)$$

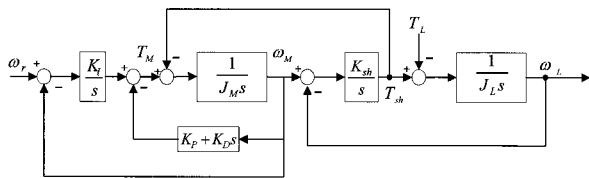
$$\frac{\omega_L}{\omega_r} = \frac{K_I \omega_a^2}{\tilde{J}_M s^2(s^2 + \frac{K_P}{\tilde{J}_M} s + \tilde{\omega}_0^2) + K_P \omega_a^2 s + K_I(s^2 + \omega_a^2)} \quad (5)$$

where

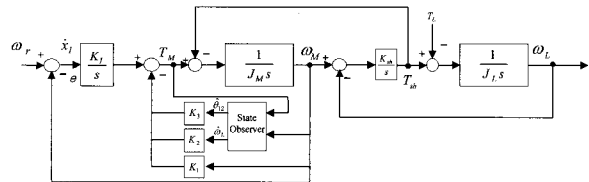
$$\tilde{J}_M = J_M + K_D, \tilde{\omega}_0 = \omega_a \sqrt{1+\tilde{K}}, \tilde{K} = \frac{J_L}{\tilde{J}_M} \quad (6)$$



(a) I-P controller



(b) I-PD controller



(c) State feedback controller

그림 3. 각 제어기를 사용한 속도제어시스템.  
Fig. 3. Speed controller system with each controller.

$$\frac{\omega_L}{\omega_r} = \frac{K_I \omega_a^2}{J_M s^2(s^2 + \frac{K_1}{J_M} s + \hat{\omega}_0^2) + (K_1 + K_2) \omega_a^2 s + K_I(s^2 + \omega_a^2)} \quad (7)$$

where

$$\hat{\omega}_0 = \sqrt{\omega_0^2 + \frac{K_3}{J_M}} \quad (8)$$

Now we introduce the weighted ITAE performance index in order to reduce overshoot and settling time of two-inertia motor system. The key idea is to calculate the performance index so as to amplify for overshoot and so as not to amplify for undershoot. To do this,  $\gamma$  of Eq.(9) should be chosen as a value between 0 and 1. The proposed method assists us in selecting pole locations with less oscillation because the parameters of closed-loop system are selected based on reducing oscillation. It is given by

$$\begin{aligned} \text{IF } e(t) > 0 \text{ THEN } I &= \int_0^{\tau} t e(t) dt \\ \text{ELSE } I &= \int_0^{\tau} t |e(t)|^{\gamma} dt \\ \text{END IF} \end{aligned} \quad (9)$$

where  $e(t) = \omega_r - CX, 0 < \gamma \leq 1$ .

In general, the closed loop system to assign system poles can be represented as follows:

$$\frac{\omega_L}{\omega_r} = \frac{\omega_1^2 \omega_2^2}{(s^2 + 2\zeta_1 \omega_1 s + \omega_1^2)(s^2 + 2\zeta_2 \omega_2 s + \omega_2^2)} \quad (10)$$

where  $\omega_i$  and  $\zeta_i$  (for  $i=1,2$ ) are the natural frequency and the damping ratio, respectively. Comparing Eq.(10) with Eq.(4), Eq.(5), and Eq.(7), respectively, the gains of all controllers are obtained as follows:

I-P Controller

$$K_P = 2(\zeta_1 \omega_1 + \zeta_2 \omega_2) J_M \quad (11)$$

$$K_I = \frac{\omega_1^2 \omega_2^2}{\omega_a^2} J_M \quad (12)$$

I-PD Controller

$$K_P = 2(\zeta_1 \omega_1 + \zeta_2 \omega_2)(J_M + K_D) \quad (13)$$

$$K_I = \frac{\omega_1^2 \omega_2^2}{\omega_a^2} (J_M + K_D) \quad (14)$$

$$K_D = \frac{\omega_a^4 J_L}{\omega_a^2(\omega_1^2 + \omega_2^2 + 4\zeta_1 \zeta_2 \omega_1 \omega_2) - \omega_1^2 \omega_2^2 - \omega_a^4} - J_M \quad (15)$$

State Feedback Controller with Integral

In case of the state feedback controller, an integrator is used to obtain the same structure as other cases. The load speed and torsion are estimated by using a state observer.

$$K_1 = 2(\zeta_1 \omega_1 + \zeta_2 \omega_2) J_M \quad (16)$$

$$K_I = \frac{\omega_1^2 \omega_2^2}{\omega_a^2} J_M \quad (17)$$

$$K_2 = \frac{2J_M}{\omega_a^2} (\omega_1 \zeta_1 (\omega_2^2 - \omega_a^2) - \omega_2 \zeta_2 (\omega_a^2 - \omega_1^2)) \quad (18)$$

$$K_3 = J_M(\omega_1^2 + \omega_2^2 + 4\zeta_1\zeta_2\omega_1\omega_2 - \frac{\omega_1^2\omega_2^2}{\omega_a^2} - \omega_0^2) \quad (19)$$

In Eq. (18), if we choose  $K_2$  as a positive constant and  $\omega_2 > \omega_1$ ,  $\zeta_1\omega_1 = \zeta_2\omega_2$ , then the relation between  $\omega_1$  and  $\omega_2$  is changed as follows:

$$\omega_2 = \sqrt{2\omega_a^2\left(1 + \frac{K_2}{K_1}\right) - \omega_1^2} > \sqrt{2\omega_a^2 - \omega_1^2} \quad (20)$$

Since  $T_M = -K_oX + K_Ix_I$ , the state equation of the closed-loop system of Fig. 3 can be represented by

$$\begin{bmatrix} \dot{X} \\ \dot{x}_I \end{bmatrix} = \begin{bmatrix} A - BK_o & BK_I \\ -C & 0 \end{bmatrix} \begin{bmatrix} X \\ x_I \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \omega_r + \begin{bmatrix} E \\ 0 \end{bmatrix} T_L \quad (21)$$

where  $X = [\omega_M \ \omega_L \ \theta_{12}]^T$ ,  $\dot{x}_I = \omega_r - CX = e$  and  $K_o = [K_1 \ K_2 \ K_3]$ .

The state observer equation is given by

$$\dot{\hat{X}} = A\hat{X} + BT_M + L_p(\omega_M - C\hat{X}) \quad (22)$$

where  $L_p$  is the observer gain.

### III. ITAE-BASED OPTIMAL PARAMETERS SELECTION

The specifications of the two-inertia motor system used are shown in Table 1. In this section, the advantage of a weighted ITAE index is evaluated. The  $\gamma$  shown in Eq.(9) was chosen as 0.7. Both the ITAE performance index values are calculated with respect to  $\zeta_1$  and  $\omega_1 / \omega_a$  for each controller, respectively.

In this paper, the poles utilized have the identical real part as discussed in [1]. The condition for these poles is given by

$$-\omega_1\zeta_1 = -\omega_2\zeta_2 \quad (23)$$

In the case of the I-P and I-PD controller, the condition for  $\omega_2$  is obtained as follows:

$$\omega_2 = \sqrt{2\omega_a^2 - \omega_1^2} \quad (24)$$

In the case of state feedback control, to meet inequality (20), we set frequency  $\omega_2$  as follows:

$$\omega_2 = \alpha\sqrt{2\omega_a^2 - \omega_1^2} \quad (25)$$

where  $\alpha$  is the positive constant, which is selected based on torsion amount. If  $\alpha$  is large, torsion increases, and vice versa.

Each controller is designed using the following procedure:

#### 표 1. 2관성 모터시스템의 기계적 파라미터.

Table 1. Mechanical parameters of a two-inertia motor system.

item	value
motor inertia [ $Kgm^2$ ]	$7.455 \times 10^{-5}$
anti-resonant frequency [ $rad/s$ ]	30
sampling time [ $ms$ ]	2

- 1) Find  $\zeta_1$ , and  $\omega_1 / \omega_a$  by using minimum ITAE value.
- 2) Set  $\alpha$  (Skip in the case of I-P, and I-PD controller).
- 3) Determine  $\omega_2$ , and  $\zeta_2$  using Eq.(23) to Eq.(25).

#### 1. Simulation comparison for step reference

##### 1.1 I-P speed controller

The ITAE values for an I-P controller with respect to inertia ratio K are shown in Fig. 4. Using optimal parameters which show minimum values, we can determine I/P gains with Eq. (11) and Eq. (12). In Fig. 4(a), it is ambiguous to select optimal parameters of  $\zeta_1$  and  $\omega_1 / \omega_a$  because the range of  $\zeta_1$  and

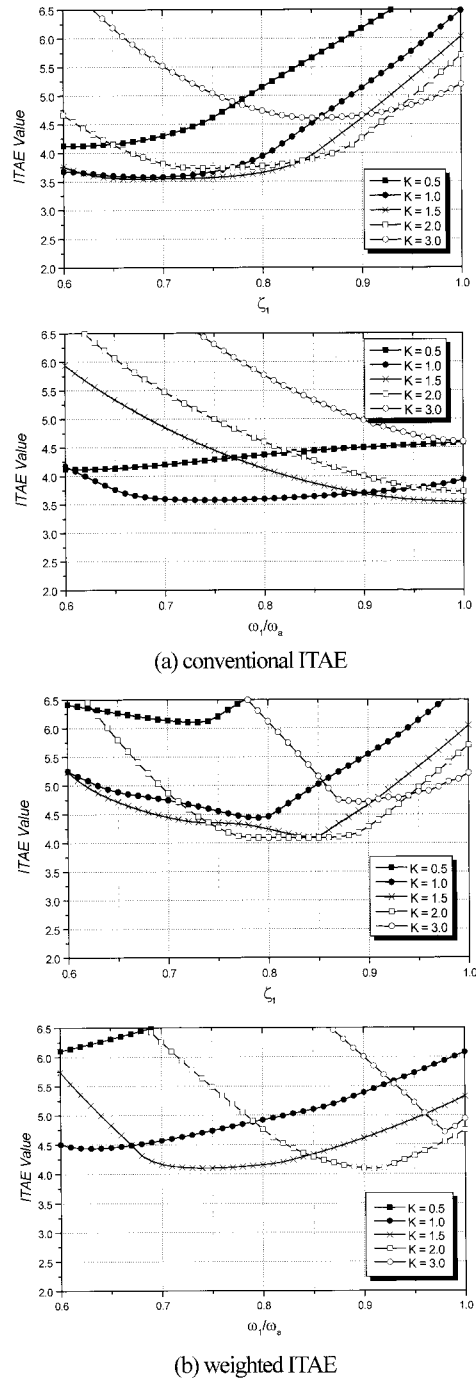
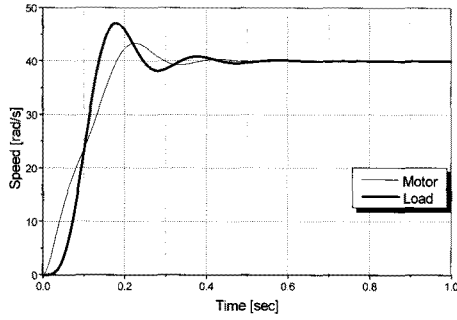
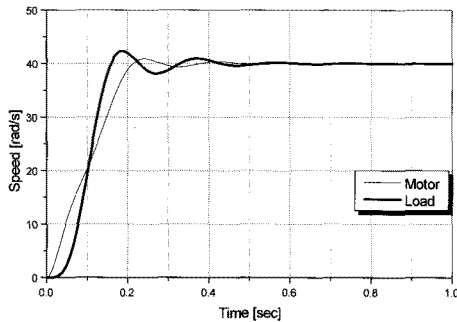


그림 4. I-P제어기 경우의 ITAE 성능지수.

Fig. 4. The ITAE performance index value for I-P controller.



(a)  $\zeta_1=0.64, \omega_1/\omega_a=0.68$  (conventional ITAE)



(b)  $\zeta_1=0.75, \omega_1/\omega_a=0.60$  (weighted ITAE)

그림 5. 속도응답 비교(K = 0.75, I-P 제어기).

Fig. 5. Speed response comparison(K = 0.75, I-P controller).

$\omega_1/\omega_a$  spread widely. However, we can easily determine optimal parameters for each inertia ratio in Fig. 4 (b). The speed responses using the I/P gains obtained above are shown in Fig. 5. The overshoot and oscillation of the case using the weighted ITAE index are smaller than those of the case using the conventional ITAE index. However, oscillation still occurs in the transient response in the I-P control.

1.2 I-PD speed controller

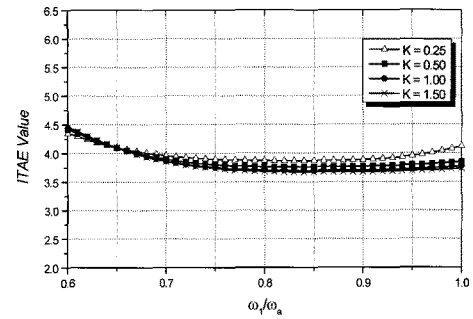
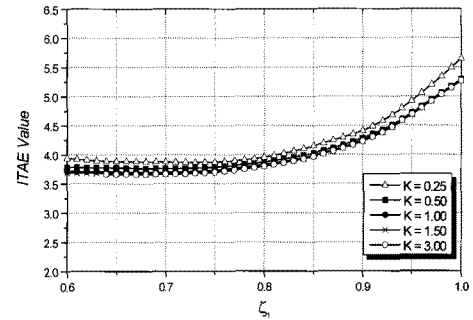
Similarly, both ITAE index values of an I-PD controller are shown in Fig. 6. In the case of the I-PD controller using the weighted ITAE index, the conspicuous minimum value appears at almost the same values of  $\zeta_1$  and  $\omega_1/\omega_a$  irrespective of the inertia ratio. The optimal parameters of  $\zeta_1, \omega_1/\omega_a$  are chosen as 0.9 and 0.73, respectively. Using Eq. (23) and Eq. (24),  $\omega_2$  will be  $1.21\omega_a$  and  $\zeta_2$  will be 0.54. Then I/P/D gains are obtained as follows:

$$K_p = 1.355\omega_a J_L, K_I = 0.4035\omega_a^2 J_L, K_D = 0.5071J_L - J_M$$

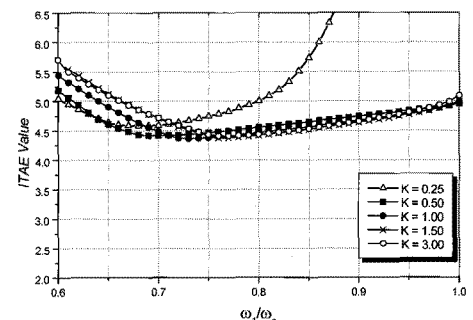
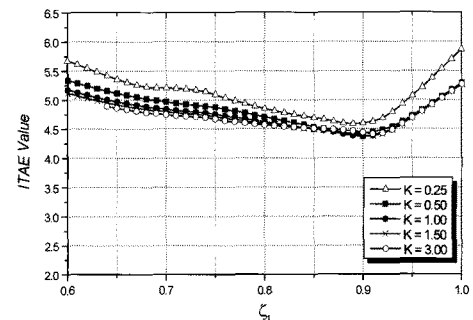
However, in the case of the conventional ITAE index, the range of optimal parameters spread widely as 0.6 ~ 0.75 for  $\zeta_1$  and 0.75~1.0 for  $\omega_1/\omega_a$ , respectively. Fig. 7 shows the speed responses when the inertia ratio,  $K$ , is 0.75. The method by the proposed weighted ITAE index reduces overshoot, settling time and torsion.

1.3 State feedback controller with integral

The graphs of both ITAE index values are shown in Fig. 8. The optimal parameters for a weighted ITAE index are almost the



(a) conventional ITAE



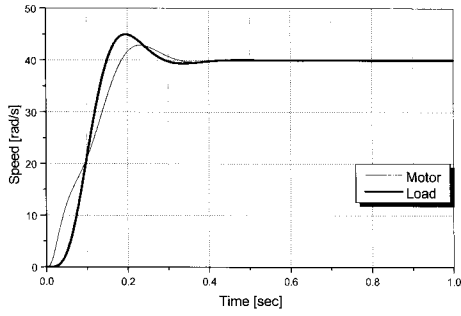
(b) weighted ITAE

그림 6. I-PD제어기 경우의 ITAE 성능지수.

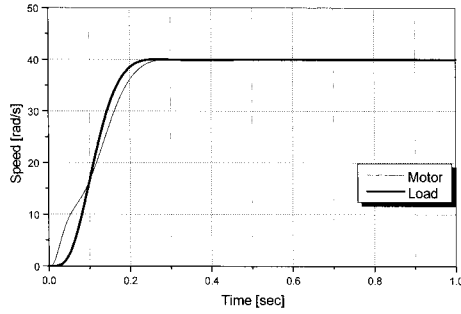
Fig. 6. The weighted ITAE performance index value for I-PD controller.

same, irrespective of the inertia ratio. This implies that the controller can be designed irrespective of inertia ratio. The optimal parameters obtained are  $\zeta_1 = 0.90, \omega_1 = 0.94\omega_a, \omega_2 = 1.057\alpha\omega_a, \zeta_2 = 0.8/\alpha$ . Then the gains of the state feedback controller from these values are obtained as follows:

$$K_1 = 3.384\omega_a J_M, K_I = 0.987\alpha^2 \omega_a^2 J_M, K_2 = 1.89(\alpha^2 - 1)\omega_a J_M, K_3 = (2.745 + 0.13\alpha^2)\omega_a^2 J_M - K_{sh}$$



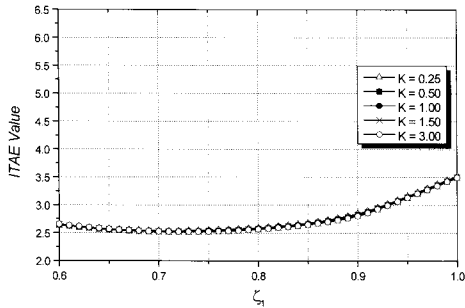
(a)  $\zeta_1=0.69, \omega_1/\omega_a=0.84$  (conventional ITAE)



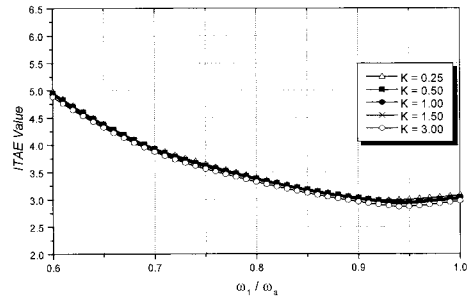
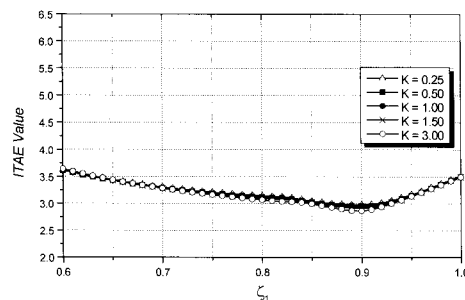
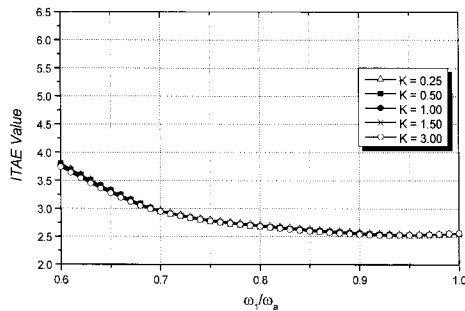
(b)  $\zeta_1=0.90, \omega_1/\omega_a=0.72$  (weighted ITAE)

그림 7. 속도 응답 비교(K=0.75, I-PD 제어기).

Fig. 7. Speed response comparison of two cases (K = 0.75, I-PD controller).



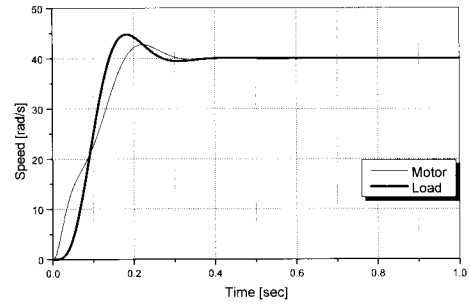
(a) using conventional ITAE



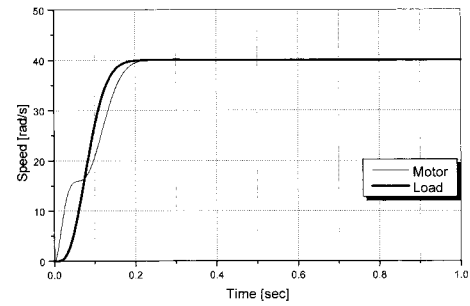
(b) using weighted ITAE

그림 8. 상태 피드백 제어기 경우의 ITAE성능지수.

Fig. 8. The ITAE performance index value for state feedback controller.



(a)  $\zeta_1=0.72, \omega_1/\omega_a=0.95$ (conventional ITAE)



(b)  $\zeta_1=0.90, \omega_1/\omega_a=0.94$ (weighted ITAE)

그림 9. 속도응답 비교(K=0.75, 상태 피드백 제어기).

Fig. 9. Speed response comparison of two cases(K = 0.75, state feedback controller).

표 2. 각 제어기에 대한 최적 파라미터.

Table 2. Optimal parameters for each controller.

Design conditions		K	0.5	0.75	1.0	1.5	2.0	
Selected Parameters	I-P	R	1.23	1.32	1.41	1.58	1.73	
		ITAE	6.10 (4.12)	4.97 (3.71)	4.44 (3.57)	4.10 (3.54)	4.09 (3.73)	
		$\zeta_1$	0.73 (0.61)	0.75 (0.64)	0.79 (0.69)	0.84 (0.67)	0.84 (0.74)	
		$\omega_1/\omega_a$	0.60 (0.62)	0.60 (0.68)	0.63 (0.74)	0.74 (1.0)	0.91 (0.99)	
		I-PD	ITAE	4.41 (3.76)	4.38 (3.72)	4.37 (3.69)	4.38 (3.67)	4.39 (3.66)
			$\zeta_1$	0.89 (0.70)	0.90 (0.69)	0.90 (0.69)	0.91 (0.68)	0.91 (0.68)
	$\omega_1/\omega_a$		0.70 (0.84)	0.72 (0.84)	0.73 (0.84)	0.75 (0.85)	0.76 (0.85)	
	SF		ITAE	2.96 (2.52)	2.94 (2.52)	2.93 (2.52)	2.91 (2.52)	2.89 (2.52)
			$\zeta_1$	0.90 (0.72)	0.90 (0.72)	0.90 (0.72)	0.90 (0.72)	0.90 (0.72)
			$\omega_1/\omega_a$	0.94 (0.95)	0.94 (0.95)	0.94 (0.95)	0.94 (0.95)	0.94 (0.94)

The range of minimum values by the conventional ITAE is 0.6 ~ 0.8 for  $\zeta_1$  and 0.9-1.0 for  $\omega_1 / \omega_a$ , respectively. The optimal parameters by the weighted ITAE are given by  $\zeta_1 = 0.9$ , and  $\omega_1 / \omega_a = 0.95$ . Here,  $\alpha$  is set at 1.5. As shown in Fig. 9, the response is greatly improved. A state feedback controller provides us the best performance compared with the I-P controller and I-PD controller and it can also be designed irrespective of the inertia ratio.

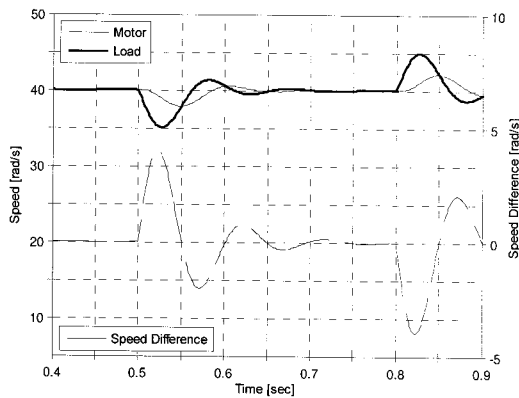
The optimal parameters for  $\zeta_1$  and  $\omega_1 / \omega_a$ , obtained by both the weighted index(values without parentheses) and conventional index(values with parentheses) in each controller are summarized in the Table 2.

2. Simulation comparison for step disturbance

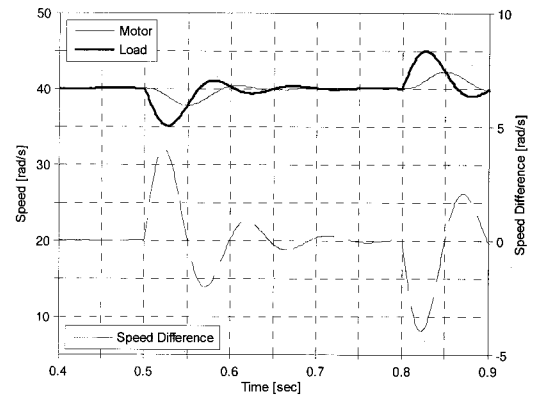
In this section, the simulations to evaluate rejection behavior for disturbance of the three controllers for step disturbance in steady state are carried out. The inertia ratio  $K$  is 0.75. The magnitude of step disturbance is 0.025[Nm] which is applied at 0.5 second and get removed at 0.8 second. All other parameters are same as one of just before section. From Fig. 10, we can see that there is no difference of performance between using conventional ITAE and using weighted ITAE except for I-PD controller. In case of I-PD controller, the case using weighted ITAE has less oscillation.

3. Discussions

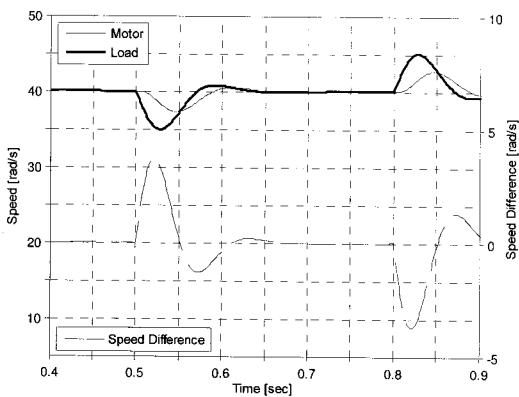
The state feedback controller shows better performance than I-P or I-PD controllers. Each bandwidth of closed loop system gets larger as the equivalent resonant frequency increases in sequence



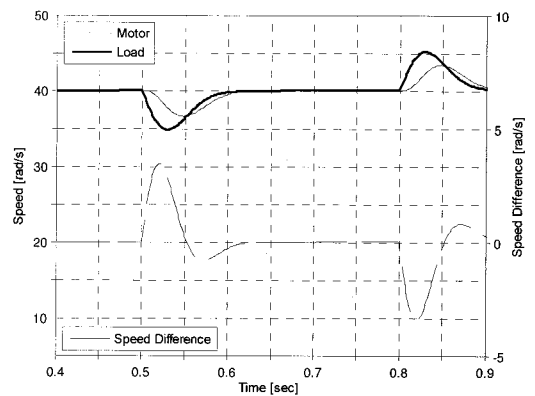
(a) I-P controller(conventional ITAE)



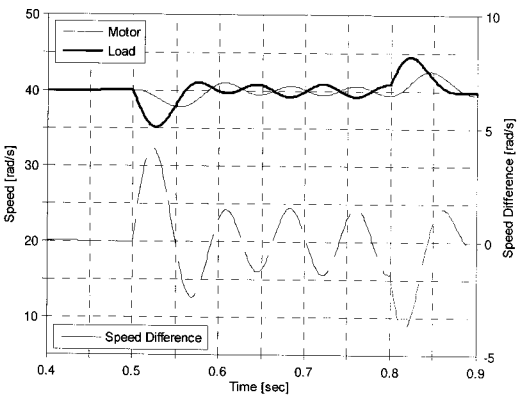
(b) I-P controller(weighted ITAE)



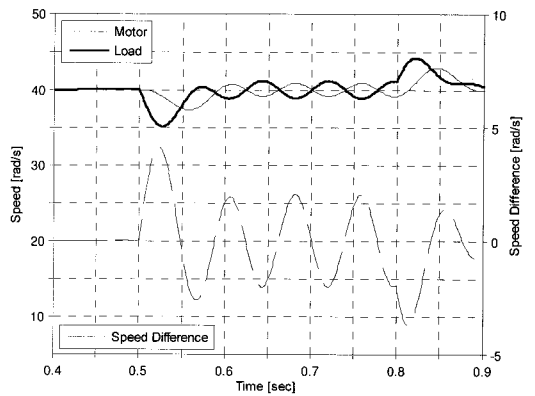
(c) I-PD controller(conventional ITAE)



(d) I-PD controller (weighted ITAE)



(e) State feed controller(conventional ITAE)



(f) State feedback controller(weighted ITAE)

그림 10. 외란 응답 비교(K=0.75).

Fig. 10. Comparison for step disturbance(K = 0.75).

I-P, I-PD, state feedback controller, which results in better response with less overshoot and less settling time. In addition, the cases using the weighted index have also better performance than those using the conventional one in each controller. However, so is not it for the step disturbance.

**IV. EXPERIMENTAL RESULTS**

In this section, three controllers designed by using weighted ITAE are evaluated through experiments.

**1. Experimental Set-up**

The three controllers are applied to the systems having two different inertia ratios, 2.75 and 0.78, respectively. The experimental unit is set up as seen in Fig. 11. Adding load inertia on the load flywheel changes the resonance ratio. The servo amplifier is used as a current mode to easily apply torque. After programming the proposed controller algorithm using C language in the host personal computer, it is sent to the DSP board and executed by DSP. The motor speed and load are measured by using an incremental encoder with  $4 \times 1000$  [pulse/revolution] and they are returned to the host PC. Here, the measured load speed is used to obtain the controlled result, but it is not used in the calculation of the torque command. The proposed algorithm calculates the torque command and it is applied to a two-inertia DC motor system as a current via a D/A converter and a PWM servo amplifier. The mechanical specifications and electrical

표 4. 모터와 서보 앰프의 전기적 사양.

Table 4. Electrical specifications of the motor and servo amplifier.

Items		value	
motor	max. voltage [V]	DC 40	
	max. speed [rpm]	5000	
	torque [Nm]	const.	0.35
		peak	2.12
	current [A]	const.	5.5
		peak	30
Resistance R [ $\Omega$ ]		0.9	
inductance L [mH]		2.5	
servo amplifier	DC supply voltage [V]	20 ~ 80	
	peak current [A]	$\pm 25$	
	max. continuous current [A]	$\pm 12.5$	
	switching frequency [KHz]	22	
	power dissipation [W]	50	

표 5. 설계된 각 제어기의 이득치.

Table 5. The designed gains of each controller.

Controller	I-P		I-PD		State feedback	
	ex. 1	ex. 2	ex. 1	ex. 2	ex. 1	ex. 2
$K_P(K_1)$	0.0100	0.0123	0.0111	0.0073	0.0101	0.0232
$K_I$	0.1176	0.2446	0.1311	0.1311	0.2611	0.9141
$K_D(K_3)$			0.00003	-0.0001	0.0373	0.9434
$K_2$					0.0070	0.0161
Observer gain $L_p$					0.4300	0.4276
					0.2301	0.0416
					-0.0057	-0.0082

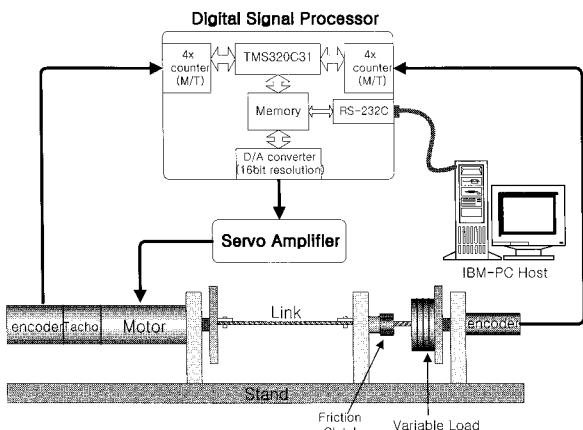


그림 11. 전체 실험 장치의 개략도.

Fig. 11. The schematic of overall experimental apparatus.

표 3. 2관성 모터 시스템의 기계적 사양.

Table 3. Mechanical specifications of the two-inertia motor system.

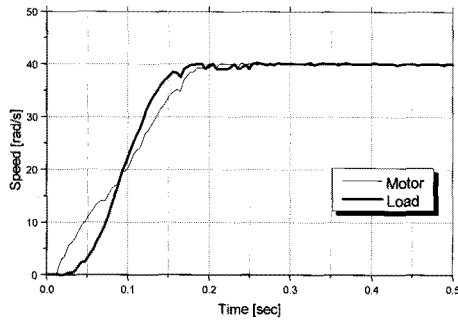
Items	experiment 1	experiment 2
motor inertia [ $Kgm^2$ ]	$7.455 \times 10^{-5}$	$1.132 \times 10^{-4}$
load inertia [ $Kgm^2$ ]	$2.047 \times 10^{-4}$	$8.878 \times 10^{-5}$
Torsion stiffness [ $Nm/rad$ ]	0.325	
Resonant frequency [ $rad/s$ ]	77.12	80.82
Anti-resonant frequency [ $rad/s$ ]	39.85	60.50
inertia ratio(K)	2.75	0.78
resonant ratio(R)	1.94	1.34
sampling time [ms]	2	

specifications of this system are shown in Table 3 and Table 4, respectively.

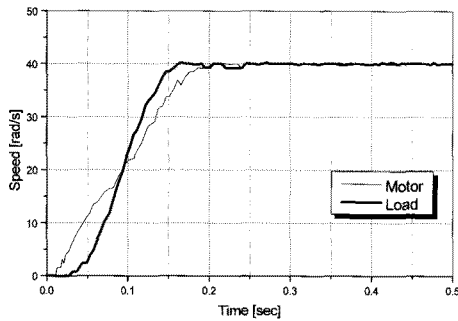
The total gains of each controller from the specification data as shown in Table 3 and Table 4 can be obtained as shown in Table 5. The experiments 1 and 2 as shown in Table 5 imply experiments, which were performed for the inertia ratios of 2.75 and 0.78, respectively.

**2. Experiments**

Fig. 12 represents the results of speed response when the inertia ratio is 2.75. In the case of when the inertia ratio is very large, even using an I-P controller, the overshoot and oscillation were not occurred. The state feedback controller provided the best performance in terms of settling time compared with other controllers. Fig. 13 shows the results when the inertia ratio decreases to 0.78. Since the inertia ratio decreased, oscillation occurred in the I-P controller. Again, using a state feedback controller provided the best performance. The settling time decreased compared to the case with a large inertia ratio. However, the torsion increased as the settling time decreased. This torsion amount can be reduced if use a small  $\alpha$ . The I-PD controller and the state feedback controller did not cause oscillation in either experiment. These results coincide with the results of the case study in the previous section.



(a) I-P controller



(b) I-PD controller

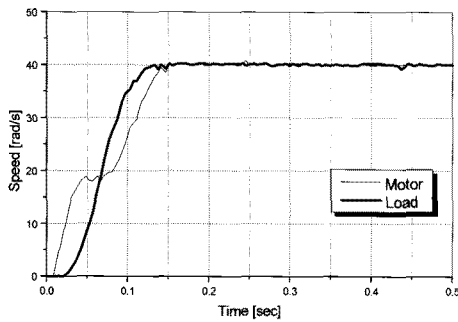
(c) State feedback controller ( $\alpha = 1.5$ )

그림 12. 속도응답(K=2.75).

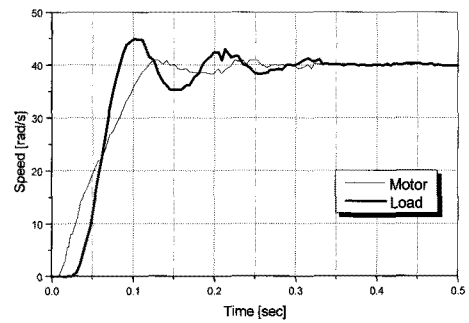
Fig. 12. Speed responses (K=2.75).

## V. CONCLUSIONS

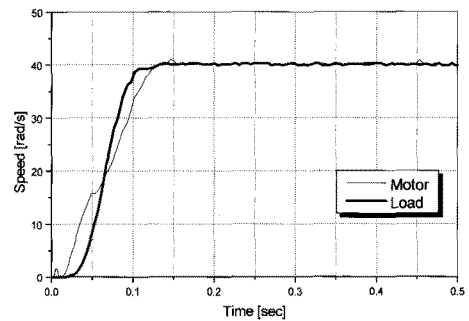
This paper described how to find the optimal parameters in order to reduce oscillation and settling time by using a weighted ITAE index when three speed controllers such as an I-P, an I-PD, and a state feedback controller are designed for two-inertia motor system. The controller designed based on a weighted ITAE index allowed us to obtain a system response with less oscillation and torsion compared to a design used a conventional ITAE index. The state feedback controller gives better performance than I-P or I-PD controllers. Each speed controller shows the responses with less overshoot and less settling time compared to results using conventional ITAE index. However, so it is not for step disturbance.

## REFERENCES

- [1] G. Zhang and J. Furusho, "Speed control of two-inertia system by P/PID control," *IEEE Trans. Industrial Electronics*, vol. 43, no. 3, pp. 603-609, 2000.
- [2] J. K. Ji and S. K. Sul, "Kalman filter and LQ based speed



(a) I-P controller



(b) I-PD controller

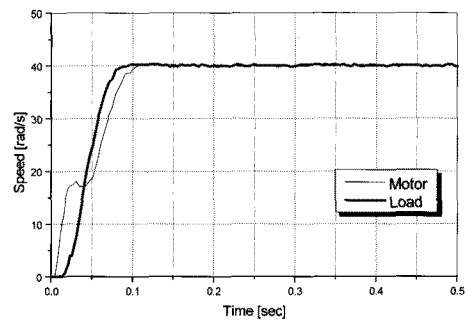
(c) State feedback controller ( $\alpha = 1.5$ )

그림 13. 속도응답(K=0.78).

Fig. 13. Speed responses (K = 0.78).

controller for torsional vibration suppression in a 2-mass motor drive system," *IEEE Trans. Industrial Electronics*, vol. 42, no. 6, pp. 564-571, 1995.

- [3] K. Sugiura and Y. Hori, 1996, "Vibration suppression in 2- and 3-mass system based on the feedback of imperfect derivative of the estimated torsional torque," *IEEE Trans. Industrial Electronics*, vol. 43, no. 1, pp. 56-64, 1996.
- [4] Y. Hori, H. Sawada, and Y. Chun, "Slow resonance ratio control for vibration suppression and disturbance rejection in torsional system," *IEEE Trans. Industrial Electronics*, vol. 46, no. 1, pp. 162-168, 1999.
- [5] S. Hashimoto, K. Hara, and K. Kamiyama, "AR-based identification and control approach in vibration suppression," *IEEE Trans. Industry Applications*, vol. 37, no. 3, pp. 806-811, 2001.
- [6] K. Ito, M. Iwasaki, and N. Matsui, "GA-based practical compensator design for a motion control system," *IEEE/ASME Trans. Mechatronics*, vol. 6, no. 2, pp. 143-148, 2001.



- [7] T. M. O'Sullivan and Christopher M. Bingham, "Enhanced servo-control performance of dual-mass systems," *IEEE Transactions on Industrial Electronics*, vol. 54, no. 3, pp. 1387-1399, June 2007.
- [8] T. O. Kowalska, "Neural-network application for mechanical variables estimation of a two-mass drive system," *IEEE Transactions on Industrial Electronics*, vol. 54, no. 3, pp. 1352-1364, June 2007.



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