On the Local Identifiability of Load Model Parameters in Measurement-based Approach

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Abstract – It is important to derive reliable parameter values in the measurement-based load model development of electric power systems. However parameter estimation tasks, in practice, often face the parameter identifiability issue; whether or not the model parameters can be estimated with a given input-output data set in reliable manner. This paper introduces concepts and practical definitions of the local identifiability of model parameters. A posteriori local identifiability is defined in the sense of nonlinear least squares. As numerical examples, local identifiability of third-order induction motor (IM) model and a Z-induction motor (Z-IM) model is studied. It is shown that parameter ill-conditioning can significantly affect on reliable parameter estimation task. Numerical studies show that local identifiability can be quite sensitive to input data and a given local solution. Finally, several countermeasures are proposed to overcome ill-conditioning problem in measurement-based load modeling.

Keywords: Local identifiability, Load modeling, Correlation matrix, Measurement-based approach, Parameter estimation

1. Introduction

Accurate load modeling is essential for power system dynamic simulation. Inaccurate load models, for instanc e, can lead to a power system being operated in mod es that result in actual system collapse or separation [1]. Given a load model, currently two major approaches are available for deriving parameter values; one is measurement-based approach and the other is component-based approach [2]-[3].

In measurement-based approach, parameter estimation task is often formulated as a nonlinear least squares problem where mismatch error between simulated model output and actual measurement is minimized [4]-[7]. Gradient-based local methods such as quasi-Newton methods are often employed for this problem. Even though each parameter of a given load model is equally important in modeling dynamic behaviors of loads, certain parameters may not be reliably estimated with given measurement data due to some reasons; for instance, parameter ill-conditioning, insensible (unobservable) parameters, and poor parameter estimation algorithm etc.

Ideally the first issue to solve in the load model parameter estimation task is to check whether or not all parameters of a given load model can be theoretically identifiable, in other words, whether or not there is any unidentifiable parameters because of inherent flaws of the model structure. This is a so-called a priori (structural) identifiability problem. Various approaches have been proposed for testing a priori global identifiability of nonlinear models [8]-[10]. However no standard algorithm exists and it still remains a challenging task.

A priori identifiability is a necessary condition for the identifiability of load model parameters using actual measurement data. This implies that the parameter estimation task using actual measurements can fail to obtain reliable parameter values despite the priori identifiability of a given load model. Hence, it is desirable to analyze a posteriori (numerical) identifiability which is to check if an obtained local solution is unique or simply one of infinite number of solutions. In case of the latter, the solution may not be reliable or may not have physical meaning.

There have been many dynamic and composite load models developed to represent aggregated electric loads of power systems. However, the identifiability issue of load model parameters has not been systematically and thoroughly studied in measurement-based load modeling. An identifiability study of simple first-order dynamic load models has been attempted in [13], where concepts and definitions of identifiability were introduced. However, it is necessary to test more complex load models such as high order induction motor load models using more sophisticated algorithms for practical use. In addition, some concepts and definitions are necessary to be enforced.

In this paper, concepts and definitions of a priori and a posteriori identifiability are briefly introduced. Based-on this definition a local identifiability of a third-order induction motor model and a composite Z-induction motor load model are studied. It is shown that parameter ill-conditioning can significantly affect on reliable parameter estimation task. In addition, numerical studies show that local identifiability can be quite sensitive to the employed measurement data and obtained local optimal solutions. Finally, several countermeasures are proposed to overcome parameter ill-conditioning problem in measurement-based load modeling.

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2. A Priori and A Posteriori Identifiability

Before introducing two identifiability definitions, let us define a general dynamic load model and output in distinguishability concept. Dynamic and composite (static plus dynamic) load models of electric power systems are described by a general nonlinear model of the following type:

$$\dot{x}(t,p) = f(x(t,p), u(t), p), \quad x_0 = x(t_0) = \Gamma(p) \tag{1}$$

$$y(t,p) = g(x(t,p), u(t), p) \tag{2}$$

$$h(x(t,p), u(t), p) \le 0 \tag{3}$$

where x is an n-dimensional state variable vector; $u \in U$ is a r-dimensional input vector of differentiable functions; y is m-dimensional output; $p \in \Theta$ is v-dimensional parameter vector; f, g, h, Γ are vectors of nonlinear functions of p and t. Equality and inequality constraints on p are represented by (3). In measurement-based load modeling, initial state x_0 is usually a function of parameter p, $\Gamma(p)$. However for some models x_0 can be specified regardless of a given parameter set.

The question of identifiability is whether or not it is possible to find a unique solution for unknown parameters of a model concerned, for a given input-output data collected in experiments performed on the real system [10], [12]. Identifiability is generally classified into two cate gories: a priori and a posteriori identifiability [14]. In order to state model identifiability, following preliminary definition is required:

Definition 1 (Output Indistinguishability) [15]: Two models $M(\hat{p})$ and $M(p)$, with the same structure $M(\cdot)$ and the same input $u \in U$, are output indistinguishable if their output y satisfy

$$y(t,\hat{p},u(t)) = y(t,p,u(t)), \quad \forall t \in R^+, \forall u \in U \tag{4}$$

2.1 A Priori Identifiability

A priori identifiability concerns the case of deterministic model and a noise-free data and is often called a structural identifiability. Even though there may be different definitions of a priori identifiability, it can be addressed for the single parameter p_i as follows [9]-[12].

1. *globally identifiable* if and only if, for almost any $\hat{p} \in \Theta$, the equation (4) has the only one solution $p_i = \hat{p}_i$.
2. *locally identifiable* if and only if, for almost any $\hat{p} \in \Theta$, the equation (4) has for p_i more than one, but a finite number of solutions;
3. *unidentifiable* if and only if, for almost any $\hat{p} \in \Theta$, the equation (4) has for p_i infinite number of solutions or no solution. Let us call the former case ∞ *unidentifiable*, the latter case, *zero unidentifiable*.
4. *input unidentifiable* if p_i is unidentifiable due to the form or shape of input signal. This is impractical to check for complex model because it involves an explicit solution for the state variables in terms of parameters, input, and time.

A priori identifiability of the model can be similarly stated [9], [11]. The priori identifiability problem is generally a nonlinear algebraic one solution of which is unfortunately nontrivial and non-unique for all but very simple models, unfortunately. Nevertheless, a priori identifiability conditions are necessary conditions for a successful estimation of model parameters from actual input-output data. It is noted that an a priori identifiable model can be rejected due to several reasons, for instance, the estimated parameter values are very poor either because of the paucity of the measured input-output data, or because of strong linkage between some parameters.

2.2 A Posteriori (Local) Identifiability

A less difficult problem than a priori (structural) identifiability but one that gives considerable useful information in practice is that of testing a local identifiability with artificially generated data or actual measurements. Hence, it is also called a numerical identifiability.

The local identifiability of a parameter in terms of the "output distinguishability" is defined as follows:

Definition 2 [16]: A parameter set Θ is said to be locally identifiable at p^* if $\exists \rho > 0$ such that the pair (p, p^*) is distinguishable for all $p \in N(p^*, \rho)$, $p \neq p^*$.

This definition states that a parameter set Θ is said to be locally identifiable at p^* if

$$\begin{aligned} & y(t,p) = y(t,p^*), \forall t \in [t_0, T] \Rightarrow p = p^*, \text{ i.e.,} \\ & y(t,p) - y(t,p^*) = \Delta y(t) = 0, \forall t \in [t_0, T] \\ & \Rightarrow p - p^* = \Delta p = 0 \end{aligned} \tag{5}$$

Equation (5) can be written in terms of output sensitivity with respect to parameters. The output sensitivity to a given parameter p^* is calculated by Taylor series expansion of (2). Neglecting higher order terms gives:

$$\Delta y(t) \approx \frac{\partial y(t)}{\partial p} \Delta p = S \Delta p \tag{6}$$

where S is an output sensitivity matrix and S_i (i^{th} column of S: output sensitivity with respect to p_i) can be computed as

$$\frac{d}{dt}\left(\frac{\partial x(t)}{\partial p_i}\right) = \frac{\partial f(x(t,p),u,p)}{\partial x}\left(\frac{\partial x(t)}{\partial p_i}\right) + \frac{\partial f(x(t,p),u,p)}{\partial p_i} \tag{7}$$

$$S_i(t) = \frac{\partial y(t)}{\partial p_i} = \frac{\partial g(x(t,p),u,p)}{\partial x}\left(\frac{\partial x(t)}{\partial p_i}\right) + \frac{\partial g(x(t,p),u,p)}{\partial p_i} \tag{8}$$

From (5) and (6), following theorem of local identifiability can be drawn (proof is referred to [18]).

Theorem 1 [18]: A parameter set Θ is said to be locally identifiable at p^* if only if S has full rank.

In similar way, a posteriori local (or numerical) identifiability at a given point p^* from real measurement data is defined as follows:

Theorem 2 [18]: The parameters p^* are numerically locally identifiable from measurement data y_m if and only if the sensitivity matrix S has full rank

Theorem 2 implies that local identifiability can be different depending on choice of measurements as well as obtained local optimal solution.

The local identifiability at a given parameter p^* as defined above is independent of the method used in parameter estimation. A method-oriented definition called "local least square identifiability" is useful in practical point of view [16] since the identification problem is often formulated as nonlinear least squares problem:

$$\min_{p\in\Theta} J(p) = \min_{p\in\Theta} \frac{1}{2}\sum_{k=1}^{q} |\varepsilon_k(p)|^2 = \min_{p\in\Theta} \frac{1}{2}\|\varepsilon(p)\|^2 \qquad (9)$$

where $\varepsilon_k(p) = y(k,p) - y_m(k)$ and $\varepsilon(p) = y(p) - y_m$ in vector form. $y_m(k)$ is k^{th} measured output. It has been proved that the local least square identifiability implies local distinguishability and vice versa [16]. In this paper parameter estimation problem is formulated in nonlinear least squares for numerical study of parameter identifiability.

It is possible that a certain parameter has little influence on the modeled output. In such case, S may have full rank, but $S^T S$ ill-conditioned. This problem can be overcome by selecting a better measurement data set in qualitative sense. However if the measurement data is limited, numerically robust algorithms or subset selection scheme [19] should be used to handle the ill-conditioned parameter. Another cause of ill-conditioned $S^T S$ is colinearity of columns of output sensitivity matrix S, which is strongly related to correlation between parameters.

Parameter Correlation Analysis

A set of parameters may be identifiable, but due to the strong interactions between the parameters as meas ured by correlations, it can be difficult to accurately e stimate those parameters individually. This phenomenon usually becomes worse in the presence of noise and measurement errors. To check this, we reduce the matrix $S^T S$ by eliminating rows and columns corresponding to the unidentifiable parameters to obtain $S_I^T S_I$. The correlation matrix (CM), φ is obtained by dividing the i, j element of $(S_I^T S_I)^{-1}$ by the square root of the product of the i^{th} and j^{th} diagonal elements [17]. Note that the values of the correlation coefficients depend on the parameter values and on the choice of points in time where the sensitivities are calculated. The diagonal elements are always 1.0. The element $\varphi(i,j)$ where $i \neq j$, represents the degree of correlation between i^{th} and j^{th} parameters. If $\varphi(i,j)$ is 1 or -1, the parameters p_i and p_j can not be identified individually, which implies that for $\varphi(i,j)$ near 1 or -1, one may have difficulties in estimating the parameters because of high correlations between them [17]. In such cases, robust numerical methods are required to converge to a local solution.

3. Test Dynamic Load Models

For local identifiability study, a third-order induction motor (IM) load model and a composite load model called Z-induction motor (Z-IM) model are considered

3.1 A Third-order Induction Motor Load Model

Induction motor load is an important component of power system dynamic loads and has significant effect on transient voltage stability [20]. By neglecting the stator transient, a third-order induction model can be derived from the detailed fifth-order model where it is assumed that induction motor is connected to an infinite bus [21]. The equations of the third-order models are:

$$\begin{cases} T_0' \dfrac{dE'}{dt} = -\dfrac{X}{X'}E' + \dfrac{X-X'}{X'}\cdot V\cdot\cos\delta \\ \dfrac{d\delta}{dt} = \omega - \omega_s - \dfrac{X-X'}{X'}\cdot\dfrac{V\cdot\sin\delta}{T_0'\cdot E'} \\ M\dfrac{d\omega}{dt} = -\dfrac{V\cdot E'\cdot\sin\delta}{X'} - T_m \end{cases} \qquad (10)$$

$$\begin{cases} P = -(VE'/X')\cdot\sin\delta \\ Q = V(V - E'\cdot\cos\delta)/X' \end{cases} \qquad (11)$$

where E', δ: voltage magnitude and angle behind transient reactance. ω_s, ω: angular velocity of stator and rotor [rad/s], X_m, X_s, X_r: magnetizing, stator and rotor reactances, $X' = X_s + X_m X_r/(X_m + X_r)$: transient reactance, $X = X_s + X_m$, $T_0' = (X_r + X_m)/\omega_s R_r$: transient open-circuit time constant, R_r: rotor resistances, M : motor inertia, T_m: load torque constant.

An initial state vector of (10), x_0 is needed to compute model output with given parameters. To obtain x_0 which is an equilibrium point, we set the derivatives of states of (10) to zero. For example the initial state δ_0 can be computed by $\delta_0 = \sin^{-1}(\arg)/2$ where $\arg = -2X\cdot X'\cdot T_m/((X-X')\cdot V^2)$. A problem arises during the parameter estimation process because it is impossible to calculate δ_0 when the magnitude of arg is greater than unity ($|\arg| > 1.0$). In order to avoid the numerical problem, nonlinear constraints can be enforced during the estimation process. Since finding initial states generally involve solving a set of nonlinear equations, one can treat initial states $(\delta_0, E_0', \omega_0)$ as simply unknown parameters to be estimated and related initial conditions can be incorporated in an objective function as regularization terms or in a set of nonlinear constraints. In this case the number of parameters to be identified increases by the number of dynamic states (in case of third-order induction motor, total number of parameters is 8).

The linearized version of IM load model in (10), (11) can be derived and is described by the following state equations:

$$\begin{aligned} \Delta\dot{x} &= A\Delta x + B\Delta u \\ \Delta y &= C\Delta x + D\Delta u \end{aligned} \tag{12}$$

where A, B, C, and D are system matrices and the detailed expression can be referred in Appendix A.

3.2 A Composite Load Model

A composite load model structure consists of static part and dynamic part. It is represented by aggregating a huge number of different load components. As a simple composite load model, let us consider the following Z-induction motor (Z-IM) model where individual static loads are represented by a single admittance $G_s + jB_s$ connected to third-order induction motor in parallel. Dynamic part of the Z-IM model is same to that of third-order induction motor represented by equation (10). Hence only difference between two models is the following model output equation:

$$\begin{cases} P = G_s V^2 - (VE'/X')\cdot\sin\delta \\ Q = B_s V^2 + V(V - E'\cdot\cos\delta)/X' \end{cases} \tag{13}$$

The derivation of linearized Z-IM model is almost similar to the case of third-order induction motor model and omitted here.

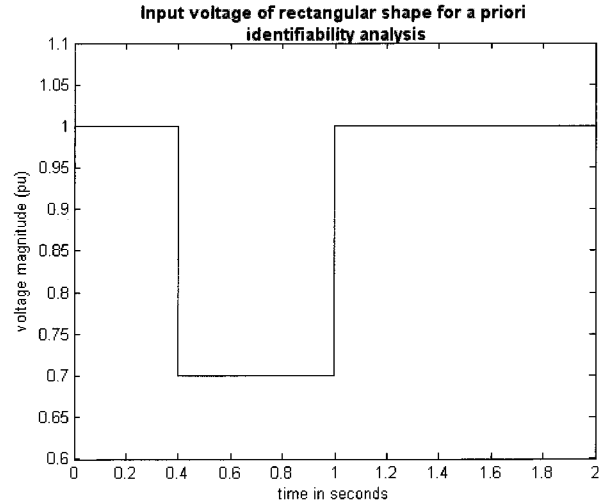
4. Numerical Studies

In this section, a local (numerical) identifiability is analyzed for a third-order induction motor and Z-IM models using an artificially generated data set and actual measurements. Singular value and individual condition number of output sensitivity matrix are analyzed to identify ill-conditioned parameters. How much parameter ill-conditioning affects the parameter estimation result is illustrated in numerical examples. In addition, colinearity of output sensitivity matrix is analyzed to check correlation between parameters. Several remedy actions are proposed to overcome parameter ill-conditioning.

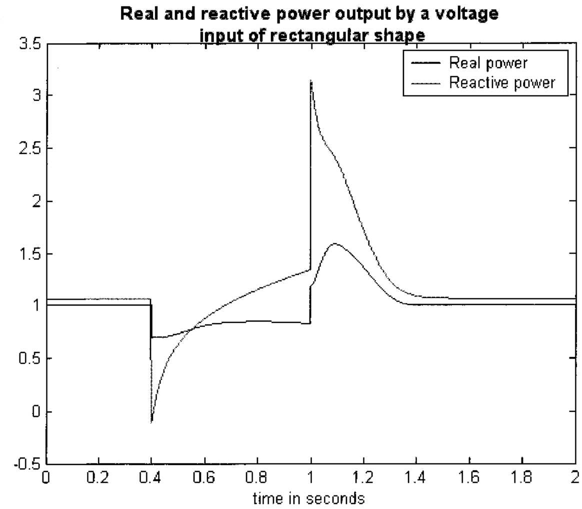
4.1 Local Identifiability Study using Artificial Data

A local identifiability study can be performed first using artificially generated data. The one advantage of employing artificial data is that one can test specific types of input and output data which may not be available from actual measurements. In doing so, unidentifiable (or unobservable) parameters and/or ill-conditioned parameters can be detected. Another advantage of local identifiability study using artificial data is that robustness of parameter estimation with respect to the level of noise can be analyzed.

In this study, a local identifiability is tested for a third-order induction motor model using an artificially generated data. To this end a designed experiment is conducted to obtain input-output data set where output is calculated by a designed noise-free input. It is assumed that a local minimum point $\hat{p}$ is $[M, T_0', X, X', T_m]^T$ = [0.005, 0.233, 1.36, 0.002, 1.0]. Rectangular-shape voltage input is applied to the third-order induction motor model with the given parameters to obtain real and reactive power outputs. Fig. 1 shows input voltage and corresponding real, reactive power output where bus voltage steps down to 0.7 p.u. which amounts to 70% of its operating point and steps up to the original value after 0.6 sec.



(a) Input voltage



(b) Real and reactive power output

Fig. 1. Rectangular-shaped input voltage and its real and reactive power outputs

Based on the designed experiment, sensitivity matrix S is obtained. In order do assess the degree of parameter ill-conditioning, singular values of S, i.e. the square roots of the eigenvalues of $S^T S$ and individual parameter condition number of each singular value, $\sigma_{\max}/\sigma_i$ are computed. In this paper, the parameter whose corresponding condition number is greater than 1.0×10^3 is considered ill-conditioned. Even though the threshold for ill-conditioning appears to be a little bit optimistic, it is observed that there is no numerical convergence problem in case of much larger condition number than 1.0×10^3.

The singular values and individual condition number are computed as shown in Table 1. In addition, each parameter is ranked in the sense of well-condition. The ranking of parameters in the sense of well-condition can be done by performing subset selection task [19] increasing the threshold value of the condition number for ill-conditioned parameters.

Table 1. Singular values of trajectory sensitivity matrix and rank of parameters of the induction motor model with a designed input-output.

Singular Values of Trajectory Sensitivity Matrix				
4274.815, 895.450, 91.741, 34.206, 31.990				
Individual Condition Number ($\sigma_{\max}/\sigma_i$)				
1.000, 4.774, 46.597, 124.974, 133.631				
Rank of Parameters				
M	X'	T_m	T_0'	X
well-conditioned ←				

As shown in Table 1, there is no unidentifiable parameter at the given point since there is no zero singular value in sensitivity matrix. In addition, condition number of each singular value is not large which means all parameters are well-conditioned and are locally identifiable. We also tested simple step voltage where the maximum of condition numbers was 460.248, which means that the rectangular-shaped voltage input and corresponding output data contain richer information than the simple step voltage input from the parameter estimation viewpoint. In order to investigate the inter-relationship between parameters, parameter correlation matrix φ is computed and shown in Table 2.

Table 2. Correlation matrix associated with the sensitivity matrix at a given point.

	M	T_0'	X	X'	T_m
M	1	-0.989	0.930	-0.862	-0.971
T_0'		1	-0.906	0.839	0.954
X			1	-0.862	-0.955
X'				1	0.855
T_m					1

Table 2 shows that the degree of correlation among parameters is relatively high, but it is observed that parameter estimation task succeeded in converging to the original parameter values with different initial guess.

When we generate model output with a given input and a set of parameters, firstly initial state x_0 should be calculated. The initial state x_0 is usually represented by a set of nonlinear function of parameters as shown in equation (1). For some dynamic models such as induction motor model in this paper, constrained nonlinear equations are necessary to solve. On the other hand, the initial state x_0 can be included in a parameter vector to be estimated.

Table 3 shows the result of sensitivity matrix analysis.

Table 3. Singular values of trajectory sensitivity matrix and rank of parameters of the induction motor model.

Singular Values of Trajectory Sensitivity Matrix							
4274.815, 895.461, 90.869, 56.511, 38.283, 30.417, 27.349, **0.752**							
Individual Condition Number ($\sigma_{\max}/\sigma_i$)							
1.0, 4.774, 47.044, 75.646, 111.665, 140.541, 156.301, **5689.910**							
Rank of Parameters							
M	X'	T_m	T_0'	E_0'	X	δ_0	ω_{r0}
well-conditioned ←→ ill-conditioned							

In this case, singular value associated with ω_{r0} is quite small compared to the others. Condition number is quite large (5689.91), which implies that the ω_{r0} considered ill-conditioned. The example of unreliable estimation will be given in section 4.2. The identifiability of the linearized third-order induction motor is analyzed in similar way. Table 4 and Table 5 show the result of local identifiability analysis with the same input-output data to the nonlinear induction motor case and parameter correlation matrix, respectively.

Table 4. Singular values of trajectory sensitivity matrix and rank of parameters of the linearized induction motor model with a given input-output.

Singular Values of Trajectory Sensitivity Matrix				
1066.785, 169.634, 31.579, 12.158, 1.132				
Individual Condition Number				
1.00, 6.289, 33.781, 87.742, 942.656				
Rank of Parameters				
M	X'	T_0'	T_m	X

Table 5. Parameter Correlation Matrix for the linearized IM model.

	M	T_0'	X	X'	T_m
M	1	-0.299	-0.192	0.103	0.029
T_0'		1	-0.446	-0.091	-0.090
X			1	0.826	0.856
X'				1	0.966
T_m					1

The maximum parameter condition number of linearized model case is a little bit larger than that of nonlinear IM model case while the rank of parameters is almost same. From the correlation matrix parameters T_m and X' show strong correlation.

When initial states $[\delta_0, E_0', \omega_0]$ are considered as parameters, unidentifiable parameters are observed. Numerical results show that the output sensitivities with respect to T_m and ω_{r0} are zeros. Careful observation reveals that two parameters do not exist in the linearized model. One can identify unidentifiable parameters; the corresponding column in sensitivity matrix has all zero elements.

Table 6. Singular values of trajectory sensitivity matrix and rank of parameters of induction motor model.

Singular Values of Trajectory Sensitivity Matrix							
1067.123, 184.475, 53.799, 29.818, 2.256, 1.047, **0.0, 0.0**							
Individual Condition Number							
1.0, 5.785, 19.835, 35.787, 473.054, 1019.631, **N/A, N/A**							
Rank of Parameters							
M	δ_0	X'	T_0'	E_0'	X	T_m	ω_{r0}
well-conditioned ◄——► ill-conditioned							

Unidentifiable parameters can be filtered by using subset selection method so as to avoid numerical divergence problem or unreliable parameter values. On the other hand, it is desirable to identify all parameter values. In order to handle unidentifiable or ill-conditioned parameters, several counter measures will be proposed in discussion.

Linearized model generally models dynamic behavior of load around a certain operating point. In measurement-based load modeling, there is still ongoing issue on whether or not linearized model should meet the following initial condition of its nonlinear model: $0 = \dot{x}(t_0, p) = f(x_0, u_0, p)$. The initial condition can be incorporated into parameter estimation task as nonlinear equality constraints. Another way to consider initial condition is to reformulate objective function (9) by adding regularization term as follows:

$$\min_p J(p) = \min_p \frac{1}{2}\left(\|\varepsilon(p)\|^2 + w\|f(x_0,u_0,p)\|^2\right) \quad (14)$$

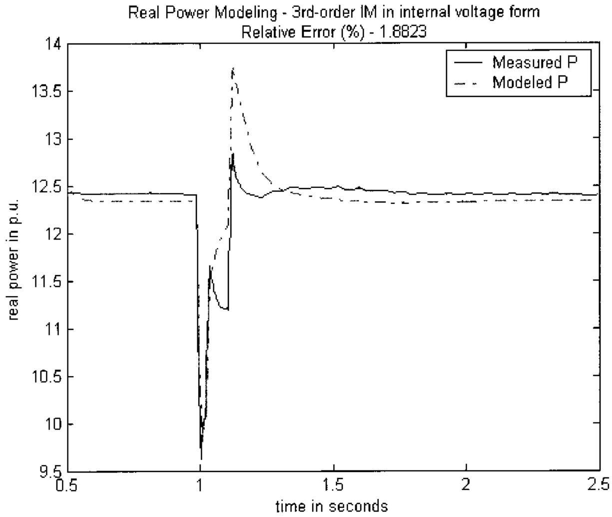
where $\varepsilon(p) = \Delta y(p) - \Delta y_m$ and w is a weighting factor. The objective function including regularization terms may result in different parameter sensitivities

4.2 Local Identifiability Study using actual measurements

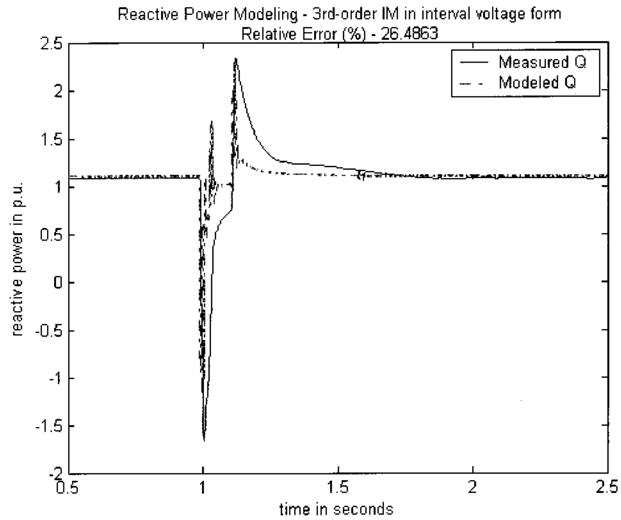
For numerical identifiability study, several actual measurement data sets obtained from a power system [4]-[5] are used. A numerical identifiability of the induction motor and the Z-IM models is tested with a given local solution point which is obtained from a set of actual measurement data taken from a power system. Model parameters are estimated using the quasi-Newton methods such as Levenberg-Marquardt method is robust and often considered to be a good method for nonlinear least squares problems [22], and the Broyden, Fletcher, Goldfarb, and Shanno (BFGS) method which is also considered reliable and well acc epted [23]. Table 7 and Fig. 2 show the estimated par ameter values and modeled real/reactive powers by the nonlinear third-order induction motor model, respectively.

Table 7. Estimated parameter values and modeling errors of induction motor model.

Estimated Parameter Values				
M	T_0'	X	X'	T_m
0.006	0.235	1.365	0.003	12.332
Modeling Error (%)		Real Power		1.88
		Reactive Power		26.49



(a) Real power modelling



(b) Reactive power modeling

Fig. 2. Modeled real and reactive powers using nonlinear 3rd-order induction motor model under study.

The results indicate that the nonlinear third-order induction motor model under study gives poor performance in modeling reactive power while real power modeling performance is acceptable. It is interesting to note that irregular oscillations are observed during the disturbance in modeled reactive power. It is noted that better local optimal solutions may exist because of the nature of the nonlinear objective function for parameter estimation.

Table 8. Singular values of trajectory sensitivity matrix and rank of parameters of IM model with a given input-output.

Singular Values of Trajectory Sensitivity Matrix				
17120.519, 5408.441, 138.408, 86.056, 16.262				
Individual Condition Number				
1.00, 3.166, 123.696, 198.946, 1052.768				
Rank of Parameters				
X'	M	T_0'	T_m	X
well-conditioned ◄——► ill-conditioned				

Compared to the results by the local identifiability study using artificial data, the local identifiability results by using actual measurements are quite different. Condition number of actual measurement case is much larger than that of artificial data case. The rank of parameters is different in the sense of well-condition while the most ill-conditioned parameter is X in both cases. Numerical identifiability study reveals that local identifiability is quite dependent of the given parameter values and input-output data.

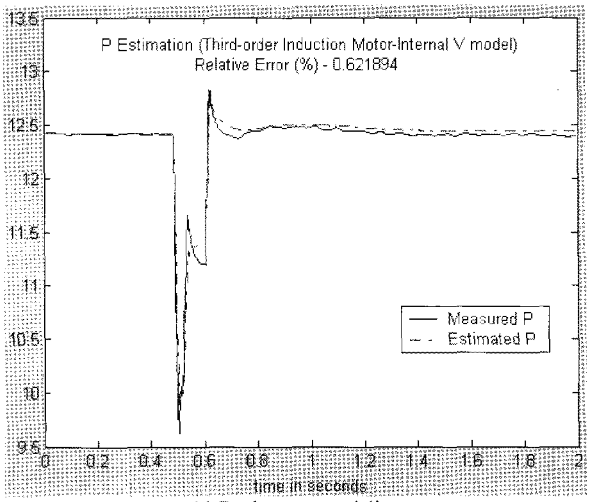
Parameters of the linearized third-order induction motor model are estimated with initial conditions incorporated into objective function as a regularization term in equation (14). Table 9 shows the parameter estimation results at two summer light loading conditions [4] with or without parameter bound constraints. The weighting factor w of regularization term is set to 1.0 for simplicity.

Table 9. Parameter estimation results of linearized IM model for two measurement data sets at summer light loading condition.

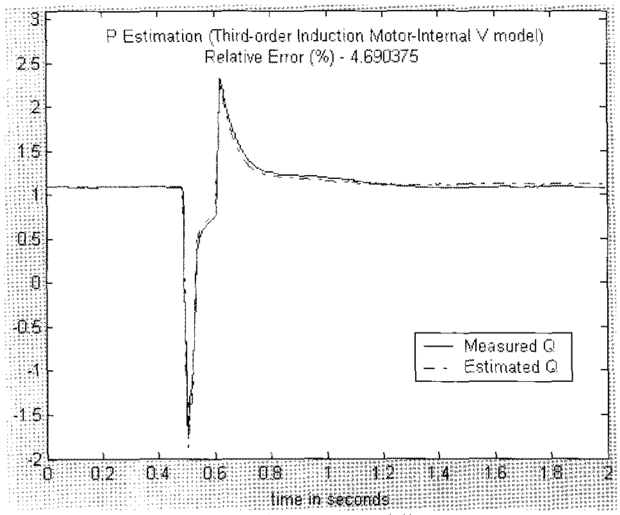
p	Summer Light #2*		Summer Light #3	
	without BC**	with BC	without BC	with BC
M	**128969.85**	10.00	**148287.68**	10.00
T_0'	0.028	0.029	0.028	0.029
X	0.029	0.035	0.032	0.038
X'	0.018	0.020	0.019	0.022
T_m	8.794	8.652	8.266	8.030
E_0'	1.485	1.442	1.471	1.427
δ_0	-0.110	-0.125	-0.113	-0.128
ω_{r0}	375.892	375.430	375.381	374.922
$\varepsilon_P(\%)$	0.383	0.418	0.395	0.451
$\varepsilon_Q(\%)$	2.340	2.258	2.071	2.052

*: measured data at summer light loading condition in [4].
**BC denotes Bound Constraint.

It is interesting to note that the estimated values of M are unrealistically large when bound constraints are not enforced. We observed that M is the most ill-conditioned parameter among three ill-conditioned parameters $[M, X, \omega_{r0}]^T$, which may cause the unrealistic parameter values of M. The second column of estimated parameters for each measurement data set denotes a local solution with bound constraints. Since output trajectory sensitivity with respect to M is very small, there is not much difference in the estimated values of other parameters and in relative mismatch errors even when M is fixed to 10.0.



(a) Real power modeling



(b) Reactive power modeling

Fig. 3. Modeled real and reactive powers using linearized 3rd-order induction motor model (constraints for initial condition are ignored).

Ill-condition problem may lead to unreasonable parameter values and may render a parameter estimation task very sensitive to initial guess. Table 10 lists local solutions of a parameter set for Z-IM model obtained by four different initial guesses.

Table 10. Local solutions obtained using different initial points for Z-IM model (SP1 case in [4])

p	Obtained local solution p^* at SP1			
	With p_0^1	With p_0^2	With p_0^3	With p_0^4
M	0.0026	0.0062	0.0062	0.0062
T_0'	**6.4206**	**17.7626**	**14.0767**	**60.1130**
X	**6.4169**	**17.7124**	**14.0293**	**60.0226**
X'	0.0359	0.0336	0.0336	0.0336
T_m	5.5516	6.0739	6.0736	6.0739
G_s	7.0089	6.4752	6.4755	6.4753
B_s	-0.2543	-0.2959	-0.3108	-0.2561
$\varepsilon_P(\%)$	1.787	1.408	1.408	1.408
$\varepsilon_Q(\%)$	11.182	13.763	13.761	13.763

p^* : obtained local solution.
p_0^i : i[th] initial point where $i = 1,\cdots,4$ here.

Table 10 indicates that the variation of estimated T_0' and X with respect to initial points is significant compared to that of the other parameters. It should be noted that modeling errors of the real and reactive powers are almost same one another for three cases (p_0^2, p_0^3, and p_0^4). A numerical identifiability analysis reveals that two parameters T_0' and X are ill-conditioned and singular values associated with these two parameters are very small. Table 11 shows the results of numerical identifiability study.

Table 11. Singular values of trajectory sensitivity matrix and rank of parameters of Z-IM model.

Singular Values of Trajectory Sensitivity Matrix						
2058.339, 639.005, 6.088, 5.640, 2.189, **0.250, 0.2e-4**						
Condition Number of Each Singular Value (σ_{max}/σ_i)						
1.00, 3.221, 338.122, 364.942, 940.341, **8247.895, 1.031e8**						
Rank of Parameters						
M	X'	B_s	G_s	T_m	X	T_0'
well-conditioned ◄——► ill-conditioned						

Table 12. Parameter Correlation Matrix at a local solution with p_0^4 in Table 11.

	M	T_0'	X	X'	T_m	G_s	B_s
M	1.0	-0.071	0.07	0.395	0.09	0.108	0.046
T_0'		1.0	-1.0	0.132	-0.037	-0.041	0.620
X			1.0	-0.132	0.039	0.041	-0.622
X'				1.0	-0.113	0.026	-0.135
T_m					1.0	0.082	-0.629
G_s						1.0	0.0
B_s							1.0

It is also interesting to note that two ill- conditioned parameters, X and T_0' are of almost same magnitude in each case. The correlation matrix shown in Table 12 indicates that two parameters have strong inter-dependency ($\varphi(2,3) = -1.0$) which implies that it is very difficult to identify each parameter separately.

Discussions

A local identifiability study using artificially generated input-output data helps us better understand the given load models. Based on the local identifiability study results, one can adjust model structure; unidentifiable parameters can be excluded from a parameter vector by fixing their values to proper ones. If there exists strong correlation ($\varphi(i,j) \approx \pm 1.0$) between parameters p_i and p_j, it is recommended to combine two parameters into one in the model structure or to fix one parameter to a typical value. For ill-conditioned parameters associated with small singular values, enforce typical parameter bounds for later use in actual parameter derivation process using measurement data.

With the prior knowledge of a local identifiability study using artificial data, load model parameters can be derived using actual measurement data. Once a local solution is obtained, a local (numerical) identifiability is tested. For ill-conditioned or unidentifiable parameters, one of the following remedy actions can be taken:

1. Exclude ill-conditioned and unidentifiable parameters from a parameter vector using a proper method such as subset selection method [19] and fix the parameter values to proper values. Then repeat parameter estimation task with the reduced parameter vector.
2. Enforce bound constraints during the estimation process. Note that constrained optimization problem generally requires more computation time.
3. Bound constraints for ill-conditioned parameters can be incorporated into an objective function as regularization terms with weighing factor. However it is generally difficult to determine the weighting factor values properly.

Since the result of local identifiability analysis is quite dependent on input data, it is important to secure high quality input-output data which has rich information enough to excite each parameter during the estimation process. In this respect, multiple measurement data sets can be used in deriving more reliable parameter values if possible [4]

5. Concluding Remarks

In this paper, concepts and definitions of a priori and a posteriori identifiability are introduced. A local (numerical) identifiability of an induction motor load model and a composite Z-induction motor load model is studied. It is shown that parameter ill-conditioning can significantly affect on reliable parameter estimation task in numerical examples. In addition, numerical studies show that local identifiability can be quite sensitive to input data and a given local solution. Finally, several countermeasures are proposed to overcome ill-conditioning problem in measurement-based load modeling.

6. Appendix

6.1 Linearization of A Third-order Induction Motor Load Model

The linearized version of the third-order induction motor model in (10) and (11) is represented as follows:

$$\Delta\dot{x} = A\Delta x + B\Delta u$$
$$\Delta y = C\Delta x + D\Delta u \quad \text{(A.1)}$$

where

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix},$$

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}, \quad D = \begin{bmatrix} d_{11} \\ d_{21} \end{bmatrix}.$$

$$a_{11} = -\frac{X}{T_0' X'}, \quad a_{12} = -\frac{(X - X')V\sin\delta}{T_0' X'}, \quad a_{13} = 0$$

$$a_{12} = \frac{(X - X')\cdot V \cdot \sin\delta}{T_0' \cdot X' \cdot E'^2}, \quad a_{22} = -\frac{(X - X')\cdot V\cdot\cos\delta}{T_0'\cdot X'\cdot E'},$$

$$a_{23} = 1$$

$$a_{31} = -\frac{V\cdot\sin\delta}{M\cdot X'}, \quad a_{32} = -\frac{V\cdot E'\cdot\cos\delta}{M\cdot X'}, \quad a_{33} = 0$$

$$b_{11} = \frac{1}{T_0'}\frac{(X - X')\cdot\cos\delta}{X'}, \quad b_{21} = -\frac{(X - X')\cdot\sin\delta}{T_0'\cdot X'\cdot E'},$$

$$b_{31} = -\frac{E'\cdot\sin\delta}{M\cdot X'}$$

$$c_{11} = -\frac{V\cdot\sin\delta}{X'}, \quad c_{12} = -\frac{V\cdot E'\cdot\cos\delta}{X'}, \quad c_{13} = 0$$

$$c_{21} = -\frac{V\cdot\cos\delta}{X'}, \quad c_{22} = \frac{V\cdot E'\cdot\sin\delta}{X'}, \quad c_{23} = 0,$$

$$d_{11} = \frac{E'\cdot\sin\delta}{X'}, \quad d_{21} = \frac{2V - E'\cdot\cos\delta}{X'}$$

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